Gated Imaging at GMRT: Pulsar Phase Prediction with TEMPO2 Polyco and PINT Polyco

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1 Introduction

Pulsar timing is the regular monitoring of the rotation of the neutron star by tracking the arrival times of the radio pulses. The precise tracking of the rotation phase allows pulsar astronomers to probe into the interior of Neutron Stars. Pulsars are rapidly rotating neutron stars that emit periodic pulses of radio emission. The pulse periods are quite stable. For pulsar a pulsar, the average of the data from many pulses gives a pulse period P. The instantaneous pulse frequency is f=1/P, and the instantaneous pulse phase ϕ is defined by $\frac{d\phi}{dt} = f$. Pulse phase is usually measured in turns of 2π radians, so $0<\phi<1$.

During timing, the average pulse profile is correlated with a template so that its phase offset can be determined. When multiplied by the instantaneous pulse period, that phase yields a time offset that can be added to a high-precision **reference point** on the profile to create the time of arrival (TOA).

The precision with which a TOA can be determined is approximately equal to the duration of a sharp pulse feature (e.g., the leading edge or the highest point on the pulse) divided by the signal-to-noise ratio of the average profile. It is usually expressed in terms of the width of the pulse features in units of the pulse period P, and the signal-to-noise ratio (S/N) such that $\sigma_{TOA}\ \alpha\ W f_P/(\mathrm{S/N})$.

In the nearly **inertial frame of the Solar System barycenter** (center of mass), the rotation period of a pulsar is nearly constant, so the time-dependent phase $\phi(t)$ of a pulsar can be approximated by the Taylor expansion. This allows us to also derive its phase in the future.

The Taylor expansion of the phase of a pulsar can be expressed as:

$$\Phi(t) = \Phi_0 + f \cdot (t - T_0) + \frac{1}{2}\dot{f} \cdot (t - T_0)^2 + \frac{1}{6}\ddot{f} \cdot (t - T_0)^3 + \dots \tag{1}$$

where:

$\Phi(t)$ is the phase of the pulsar at time t,

Φ_0 is the initial phase at the reference epoch T_0,

f is the pulse frequency,

$\dot{f}$ is the first derivative of frequency with respect to time,

$\ddot{f}$ is the second derivative of frequency with respect to time,

It is important to note that the above Taylor expansion works at the inertial frame thus we need to perform some corrections. This is because we are observing from a **Topocentric** frame of reference thus These corrections include effects due to the rotation and revolution of the Earth, the effect of the Earth-Moon system on the position of the Earth, and the effect of all the planets in the Solar System. Relativistic corrections for the clock on the Earth are also included, as are corrections for dispersion delay at the Doppler corrected frequency of observation Y.Gupta, Ch17, NCRA blue book These corrections are explained in detail in [2].

1.1 Objectives

Once the above-mentioned points have been addressed we are now in a position to list the objectives for the phase prediction of Pulsars using TEMPO2 and PINT The present report aims to summarize the following objectives.

To predict the phase of a Pulsar at a given observation time at GMRT

1. Using TEMPO2 Polyco
   (a) Polyco command and its features
   (b) Tests conducted (A summary)
   (c) Algorithm to get the phase at a given observation time at GMRT
2. Using PINT Polyco (in progress)
   (a) Brief introduction to PINT
   (b) Some useful features
   (c) Tests conducted (A summary)
   (d) Algorithm to get the phase at a given observation time at GMRT
3. Comparison of the phase prediction results, methods, and flexibility.
4. Conclusion and Key Takeaways.

2 To predict the phase of a pulsar at a given observation time at GMRT

2.1 Using TEMPO2 Polyco

- The polyco function in tempo2 is used to predict the phase and frequency of a Pulsar for a given range of MJD when the observatory location is given.

2.1.1 Polyco command and its features

The command is: `-polyco "mjd1 mjd2 nspan ncoeff maxha site_code freq"` The reference is given using .par file for a Pulsar **tempo2 command**: `tempo2 -f J0835-4510.par -polyco "57000 57001 300 3 10 pks 1368.6020 "-tempo1` tempo2 allows the output to be printed in tempo1 format which is given by "-tempo1" feature . This format been followed throughout our study.

