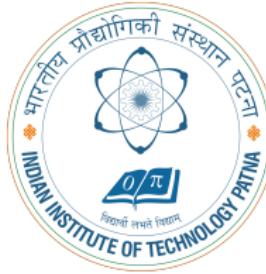


Chapter Sums: Numerical Problems

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Problem 1: Transmission Time Calculation I

Problem: An 802.11ac AP transmits a 2000-byte data frame to a client using a PHY rate of 867 Mbps. Ignoring MAC overhead, preambles, and waiting times, calculate the transmission time of the data portion over the air.

Solution Steps:

- ① Convert packet size to bits: $2000 \text{ bytes} \times 8 \text{ bits/byte} = 16,000 \text{ bits}$
- ② Time = Size / Rate
- ③ Rate = 867 Mbps = $867 \times 10^6 \text{ bps}$
- ④ Time = $\frac{16,000}{867 \times 10^6} = 1.845 \times 10^{-5} \text{ seconds}$
- ⑤ Convert to microseconds: $1.845 \times 10^{-5} \times 10^6 = 18.45 \mu\text{s}$

Model Answer: The raw data transmission takes approximately **18.45 μs** .

Problem 2: Protocol Efficiency Calculation I

Problem: An 802.11n transmission has the following timing parameters:

- DIFS: $34 \mu\text{s}$
- PHY/MAC headers + preamble: $40 \mu\text{s}$
- Data transmission time: $27.7 \mu\text{s}$
- SIFS: $16 \mu\text{s}$
- ACK transmission time: $20 \mu\text{s}$

Calculate the total airtime and protocol efficiency.

Solution Steps:

- ① Total time = DIFS + Headers + Data + SIFS + ACK
- ② Total = $34 + 40 + 27.7 + 16 + 20 = 137.7 \mu\text{s}$
- ③ Efficiency = Data time / Total time = $27.7/137.7 = 0.201 = 20.1\%$

Model Answer: Total airtime = **137.7 μs** , Efficiency = **20.1%**.

Problem 3: Capacity vs. Coverage Design I

Problem: You are designing Wi-Fi for a $60m \times 40m$ exhibition hall. Option A uses 2 APs with 867 Mbps each. Option B uses 8 APs with 433 Mbps each. Which design provides better aggregate throughput for 100 simultaneous clients? Why?

Solution Steps:

- ① Option A aggregate capacity: $2 \times 867 = 1734$ Mbps (theoretical)
- ② Option B aggregate capacity: $8 \times 433 = 3464$ Mbps (theoretical)
- ③ Client density: $100 \text{ clients} / 2 \text{ APs} = 50 \text{ clients/AP}$ vs. $100/8 = 12.5 \text{ clients/AP}$
- ④ With CSMA/CA, more clients per AP = more collisions and backoff

Model Answer: Option B provides $2\times$ theoretical capacity and $4\times$ better client distribution, resulting in significantly higher real-world throughput.

Problem 4: Overhead Dominance Analysis I

Problem: Compare the efficiency of transmitting a 64-byte VoIP packet vs a 1500-byte data frame at 54 Mbps. Assume:

- DIFS: $34 \mu s$, SIFS: $16 \mu s$
- PHY preamble: $20 \mu s$
- ACK: 14 bytes at 6 Mbps
- MAC header: 30 bytes
- Average backoff: $67.5 \mu s$

Solution Steps:

- ① VoIP: Data = $(64 + 30) \times 8/54 = 13.93 \mu s$, ACK = $14 \times 8/6 + 20 = 38.67 \mu s$
- ② Total VoIP = $34 + 67.5 + 20 + 13.93 + 16 + 38.67 = 190.1 \mu s$, Efficiency = $13.93/190.1 = 7.3\%$

Problem 4: Overhead Dominance Analysis II

- ③ Data: Data = $(1500 + 30) \times 8/54 = 226.67 \mu s$, Total = $34 + 67.5 + 20 + 226.67 + 16 + 38.67 = 402.84 \mu s$, Efficiency = $226.67/402.84 = 56.3\%$

Model Answer: VoIP efficiency = 7.3%, Data efficiency = 56.3%. Small packets are 7.7× less efficient.

