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ETC3530

ETC3530

Life Insurance Mathematics

人寿保险数学

Week 1

第 1 周

Life Assurance Contracts and Life Annuity Contracts

人寿保险合同和人寿年金合同

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Life Tables and Survival Models Revision

生命表和生存模型修订

Life Insurance Contracts - Background

人寿保险合同 - 背景

Life Assurance Contracts

人寿保险合同

Whole of Life Assurance

终身寿险

Term Assurance

定期保证

Pure Endowment

纯禀赋

Endowment Assurance

捐赠保证

Life Annuity Contracts

人寿年金合约

Whole Life Annuities

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临时年金

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Continuous-time Calculations

连续时间计算

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Go over 2430 Life Table material - Lecture 9 Semester 1.

回顾 2430 生命表材料 - 第 1 学期第 9 课。

Complete overview not given here.

这里没有给出完整的概述。

Important material:

重要材料：

Definition of survival and death probabilities.

生存和死亡概率的定义。

Construction of life tables and use in calculating

生命表的构建和计算中的应用

survival/death probabilities

生存/死亡概率

Survival models (force of mortality), links to life tables and

生存模型（死亡的力量），生命表的链接和

survival probabilities

生存概率

Curtate and complete future lifetime random variables and

整理和完成未来的生命周期随机变量和

calculation of their expectation.

计算他们的期望。

Definitions

定义

t px the probability that a life aged x survives another t years.

t px 年龄 x 的生命再存活 t 年的概率。

t qx the probability that a life aged x dies within the next t years. Note: t qx +t px = 1

t qx 一个年龄为 x 的生命在接下来的 t 年内死亡的概率。注：t qx +t px = 1

µx - annual rate at which people are dying at exact age x (it is a probability rate; e.g. 1 person out of a 1000 per day).

µx - 人们在确切年龄 x 死亡的年率（它是概率；例如，每天 1000 人中有 1 人）。

µx dt – probability of a life aged x dying over the small time interval (x, x + dt ). Note: when we multiply µx by dt the quantity goes from being a probability rate to a probability.

µx dt – 在小时间间隔 (x, x + dt ) 内衰老 x 的生命死亡的概率。注意：当我们将 µx 乘以 dt 时，数量从概率变为概率。

µx is referred to as the force of mortality; it is:

µx 被称为死亡的力量；这是：

t qx

tqx

µx = lim

x = 限制

t→0+

t→0+

t

t

Life Tables

生命表

lx - is the expected number of survivors at age x out of a population starting from a particular age, usually birth, l0

lx - 是从特定年龄开始的人口中 x 年龄的预期幸存者数量，通常是出生，l0

The life table has entries for discrete integer values of x ; not continuous.

生命表具有 x 的离散整数值的条目；不连续。

►

►

t px = lx +t

t 像素 = lx +t

lx

lx

Interpretation: number of survivors at time x + t out of the population at time x .

解释：在时间 x + t 的幸存者人数在时间 x 的人口中。

►

►

t qx = 1 − t px = lx − lx +t

t qx = 1 - t px = lx - lx +t

lx

lx

Interpretation: Expected number of deaths from time x to time x + t , out of population at time x .

解释： 从时间 x 到时间 x + t 的预期死亡人数，在时间 x 的人口之外。

Curtate Lifespan

由寿命提供

The random variable Kx representing the whole number of years before death for someone already aged x is known as the curtate lifespan.

代表已满 x 岁的人死亡前的整数年数的随机变量 Kx 被称为 curtate lifespan。

For 0kωx (x = ω is the largest age for which lx is non-zero), we have:

对于 0kωx（x = ω 是 lx 不为零的最大年龄），我们有：

≤≤−

≤≤−

P{Kx = k} =lx +k − lx +k +1

P{Kx = k} =lx +k - lx +k +1

lx

lx

=lx +k lx +k − lx +k +1

=lx +k lx +k - lx +k +1

lx

lx

= k px qx +k

= k 像素 qx +k

lx +k

lx +k

Note: Kx does not take continuous values; it is a discrete random variable and k px qx+k is the probability mass function for Kx .

注：Kx 不取连续值；它是一个离散随机变量，k px qx+k 是 Kx 的概率质量函数。

As the above shows, the probability mass function for Kx can be computed from either the life table or knowledge of t px and t qx .

如上所示，Kx 的概率质量函数可以从生命表或 t px 和 t qx 的知识中计算出来。

Consider the probability that a life aged x , will survive another n years but then die during the subsequent m years. This can be expressed in terms of the complete lifetime random variable Tx :

考虑一个年龄为 x 的生命将再存活 n 年但在随后的 m 年中死亡的概率。这可以用完整的生命周期随机变量 Tx 来表示：

n|mqx = P{n < Tx ≤ n + m}

n|mqx = P{n < Tx ≤ n + m}

This can be written as

这可以写成

n|m

n|米

qx = lx +n − lx +n+m

qx = lx +n - lx +n+m

lx

lx

Numerator: number of expected deaths between year x + n

分子：x + n 年之间的预期死亡人数

and year x + n + m.

和 x + n + m 年。

Denominator: expected number of individuals in population at time x

分母：在时间 x 时人口中的预期个体数量

In terms of standard survival and death probabilities, we have:

就标准生存和死亡概率而言，我们有：

►

►

n|mqx = npx − n+mpx

n|mqx = npx - n+mpx

Probability that life survives another n years, but not another n + m years.

生命再存活 n 年，但不能再存活 n + m 年的概率。

►

►

n|mqx = n+mqx − nqx

n|mqx = n+mqx - nqx

Probability that life dies within the next n + m years but not within the next n years.

生命在未来 n + m 年内死亡但不会在未来 n 年内死亡的概率。

Note:

笔记：

In the special case where m = 1 (i.e. lives another n years, but dies within the year after); we drop the m = 1 and write:

在 m = 1 的特殊情况下（即再活 n 年，但在后一年内死亡）；我们去掉 m = 1 并写：

n|1qx = n|qx

n|1qx = n|qx

Satisfy yourself that the curtate lifetime probability distribution can be expressed in terms of the new notation as:

满足你自己，curtate寿命概率分布可以用新符号表示为：

P{Kx = n} = n qx = lx +n − lx +n+1

P{Kx = n} = n qx = lx +n - lx +n+1

|lx

|lx

Discounting

打折

If the interest rate per annum is i%, recall that the present value of a cashflow C at time t years, is given by:

如果年利率为 i%，回想一下 t 年现金流量 C 的现值由下式给出：

PVC(0) =C= Cvt

PVC(0) =C= Cvt

1i t

1 吨

( + )

( + )

We define the unit time discount factor

我们定义单位时间折扣因子

v = 1

v = 1

1 + i

1 + 我

Annuity Contracts

年金合同

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Brief Introduction

简单的介绍

Life insurance policies are contracts entered into by a life insurance company with a policyholder. The benefits of these contracts to policyholders (and shareholders) are contingent on the life of the policyholder.

人寿保险单是人寿保险公司与投保人签订的合同。这些合同对投保人（和股东）的好处取决于投保人的寿命。

Assurance contracts

保证合同

Annuity contracts

年金合同

Annuity Contracts

年金合同

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Assurance contracts - what is classically associated or thought of as life insurance.

保证合同 - 经典相关或被认为是人寿保险的东西。

The benefit paid by an assurance contract is an amount called the sum assured.

保证合同支付的利益是一个称为保额的金额。

There are 4 main types of assurance contracts

有4种主要类型的保证合同

A Whole of Life Assurance. The sum assured will be paid

一个完整的人寿保障。将支付保额

on the policyholder’s death.

关于投保人的死亡。

A Term Assurance. Sum assured is paid on or after death,

期限保证。保额在死亡时或之后支付，

provided death occurs during a specified period called the term of the contract.

如果死亡发生在称为合同期限的特定时期内。

A Pure Endowment. Sum assured paid at the end of a fixed

纯粹的禀赋。在固定期限结束时支付的保额

term, provided the policyholder is alive.

期限，前提是投保人还活着。

An Endowment Assurance. A combination of a term

捐赠保证。一个术语的组合

assurance and a pure endowment. Sum assured is payable either on death during the term or on survival to the end of the term. Sum assured on death or survival need not be the same (usually are).

保证和纯粹的禀赋。保额可在期限内死亡或生存至期限结束时支付。死亡或生存的保额不必相同（通常是相同的）。

Pays a sum assured, S, on the policyholder’s death.

在投保人死亡时支付保额 S。

Assumptions and scope in this lecture:

本讲座的假设和范围：

We will not yet consider the premiums paid by the

我们还不会考虑保险公司支付的保费

policyholder; only payments made to the policyholder.

投保人；只支付给投保人。

We consider the benefit payable to a life currently aged x .

我们考虑支付给当前年龄 x 的人的利益。

We suppose that the sum assured is payable, not on death,

我们假设保额是支付的，而不是死亡时，

but at the end of the year of death. The payment is made at time Kx + 1. (Recall the curtate future lifespan, Kx is the whole number of years remaining for a life aged x before dying.)

但在死亡之年结束时。付款是在时间 Kx + 1 时支付的。（回想一下简单的未来寿命，Kx 是在死前 x 岁的生命剩余的年数。）

The timing of the payment is no longer certain, so the present value is no longer deterministic (i.e. not known in advance).

付款时间不再确定，因此现值不再具有确定性（即事先不知道）。

The present value at time 0 of a certain payment of 1 to be made at time t is vt .

在时间 t 支付的某笔 1 在时间 0 的现值为 vt 。

If the time of the payment is not certain, but a random variable H, then the present value is v H , also a random variable.

如果付款时间不确定，而是一个随机变量H，那么现值为v H ，也是一个随机变量。

An uncertain payment time is the situation that needs to be addressed for whole life assurance.

不确定的付款时间是终身寿险需要解决的情况。

We can no longer know the present value of the payment with certainty, but can evaluate on average, across many policyholders, what that payment will be.

我们不再能够确定地知道付款的现值，但可以平均评估许多投保人的付款金额。

Let W = Sv H denote the present value of the whole life assurance benefit payment, we need to find:

令 W = Sv H 表示终身寿险给付的现值，我们需要找到：

E{W} = E{Sv H }

E{W} = E{Sv H }

, where:

， 在哪里：

S is the sum assured.

S 是保额。

H is Kx + 1.

