ORBIT OF A COMET

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The write-up is about my approach to determine whether the comet is moving in an elliptical or hyperbolic orbit around the sun, and what the time period is (if elliptic) or the distance of closest approach to the sun is (if hyperbolic).

The problem statement is:
In this problem, we want to study the orbit of a comet, whose astrometry data is provided. For this problem, make the following three important simplifying assumptions:

- The orbit of the earth is perfectly circular with radius = 1AU and time period = 1 year.
- The orbit of the comet is coplanar to the orbit of the earth.
- Gravitational effects of the earth on the comet can be neglected.

As usual, Kepler's laws of planetary motion are obeyed by all orbits. An astronomer (sitting on earth, naturally) has measured the distance of a certain comet from earth and the angle between the comet and the sun, as seen from earth (where a negative sign indicates that the comet is to the west of the sun). The time of observation after the first observation (in years), distance between earth and the comet (in AU) and the angle between the sun and the comet (in radians) is given in the data file CometData.csv.
Your task, should you choose to attempt this problem, is to help the astronomer to determine whether the comet is moving in an elliptical or hyperbolic orbit around the sun, and what the time period is (if elliptic) or the distance of closest approach to the sun (if hyperbolic).
You should submit a small writeup about your approach/method to this problem (can be a scanned handwritten document) and a jupyter notebook that implements this approach. You should explicitly calculate the eccentricity, and time period or distance of closest approach of the orbit (and include this value at the end of your writeup). Additionally, make a scatterplot of the position of the comet with respect to the sun.

PROCESS

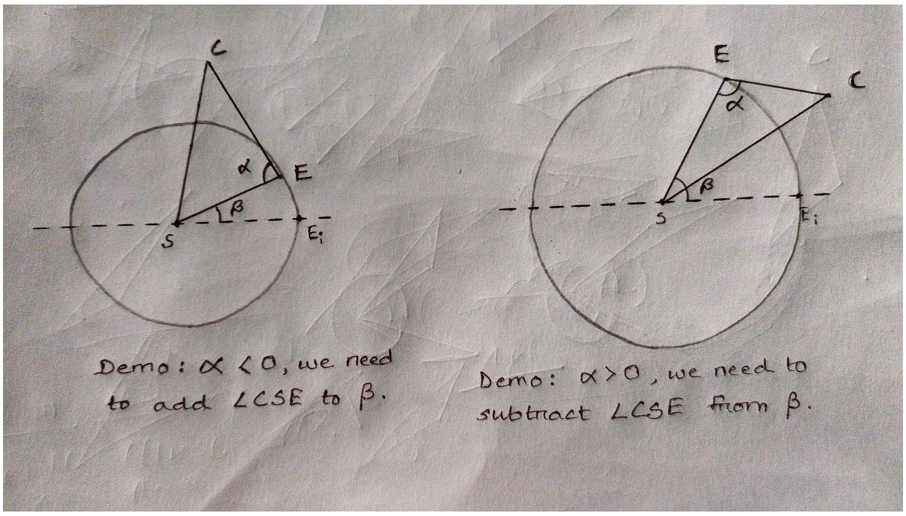
1. **Goal:**
   To create arrays that store the given data: the time of observation after the first observation (in years), distance between earth and the comet (in AU) and the angle between the sun and the comet (in radians) from the data file **CometData.csv**.
   Implementation:
   This was done using pandas dataframe.

2. **Goal:**
   To find the angle that the earth has covered in its orbit around the sun from the time given. I choose the value to be in the range $(-\pi, \pi]$.
   Implementation:
   In one year, 2π angle is covered by the earth. After every year the earth again starts from angle 0. So, I used this periodicity to find the angle covered by taking only the fractional part of the year. A function ($from\_date$) was made for that.

3. **Goal:**
   To calculate the distance of comet from the sun and the angle made by the comet at the sun from the sun-initial earth position.
   Implementation:
   This is done using cosine rule. I have defined two functions for that.
   For a triangle ABC with sides a,b and c, I have:

$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos A \\
b^2 &= a^2 + c^2 - 2ac\cos B \\
c^2 &= a^2 + b^2 - 2ab\cos C \\
\mathrm{Cos}\, A &= \frac{b^2 + c^2 - a^2}{2bc} \\
\mathrm{Cos}\, B &= \frac{a^2 + c^2 - b^2}{2ac} \\
\mathrm{Cos}\, C &= \frac{a^2 + b^2 - c^2}{2ab}
\end{aligned}
$$

   As per the diagram, first length of CS is found using the first function($get\_distance$). Then using the second function(get_angle), the angle CSE is found. Then I looked if the comet is towards west or east of the sun (from the sign of given angles). If the sign is non-negative, angle CSE is subtracted from earth's angle, and vice-versa.