Problem 5: Channel Overlap Calculation I

Problem: In 2.4 GHz band, channel 6 uses 2.426-2.448 GHz (center 2.437 GHz). Channel 7 uses 2.431-2.453 GHz (center 2.442 GHz). Calculate the overlap in MHz and as a percentage.

Solution Steps:

- ① Channel 6 range: 2.426 to 2.448 GHz (22 MHz wide)
- ② Channel 7 range: 2.431 to 2.453 GHz (22 MHz wide)
- ③ Overlap: from 2.431 to 2.448 GHz = 17 MHz
- ④ Percentage: $17/22 \times 100\% = 77.3\%$

Model Answer: Overlap = 17 MHz or 77.3% of channel bandwidth.

Problem 6: MCS and Airtime Inequality I

Problem: Client A uses MCS 9 (650 Mbps) and Client B uses MCS 0 (6.5 Mbps). Both download 50 MB files. How much longer does Client B take? If they share airtime equally, what effective throughput does each get?

Solution Steps:

- ① File size: $50 \text{ MB} = 50 \times 8 = 400 \text{ Mb}$
- ② Time A = $400/650 = 0.615 \text{ seconds}$
- ③ Time B = $400/6.5 = 61.54 \text{ seconds}$
- ④ Ratio = $61.54/0.615 = 100 \times \text{longer}$
- ⑤ With equal airtime sharing: A gets $50\% \text{ airtime} = 0.5 \times 650 = 325 \text{ Mbps}$, B gets $0.5 \times 6.5 = 3.25 \text{ Mbps}$

Model Answer: Client B takes **100×** longer. With equal airtime sharing, A gets **325 Mbps**, B gets **3.25 Mbps**.

Problem 7: Beacon Overhead Calculation I

Problem: An AP transmits beacons every 100 TU (102.4 ms) at 1 Mbps. Beacon size is 200 bytes. Calculate:

- ① Percentage of airtime consumed by beacons
- ② If 5 APs share the same channel, what's the combined overhead?

Solution Steps:

- ① Beacon time = $200 \times 8/1 = 1600 \mu s = 1.6 \text{ ms}$
- ② Beacon interval = 102.4 ms
- ③ Percentage = $(1.6/102.4) \times 100\% = 1.56\%$
- ④ 5 APs = $5 \times 1.56\% = 7.8\%$

Model Answer: Single AP overhead = 1.56%, 5 APs = 7.8% (ideal non-colliding case).

Problem 8: TCP Throughput Estimation I

Problem: An 802.11ac link has a PHY rate of 1.3 Gbps. Protocol efficiency is 65% and contention factor reduces throughput by another 20% in a moderately loaded network. What is the expected TCP throughput?

Solution Steps:

- ① $\text{Throughput} = \text{PHY rate} \times \text{Protocol efficiency} \times (1 - \text{Contention factor})$
- ② $= 1300 \times 0.65 \times 0.8 = 1300 \times 0.52 = 676 \text{ Mbps}$

Model Answer: Expected TCP throughput = **676 Mbps.**

Problem 9: IFS Timing Calculation I

Problem: In 802.11ac (5 GHz), SIFS = 16 μs , Slot Time = 9 μs .

Calculate:

- ① DIFS duration
- ② Total wait time with backoff of 4 slots

Solution Steps:

- ① $\text{DIFS} = \text{SIFS} + (2 \times \text{Slot Time}) = 16 + (2 \times 9) = 16 + 18 = 34 \mu\text{s}$
- ② Backoff time = $4 \times 9 = 36 \mu\text{s}$
- ③ Total wait = DIFS + Backoff = $34 + 36 = 70 \mu\text{s}$

Model Answer: DIFS = 34 μs , Total wait = 70 μs .

Problem 10: Backoff Window Calculation I

Problem: 802.11ac has CWmin = 15, CWmax = 1023, Slot Time = 9 μs .

- ① Initial backoff range (slots and time)?
- ② After 3 collisions, what's the new CW and backoff range?