2.1.2 Output information

Part of the output for the command above is given below

```
0835-4510   8-Dec-14  100000.00   56999.41666666660           67.844630 -0.494 -2.938
    614212053.677884   11.188233045354  pks  300    3  1368.602
  6.09452283345319936e-05 -1.77295728075877908e-02  4.47987820042647483e-07
```

The information is stored in the following format where L1, L2 and L3 refer to the line numbers in the table format given in 2.1.2 respectively:

L1 PSRNAME Date of Obs Time (hhmmss.ss) TMID (MJD) DM Doppler shift (10^{-4}) Log_{10} of fit rms residual

L2 Reference Phase (RPHASE) Reference rotation frequency (F0) Observatory Data span (minutes)

Number of coefficients Frequency (MHz)

L3 Coefficient 1 Coefficient 2 Coefficient 3

Tempo Style Polyco File

The polynomial ephemerides are written to file `polyco_new.dat`. Entries are listed sequentially within the file. The file format is:

```
Line  Columns     Item
----  -------   -----------------------------------
 1       1-10   Pulsar Name
        11-19   Date (dd-mmm-yy)
        20-31   UTC (hhmmss.ss)
        32-51   TMID (MJD)
        52-72   DM
        74-79   Doppler shift due to earth motion ($10^{-4}$)
        80-86   $\log_{10}$ of fit rms residual in periods
 2       1-20   Reference Phase (RPHASE)
        21-38   Reference rotation frequency (F0)
        39-43   Observatory number
        44-49   Data span (minutes)
        50-54   Number of coefficients
        55-75   Observing frequency (MHz)
        76-80   Binary phase
 3*      1-25   Coefficient 1 (COEFF(1))
        26-50   Coefficient 2 (COEFF(2))
        51-75   Coefficient 3 (COEFF(3))
```

* Subsequent lines have three coefficients each, up to NCOEFF.

The pulse phase and frequency at time T are then calculated as:

$$\begin{aligned} \Delta T &= 1440(T - TMID) \\ \phi &= \phi_0 + 60\Delta T f_0 + \mathrm{COEFF}[1] + \mathrm{COEFF}[2]\Delta T + \mathrm{COEFF}[3]\Delta T^2 + \ldots \\ f(\mathrm{Hz}) &= f_0 + \frac{1}{60}\left(\mathrm{COEFF}[2] + 2\mathrm{COEFF}[3]\Delta T + 3\mathrm{COEFF}[4]\Delta T^2 + \ldots\right) \end{aligned} \tag{2}$$

2.2 Sanity Tests on Polyco

The polyco command gives polynomial phases for a range of MJD values depending upon what is input parameters given like MJD start, Tspan, and most importantly what is the max HA range. The phase at some particular time can then be calculated using 2 where TMID will be a suitable MJD(close to T) for which the coefficients are valid. These phase values needed to be checked so there were multiple tests that were conducted on the phases to understand the nature of the phase calculated and what assumptions are included in the polyco code. A separate document called **polyco_key** is being maintained which describes the working of the polyco code in detail.

The detailed tests with results are given in my weekly reports and jupyter notebooks I will just list out the relevant conclusions in this report for now.

What is the reference file used for polyco?

-polyco uses the .par file given in the input of the command line in the current case we have J0835-4510.par.

2.2.1 What is the unit of the output phase?

The output phase is an absolute phase which is unitless

2.2.2 What is the reference phase i.e, what MJD is considered as 0.0 phase ?

It was observed that the reference phase in TEMPO2 polyco is an MJD very close to the PEPPOCH (This is the current understanding)

2.2.3 Why is the output MJD range printed by polyco?

TEMPO2 does not usually give the phase values at any desired MJD . This can be observed by the user when he check the polyco_new.dat file. It was also observed that the in certain instants the TSPAN between the MJD values was also not maintained. To study this the polyco code was thoroughly studied. There are a few important takeaways from the study :

1. The Polyco uses the observation coordinates and calculates the local HA , LST these values were checked and compared with astropy libraries
2. The Polyco program gives the phase values at MJD for a full range of +MAXHA to -MAXHA . where MAXHA is the parameter given to polyco by the user.
3. When polyco is asked to check for phases between MJD_start and MJD_end it will calculate the values starting from an **offset**(calculated within the polyco code) to MJD_start which depends on the difference between local HA and MAXHA and the tspan/2 with some rounding off.
4. It was found that when this offset is large which is almost close to half of a day it will go ahead to calculate the values from there which could be not useful but they lie in the HA range according to the polyco code giving us some extra values.