H 是 Kx + 1。

or;

或者;

E{W} = E{Sv Kx +1} = SE{v Kx +1}(1)

E{W} = E{Sv Kx +1} = SE{v Kx +1}(1)

In ETC2430, we looked at the expectation of the random variable Kx ’

在 ETC2430 中，我们查看了随机变量 Kx 的期望值

∞∞

∞∞

E{Kx } = Σ kP(Kx = k ) = Σ lx +k

E{Kx } = Σ kP(Kx = k ) = Σ lx +k

k =0

k =0

k =1

k =1

lx

lx

Now we are interested in the expectation of functions of Kx

现在我们对 Kx 函数的期望感兴趣

for any function g:

对于任何函数 g：

∞

∞

Σ

Σ

E{g(Kx )} =g(k )P(Kx = k )(2)

E{g(Kx )} =g(k )P(Kx = k )(2)

k =0

k =0

NOTE: In general, when evaluating expectation of a function, g, of any random variables X :

注意：通常，在评估任何随机变量 X 的函数 g 的期望时：

E{g(X )} = g(E{X})

E{g(X)} = g(E{X})

Example: Suppose we have a random variable X with probability mass function P(X = 1) = 1 , P(X = 2) = 1 and

示例：假设我们有一个随机变量 X，其概率质量函数为 P(X = 1) = 1 ， P(X = 2) = 1 和

42

42

P(X = 3) = 1 , what are E{X} and E{X 2}?

P(X = 3) = 1 ，什么是 E{X} 和 E{X 2}？

4

4

Solution:

解决方案：

E{X} = 1 × P(X = 1) + 2 × P(X = 2) + 3 × P(X = 3)

E{X} = 1 × P(X = 1) + 2 × P(X = 2) + 3 × P(X = 3)

= 0.25 + 1 + 0.75 = 2

= 0.25 + 1 + 0.75 = 2

E{X 2} = 12 × P(X = 1) + 22 × P(X = 2) + 32 × P(X = 3)

E{X 2} = 12 × P(X = 1) + 22 × P(X = 2) + 32 × P(X = 3)

= 0.25 + 2 + 2.25 = 4.5

= 0.25 + 2 + 2.25 = 4.5

Clearly E{X 2} = 4.5 /= (E{X})2 = 4

显然 E{X 2} = 4.5 /= (E{X})2 = 4

The same situation applies when g(X ) = v X , the expectation cannot be found by evaluating g(E{X}).

同样的情况适用于 g(X ) = v X 时，无法通过计算 g(E{X}) 来找到期望。

In general E g(X ) is evaluated by taking the weighted sum of probabilities as per equation (2), where the probability of each outcome is weighted by the function value g(X ) for that outcome.

一般来说，E g(X ) 是通过根据等式 (2) 取概率的加权和来评估的，其中每个结果的概率由该结果的函数值 g(X ) 加权。

{}

{}

The variance (square of the standard deviation) is a useful quantity to give an idea of the spread of the present value random variable:

方差（标准差的平方）是一个有用的量，可以用来了解现值随机变量的分布：

var{X} = E{(X − E{X})2} = E{X 2} − E{X}2(3)

var{X} = E{(X - E{X})2} = E{X 2} - E{X}2(3)

For functions of a random variable

对于随机变量的函数

var{g(X )} = E{(g(X )−E{g(X )})2} = E{g(X )2}−E{g(X )}2

var{g(X )} = E{(g(X )−E{g(X )})2} = E{g(X )2}−E{g(X )}2

(4)

(4)

The following scaling properties are used to scale expected valua and variance of a standardised sum assured payment of 1 to a payment of amount S. If S is some constant value:

以下缩放属性用于将标准化保额支付 1 的预期价值和方差缩放为金额 S。如果 S 是某个恒定值：

E{Sg(X )} = SE{g(X )}(5)

E {Sg (X)} = SE {g (X)} (5)

var{Sg(X )} = S2var{g(X )}(6)

var{Sg(X)} = S2var{g(X)}(6)

Equation (5) tells us that multiplying a random variable by a constant S, multiplies it’s expectation also by S.

等式 (5) 告诉我们，将随机变量乘以常数 S，也将其期望乘以 S。

Equation (6) tells us that multiplying a random variable by a constant S, multiplies it’s variance by the square of S.

等式 (6) 告诉我们，将随机变量乘以常数 S，将其方差乘以 S 的平方。

To evaluate the EPV of a whole life assurance, we use

为了评估终身寿险的 EPV，我们使用

g(k ) = vk +1 in (2) to obtain:

g(k ) = vk +1 在 (2) 中得到：

E{v Kx +1} =

E{v Kx +1} =

=

=

∞

∞

vP(Kx = k )

vP(Kx = k )

Σ k +1

SK +1

k =0

k =0

Σ vk +1 lx +k − lx +k +1

Σ vk +1 lx +k − lx +k +1

∞

∞

k =0lx

k =0lx

∞

∞

Σ

Σ

=vk +1k|qx(7)

=vk +1k|qx(7)

k =0

k =0

k|qx is the probability that a life aged x years survives k years, but dies before time k + 1.

k|qx 是 x 岁的生命存活 k 年，但在时间 k + 1 之前死亡的概率。

Equation (7) is constructued by:

等式 (7) 由以下公式构成：

assume Kx takes the value k , present value of benefit will be vk +1 in this case.

假设 Kx 取值 k ，在这种情况下，收益的现值为 vk +1。

multiply the PV of the benefit by P(Kx = k ).

将收益的 PV 乘以 P(Kx = k )。

sum over all possible values of k

对所有可能的 k 值求和

The summation (7) is given a special definition in actuarial notation:

求和 (7) 在精算符号中给出了特殊定义：

Σ k +1

SK +1

∞

∞

Ax =vk|qx(8)

斧头 =vk|qx(8)

k =0

k =0

The sum is written as Σ∞k 0 for brevity, instead of Σω−x−1.

为简洁起见，总和写为 Σ∞k 0，而不是 Σω−x−1。

k =0

k =0

=

=

Recall lx = 0 for x ≥ ω, k px = 0, and hence k|qx = 0 for

回想一下，对于 x ≥ ω，lx = 0，k px = 0，因此对于

k ≥ ω − x .

k ≥ ω - x 。

As stated in (1), using (5) we have that

如（1）中所述，使用（5）我们有

E W = E Sv Kx +1 = SE v Kx +1 = SAx , so if sum assured is S, then the EPV of the benefit is SAx .

E W = E Sv Kx +1 = SE v Kx +1 = SAx ，因此如果保额为 S，则福利的 EPV 为 SAx 。

{}{}{}

{}{}{}

Example A whole life assurance pays a benefit of $50, 000 at the end of the policy year of death of a life now aged exactly 90. Mortality is assumed to follow the life table given below:

示例 终身人寿保险在保单年度结束时支付 $50, 000 的利益，现在正好 90 岁的人死亡。假设死亡率遵循下面给出的生命表：

Calculate the expected present value of this benefit using an effective rate of interest of 5% pa.

使用每年 5% 的有效利率计算该收益的预期现值。

Expected Present Value

预期现值

Solution:

解决方案：

2

2

Σ

Σ

50000A90= 50000vk+1k|q90

50000A90= 50000vk+1k|q90

k =0

k =0

= 50000 Σ vk +1 l90+k − l90+k +1

= 50000 Σ vk +1 l90+k − l90+k +1

k =0

k =0

2

2

l90

l90

= 50000( 1 25 + 135 + 140

= 50000( 1 25 + 135 + 140

1.051001.0521001.053100

1.051001.0521001.053100

= $45, 054.53

= $45, 054.53

Evaluating Ax

评估斧头

The life assurance function Ax is normally evaluated in one of two ways:

寿命保证函数 Ax 通常以以下两种方式之一进行评估：

Using tables, e.g. AM92 tables found in the “Formulae and Tables for Examinations”. Note that the tables only provide Ax values for certain rates i .

使用表格，例如AM92 表格可在“考试公式和表格”中找到。请注意，这些表格仅提供特定比率 i 的 Ax 值。

Direct calculation from a life table using a computer. Any

使用计算机从生命表中直接计算。任何

rate of interest can be used, including a rate that varies over time.

可以使用利率，包括随时间变化的利率。

Example of using tables: Find A40 using AM92 at 6%. Solution: go to page 102 of tables to find A40 = 0.12313.

使用表格的示例：使用 AM92 在 6% 处查找 A40。解决方法：到第 102 页表格找到 A40 = 0.12313。

Observation from AM92 tables: Ax seems to increase with age. E.g. A30 = 0.07328 and A40 = 0.12313. The greater the age x , the more likely the policyholder is going to die in the near future, therefore we are discounting the benefit over a shorter period, and hence will obtain a higher expected present value.

来自 AM92 表的观察：斧头似乎随着年龄的增长而增加。例如。 A30 = 0.07328 和 A40 = 0.12313。年龄 x 越大，保单持有人在不久的将来死亡的可能性就越大，因此我们在更短的时间内折现收益，因此将获得更高的预期现值。

Why calculate the variance? An insurance company will not only be interested in the expected present value of the claim, but the amount of variability in payments it can expect to see.

为什么要计算方差？保险公司不仅对索赔的预期现值感兴趣，而且对预期的付款变化量感兴趣。

Using (4), we have:

使用（4），我们有：

var{v Kx +1} = E{(v Kx +1)2} − E{VKx +1}2

var{v Kx +1} = E{(v Kx +1)2} - E{VKx +1}2

∞

∞

Σ

Σ

=(vk +1)2k|qx − (Ax )2

=(vk +1)2k|qx - (Ax )2

k =0

k =0

However, since (vk +1)2 = (v 2)k+1, the term∞ (v 2)k+1 q , is the same as evaluating Ax =∞k =0 vk +1k|qx , but with v replaced by v 2.

然而，由于 (vk +1)2 = (v 2)k+1，∞ (v 2)k+1 q 项与计算 Ax =∞k =0 vk +1k|qx 相同，但使用 v由 v 2 代替。

Σk =0k| x

Σk =0k| X

Σ

Σ

Replacing v by v 2 means we are using a new interest rate i∗ for which v∗ = v 2, i.e.

将 v 替换为 v 2 意味着我们正在使用一个新的利率 i∗，其中 v∗ = v 2，即

1

1

1 + i∗

1 + i∗

1

1

=(1 + i)2

=(1 + i)2

=⇒ i∗= (1 + i)2 − 1 We use the definition:

=⇒ i∗= (1 + i)2 − 1 我们使用以下定义：

Σ22 k +1

Σ22 k +1

, to write:

， 来写：

∞

∞

Ax =(v )k|qx(9)

Ax =(v )k|qx(9)

k =0

k =0

var{v Kx +1} = 2Ax − (Ax )2

var{v Kx +1} = 2Ax - (Ax )2

where if Ax is evaluated at the rate i, then 2Ax simply denotes

其中，如果 Ax 以速率 i 计算，则 2Ax 仅表示

Ax evaluated at a rate (i + 1)2 − 1.