Solution Steps:

- ① Initial: CW = 15, slots = 0-15, time = 0 to $15 \times 9 = 0 - 135 \mu\text{s}$
- ② After 1st: CW = $2 \times (15 + 1) - 1 = 31$
- ③ After 2nd: CW = $2 \times (31 + 1) - 1 = 63$
- ④ After 3rd: CW = $2 \times (63 + 1) - 1 = 127$
- ⑤ Slots = 0-127, time = 0 to $127 \times 9 = 0 - 1143 \mu\text{s}$

Model Answer: Initial: 0 – 15 slots (0 – 135 μs), After 3 collisions:
0 – 127 slots (0 – 1143 μs).

Problem 11: Collision Probability Calculation I

Problem: Two stations use 802.11ac with $CW_{min} = 15$. What is the probability they select the same backoff slot? What is this probability after one collision?

Solution Steps:

- ① Initial: $CW = 15$, so 16 possible slots (0-15)
- ② Probability both pick same slot = $1/16 = 0.0625 = 6.25\%$
- ③ After collision: $CW = 31$, 32 possible slots
- ④ Probability = $1/32 = 0.03125 = 3.125\%$

Model Answer: Initial collision probability = **6.25%**, after collision = **3.125%**.

Problem 12: Complete Frame Transmission Time I

Problem: Calculate total transmission time for a 1000-byte frame at 130 Mbps using 802.11ac parameters:

- DIFS: $34 \mu s$, SIFS: $16 \mu s$, Slot: $9 \mu s$
- PHY preamble: $20 \mu s$, MAC header: 36 bytes
- ACK: 14 bytes at 6 Mbps (basic rate)
- Average backoff: 7.5 slots

Solution Steps:

- ① Data: $(1000 + 36) \times 8 = 8288 \text{ bits}$, time = $8288/130 = 63.75 \mu s$
- ② Data with preamble: $63.75 + 20 = 83.75 \mu s$
- ③ ACK: $14 \times 8/6 = 18.67 \mu s$, with preamble: $18.67 + 20 = 38.67 \mu s$
- ④ Backoff: $7.5 \times 9 = 67.5 \mu s$
- ⑤ Total = $34 + 67.5 + 83.75 + 16 + 38.67 = 239.92 \mu s$

Model Answer: Total transmission time = **239.92 μs .**

Problem 13: Rate Anomaly Impact I

Problem: A network has 1 client at 6 Mbps and 4 clients at 54 Mbps. Each gets equal airtime. Calculate:

- ① Throughput per client
- ② Total network throughput
- ③ Compare to ideal case (all at 54 Mbps)

Solution Steps:

- ① Total clients = 5, each gets 20% airtime
- ② Slow client: $0.2 \times 6 = 1.2$ Mbps
- ③ Fast clients: $0.2 \times 54 = 10.8$ Mbps each
- ④ Total = $1.2 + 4 \times 10.8 = 1.2 + 43.2 = 44.4$ Mbps
- ⑤ Ideal (all 54 Mbps): $5 \times 0.2 \times 54 = 54$ Mbps
- ⑥ Loss = $54 - 44.4 = 9.6$ Mbps (18% reduction)

Model Answer: Slow client = **1.2 Mbps**, Fast clients = **10.8 Mbps** each, Total = **44.4 Mbps**, **18%** reduction from ideal.

Problem 14: RTS/CTS Efficiency Analysis I

Problem: Compare efficiency for 1500-byte frames at 130 Mbps:

- Without RTS/CTS: Total time = 240 μs (from Problem 12)
- With RTS/CTS: Add RTS (44 μs), CTS (38 μs), 2 extra SIFS (32 μs)
- Collision probability without RTS/CTS: 10%, collision causes retransmission

Solution Steps:

- Without RTS/CTS: 10% retransmissions = $240/0.9 = 266.7 \mu s$ effective
- With RTS/CTS: Add $44 + 38 + 32 = 114 \mu s$, total = $240 + 114 = 354 \mu s$
- Without RTS: 266.7 μs , With RTS: 354 μs , RTS adds 33% overhead

Model Answer: Without RTS/CTS (with collisions): **266.7 μs** , With RTS/CTS: **354 μs** . RTS/CTS is beneficial when collision probability exceeds $\sim 32\%$.