2.2.4 How does the phase interpolation depend on the number of coefficients? How does the error vary? What is the sufficient no. of coefficients for the gating?

Given that it is not possible to get the phase at any given instant in polyco it is important to understand the interpolation formulae and the phases calculated using the phase formulae and how it change with the no. of coefficients and what factors from our major input parameters is affecting the errors and its variation with no. of coefficients. A proper juypter notebook3 has been made for all the tests performed so one can redo the tests with different values if required.

key takeaways :

- Phase error goes down significantly with no. of coefficients , phase error was checked till 5th coefficient

- There are variations of the order 10^{-3} were observed till 3 coefficients
- The phase error was observed to be proportional to the dt till the 3rd coeff however it wasn't significant however large dt is not advisable.
- No variation in the trend of the phase variation with no. of coefficients was observed with respect to change in observing frequency

Phase error = Rphase_tmid2 - phase(dt, no. of Coeff) where Rphase_tmid2 is the phase output by polyco at an MJD (tmid+dt)

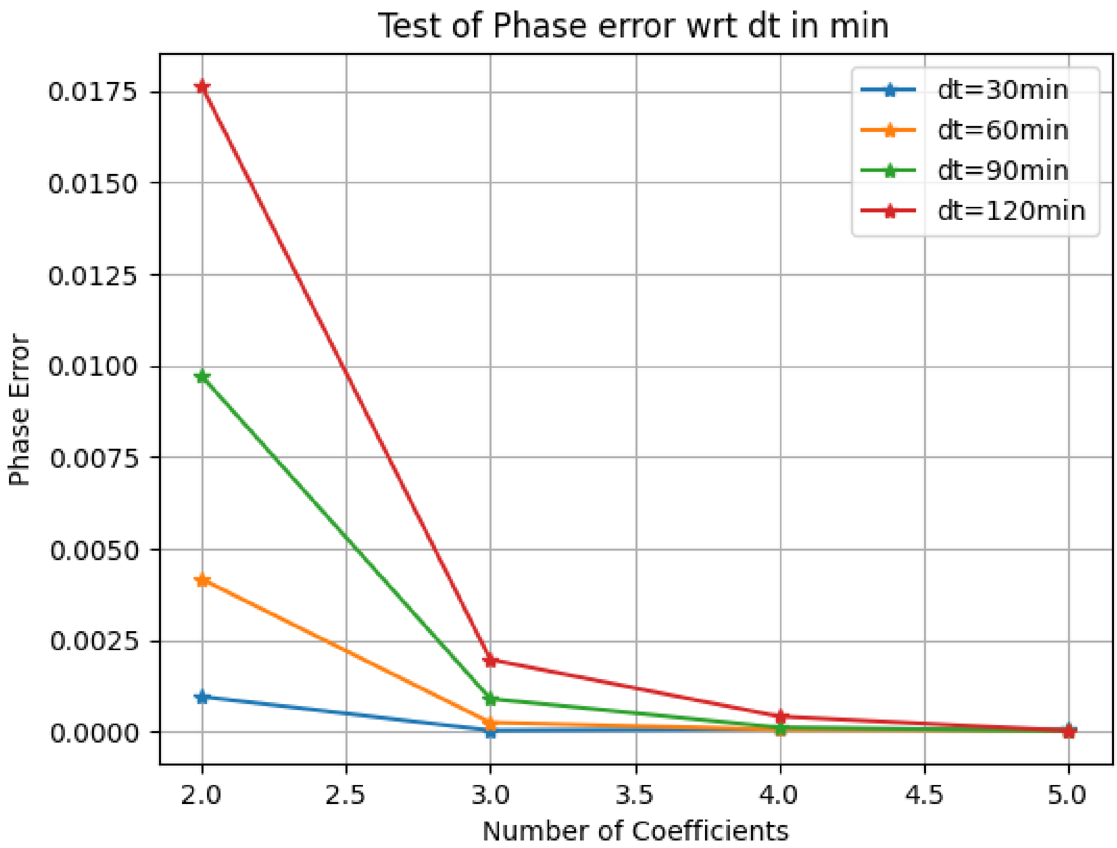

Figure 1: The variation of phase error with no. of coefficients as the dt is varied