Ax 以 (i + 1)2 - 1 的速率评估。

Using (6), we also have:

使用 (6)，我们还有：

var{Sv Kx +1} = S2var{v Kx +1}

var{Sv Kx +1} = S2var{v Kx +1}

= S2[2Ax − (Ax )2]

= S2[2Ax - (Ax )2]

Example Vivian, aged exactly 30 buys a whole life assurance with a sum assured of $50, 000 payable at the end of the year of death. Calculate the standard deviation of the present value of this benefit using AM92 Ultimate mortality and 6% pa interest. Solution On page 102 of AMT92, we find A30 = 0.07328 and 2A30 = 0.01210. The standard deviation (square root of the variance) is:

例如，年仅 30 岁的 Vivian 购买了一份保额为 $50, 000 的终身人寿保险，在死亡当年年底支付。使用 AM92 最终死亡率和 6% 的年利率计算此福利现值的标准差。解决方案 在 AMT92 的第 102 页，我们发现 A30 = 0.07328 和 2A30 = 0.01210。标准差（方差的平方根）为：

q500002[2A30 − (A30)2] = 50000√0.01210 − 0.073282 = $4, 102

q500002[2A30 - (A30)2] = 50000√0.01210 - 0.073282 = $4, 102

Using life table entries, we can calculate the terms vk +1k|qx in

使用生命表条目，我们可以计算项 vk +1k|qx 在

for Ax and the terms (v 2)k+1k|qx in (9) for 2Ax .

用于 Ax 和 (9) 中的 (v 2)k+1k|qx 用于 2Ax 。

Take starting age x , and for k = 0, 1, . . . , ω − x − 1.

取起始年龄 x ，对于 k = 0, 1, 。 . . , ω - x - 1。

Evaluate k px = lx +k

计算 k px = lx +k

lx

lx

Evaluate qx+k = lx +k −l lx +k +1

计算 qx+k = lx +k -l lx +k +1

x +k

x +k

Calculate k|qx = k px × qx+k .

计算 k|qx = k px × qx+k 。

Calculate k|qx vk +1 and k|qx v 2(k +1).

计算 k|qx vk +1 和 k|qx v 2(k +1)。

Sum all k|qx vk +1 to get Ax , i.e. apply equation (8).

将所有 k|qx vk +1 相加得到 Ax ，即应用等式（8）。

Sum all k|qx v 2(k +1) to get 2Ax , i.e. apply equation (9).

将所有 k|qx v 2(k +1) 相加得到 2Ax ，即应用等式 (9)。

Example Using life table data from Victoria (ABS 2016-2018), calculate A50 and 2A50.

示例 使用 Victoria (ABS 2016-2018) 的生命表数据，计算 A50 和 2A50。

See the worksheet “AssuranceCalculationExample” in the lecture work book.

请参阅讲座工作簿中的工作表“AssuranceCalculationExample”。

Discussion

讨论

In practice, evaluation of Ax and 2Ax is carried out directly from the life table.

在实践中，Ax 和 2Ax 的评估是直接从寿命表中进行的。

Time dependent (and stochastic) interest rate models may be used.

可以使用与时间相关的（和随机的）利率模型。

Mortality changes over time, e.g. pandemics.

死亡率随时间而变化，例如流行病。

Description and Definitions

描述和定义

Definition A term assurance contract is a contract to pay a sum assured on or after death, provided death occurs during a specified period, Kx < n.

定义 定期保证合同是在死亡时或死亡后支付保额的合同，前提是死亡发生在指定期间，Kx < n。

n is the term of the contract.

n 是合同的期限。

If the life survives to the end of the term of the contract, no payment will be made to the policyholder.

如果生命持续到合同期限结束，则不会向保单持有人付款。

For a sum assured of S, the present value of the term assurance benefit can be expressed as:

对于 S 的保额，定期保障利益的现值可以表示为：

Sv Kx +1if Kx < n

Sv Kx +1 如果 Kx < n

(

(

F

F

=

=

0if Kx ≥ n

0如果 Kx ≥ n

(10)

(10)

Expected Present Value

预期现值

Using (3);

使用（3）；

E{F} =

E{F} =

n−1

n-1

SvP(Kx = k ) + 0 × P(Kx ≥ n)

SvP(Kx = k ) + 0 × P(Kx ≥ n)

Σk +1

Σk +1

k =0

k =0

n−1

n-1

Σ

Σ

= Svk +1k|qx + 0 × npx k =0

= Svk +1k|qx + 0 × npx k =0

n−1

n-1

Σ k +1

SK +1

= Svk|qx k =0

= Svk|qx k =0

= SA1

= SA1

x :n

x:n

(actuarial notation)

（精算符号）

Actuarial Notation A1

精算符号 A1

x :n

x:n

The notation:

符号：

A1

A1

x :n

x:n

n−1

n-1

=vk|qx(11)

=vk|qx(11)

Σ k +1

SK +1

k =0

k =0

, is defined to be the EPV of 1, payable at the end of the year of death of a life currently aged x , provided death occurs in the next n years.

, 定义为 1 的 EPV，如果在接下来的 n 年内发生死亡，则在当前年龄为 x 的生命的死亡当年年底支付。

The terms x and n are said to be “statuses” of the payment.

术语 x 和 n 被称为付款的“状态”。

Actuarial Notation A1

精算符号 A1

x :n

x:n

x is called the life status. The number 1 is positioned above x to indicate that payment is made only when the life status fails (i.e. dies). The number 1 means that the life status has to fail first.

x称为生命状态。数字 1 位于 x 上方，表示仅在生命状态失败（即死亡）时付款。数字 1 表示生命状态必须先失败。

n is the term status and it remains active as long as the duration of time from the valuation date does not exceed n years.

n 是期限状态，只要从估值日期起的持续时间不超过 n 年，它就会保持活动状态。

For a payment to be made, the life status must fail before the term status.

要进行付款，生活状态必须在期限状态之前失败。

Second moment and variance

二阶矩和方差

Similar to the derivation of the variance for whole of life assurances, we can show

类似于整个人寿保证的方差推导，我们可以证明

E F 2 = (S2)2A1

E F 2 = (S2)2A1

{}

{}

x :n

x:n

where,

在哪里，

2A1

2A1

x :n

x:n

n−1

n-1

=vk|qx k =0

=vk|qx k =0

Σ 2(k +1)

P 2(k +1)

or equivalently A1

或等效的 A1

x :n

x:n

is calculated at rate of interest (1 + i)2 − 1.

以利率 (1 + i)2 - 1 计算。

We then have,

然后我们有，

x :n

x:n

x :n

x:n

var{F} =

其中{F} =

S2 "2A1

S2" 2A1

— A1

— A1

2#

2#

Evaluation of EPV

EPV的评估

Evaluation of the term assurance EPV A1

评估期限保证 EPV A1

x :n

x:n

is generally carried

一般是带的

out directly from the life table. There are two approaches generally used:

直接从生命表中取出。一般采用两种方法：

The terms vk +1k|qx in (11) can be calculated in the same way as for the whole life assurance and then simply summed. The difference with whole life is that the summation stops at k = n − 1 instead of continuing to

(11) 中的项 vk +1k|qx 可以用与终身寿险相同的方式计算，然后简单地求和。与整个生命的不同之处在于求和在 k = n - 1 处停止，而不是继续

k = ω − x − 1 (the end of the life table).

k = ω - x - 1（生命表的尽头）。

If the whole life assurance function is known for ages x and

如果整个生命保障函数已知年龄 x 并且

x + n, i.e. Ax and Ax+n are known, there is a relation that

x + n，即 Ax 和 Ax+n 是已知的，有一个关系

links these quantities to A1 .

将这些量与 A1 联系起来。

x :n

x:n

Example: A term assurance contract for a sum assured of

示例：保额的定期保证合同

$100, 000 is written on a female life aged 50 with a term of 4 years. Using the Victoria 2016-2018 life tables as the mortality basis, and assuming an interest rate of 5%, calculate the expected present value and standard deviation of this term assurance policy.

$100, 000 写在 50 岁女性寿命 4 年。以维多利亚州 2016-2018 年寿险表为死亡率基础，假设利率为 5%，计算本期保单的预期现值和标准差。

Solution: Using the relevant life table entries from the Female Victoria table, we calculate as follows:

解决方案：使用女性维多利亚表中的相关生命表条目，我们计算如下：

For k = 0, 1, 2, 3, we calculate:

对于 k = 0、1、2、3，我们计算：

k p50 = l50+k and q50+k = l50+k −l50+k +1

k p50 = l50+k 和 q50+k = l50+k -l50+k +1

l50

l50

l50+k

l50+k

k|q50 = k p50 × q50+k

k|q50 = k p50 × q50+k

Using v = 1/1.05, calculate k|q50vk +1, and k|q50v 2(k +1)

使用 v = 1/1.05，计算 k|q50vk +1 和 k|q50v 2(k +1)

Using these calculated terms; we obtain

使用这些计算的术语；我们获得

A 1

1

50:4

50:4

3

3

=vk +1k|qx k =0

=vk +1k|qx k =0

Σ

Σ

, and

， 和

2A 1

2A 1

50:4

50:4

= 0.0015567 + 0.0016308 + 0.0016855 + 0.0017397

= 0.0015567 + 0.0016308 + 0.0016855 + 0.0017397

= 0.0066126

= 0.0066126

3

3

Σ

Σ

=v 2(k +1)k|qx k =0

=v 2(k +1)k|qx k =0

= 0.0014825 + 0.0014791 + 0.0014560 + 0.0014313

= 0.0014825 + 0.0014791 + 0.0014560 + 0.0014313

= 0.0058489

= 0.0058489

The EPV is given by:

EPV 由下式给出：

E F = 100, 000A 1

E F = 100, 000A 1

{ }×

{ }×

50:4

50:4

The standard deviation is:

标准差为：

= 100, 000 × 0.0066126 = $661.26

= 100, 000 × 0.0066126 = $661.26

√"# 1

√"# 1

√

√

var{F} = 100, 000

var{F} = 100,000

2A1

2A1

x :n

x:n

−

−

A1

A1

x :n

x:n

2

2

2

2

= 100000 ×0.0058489 − 0.00661262

= 100000 ×0.0058489 − 0.00661262

= 7619.20

= 7619.20

Evaluation of EPV and variance using whole life quantities

使用全寿命量评估 EPV 和方差

To calculate with reference to the whole life assurance

参照终身寿险计算

quantities; we note that the summation in A1

数量；我们注意到 A1 中的总和

x :n

x:n

is a truncation of

是一个截断

the terms in the summation of Ax .