Problem 15: Slot Time and Network Scale I

Problem: 802.11b uses $20 \mu\text{s}$ slot time. What is the maximum one-way propagation delay this accommodates? If speed of light in air is $3 \times 10^8 \text{ m/s}$, what's the maximum network diameter?

Solution Steps:

- ① Slot time includes $2 \times$ propagation delay + PHY switching
- ② For $20 \mu\text{s}$ slot, propagation delay budget $10 \mu\text{s}$ (round trip $20 \mu\text{s}$)
- ③ One-way propagation = $10 \mu\text{s} = 10 \times 10^{-6} \text{ s}$
- ④ Distance = speed \times time = $3 \times 10^8 \times 10 \times 10^{-6} = 3000 \text{ m} = 3 \text{ km}$

Model Answer: Maximum one-way propagation = $10 \mu\text{s}$, maximum network diameter = 3 km .

Problem 16: CWmin Optimization I

Problem: For a network with 25 active stations, which CWmin value minimizes collisions? Calculate collision probability for CWmin=15 vs CWmin=31.

Solution Steps:

- ① For N stations, approximate collision probability =

$$1 - (1 - 1/(CW + 1))^{N-1}$$

- ② CWmin=15 (16 slots):

$$P_c = 1 - (1 - 1/16)^{24} = 1 - (0.9375)^{24} = 1 - 0.22 = 0.78 = 78\%$$

- ③ CWmin=31 (32 slots):

$$P_c = 1 - (1 - 1/32)^{24} = 1 - (0.96875)^{24} = 1 - 0.47 = 0.53 = 53\%$$

Model Answer: CWmin=31 gives 53% collision probability vs 78% for CWmin=15. Larger CWmin recommended for dense networks.

Problem 17: NAV Duration Calculation I

Problem: A station sends a DATA frame of 1500 bytes at 130 Mbps. Calculate the Duration field value that should be set in the DATA frame header.

Given: SIFS = 16 μ s, ACK at 6 Mbps = 38.67 μ s

Solution Steps:

- ① Duration field covers the time until end of ACK
- ② Duration = SIFS + ACK transmission time
- ③ $= 16 + 38.67 = 54.67 \mu s$

Model Answer: Duration = 54.67 μ s (rounded to integer in actual implementation).

Problem 18: Retry Limit Impact I

Problem: A VoIP packet (160 bytes payload + 40 bytes IP/UDP + 30 bytes MAC) is transmitted at 54 Mbps. Short Retry Limit = 7. If packet error rate is 20% per transmission, what's the probability the packet is discarded?

Solution Steps:

① Probability of success on any attempt = 0.8

② Probability of failure on all 7 attempts =
 $(0.2)^7 = 0.0000128 = 0.00128\%$

③ Probability of eventual success =
 $1 - 0.0000128 = 0.9999872 = 99.99872\%$

Model Answer: Discard probability = **0.00128%** (extremely low).

Problem 19: Hidden Node Throughput Impact I

Problem: Two hidden stations each attempt to transmit with probability 0.3 per slot. What is the collision probability? What is the throughput relative to collision-free case?

Solution Steps:

- ① Collision probability = $P(A) \times P(B) = 0.3 \times 0.3 = 0.09 = 9\%$
- ② Successful transmission probability =
$$P(A)(1 - P(B)) + P(B)(1 - P(A)) = 0.3 \times 0.7 + 0.3 \times 0.7 = 0.42 = 42\%$$
- ③ Relative throughput = $0.42/0.3 = 1.4$? Wait, that's not right.
- ④ Actually, with no collisions, one station would get 0.3 throughput.
- ⑤ With collisions, total successful transmissions = 0.42
- ⑥ Throughput ratio = $0.42/0.6 = 0.7 = 70\%$ of ideal two-station case

Model Answer: Collision probability = 9%, Throughput = 70% of collision-free.

Problem 20: Deauthentication Attack Impact I

Problem: A deauthentication frame is 30 bytes, transmitted at 1 Mbps. An attacker sends 100 deauth frames per second. What percentage of airtime is consumed? If each deauth forces a client to reauthenticate (taking 5 ms), what's the client availability reduction?