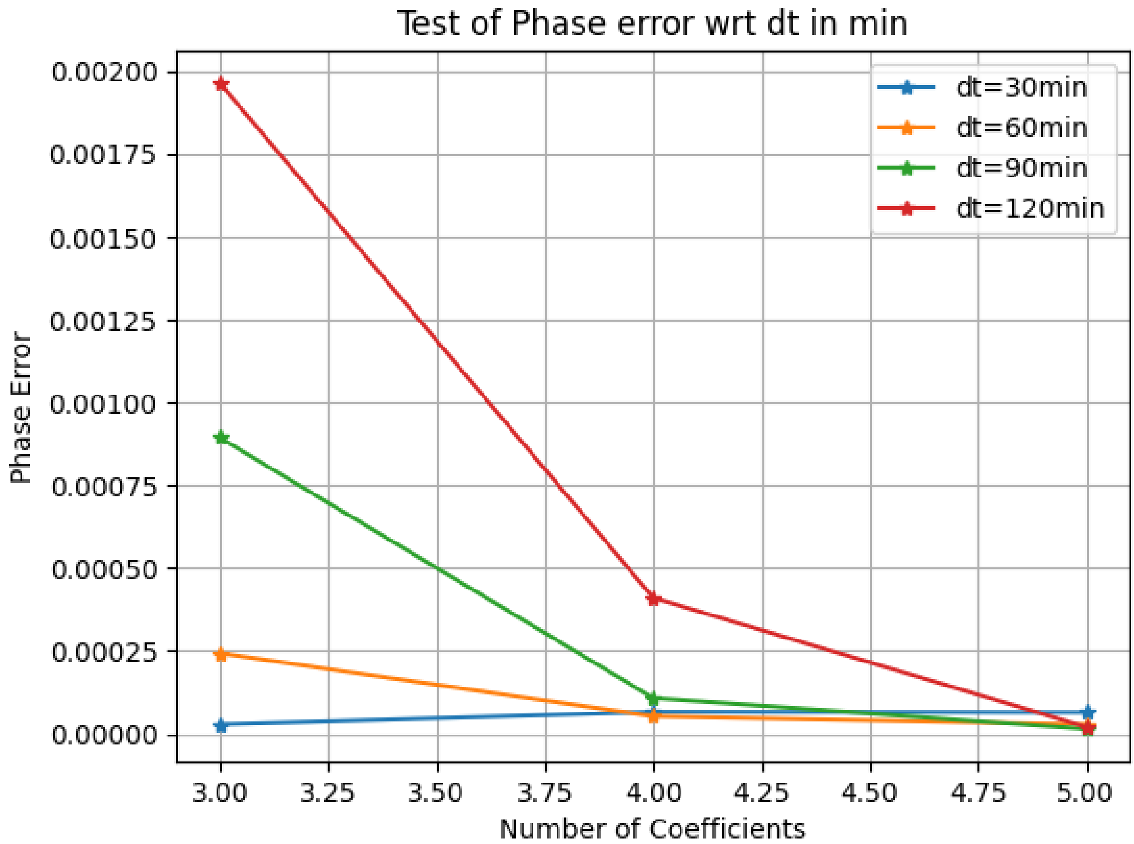

Figure 2: The same plot as above but indexed from coeff 3 to 5 to observe the variation in trend