Ax 的总和中的项。

n−1∞∞

n−1∞∞

A1=

A1=

Σ k|qx vk +1 = Σ k|qx vk +1 − Σ k|qx vk +1

Σ k|qx vk +1 = Σ k|qx vk +1 - Σ k|qx vk +1

x :n

x:n

k =0

k =0

∞

∞

Σ

Σ

k =0

k =0

k =n

k = n

= Ax −k|qx vk +1 k =n

= Ax -k|qx vk +1 k =n

∞

∞

Σk +1

Σk +1

= Ax −k px qx +k v

= Ax -k px qx +k v

k =n

k = n

∞

∞

nΣ k n+1

nΣ k n+1

= Ax − v npxv −k−npx +nqx +n+k−n

= Ax - v npxv -k-npx +nqx +n+k-n

k =n

k = n

Evaluation of EPV and variance using whole life quantities

使用全寿命量评估 EPV 和方差

If we make the substitution r = kn inside the summation, we get

如果我们在求和中进行替换 r = kn，我们得到

−

−

A1

A1

x :n

x:n

Interpretation A1

解释 A1

∞

∞

= Ax − v npxvr px +nqx +n+r

= Ax - v npxvr px +nqx +n+r

Σnr +1

Σnr +1

r =0

r =0

∞

∞

Σnr +1

Σnr +1

= Ax − v npxvr |qx +n

= Ax - v npxvr |qx +n

r =0

r =0

= Ax − vnnpx Ax +n

= Ax - vnnpx Ax +n

is the difference between Ax (no restriction

是斧头之间的区别（没有限制

x :n

x:n

on when death can occur) and, Ax+n (no restriction on when

关于何时可能发生死亡）和，Ax+n（对何时发生没有限制

death occurs but starting in n years time) discounted by back by n years and multiplied by the probability that the life survives n years from x .

死亡发生但开始于 n 年时间) 向后贴现 n 年并乘以生命从 x 存活 n 年的概率。

Description and Definitions

描述和定义

Definition: A pure endowment contract pays a sum assured S after n years provided a life aged x is still alive. The payment, G, is 0 if the policyholder dies within n years, but S if the policyholder survives to the end of the term.

定义：如果 x 岁的人还活着，纯养老合同在 n 年后支付保额 S。如果投保人在 n 年内死亡，则赔付 G 为 0，如果投保人存活到期限结束，则赔付为 S。

G0if Kx < n Svnif Kx ≥ n

G0if Kx < n Svnif Kx ≥ n

(=

(=

(12)

(12)

E{G} = SvnP(Kx ≥ n) + 0 × P(Kx < n)

E{G} = SvnP(Kx ≥ n) + 0 × P(Kx < n)

= Svnnpx

= svnnpx

= SA 1

= 1

x :n

x:n

The actuarial notation:

精算符号：

A

A

1 = vnnpx(13)

1 = vnnpx(13)

signifies

表示

x :n

x:n

that the benefit is only paid when the term of n years ends,

该福利仅在 n 年期限结束时支付，

i.e. the moment that the n status fails. That is why the number is placed above n .

即 n 状态失败的那一刻。这就是为什么将数字放在 n 之上的原因。

The number placed above n is 1, meaning that the term status needs to be the first of the two terms to fail.

n 上方的数字是 1，这意味着术语状态需要是两个术语中的第一个才能失败。

Example At a certain company, the probability of each employee leaving during any given year is 5%, independent of the other employees. Those who remain with the company for 25 years are given $1, 000, 000. Calculate the EPV of this payment to a new starter, assuming an effective interest rate of 7% pa and ignoring the possibility of death.

示例 在某家公司，每位员工在任何给定年份离职的概率为 5%，与其他员工无关。那些在公司工作了 25 年的人将获得 1,000,000 美元。计算这笔支付给新启动者的 EPV，假设有效利率为每年 7% 并忽略死亡的可能性。

Solution Think of leaving the company as “death” or

解决方案 将离开公司视为“死亡”或

non-survival, and remaining with the company as being still alive or survival. The probability of remaining with the company in any given year is 0.95, so the probability of remaining 25 years is 0.9525 (i.e. 25px = 0.9525, and is independent of x ).

非生存，并留在公司作为仍然活着或生存。在任何给定年份留在公司的概率为 0.95，因此剩余 25 年的概率为 0.9525（即 25px = 0.9525，并且与 x 无关）。

Therefore, the EPV is:

因此，EPV 为：

E{G} = 1, 000, 000 × v 2525px

E{G} = 1,000,000 × v 2525px

1 000 00010 9525

1 000 00010 9525

=,,×× .

=,,×× .

1.0725

1.0725

= $51, 109

= $51, 109

Following the same pattern of derivation as for the whole life assurance:

遵循与终身寿险相同的派生模式：

var

曾是

{G} =

{G} =

2#

2#

S2 "2A 1 − A

S2 "2A 1 - A

1

1

x

x

where

在哪里

:nx :n

:nx:n

2A 1 = v 2nnpx(14)

2A 1 = v 2nnpx(14)

x :n

x:n

which, as before, is equivalent to evaluating A 1 at a rate

和以前一样，这相当于以一个速率评估 A 1

(1 + i)2 − 1.

(1 + i)2 - 1。

x :n

x:n

Example.Calculate the standard deviation of the payment described in the previous example.

示例。计算前面示例中描述的付款的标准差。

Solution We already have A

解决方案 我们已经有一个

1 = 0.051109 from the previous

1 = 前一个的 0.051109

x :n

x:n

part. To evaluate 2A

部分。评估 2A

1 , we replace v = 1 1

1 , 我们替换 v = 1 1

by v =1 2 .

将 v =1 2 。

Therefore;

所以;

x :n

x:n

.07

.07

1.07

1.07

, so

， 所以

2A 1 =

2A 1 =

x :n

x:n

125

125

1.072

1.072

× 0.9525

× 0.9525

= 0.009417

= 0.009417

var{G} =(1, 000, 000)2 × [0.009417 − 0.0511092]

var{G} =(1,000,000)2 × [0.009417 - 0.0511092]

√q

√q

= $82, 490

= $82, 490

Description and Definitions

描述和定义

An endowment assurance is a combination of a term assurance and a pure endowment.

禀赋保证是期限保证和纯禀赋的结合。

Definition An endowment assurance contract pays a sum assured S to a life now aged x at the end of the year of death, if death occurs during the next n years, or after n years if the life is then alive.

定义 如果在接下来的 n 年内发生死亡，或者如果该生命还活着，则在 n 年后，养老保险合同将在死亡当年年底向现在年龄 x 的生命支付保额 S。

The present value H of the payment is:

付款的现值 H 为：

or equivalently,

或等效地，

HSv Kx +1if Kx < n Svnif Kx ≥ n

HSv Kx +1如果 Kx < n Svnif Kx ≥ n

H = vmin{Kx +1,n}

H = vmin{Kx +1,n}

(

(

=

=

(15)

(15)

Expected Present Value

预期现值

n−1

n-1

Σ k +1n

S k +1n

E{H} = SvP(Kx = k ) + Sv P(Kx ≥ n)

E{H} = SvP(Kx = k ) + Sv P(Kx ≥ n)

k =0 n−1

k =0 n−1

Σ k +1n

S k +1n

= Svk|qx + Sv npx

= Svk|qx + Sv npx

k =0

k =0

= SA1

= SA1

x :n

x:n

= SAx:n

= SAx: n

+ SA

+ 南非

1

1

x :n

x:n

The symbol Ax:n has no number above either status, meaning that payment is made on the first of the two statuses to fail, regardless of order.

符号 Ax:n 在任一状态上方都没有数字，这意味着无论顺序如何，都会在两种失败状态中的第一个状态下进行付款。

The quantity,

数量，

n−1

n-1

Σ k +1n

S k +1n

Ax:n =vk|qx + v npx(16)

轴：n =vk|qx + v npx(16)

k =0

k =0

can be interpreted as the EPV of 1, paid after n years, or at the end of the year of death for a life aged x , whichever occurs first.

可以解释为 1 的 EPV，在 n 年后支付，或在死亡年份结束时支付 x 岁的生命，以先到者为准。

Expected Present Value

预期现值

Similarly to before, we find the variance is given by

与之前类似，我们发现方差由下式给出

where

在哪里

::

::

n−1

n-1

Σ

Σ

var{H} = S2 h2Ax n − Ax n 2i

var{H} = S2 h2Ax n - Ax n 2i

2Ax :n =v 2(k +1)k|qx + v 2nnpx k =0

2Ax :n =v 2(k +1)k|qx + v 2nnpx k =0

which is again the same as evaluating Ax:n at interest rate

这再次与以利率评估 Ax:n 相同

(1 + i)2 − 1.

(1 + i)2 - 1。

Annuity Contracts

年金合同

Life Tables and Survival Models Revision

生命表和生存模型修订

Life Insurance Contracts - Background

人寿保险合同 - 背景

Life Assurance Contracts

人寿保险合同

Whole of Life Assurance

终身寿险

Term Assurance

定期保证

Pure Endowment

纯禀赋

Endowment Assurance

捐赠保证

Life Annuity Contracts

人寿年金合约

Whole Life Annuities

终身年金

Temporary Annuities

临时年金

Deferred Annuities

递延年金

Guaranteed Annuities

保证年金

Continuous-time Calculations

连续时间计算

Definition: A life annuity contract provides payments at stated times provided a life is still alive.

定义：人寿年金合同在规定的时间提供付款，前提是人还活着。

There are 4 main types of annuity contracts

年金合同主要有4种类型

Whole life level annuity. Payments made for the whole of

终身年金。支付的全部款项

life with level payments.

生活水平支付。

Temporary level annuity. Level payments are made only

临时级年金。仅进行水平付款

during a limited term (provided of course the life is still alive).

在有限的期限内（当然前提是生命还活着）。

Guaranteed annuity. Annuities under which payments are

保证年金。支付的年金

made for the whole of life, or for a given term if longer.

为整个生命或特定期限（如果更长）而制造。

Deferred annuity. Any life annuity under which the start of

延期年金。任何开始年金的人寿年金

payment is deferred.

付款被推迟。

Many combinations or product possibilities are encountered in practice. The income form may be:

在实践中会遇到许多组合或产品可能性。收入形式可能是：

level, e.g. $X per annum.

水平，例如$X 每年。

increasing, e.g. starting at $Y pa, but increasing at 5% per annum.

增加，例如从 $Y pa 开始，但以每年 5% 的速度增长。

may be tied to the value of an investment account (variable annuities).

可能与投资账户的价值（可变年金）挂钩。

The term may be:

该术语可能是：

For a whole of a person’s life.

对于一个人的一生。

a limited term, e.g. 5 years.

一个有限的期限，例如5年。

a minimum term, e.g. at least 5 years or the length of life, whichever is greater.

最低期限，例如至少 5 年或生命长度，以较大者为准。

deferred, e.g. $Z per annum paid from the policyholder’s 60th birthday, or paid from retirement.

延期，例如$Z 从投保人 60 岁生日起支付，或从退休后支付。

Example from industry: MyNorth Guarantees offered by AMP https://www.northonline.com.au/guarantees

行业示例：AMP 提供的 MyNorth 保证 https://www.northonline.com.au/guarantees

A line of variable annuity type products, that pay an income (post expiry of a term), dependent on the highest level taken by an investment account.