Solution Steps:

- ① Deauth frame time = $30 \times 8/1 = 240 \mu s$ (ignoring preamble)
- ② With 802.11b preamble ($192 \mu s$): total = $240 + 192 = 432 \mu s$
- ③ $100 \text{ frames/second} = 100 \times 432 = 43,200 \mu s = 43.2 \text{ ms}$
- ④ Percentage = $(43.2/1000) \times 100\% = 4.32\%$
- ⑤ Each deauth forces 5 ms reauthentication
- ⑥ $100 \text{ deauth/sec} \times 5 \text{ ms} = 500 \text{ ms reauth time per second}$
- ⑦ Client availability = $(1000 - 500)/1000 = 50\%$

Model Answer: Attack consumes **4.32%** airtime, reduces client availability to **50%**.

Problem 21: Fragmentation Overhead Analysis I

Problem: A 2304-byte MSDU is fragmented into 3 fragments of 768 bytes each. Each fragment adds 30-byte MAC header and 20 μs PHY preamble. Compare total overhead with unfragmented transmission (same header/preamble).

Solution Steps:

- ① Unfragmented: 1 header (30 bytes) + 1 preamble (20 μs)
- ② Overhead = $30 \times 8 / 130 + 20 = 1.85 + 20 = 21.85 \mu s$
- ③ Fragmented: 3 headers + 3 preambles = $3 \times 21.85 = 65.55 \mu s$
- ④ Additional overhead = $65.55 - 21.85 = 43.7 \mu s$ (3 \times increase)

Model Answer: Fragmentation adds **43.7 μs** (200%) overhead, but reduces retransmission cost from 2304 to 768 bytes on error.

Problem 22: Sequence Number Wrap Time I

Problem: 802.11 uses 12-bit sequence numbers (0-4095). If a station transmits 1500-byte frames at 130 Mbps continuously, how long until the sequence number wraps?

Solution Steps:

- ① Frame transmission time = $83.38 \mu s$ (from Problem 22)
- ② 4096 frames take $4096 \times 83.38 \mu s = 341,524 \mu s = 341.5 ms$
- ③ Approximately 0.34 seconds

Model Answer: Sequence number wraps after **0.34 seconds** of continuous transmission at max rate.

Problem 23: Beacon Interval in TUs I

Problem: An AP is configured with Beacon Interval = 75 TU. How many milliseconds between beacons? If beacon size is 180 bytes at 1 Mbps, what's the airtime percentage?

Solution Steps:

- ① $1 \text{ TU} = 1024 \mu\text{s} = 1.024 \text{ ms}$
- ② $\text{Beacon interval} = 75 \times 1.024 = 76.8 \text{ ms}$
- ③ $\text{Beacon time} = 180 \times 8/1 = 1440 \mu\text{s} = 1.44 \text{ ms}$
- ④ $\text{Percentage} = (1.44/76.8) \times 100\% = 1.875\%$

Model Answer: Beacon interval = **76.8 ms**, overhead = **1.875%**.

Problem 24: Listen Interval and Sleep Time I

Problem: A station sets Listen Interval = 10. Beacon Interval = 100 TU. How long does the station sleep between wake-ups? If it sleeps 99.9% of the time, what's the power savings factor? [awake current 100 mA, sleep 1 mA]

Solution Steps:

- ① Beacon interval = $100 \times 1024 \mu s = 102.4 \text{ ms}$
- ② Listen Interval = $10 \times 102.4 = 1024 \text{ ms} = 1.024 \text{ seconds}$
- ③ Sleep time = $1.024 \text{ s} - \text{wake time}$
- ④ If 99.9% sleep, awake = 0.1% = 1.024 ms
- ⑤ Power consumption ratio = sleep current / awake current
- ⑥ If awake current 100 mA, sleep 1 mA, average =
 $(0.001 \times 1024 + 100 \times 1.024)/1024 \approx 1.1 \text{ mA}$
- ⑦ Savings factor vs always awake = $100/1.1 \approx 90\times$

Model Answer: Sleep interval = **1.024 s**, power savings $\sim 90\times$.

