AUTOMATIC CONTROL
KTH
Applied Estimation EL2320
Exam 8:00-12:00 April 10, 2015

Aids: None, no books, no notes, nor calculators

Observe:

- Name and person number on every page
- Answers should be in English (or Swedish)
- Only write on one side of the sheets
- Specify the total number of handed in pages on the cover
- Be careful to label each answer with the question number and letter
- Be careful to only state things you know are true as one incorrect statement can negate an otherwise correct or partially correct answer.
- That said also be sure to answer the question asked.
- Motivate answer and clearly state any additional assumptions you may need to make.

Results: The results will be available on 'mina sidor', in a few weeks
Any promblems with the recording of grades, STEX Studerandexpeditionen, Osquldasv. 10.

Responsible: John Folkesson, 08-790-6201

Good to know (what all the symbols mean you should be able to figure, this is just in case you forgot how to put them together):

$$G(\mathbf{x}, \mu, \Sigma) = \frac{1}{\sqrt{|2\pi\Sigma|}} \exp \frac{1}{2}(\mathbf{x} - \mu)^T \Sigma^{-1} (\mathbf{x} - \mu) \tag{1}$$

Typical Non-linear system equations:

$\mathbf{x}_t = \mathbf{g}(\mathbf{u}_t, \mathbf{x}_{t-1}) + \varepsilon_t$

$\mathbf{z}_t = \mathbf{h}(\mathbf{x}_t) + \delta_t$

'Kalman Gain'

$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$

Predict phase:

$\bar{\Sigma}_t = R_t + G_t \Sigma_{t-1} G_t^T$

$\bar{\mu}_\mathbf{t} = \mathbf{g}(\mathbf{u}_t, \mu_{t-1})$

Update:

$\Sigma_t = \bar{\Sigma}_t - K_t H_t \bar{\Sigma}_t$

$\mu_t = \bar{\mu}_\mathbf{t} + K_t \eta_t$

Questions

1. (6p) For a joint probability density function (pdf) given by:

   $p(x, y) = k(x + y)$ for both x and $y \in (0, 1)$ with k a constant

   a) What is the value of k? (1p)

   b) What is the pdf of x, $p(x) =$? (1p)

   c) What is the conditional probability $p(y|x)$? (1p)

   d) What is the variance of x? (1p)

   e) What is the expected value of y given x=1? (1p)

   f) Are x and y independent? (1p)

2. (6p) You are playing a game where you bet on the color of hidden card drawn from a set of four playing cards consisting of 2 red cards and 2 black cards. At the start of the game the cards are mixed and top card is removed and hidden. Let us denote the random variable of the hidden card's color as H which then can take values $\{R, B\}$

   a) What is the probability of a red card having been hidden, $P(H = R)$? (1p)

   b) From the remaining three cards one is turned over. Call its color variable C_1 What is the probability of it being red card, $P(C_1 = R)$? (1p)

   c) You see the card is indeed red. What is the probability of the hidden card being red now $P(H = R|C_1 = R)$? Please explain why in two ways. First by strating from the definition of conditional probability, and then by a simple card counting argument. (2p)

   d) A second card is now revealed. You see the card is black. What is the probability of the hidden card being red now $P(H = R|C_2 = B, C_1 = R)$? Please explain why in two ways. First by a simple card counting argument then by using Bayes theorem as in Bayes filtering, to write an expression using $P(H = R|C_1 = R)$, along with some of these quantities:

   $P(H = R|C_2 = R)$,

   $P(C_2 = B|C_1 = R)$,

   $P(C_2 = B, C_1 = R)$,

   $P(C_2 = B|C_1 = R, H = R)$,

   $P(C_2 = B|C_1 = R, H = R)$,

   $P(C_2 = R|C_1 = B, H = R)$,

   $P(C_1 = R, H = R)$. (2p)