一系列可变年金类型的产品，根据投资账户的最高水平支付收入（期限届满后）。

The value of the protected balance (amount due to policyholder) is set to be the greeater of the highest value taken by the investment on policy anniversaries or the initial value of the investment.

受保护余额的价值（应付保单持有人的金额）被设置为在保单周年纪念日投资获得的最高价值或投资的初始价值中的较大者。

Various vesting options, annuity payment terms on policy expiry.

各种归属选项，保单到期时的年金支付条款。

The value of such policies depends on the investment outcome, (and life of policyholder, and customer behaviour,

此类保单的价值取决于投资结果（以及保单持有人的寿命和客户行为，

e.g. vesting).

例如归属）。

Computational Monte Carlo methods used to price and hedge such policies.

用于对此类保单进行定价和对冲的计算蒙特卡罗方法。

Insurer must administer a hedging program to cover the possible shortfall that may occur between the amount due to the policyholder and the investment account value. (squiggly blue line: no liability for insurer, squiggly red line: shortfall that insurer must cover).

保险公司必须实施对冲计划，以弥补保单持有人应付金额与投资账户价值之间可能出现的差额。 （波浪状的蓝线：保险公司不承担责任，波浪状的红线：保险公司必须承担的差额）。

Industry has expended since introduction of North, varied success (perception of high fees, rising equity markets).

自从引入 North 以来，行业已经扩张，取得了不同程度的成功（对高额费用的认识，股票市场的上涨）。

We assume payments are made in arrears starting at the end of the first policy anniversary - an immediate whole life annuity.

我们假设从第一个保单周年日结束时开始拖欠付款 - 即期的终身年金。

The word “immediate” here is used to distinguish it from a deferred annuity where payments start some years in the future. (This is different to the usage in the term immediate annuity where first payment is made within the first year).

这里的“立即”一词用于将其与延期年金区别开来，后者在未来几年开始付款。 （这与立即年金一词的用法不同，即在第一年内首次付款）。

Consider an annuity contract to pay 1 at the end of each future year, provided a life aged x is then alive.

假设一个年金合同在未来每一年结束时支付 1，假设一个年龄为 x 的人当时还活着。

Example, suppose for a life aged x , the person dies in the third year after taking out the annuity, i.e. death occurs between time 2 and time 3.

例如，假设一个人 x 岁，在取出年金后的第三年死亡，即死亡发生在时间 2 和时间 3 之间。

Here the life has survived for two complete years before dying,

生命在这里存活了整整两年才死去，

Kx = 2, and the present value of the annuity is a2 .

Kx = 2，年金现值为a2。

In general:

一般来说：

If a life aged x dies between the ages x + k and x + k + 1 for (k = 0, . . . , ω − x − 1), then the PV of the annuity is ak , since k is just the curtate future lifespan,

如果年龄 x 的生命在 (k = 0, . . . , ω - x - 1) 的年龄 x + k 和 x + k + 1 之间死亡，则年金的 PV 为 ak ，因为 k 只是 curtate未来的寿命，

PV = aKx

PV = aKx

We have considered the above for unit payments of 1, if the level payment is S, then the present value is given by SaKx .

我们已经考虑了上述单位支付 1，如果支付水平为 S，则现值由 SaKx 给出。

Since we know the distrubtion of Kx , i.e. we know how to compute P(Kx = k ) = k|qx , we can compute moments of aKx , in particular the expectation (first moment) and variance (derived from the second moment).

由于我们知道 Kx 的分布，即我们知道如何计算 P(Kx = k ) = k|qx ，我们可以计算 aKx 的矩，特别是期望（第一矩）和方差（从第二矩导出）。

Expected Present Value

预期现值

We introduce the notation for the EPV of a whole life annuity:

我们介绍终身年金 EPV 的符号：

∞

∞

Σ

Σ

ax = E{aKx } =ak P(Kx = k )

ax = E{aKx } =ak P(Kx = k )

k =0

k =0

ax is the expected present value (NOT the present value) of an immediate annuity of 1 unit paid in arrears for as long as the the life status x remains in the alive state.

ax 是在生命状态 x 保持有效状态时，拖欠支付的 1 个单位的即时年金的预期现值（不是现值）。

Note the difference to an which is the present value of an immediate annuity of 1 unit paid in arrears for as long as the term status n remains active, that is until n years have elapsed.

请注意与 an 的差异，即只要期限状态 n 保持有效，即直到 n 年过去，即期年金 1 个单位的拖欠支付的现值。

Expected Present Value

预期现值

To derive an expression for ax , we note again that

为了导出 ax 的表达式，我们再次注意到

P(Kx = k ) = k|qx , and write

P(Kx = k ) = k|qx ，然后写

∞

∞

Σ

Σ

ax= E{aKx } =ak k|qx

ax= E{aKx } =ak k|qx

k =0

k =0

v +  k|qx

v +  k|qx

Σ kΣ−1

Σ kΣ−1

∞

∞

=

=





k =1

k =1

j =0

j =0

j 1

j 1

In the above step, we have:

在上述步骤中，我们有：

Used the fact that ak = v + v 2 + · · · + vk = Σk−1 v j +1.

使用了 ak = v + v 2 + · · · + vk = Σk−1 v j +1 的事实。

j =0

j =0

Note that we used a different summation variable, j, for the annuity payments, to distinguish it from the summation variable across possible curtate lifespans which is k .

请注意，我们为年金支付使用了不同的求和变量 j，以将其与跨可能的简单寿命的求和变量（即 k）区分开来。

Also used the fact that a0 = 0 which allowed us to drop the term for k = 0 in the summation.

还使用了 a0 = 0 的事实，这允许我们在求和中删除 k = 0 的术语。

Expected Present Value

预期现值

Writing out the sum more fully

更完整地写出总和

ax = v × 1|qx

ax = v × 1|qx

+ (v + v 2) × 2|qx

+ (v + v 2) × 2|qx

+ (v + v 2 + v 3) × 3|qx

+ (v + v 2 + v 3) × 3|qx

.

.

, and reversing the order of summation by collecting powers of

, 并通过收集 的幂来颠倒求和的顺序

v :

在 ：

ax = v 1|qx + 2|qx + · · · + v 2 2|qx + 3|qx + · · · + · · ·

ax = v 1|qx + 2|qx + ···+ v 2 2|qx + 3|qx +···+···

∞

∞

∞

∞

= v Σ k|qx + v 2 Σ k|qx + · · ·

= v Σ k|qx + v 2 Σ k|qx + ···

So:

所以：

k =1

k =1

k =2

k =2

a = Σ∞  Σ∞

a = Σ∞  Σ∞

x

x

k|qx  vj+1

k|qx  vj+1

j=0k =j+1

j=0k =j+1

Note that∞k =1 k|qx is the probability that the life dies at some point after time 1 year, this is equal to the probability that the life is still alive at time 1 year, i.e. 1px . In general,∞k =j +1 k|qx is equal to j+1px . Hence;

请注意，∞k =1 k|qx 是生命在 1 年后的某个时间点死亡的概率，这等于生命在 1 年的时间仍然活着的概率，即 1px 。一般来说，∞k =j +1 k|qx 等于 j+1px 。因此;

Σ

Σ

Σ

Σ

∞∞

∞∞

ax = Σ j +1px vj +1 = Σ j px vj

ax = Σ j +1px vj +1 = Σ j px vj

Interpretation:

解释：

j =0

j =0

j =1

j =1

Each payment is conditional on whether the policyholder is alive or not at the time payment is due.

每笔付款都取决于保单持有人在付款到期时是否还活着。

The present value of the annuity payment made at the time

当时支付的年金的现值

j is vj .

j 是 vj 。

The expected present value of this payment is vj multiplied by the probability that the policyholder is still alive at this time, j px .

这笔付款的预期现值是 vj 乘以投保人此时还活着的概率 j px 。

Summing over all future years gives the expected present

对所有未来年份求和得出预期的现在

value of all future annuity payments.

所有未来年金支付的价值。

Example: A whole life annuity, paying $1 per annum, is payable in arrears to a life aged 90. The interest rate is 5% per annum. Mortality is assumed to follow the life table below. What is the expected present value of this annuity?

示例：终身年金，每年支付 1 美元，将拖欠 90 岁的人。利率为每年 5%。假设死亡率遵循下面的生命表。该年金的预期现值是多少？

Solution Use the formula ax =∞j =1 j px v j , the expected present value is:

解 使用公式 ax =∞j =1 j px v j ，预期现值为：

Σ

Σ

a90 = vp90 + v 22p90 = v l91 + v 2 l92

a90 = vp90 + v 22p90 = v l91 + v 2 l92

l90l90

l90l90

(Note: j p90 = 0 for j3). Using the values from the above table:

（注：对于 j3，j p90 = 0）。使用上表中的值：

≥

≥

a90 = 1.05−1 × 75 + 1.05−2 × 40 = 1.07710

a90 = 1.05−1 x 75 + 1.05−2 x 40 = 1.07710

100100

100100

The EPV of a whole life annuity is also related to the EPV of a

终身年金的 EPV 也与

i

i

whole life assurance. Since an

终身保障。由于一个

a

a

= 1−vn :

= 1−vn：

1 − v Kx

1 - v Kx

x = E{aKx } = E{i}

x = E{aKx } = E{i}

= E 1 1 −

= E 1 1 -

i

i

v Kx +1

v Kx +1

v

v

=1 (v − E{v Kx +1})

=1 (v − E{v Kx +1})

iv

iv

1

1

=d (v − Ax )

=d (v - Ax)

where d = iv and Ax = E v Kx +1 is the EPV of a unit whole life assurance.

其中 d = iv 和 Ax = E v Kx +1 是单位终身寿险的 EPV。

{}

{}

Question to ponder: why is vAx (and hence ax ) always positive?

思考的问题：为什么 vAx （因此 ax ）总是积极的？

−

−

The variance can also be calculated by utilising previous results on assurances

方差也可以通过利用以前的保证结果来计算

var

曾是

{aKx } = var{

{aKx } = 变量{

1v Kx

1v Kx

i}

一世}

−

−

=1 var{v Kx }

=1 变量{v Kx }

i2

i2

=1 var

=1 是

i2

i2

v Kx +1

v Kx +1

v}

在}

{

{

1 var v Kx +1

1 var v Kx +1

={}

={}

i2v 2

i2v 2

=1 h2Ax − (Ax )2i

=1 h2Ax - (Ax )2i

d 2

d 2

(Recall the meaning of the superscript 2: an assurance function (Ax ) evaluated at rate of interest (i + 1)2 − 1.

（回想上标 2 的含义：保证函数 (Ax ) 以利率 (i + 1)2 - 1 计算。

Notes on steps in the derivation:

推导步骤的注意事项：

In the second line we have used the fact that

在第二行中，我们使用了以下事实

var a + X = var X when a is a constant and X is a random variable.