Problem 25: PS-Poll Efficiency I

Problem: Legacy power save uses one PS-Poll per buffered frame. PS-Poll is 20 bytes at 1 Mbps (basic rate) plus PHY preamble 192 μs (802.11b). Compare to retrieving 5 buffered frames with individual PS-Polls vs block transfer.

Solution Steps:

- ① PS-Poll time = $20 \times 8/1 + 192 = 160 + 192 = 352 \mu\text{s}$
- ② 5 PS-Polls = $5 \times 352 = 1760 \mu\text{s}$
- ③ Data frames: $5 \times (\text{data} + \text{ACK})$ time
- ④ If AP could send all 5 in one TXOP, save 4 PS-Polls = $1408 \mu\text{s}$

Model Answer: PS-Poll overhead = **352 μs** each. Block transfer saves **1.408 ms** for 5 frames.

Problem 26: Maximum MSDU Size Calculation I

Problem: 802.11 MSDU maximum is 2304 bytes. With 30-byte MAC header, 8-byte LLC/SNAP header, and 4-byte FCS, what's the maximum PHY payload size? What's the airtime at 130 Mbps including 20 μs preamble?

Solution Steps:

- ① MSDU = 2304 bytes
- ② MAC header = 30 bytes
- ③ FCS = 4 bytes
- ④ Total MPDU = $2304 + 30 + 4 + 8 = 2346$ bytes
- ⑤ Bits = $2346 \times 8 = 18,768$ bits
- ⑥ Data time = $18768/130 = 144.37 \mu s$
- ⑦ With preamble = $144.37 + 20 = 164.37 \mu s$

Model Answer: Maximum MPDU = **2346 bytes**, airtime = **164.37 μs** .

Problem 27: A-MSDU Aggregation Efficiency I

Problem: An A-MSDU aggregates 5×1500 -byte frames into one MPDU. MAC header is 30 bytes, A-MSDU subframe headers add 14 bytes per frame. Calculate efficiency gain vs sending 5 separate frames. $20 \mu\text{s}$ preamble.

Solution Steps:

- ① Separate: $5 \times (1500 + 30) = 7650$ bytes data + headers, plus 5 preambles
- ② Separate airtime =
$$5 \times (7650 \times 8/130 + 20) = 5 \times (470.77 + 20) = 5 \times 490.77 = 2453.85 \mu\text{s}$$
- ③ Aggregated: $5 \times 1500 = 7500$ bytes payload, $5 \times 14 = 70$ bytes subheaders, + 30 MAC header
- ④ Total = $7500 + 70 + 30 = 7600$ bytes = 60,800 bits
- ⑤ Airtime = $60800/130 + 20 = 467.69 + 20 = 487.69 \mu\text{s}$

Problem 27: A-MSDU Aggregation Efficiency II

- ⑥ Gain = $2453.85/487.69 = 5.03 \times$ faster!

Model Answer: A-MSDU aggregation provides **5.03×** throughput improvement for 5 frames.

Key Formulas Reference I

- **Transmission time:** $T = \frac{\text{bits}}{\text{PHY rate}}$
- **Protocol efficiency:** $\eta = \frac{T_{\text{data}}}{T_{\text{total}}}$
- **DIFS:** DIFS = SIFS + 2 × SlotTime
- **Backoff time:** $T_{\text{backoff}} = \text{random}(0, \text{CW}) \times \text{SlotTime}$
- **CW after collision:** $\text{CW}_{\text{new}} = \min(2 \times (\text{CW}_{\text{old}} + 1) - 1, \text{CW}_{\text{max}})$
- **Collision probability:** $P_c \approx 1 - (1 - \frac{1}{\text{CW}+1})^{N-1}$
- **RTS Duration:** $T_{\text{RTS}} = T_{\text{CTS}} + T_{\text{DATA}} + T_{\text{ACK}} + 3 \times \text{SIFS}$
- **Throughput:**
 $\text{TCP Throughput} \approx \text{PHY Rate} \times \text{Efficiency} \times (1 - \text{Contention})$