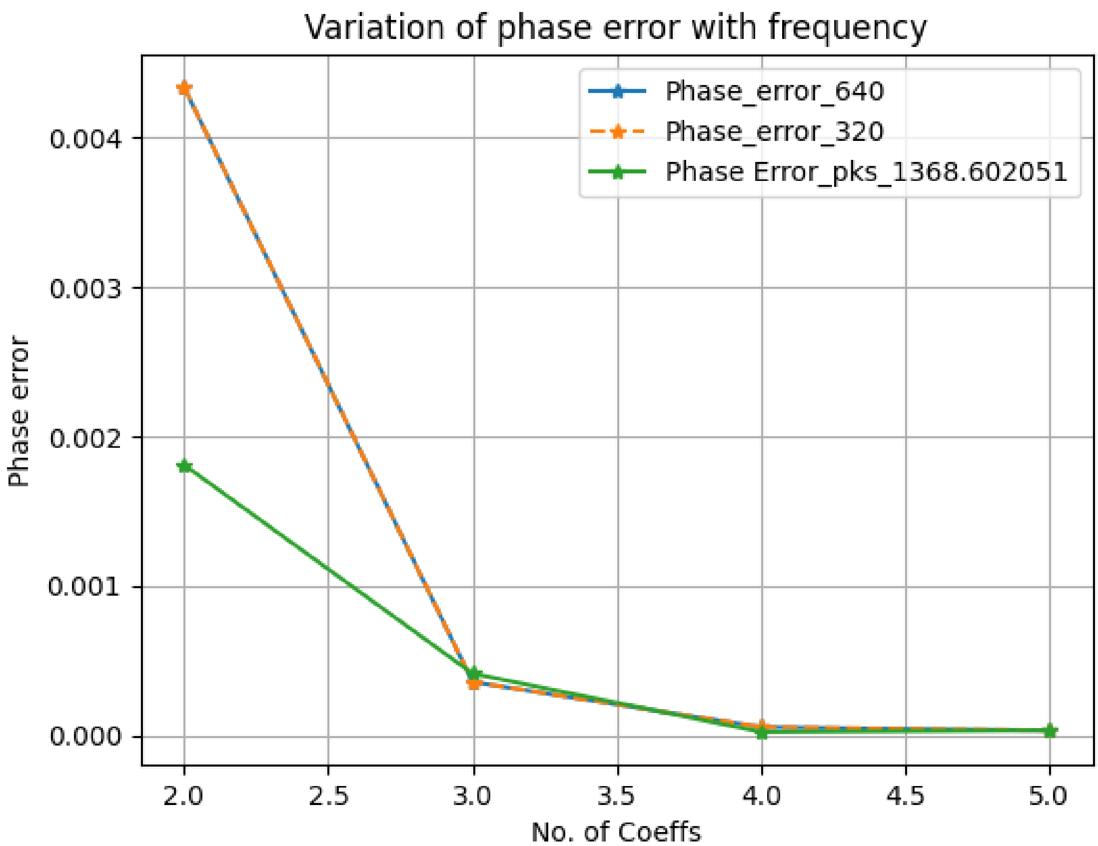

Figure 3: The trend for the phase error does not vary with frequency for the same observatory however a trend change was observed for a different observatory location which is only significant to the 2nd coefficient

2.2.5 Variation of phase with frequency

The polyco output gives coefficients that can predict the phase of the pulsar at any given MJD value . With the help (Freqsweep_Phase_calculator) we can generate these phase values for a range of frequencies

The Phase of pulsar changes with the frequency of observation for a given epoch this delay is due to the frequency-dependent propagation of radio waves through the ionized medium The dispersion relation is defined as follows [1]:

$$\phi(t, f) \approx \phi(t, f_0) - \nu(t, f_0) \left(\frac{1}{f^2} - \frac{1}{f_0^2} \right) D \tag{3}$$

where,

$$\begin{aligned}
f &: \text{Frequency} \\
f_0 &: \text{Observing frequency} \\
\phi(t, f_0) &: \text{Initial phase at reference frequency} \\
\nu(t, f_0) &: \text{Frequency of pulsar at } f_0 \\
D &: \mathrm{DM}/k_d \\
k_d &: 4.149\,\mathrm{GHz}^2\,\mathrm{pc}^{-1}\,\mathrm{cm}^3\,\mathrm{ms}
\end{aligned}$$

The Phase output as a function of frequency was compared to the output generated by the phases calculated using the Polyco formula 2

It was verified that the phase output by polyco followed the dispersion relation using the following steps

1. The absolute phase output for polyco and the phases are calculated using dispersion relation which we will label as ϕ_{polyco} and ϕ_{disp} respectively.