当 a 为常数且 X 为随机变量时，var a + X = var X。

{}{ }

{}{ }

The second and fourth lines are using

第二行和第四行正在使用

var{aX} = a2var{X} for constant a and random variable

var{aX} = a2var{X} 用于常量 a 和随机变量

x .

X 。

The final line uses the result from Lecture 10 slide 30 for the variance of a unit assurance var{v Kx +1}.

最后一行使用 Lecture 10 slide 30 的结果作为单位保证 var{v Kx +1} 的方差。

Example: A whole life annuity of $100 pa is payable annually in arrears to a life aged 65. Calculate the standard deviation of the benefits from this annuity, assuming AM92 Ultimate mortality and an annual effective interest rate of 4%.

示例：每年 100 美元的终身年金支付给 65 岁的人。计算该年金收益的标准差，假设 AM92 最终死亡率和 4% 的年实际利率。

Solution: The variance of the benefits from this annuity is:

解：该年金收益的方差为：

var{100a} =h2A65 − (A65)2i

var{100a} =h2A65 - (A65)2i

100

100

K65

K65

d 2

d 2

using values from the tables (page 97), and recalling that

使用表格中的值（第 97 页），并回顾

d = iv , this gives the variance as:

d = iv ，这给出了方差：

1002

1002

h0 30855

h0 30855

0 52786 2i

0 52786 2i

449 69 2

449 69 2

= (

= (

.

.

)

)

So we have a standard deviation of $449.69.

所以我们的标准差是 449.69 美元。

(0.04/1.04)2

(0.04/1.04)2

.

.

— (

— (

.

.

)

)

Consider an annuity contract to pay 1 at the start of the current and future years, provided a life now aged x is then alive.

考虑在当前和未来年份开始时支付 1 的年金合同，假设现在年龄为 x 的生命还活着。

Recall a¨n is the present value of an annuity due with n

回想一下，a¨n 是 n 到期的年金的现值

payments at the start of each of the next n years, so payments times are at 0, 1, . . . , n1 .

在接下来 n 年的每一年开始时付款，因此付款时间为 0, 1, . . . , n1 。

{− }

{− }

For a whole life annuity in advance:

提前领取终身年金：

PV = a¨Kx +1

PV = a¨Kx +1

This provides:

这提供了：

1 at age x (at the start of the first year), certain to be paid.

1 在 x 岁时（在第一年开始时），肯定会得到报酬。

a further payment of 1 at the start of each subseuquent policy year, as long as policyholder is alive. (Note the last payment will be at time Kx . Don’t get confused by the index Kx + 1, which indicates the number of payments.

只要保单持有人还活着，在随后的每个保单年度开始时再支付 1。 （注意最后一次付款将在时间 Kx 。不要被索引 Kx + 1 混淆，它表示付款次数。

Introduce notation a¨x , meaning EPV of unit annuity payments made in advance for as long as the life survives.

引入符号 a¨x ，表示只要生命存在，就提前支付单位年金的 EPV。

a¨x= E{a¨Kx +1 }

a¨x= E{a¨Kx +1 }

Σ

Σ

k

k

∞

∞

Σ

Σ

=

=

k =0j =0

k =0j =0





v j  k|qx

v j  k|qx

= Σ Σ∞

= Σ Σ∞

∞

∞





q  v j

q  v j

j =0

j =0

∞

∞

Σ

Σ

k| x

k| X

k =j

k =j

=j px v j j =0

=j 像素 v j j =0

Evaluate the following using the ABS Australian Mortality Tables 2016-2018, and assuming the effective annual interest rate is 5%

使用 ABS 澳大利亚死亡率表 2016-2018 评估以下内容，并假设有效年利率为 5%

a). a¨30 using Victoria female mortality. b). a¨75 using Victoria male mortality.

一个）。 a-30 使用维多利亚女性死亡率。乙）。 a-75 使用维多利亚男性死亡率。

Solution:

解决方案：

Use the life table lx to evaluate the required j px = lx +j , then

使用生命表 lx 来评估所需的 j px = lx +j ，然后

lx

lx

multiply by the discount factor v j and sum. See Week 1 work book.

乘以折扣因子 v j 和总和。请参阅第 1 周工作手册。

Answers: a¨30 = 19.34, and a¨75 = 9.31

答案：a-30 = 19.34，a-75 = 9.31

Variance

方差

Recall that the present value of an annuity in advance is given

回想一下，预先给定年金的现值

n

n

by a¨n = 1−v , where d = iv , using this to evaluate the variance

通过 a¨n = 1−v ，其中 d = iv ，使用它来评估方差

d

d

gives:

给出：

var{a¨Kx +1 } = var{

var{a¨Kx +1 } = var{

1v Kx +1

1v Kx +1

d}

d}

−

−

=1 var{v Kx +1}

=1 变量{v Kx +1}

d 2

d 2

=1 h2Ax − (Ax )2i

=1 h2Ax - (Ax )2i

d 2

d 2

Background

背景

A temporary annuity has payments that are limited to a specified term, unlike whole life which is not. The payments in a temporary annuity will stop even if the life is still alive.

临时年金的付款仅限于特定期限，而终身年金则没有。即使生命还活着，临时年金的支付也会停止。

Definition:

定义：

A temporary annuity in arrears is a contract that pays 1 at the end of each of the next n years, provided a life now aged x is then alive. Denote the present value X of a temporary annuity payable in arrears by:

拖欠的临时年金是在未来 n 年的每一年结束时支付 1 的合同，前提是现在 x 岁的人还活着。用以下方式表示拖欠应付的临时年金的现值 X：

(

(

X =aKxif Kx < n

X =aKxif Kx < n

anif Kx ≥ n

动画 Kx ≥ n

So, X = amin{Kx ,n} .

所以，X = amin{Kx ,n} 。

E{amin{Kx ,n} } =

E{amin{Kx ,n} } =

∞

∞

amin k,n P(Kx = k )

所有 k,n P(Kx = k)

Σ{}

Σ{}

k =0

k =0

n−1∞

n−1∞

= Σ ak P(Kx = k ) + an Σ P(Kx = k )

= Σ ak P(Kx = k ) + Σ P(Kx = k )

k =0

k =0

n−1

n-1

Σ

Σ

k =n

k = n

=ak P(Kx = k ) + an P(Kx ≥ n)

=ak P(Kx = k ) + an P(Kx ≥ n)

k =0 n−1

k =0 n−1

Σ

Σ

=k|qx ak + npx an

=k|qx 和 + npx 一个

k =0

k =0

In actuarial notation E{amin Kx ,n } is denoted by ax:n , and it can be shown that:

在精算记法中，E{amin Kx ,n } 用 ax:n 表示，可以证明：

{}

{}

ax :n= E{amin{Kx ,n} }

斧头：n= E{amin{Kx ,n} }

n−1

n-1

Σ

Σ

=k|qx ak + npx an

=k|qx 和 + npx 一个

k =1 n

k =1 大约

Σ

Σ

=k px vk k =1

=k 像素 vk k =1

The last step follows the same pattern as for deriving the simplification of ax ; i.e. express an as a sum of the PV of the payments made, and then reverse the order of summation to write as a sum of powers of v . This is left as an exercise.

最后一步遵循与推导 ax 的简化相同的模式；即，将 an 表示为已付款 PV 的总和，然后将求和顺序颠倒以写为 v 的幂的总和。这留作练习。

The expression seems logical given that ax =∞k =1 k px vk for the whole life annuity, and this is the same summation but only allowing for the first n terms.

考虑到整个终身年金的 ax =∞k =1 k px vk，这个表达式似乎是合乎逻辑的，这是相同的求和，但只允许前 n 项。

Σ

Σ

The notation used, ax:n , follows the same logic as for the assurance Ax:n only it relates to a series of payments instead of a single payment. It says that the series of annuity payments are made in arrears until either the life status, x, or the term status n fails.

使用的符号 ax:n 遵循与保证 Ax:n 相同的逻辑，只是它涉及一系列支付而不是单次支付。它表示，一系列年金支付被拖欠，直到生命状态 x 或期限状态 n 失败。

Example:

例子：

Calculate the value of a40:3 when the effective annual rate of interest is 6% and:

计算实际年利率为 6% 时 a40:3 的值，并且：

lx = 100 − xat all ages x ≤ 100

lx = 100 - x 在所有年龄 x ≤ 100

Solution;

解决方案;

Using the formula:

使用公式：

a= Σ v j j p40 = v l41 + v 2 l42 + v 3 l43

a= Σ v j j p40 = v l41 + v 2 l42 + v 3 l43

j =1

j =1

3

3

40:3

40:3

l40

l40

l40

l40

l40

l40

= 1.06−1 × 59 + 1.06−2 × 58 + 1.06−3 × 57

= 1.06−1 × 59 + 1.06−2 × 58 + 1.06−3 × 57

60

60

= 2.58564

= 2.58564

6060

6060

Variance

方差

The variance of the temporary annuity in arrears can in fact be expressed in terms of the moments of the endowment assurance, Ax:n . Specifically,

拖欠临时年金的方差实际上可以用养老保障的时刻 Ax:n 来表示。具体来说，

var{a} =h2A− (A)2i

var{a} = h2A− (A)2i

1

1

min{Kx ,n}

最小{Kx ,n}

d 2

d 2

x :n+1

x :n+1

x :n+1

x :n+1

The proof of this will be easy to see once it is worked out for the temporary annuity-due.

一旦计算出临时年金到期，这一点的证明将很容易看到。

Background

背景

Definition: A temporary immediate annuity in advance (or annuity-due) has payments made in advance and limited to a specified term.

定义：预付临时即时年金（或到期年金）提前支付并限制在特定期限内。

The present value, Y is given by:

现值 Y 由下式给出：

Y=a¨Kx +1if Kx < n

Y=a¨Kx +1如果 Kx < n

(

(

a¨nif Kx ≥ n

a¨nif Kx ≥ n

= a¨min{Kx +1,n}

= a¨min{Kx +1,n}

The contract pays 1 at the start of each of the next n years, provided a life now aged x is alive.

假设现在 x 岁的生命还活着，合约在接下来的 n 年开始时支付 1。

The expected present value is given the notation a¨x :n

预期现值用符号 a¨x :n

a¨x :n = E{a¨min{Kx +1,n} }

a¨x :n = E{a¨min{Kx +1,n} }

∞

∞

Σ{}

Σ{}

=a¨min k +1,n P(Kx = k )

=a¨min k +1,n P(Kx = k )

k =0

k =0

n−1∞

n−1∞

= Σ a¨k +1 P(Kx = k ) + a¨n Σ P(Kx = k )

= Σ a¨k +1 P(Kx = k ) + a¨n Σ P(Kx = k )

k =0

k =0

n−1

n-1

Σ

Σ

=k|qx a¨k +1 + npx a¨n

=k|qx a¨k +1 + npx a¨n

k =0

k =0

k =n

k = n

Σn−1 Σkj Σn−1 j 

Σn−1 Σkj Σn−1 j 

k =0

k =0

j =0

j =0

j =0

j =0

=

=





v  k|qx + 

v  k|qx + 

v  npx

v  npx

Σn−1 Σn−1

Σn−1 Σn−1

k|qx + npx  v j

k|qx + npx  v j

j =0k =j

j =0k =j





=

=

n−1

n-1

Σ

Σ

=j px v j j =0

=j 像素 v j j =0

EPV of temporary annuity-due is a similar result to whole life annuity due where the EPV is∞j =0 j px v j , but with payments only continuing up to time n − 1.