3. (7p) This question is on the Kalman Filter. You are on a bicycle and trying as best you can to hold a steady constant speed. You glance down at your speedometer to see that your speed is 5.0 m/s. Your speedometer is an old fashion analog type with a needle that points to number on the face of a disk, sort of like a clock face. The sensor is in this case essentially way more accurate than you can read it with all the the sweat and sun. You estimate a that your reading is therefore Gaussian distributed with standard deviation 0.5 m/s.

   a) What is the posterior distribution of your speed $p(v)$ (1p)? Be sure to quantify all parameters.

   b) You check again after exactly 10 seconds and read 5.2 m/s. You quickly decide to fuse these two reading in your head using a Kalman filter. In order to estimate your current belief over your speed v what other physical process do you need to model? (2p)

   c) Make the model you indicated in (b) and then use it to estimate the belief using the Kalman filter after the measurement. Be sure to indicate any quanties that you need to 'make up' in order to answer. Both tell me the values and what they are. Do not assume there are standard notation for Kalman filters but be clear what all symbols are by starting from basic formulas and defining them.(4p)

4. (6p) The example of robot localization was used in the labs, lectures and book. The state is given by $\mathbf{x} = (x, y, \theta)$, that is the two Cartesian coordinates and the angle between the robot direction and the x axis. The motion model is one of a constant velocity of $10m/s$ in the direction of the robot. Over one second intervals, Gaussian 'white' noise disturbs this dynamic with uncorrelated variance in both x and y of 0.01 and in θ of 0.0001.

   a) Convert the above text to formulas that describe the motion process including noise and all known parameters.(1p)

   b) What is the 'G' Jacobian matrix needed for a EKF prediction based on your model in (a). (1p)

   c) Assume that the robot is known to be at the origin facing an unknown direction. A beacon at the origin allows the robot to measure its distance to the origin every 5 seconds. The robot travels for 5 seconds and is now about to take its first measurement. Describe the general shape of the robot belief. A drawing or picture will help convince me you really get it. Indicate the scales only approximately. (1p)

   d) Assuming independent Gaussian noise on the measurements with variance $1m^2$. Convert the above text to formulas that describe the measurement model including noise and all known parameters. (1p)

   e) Now the robot takes the measurement and gets 49 m. What does the posterior distribution look like after this measurement? Again just give a general description with approximate scales. (1p)

   f) What single fact in the problem as stated above (two words from my text) makes it impossible to use the EKF to even roughly estimate the pose of the robot using the given state vector. (1p)

5. (13 p) These questions are on the particle filter, PF used for state estimation.

   a) How does one estimate the value of the posterior probability density function (pdf) over the state at some particular point in the state space. Give two specific suggestions. (2p)

   b) What is needed to carry out the diffusion step. (1p)

   c) Describe the difference between target and proposal distributions. (2p)

   d) Why do we compute weights? (2p)

   e) Why do we re-sample? (2p)

   f) What happens when a spurious measurement is used in the estimation. That is what is the sequence of steps that lead it to effect the particle distribution over the state space. (2p)

   g) What does the KLD-sampling do? (2p)

6. (12p)

   a) For the extended Kalman filter, EKF, increasing the covariance estimates of the noise for both the dynamics and the measurements as well as the a priori distribution's by a factor of 10, (that is, multiply R, Q and Σ by 10) will clearly change the model dramatically. What effect does it have on the estimate? Be sure to explain how each step of the algorithm will contribute to the total effect. (3p)

   b) What will the be the effect on the particle, PF, estimate of the same change as in (a)? Again walk me through each step. (3p)

   c) What is it that decides whether or not a UKF will be significantly better than an EKF for a given estimation problem? (2p)

   d) In what situation might the iterative EKF be better than the EKF. State the phase in the algorithm and the way the situation impacts that phase in the two algorithms. (2p)

   e) If all the assumptions of the Kalman filter are valid for an estimation problem could it still be better to use the PF? why/why not ? (2p)