2. The relationship between phase changes and frequency is generated by two different methods: polyco and dispersion relation. we observe a change in phase for a range of frequencies(320 MHz to 610 MHz) at a given TMID value in the polyco output taking $f_o = 320MHz$ as a reference frequency to calculate the phase change.[1]

[1]Please ignore the title of the 4

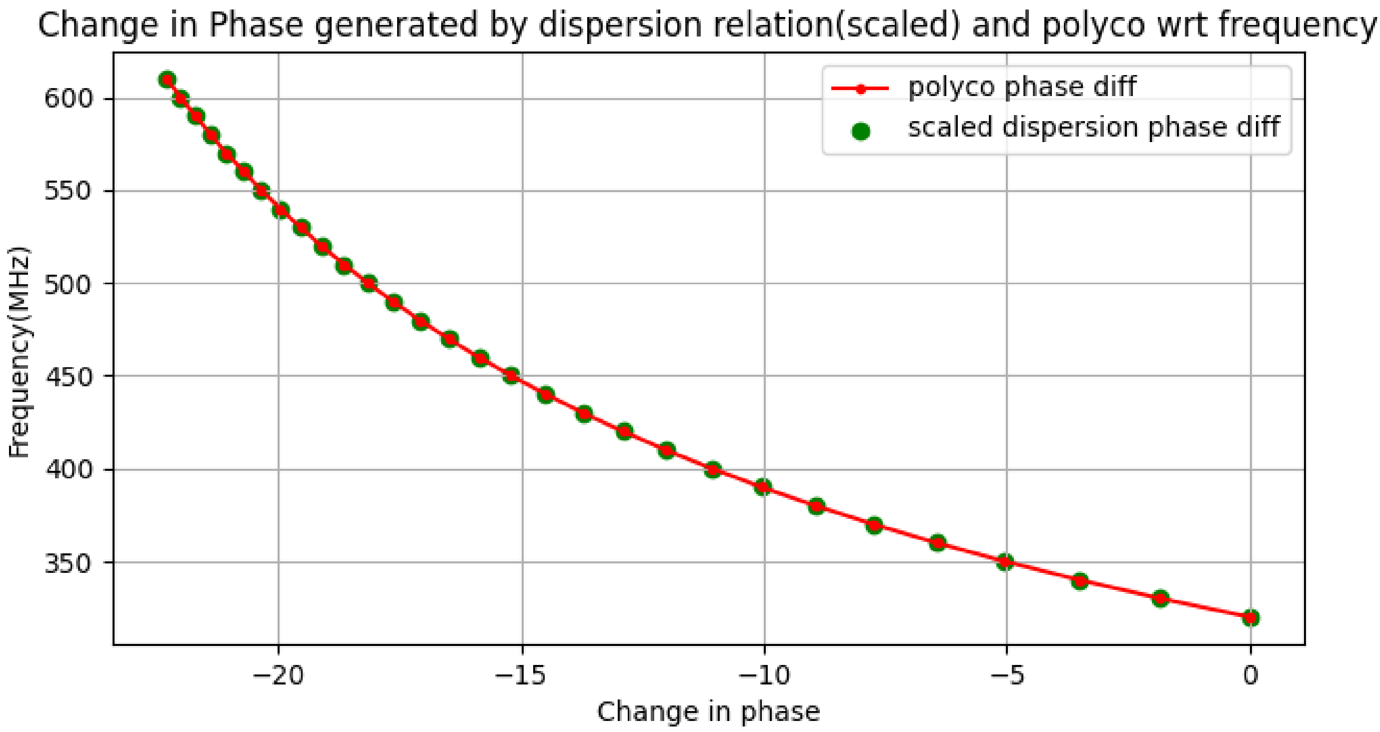

Figure 4: The plot shows an agreement with the change in phase values generated by polyco and dispersion formulae