临时年金到期的 EPV 与终生年金到期的结果相似，其中 EPV 为∞j =0 j px v j ，但付款仅持续到时间 n - 1。

Σ

Σ

Note: a¨x :n − ax:n /= 1 unlike whole life annuties where

注意： a¨x :n − ax:n /= 1 不像终身年金

a¨x − ax = 1.

a¨x - ax = 1。

A relationship which holds is a¨x :n+1 − ax:n = 1. (See tutorial questions).

一个成立的关系是 a¨x :n+1 − ax:n = 1。（见教程问题）。

Example A 35 year old male purchases an endowment assurance with a term of 30 years. The premiums for the policy are payable annually in advance while the policy is in force, and each premium is $2, 500. Calculate the expected present value of the premiums paid, using the Victoria ABS Mortality Tables. Solution

示例 一名 35 岁的男性购买了一份为期 30 年的养老保险。保单的保费在保单生效期间每年提前支付，每笔保费为 2 美元，500 美元。使用维多利亚 ABS 死亡率表计算已付保费的预期现值。解决方案

Note: this question is about the present value of the premiums paid for the endowment assurance, and not the present value of the endowment assurance itself. The premiums paid are like a temporary immediate annuity-due, with a term of 30 years.

注意：这个问题是关于为养老保险支付的保费的现值，而不是养老保险本身的现值。所支付的保费就像临时的即时年金到期，期限为 30 年。

See lecture work book in the sheet

请参阅工作表中的讲座工作簿

“TemporaryAnnuityDueExample”; a¨35:30 is calculated via the

“临时年金到期示例”； a¨35:30 是通过计算

formula Σ29 j p35v j to give 17.6254. Then

公式 Σ29 j p35v j 给出 17.6254。然后

j =0

j =0

2500a¨35:30 = 2500 × 17.6254 = $44, 063.51

2500a¨35:30 = 2500 × 17.6254 = $44, 063.51

n

n

Note that since a¨n = 1−v , we have

请注意，由于 a¨n = 1−v ，我们有

d

d

1 − v min{Kx +1,n}

1 − v min{Kx +1,n}

var{a¨min{Kx +1,n} } = var{d}

var{amin{Kx +1,n} } = var{d}

=1 var{v min{Kx +1,n}}

=1 var{v min{Kx +1,n}}

d 2

d 2

=1 2Ax n − (Ax n )2

=1 2Ax n - (Ax n )2

d 2

d 2

:

:

:

:

The very last line uses the result for the variance of an endowment assurance.

最后一行将结果用于捐赠保证的方差。

Note that this now allows us to derive the previously stated result for the variance of a temporary annuity in arrears. Since ax:n = a¨x :n+1 − 1, replacing n with min{Kx , n} gives:

请注意，这现在允许我们推导出先前所述的拖欠临时年金方差的结果。由于 ax:n = a¨x :n+1 − 1，将 n 替换为 min{Kx , n} 给出：

var{amin{Kx ,n} } = var{a¨min{Kx +1,n+1} − 1}

var{amin{Kx ,n} } = var{amin{Kx +1,n+1} - 1}

= var{a¨min{Kx +1,n+1} }

= var{aamin{Kx +1,n+1} }

x :n+1

x :n+1

h=

h=

1 2A

1 2A

x :n+1

x :n+1

d 2

d 2

— (A

- （一个

)2i

2i

Background

背景

Definition: Deferred life annuities are annuities under which payment does not begin immediately but is deferred for one or more years.

定义：延期人寿年金是指不立即开始支付而是延期一年或多年的年金。

Consider, an annuity paying 1 per annum payable annually in arrears to a life now aged x , deferred for n years; if X is the present value of the annuity, then:

考虑，每年支付 1 的年金，每年支付给现在年龄为 x 的人，延期 n 年；如果 X 是年金的现值，则：

= (

= (

X0 if Kx ≤ n vnaK −nif Kx > n

X0 如果 Kx ≤ n vnaK −nif Kx > n

x

x

(

(

0if Kx ≤ n

0如果 Kx ≤ n

=a− aif Kx > n

=a− aif Kx > n

Kxn

Kn

= vna

= vna

max{Kx −n,0}

最大{Kx -n,0}

The approach taken to get the expected value for the deferred annuity in arrears is to try and write the present value of the deferred annuity in terms of a whole life annuity (whose payments last for the whole of life) and a temporary annuity. Let

获得拖欠延期年金预期值的方法是尝试将延期年金的现值写入终生年金（其支付持续终生）和临时年金。让

Y = a

和 = 到

so that E{Y} = ax , and

使得 E{Y} = ax ，并且

Z = a

Z = 一个

so that E{Z} = ax:n .

所以 E{Z} = ax:n 。

=aKxif Kx ≤ n

=aKxif Kx ≤ n

if Kx > n

如果 Kx > n

Kx

Kx

aKx

aKx

(

(

=aKxif Kx ≤ n

=aKxif Kx ≤ n

Kx

Kx

(

(

anif Kx > n

动画 Kx > n

Using the above for Y and Z , we have:

使用上面的 Y 和 Z ，我们有：

(−

(−

YZ=aKx − aKx = 0if Kx ≤ n

YZ=aKx - aKx = 0 如果 Kx ≤ n

aKx − anif Kx > n

aKx - 动画 Kx > n

= X

= X

Therefore;

所以;

E{X} = E{Y − Z} = E{Y} − E{Z} = ax − ax:n

E{X} = E{Y - Z} = E{Y} - E{Z} = ax - ax:n

Interpretation: the value of the deferred annuity is equal to a series of payments paid for the whole of life, minus the value of the payments that will not be made for the first n years.

解释：递延年金的价值等于终生支付的一系列款项，减去前n年不支付的款项的价值。

The actuarial notation for the expected present value of a deferred whole life annuity is:

递延终身年金的预期现值的精算符号为：

E{X} = n|ax = ax − ax:n

E{X} = n|ax = ax − ax:n

The following expression can also be derived:

还可以推导出以下表达式：

n|ax = vnnpx ax +n

n|ax = vnnpx 斧头 +n

Intepretation of result: the expected present value of the deferred annuity is equal to the expected present value of life annuity for a survivor to age x + n, discounted back n years to allow for interest, multiplied by the probability, npx , that the policy holder survives to age x + n.

结果解释：递延年金的预期现值等于幸存者到 x + n 岁的人寿年金的预期现值，贴现 n 年以考虑利息，乘以概率 npx ，该保单持有人活到 x + n 岁。

With a similar definition for annuities paid in advance, the following relationship also holds:

对于预付年金的类似定义，以下关系也成立：

n|a¨x = vnnpx a¨x +n

n|a¨x = vnnpx a¨x +n

(note also that ax = 1|a¨x )

（还要注意 ax = 1|a¨x ）

Example: A 50-year old Victorian woman purchases a deferred annuity to provide herself with an income of $15, 000 paid annuallly in advance from age 70 until death. Calculate the expected present value of the benefits from this annuity, using the ABS Victoria life table and assuming an interest rate of 4% per annum effective.

示例：一名 50 岁的维多利亚妇女购买延期年金，为自己提供 15,000 美元的收入，从 70 岁到去世每年提前支付。使用 ABS Victoria 生命表并假设有效年利率为 4%，计算该年金收益的预期现值。

Solution: See lecture work-book, sheet ”DeferredAnnuityExample”. From the Victorian female life table, a¨70 = 13.01762, and 20p50 = 0.922893, so we have:

解决方案：参见讲座工作簿，表格“DeferredAnnuityExample”。从维多利亚时代的女性生命表中，a¨70 = 13.01762，并且 20p50 = 0.922893，所以我们有：

1500020|a¨50= 15000v 2020p50a¨70

1500020|a¨50= 15000v 2020p50a¨70

= 15000v 20 l70 a¨70

= 15000v 20 l70 a¨70

l50

l50

= 15000 × 1.04−20 × 0.922893 × 13.01762

= 15000 × 1.04−20 × 0.922893 × 13.01762

= $82, 244.59

= $82, 244.59

Background

背景

A guaranteed annuity is a whole life annuity (in arrears) with the payments having a minimum specified term.

保证年金是终生年金（拖欠），付款有最低规定期限。

A guaranteed annuity-due has payments that are made in advance and also have a minimum specified term.

保证年金到期具有提前支付的款项，并且还具有最低规定期限。

Widely used in industry; reason being that insurers wish to avoid bad publicity should a policyholder die soon after uptake of a policy.

广泛应用于工业；原因是如果保单持有人在投保后不久去世，保险公司希望避免不良宣传。

Consider a guaranteed annuity that pays 1 at the start (so an annuity in advance) of each future year for the next n years, and at the start of each subsequent year provided the life is then alive. If a life aged x dies within n years, then exactly n payments will be made. If the policyholder lives for longer than n years, then Kx + 1 payments will be made, so the present value is:

考虑一个保证年金，它在未来 n 年的每一年开始时支付 1（因此提前年金），并且在随后的每一年开始时支付 1，前提是生命还活着。如果一个年龄为 x 的生命在 n 年内去世，那么将恰好支付 n 次付款。如果投保人的寿命超过 n 年，则将支付 Kx + 1 付款，因此现值为：

PV=a¨nif Kx < n

PV=a¨nif Kx < n

(

(

if Kx ≥ n

如果 Kx ≥ n

a¨Kx +1

a¨Kx +1

= a¨max{Kx +1,n}

= a¨max{Kx +1,n}

The expected present value for a guaranteed annuity in advance is denoted by a¨x :n , where the notation x : n means that payments continue until both statuses - the life and the term fail. We have the result:

提前保证年金的预期现值由 a¨x :n 表示，其中符号 x : n 表示付款将持续到两种状态 - 寿命和期限都失败。我们有结果：

a¨x :n= E{a¨max{Kx +1,n}}

a¨x :n= E{a¨max{Kx +1,n}}

= a¨n + vnnpx a¨x

= a¨n + vnnpx a¨x

= a¨n + n|a¨x

= a¨n + n|a¨x

Similarly, for a guaranteed annuity in arrears, we have, the expected present value:

同样，对于拖欠的保证年金，我们有预期现值：

ax :n = an + n|ax

斧头：n = an + n|ax

Example: Calculate a60:10 for a Victorian Male using ABS mortality, and an interest rate of 6%.