RESULTS

Now, I have r and θ, I can use the data to fit this into general equation of conic of the form:

$$
r(\theta) = \frac{r_0}{1 + e\cos(\theta - \theta_0)}
$$

where, e is eccentricity, r_0 is length of semi-latus-rectum, where the focus of this conic section is at the origin and θ_0 is the angle of tilting.
From there, I got the values of the parameters using **lmfit** library.
I found the following things:

- eccentricity: ≈ 0.92 i.e. the orbit is elliptical (since, 0¡e¡1).
- semi-latus-rectum: ≈ 3.072 AU i.e. semi-major axis: ≈ 20 AU (since, a=$\dfrac{r_0}{1-e^2}$).
- time period: ≈ 89.5 years. Since, Kepler's third law states:

$$T^2 \propto a^3 \implies \frac{T^2_{earth}}{T^2_{comet}} = \frac{a^3_{earth}}{a^3_{comet}}$$

Since earth's radius is 1 AU and time period is 1 year,

$$T_{comet} = \sqrt{a^3_{comet}}$$

A model efficiency report is also presented.

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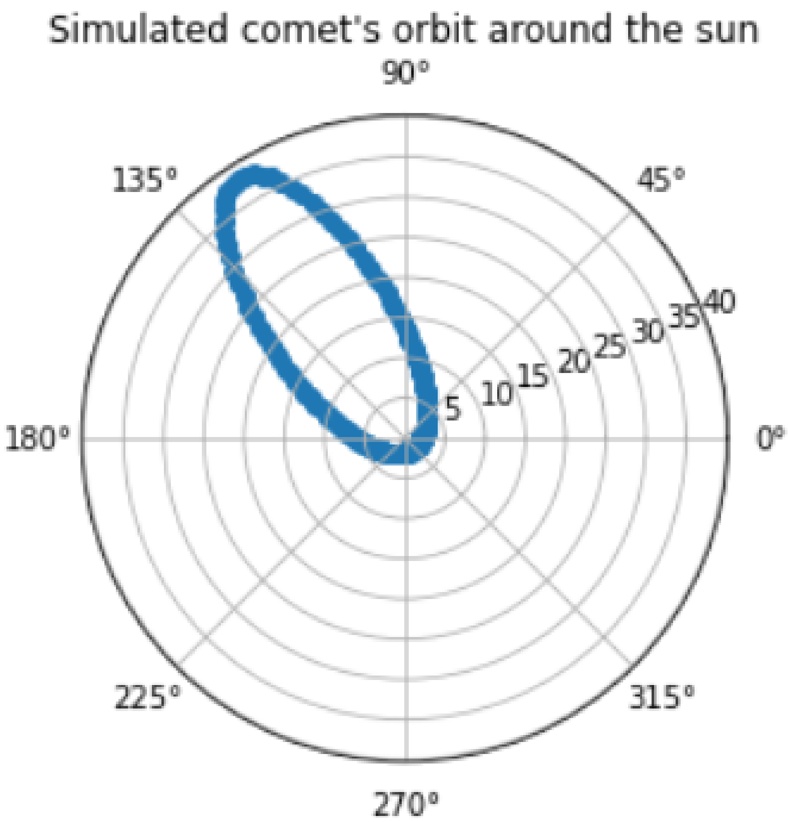
[[Variables]]
    l:        3.07200000 +/- 1.2514e-13 (0.00%) (init = 10)
    e:        0.92000000 +/- 4.2101e-15 (0.00%) (init = 0)
    theta_0: -1.00000000 +/- 2.5506e-14 (0.00%) (init = 0)
[[Correlations]] (unreported correlations are < 0.100)
    C(l, theta_0) =  0.981
    C(e, theta_0) =  0.911
    C(l, e)       =  0.817
```

I got the following simulated orbit of the comet around the sun.



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