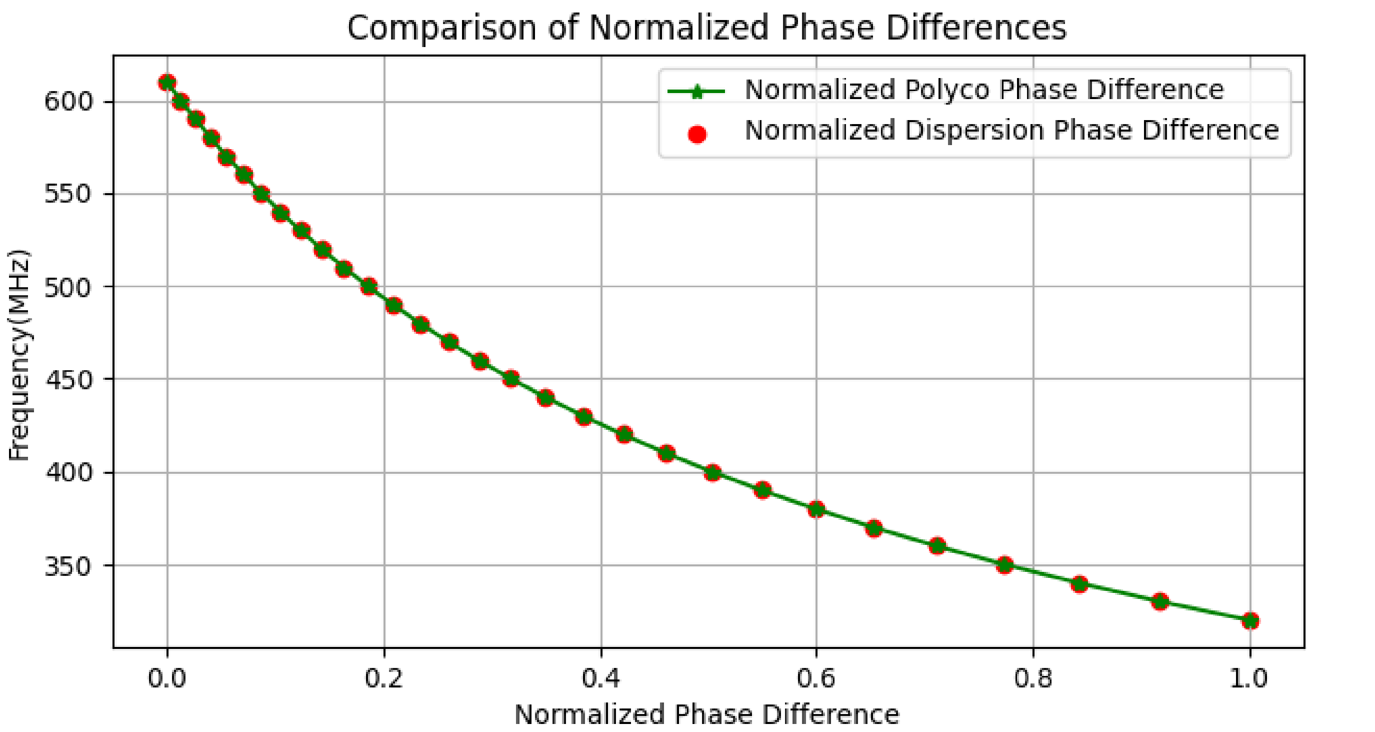

Figure 5: The Normalised values $\phi^N_{disp}(f_o)$ and $\phi^N_{polyco}(f_o)$ show an agreement in the plot

3. The normalization of the change of phase values for $\Delta\phi_{disp}$ and $\Delta\phi_{polyco}$

$$\Delta\phi^N_{disp}(f_o) = \frac{\Delta\phi_{disp}(f_o) - \min(\Delta\phi_{disp})}{\max(\Delta\phi_{disp}) - \min(\Delta\phi_{disp})} \tag{4}$$

$$\Delta\phi^N_{polyco}(f_o) = \frac{\Delta\phi_{polyco}(f_o) - \min(\Delta\phi_{polyco})}{\max(\Delta\phi_{polyco}) - \min(\Delta\phi_{polyco})} \tag{5}$$