示例：使用 ABS 死亡率和 6% 的利率计算维多利亚时代男性的 a60:10。

Solution: See lecture work book (”GuaranteedAnnuityExample” worksheet).

解决方案：参见讲座工作簿（“GuaranteedAnnuityExample”工作表）。

a¨60:10=

a¨60:10=

=

=

a¨10 + 10|a¨60

a¨10 + 10|a¨60

a¨10 + v 1010p60a¨70

a¨10 + in 1010p60a¨70

1 − v 10

1 - 在 10

=

=

d

d

+ v 10 l70 a¨70

+ v 10 l70 a¨70

l60

l60

11.06−10

11.06−10

−

−

=0.06/1.06

=0.06/1.06

+ 1.06

+ 1.06

−10

−10

85099

85099

× 93044 × 10.33308

× 93044 × 10.33308

= 13.07894

= 13.07894

Annuity Contracts

年金合同

Life Tables and Survival Models Revision

生命表和生存模型修订

Life Insurance Contracts - Background

人寿保险合同 - 背景

Life Assurance Contracts

人寿保险合同

Whole of Life Assurance

终身寿险

Term Assurance

定期保证

Pure Endowment

纯禀赋

Endowment Assurance

捐赠保证

Life Annuity Contracts

人寿年金合约

Whole Life Annuities

终身年金

Temporary Annuities

临时年金

Deferred Annuities

递延年金

Guaranteed Annuities

保证年金

Continuous-time Calculations

连续时间计算

In ETC2430, we considered payments to be made at the end of the year of death for assurances, and annuties to be paid yearly contingent on the individual being alive at the time of a payment.

在 ETC2430 中，我们认为应在死亡当年年底支付保证金，每年支付年金取决于个人在支付时是否还活着。

The product definitions allowed us to utilise the curtate future lifespan random variable, Kx which is discrete.

产品定义允许我们利用有限的未来寿命随机变量 Kx，它是离散的。

The extension to continuous time, where benefits could be paid immediately and annuities are paid continuously can be done with another random variable that we briefly studied, Tx , the complete future lifetime of a life aged x .

延长连续时间，即可以立即支付福利并连续支付年金，可以使用我们简要研究过的另一个随机变量 Tx 来完成，即 x 岁生命的完整未来寿命。

For assurance products, if the benefit is payable:

对于保障产品，如果支付福利：

at the end of the year of death, time of payment is Kx + 1. (discrete)

死亡年份结束时，支付时间为 Kx + 1。（离散）

immediately on death, time of payments is Tx . (continuous)

立即死亡，付款时间为 Tx 。 （连续的）

In continuous time, where benefits are paid immediately on death, the random present value for the payment will be

在连续时间内，在死亡时立即支付福利，支付的随机现值为

v Tx . In discrete time it was v Kx +1.

v 发送。在离散时间内，它是 v Kx +1。

Mathematically, the difference in EPV is that v Kx +1 is a discrete random variable, whereas v Tx is a continuous random variable.

在数学上，EPV 的区别在于 v Kx +1 是离散随机变量，而 v Tx 是连续随机变量。

Hence, we used the probability mass function of Kx for the former, and will need the probability density function(pdf) of Tx for the latter.

因此，我们对前者使用了 Kx 的概率质量函数，而对后者则需要 Tx 的概率密度函数 (pdf)。

Whole of life Assurances: Expectation

终身寿险：期望

Recall from ETC2430 (2022) Lecture 10 (slide 25): The expectation of v Kx +1 is given by:

回顾 ETC2430（2022）第 10 讲（幻灯片 25）：v Kx +1 的期望由下式给出：

Ev Kx +1= ∞

房子 Kx +1= ∞

,,Σ

,,S

k =0

k =0

vk+1P [Kx

vk+1P [Kx

∞

∞

= k ] =

= k] =

Σ

Σ

k =0

k =0

vk +1k|qx

vk +1k|qx

= Ax

= 斧头

Also recall, the pdf of Tx , which we denote by fTx (t ) is given by t px µx+t (Lecture 9 slide 6).

还记得，我们用 fTx (t) 表示的 Tx 的 pdf 由 t px µx+t 给出（第 9 讲幻灯片 6）。

Hence to get the continuous time version, the expectation of v Tx can be written as:

因此，为了获得连续时间版本，v Tx 的期望可以写成：

E ,v Tx , = ∫ ∞ vt fT (t )dt = ∫ ∞ vt t px µx+t dt = A¯x .

E ,v Tx , = ∫ ∞ vt fT (t )dt = ∫ ∞ vt t px µx+t dt = A¯x 。

x

x

00

00

Whole of life Assurances: Expectation

终身寿险：期望

Discrete: The benefit is payable at the end of year of death: Ax =∞k =0 vk +1k qx

离散：在死亡年份结束时支付给付：Ax =∞k =0 vk +1k qx

Σ|

小号|

k qx : if the policyholder dies in the year [k, k + 1),

k qx : 如果投保人在 [k, k + 1) 年内死亡，

|

|

a benefit of 1 is paid at the end of the year, at k + 1,

在年底支付 1 的福利，在 k + 1 时，

and is discounted back to time 0 by vk +1.

并且被 vk +1 折回到时间 0。

summed for all possible k .

对所有可能的 k 求和。

Continuous In case the benefit is payable immediately on death:

持续 如果福利在死亡时立即支付：

∫

∫

A¯x =∞ vt t px µx +t dt

A¯x =∞ vt t px µx +t dt

0

0

if the policyholder is alive at time t ( t px ),

如果投保人在时间 t ( t px ) 还活着，

and immediately dies at age x + t (µx+t dt ),

并在 x + t (µx+t dt ) 岁时立即死亡，

an amount of 1 is paid immediately, at t ,

在 t 时立即支付 1 的金额，

and is discounted back to time 0 by vt .

并且被 vt 折回到时间 0。

integrated across all possible t .

整合所有可能的 t 。

Whole of life Assurances: Variance

终身寿险：差异

Derivation of the variance follows along the lines of the discrete-time case:

方差的推导遵循离散时间情况：

var ,v Tx , = E ,(v Tx )2, − E ,v Tx , 2

var ,v Tx , = E ,(v Tx )2, - E ,v Tx , 2

= E ,(v 2)Tx , − A¯x 2

= E ,(v 2)Tx , - A¯x 2

= ∫ ∞(v 2)t t px µx+t dt − A¯x 2

= ∫ ∞(v 2)t t px µx+t dt − A¯x 2

0

0

= 2A¯x − A¯x 2

= 2A¯x − A¯x 2

2A¯x is just A¯x calculated at the rate of interest

2A¯x 就是按利率计算的 A¯x

ij = (1 + i)2 − 1.

ij = (1 + i)2 - 1。

Term Assurance

定期保证

The random variable for the benefit payable by a unit term assurance:

单位定期保险应付给付的随机变量：

(

(

F¯ =

F¯ =

The expectation is:

期望是：

v Txif Tx < n

v Txif Tx < n

0if Tx ≥ n

0如果 Tx ≥ n

A¯ 1

A¯ 1

x :n

x:n

= E F¯ } =

= E F¯ } =

∞ vt f

∞ 作为 f

0

0

∫

∫

Tx (t )dt =

Tx (t)dt =

n vt

n vt

0

0

∫

∫

t px µx +t dt

t px µx +t dt

The variance is:

方差为：

var F¯ } =

是 F¯ } =

2A¯

2A¯

1

1

x :n

x:n

— A¯

— A¯

2

2

1

1

x :n

x:n

Endowment Assurance

捐赠保证

The random variable for the benefit payable by a unit endowment assurance:

单位养老保障应付福利的随机变量：

H¯ =

H¯ =

The expectation is:

期望是：

v Txif Tx < n

v Txif Tx < n

vnif Tx ≥ n

vnif Tx ≥ n

(

(

A¯E F¯ }

A¯E F¯ }

x :n =

x:n =

∫ ∞ vt ft dt∫ n vt p

∫ ∞ vt ft dt∫ n vt p

0

0

0

0

dtvn p

电视节目

The variance is:

方差为：

::

::

=

=

Tx ( )

发送 ( )

=

=

t

t

x µx +t

x µx +t

+

+

n

n

x

x

var H¯ } = 2A¯x n − A¯x n 2

var H¯ } = 2A¯x n − A¯x n 2

Recall from ETC2430: Random PV of an annuity payable yearly in advance (provided individual is alive at time of payment):

从 ETC2430 召回：每年提前支付的年金的随机 PV（假设个人在支付时还活着）：

Kx

Kx

ΣavK +1

ΣavK +1

¨=k .

¨=k。

x

x

k =0

k =0

Random PV of an annuity payable continuously (at rate of

连续应付年金的随机 PV（按

$1 pa), till time of death:

每年 1 美元），直至死亡：

Tx

发送

∫

∫

vt dt.

vt dt。

a¯Tx =

a¯Tx =

0

0

Mathematically, moving to expectations is done in a similar way. We do this for a whole life annuity

从数学上讲，转向期望是以类似的方式完成的。我们这样做是为了终身年金

Whole life annuity - discrete case (Lecture 11 slide 22):

终身年金 - 离散案例（第 11 讲幻灯片 22）：

k +1

k +1

E ,a¨

和

= ∞ a¨

= ∞ a¨

k =0

k =0

Kx +1

Kx +1

,Σ

,S

P [Kx

P [Kx

∞

∞

= k ] =

= k] =

Σ

Σ

k =0

k =0

vk k px

vk k 像素

= a¨x .

= a¨x .

Whole life annuity - continuous case:

终身年金-连续案例：

E ,a¯Tx , = ∫0∞ a¯t fTx (t )dt = ∫0∞ vt t px dt = a¯x .

E ,a¯Tx , = ∫0∞ a¯t fTx (t )dt = ∫0∞ vt t px dt = a¯x 。

Discrete case:

离散案例：

∞

∞

Σ k

SK

a¨x =v k px .

a¨x =v k px 。

k =0

k =0

if the policyholder is alive at any time k0 ( k px ),

如果投保人在任何时候都活着 k0 ( k px ),

≥

≥

an amount of 1 is paid at time k ,

在时间 k 支付 1 的金额，

and is discounted back to time 0 by vk .

并且被 vk 折回到时间 0。

Continuous case:

连续案例：

a¯x = ∫0∞ vt t px dt.

a¯x = ∫0∞ vt t px dt。

if the policyholder is alive at any time t0 ( t px ),

如果投保人在任何时间 t0 ( t px ) 还活着，

≥

≥

a (infinitesimally small) amount dt is paid at time t ,

在时间 t 支付了一个（无穷小）金额 dt，

and is discount back to time 0 by vt .

并且被 vt 折回到时间 0。