4. This normalization process ensures that both sets of phase differences are plotted on the same scale between 0 and 1

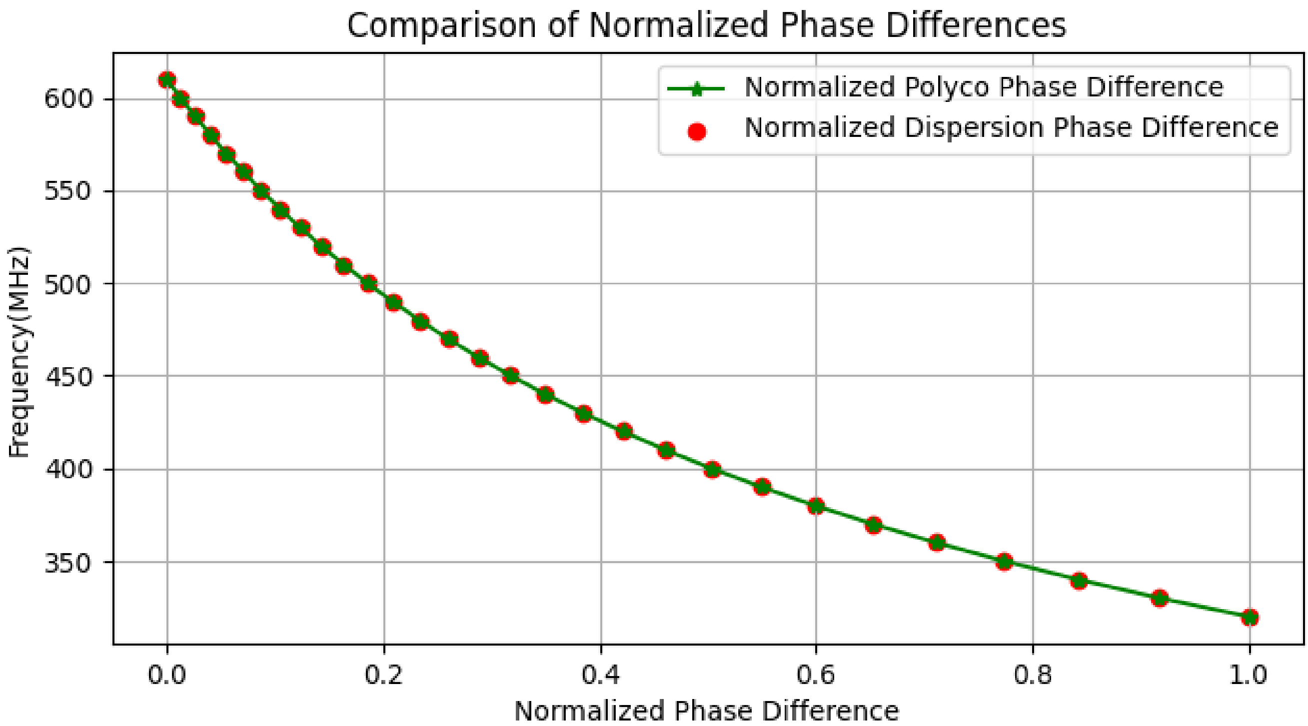

Figure 6: The Normalised values $\phi^N_{disp}(f_o)$ and $\phi^N_{polyco}(f_o)$ show an agreement in the plot

2.3 Algorithm to get the phase at a given observation time at GMRT

We need to figure out a way to get the phase of a pulsar given the Date time and location of the observer. Polyco cannot directly give the phase at any given instant so it was decided that we will interpolate the phase between a window of MJDs[2]. The algorithm goes as follows

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Algorithm 1 Phase Calculation Algorithm

- **Input:** Date: 30/12/2023; Time: 3:10:00 (IST); Location: GMRT
- **Output:** Phase for Vela at GMRT at Time : 3:10:00

Procedure:

1: Get Transit time for Vela at GMRT for a given date in **UTC**
2: Decide a suitable Hour Angle (HA) range preferably using 12 Hours and keep the time span as 60 min
3: Get the Modified Julian Date (MJD) values in the bracket of the desired time
4: Use phase interpolation to get the phase at given observation time(UTC) **??**
5: Report the calculated phase

To check the output of the exercise please check this link : phase at observation time

[2]Polyco gives output MJD in UTC: IST = 5:30 +UTC

2.4 Pint: A python based timing tool

Pint offers a great deal of flexibility in reading the polyco tables which can be used by us for the phase calculation. The convention in which the phase is printed is different but there is a lot of documentation for pint which makes it really user friendly few things that we can do :

- Use pint to read the polyco table to make it more accessible for our use
- There is a direct option of calculating phases which could make our life easier wrt to coding.
- When observed pint polyco clearly takes the TZRMJD as the 0 phase reference whereas in tempo2 we do not clearly understand what is the reference value that is where is the 0 of the phase . what we know is that it is close to PEPOCH.

Other Sections on PINT will be added soon

3 Codes

Please check the respective Python notebooks for an explanation the code exercises have been made as reader-friendly as possible

- Directory to the code that is used to generate polyco files at multiple parameters. Paramter sweep frequency : Please check the python notebook for explanation.
- Tempo2 Polyco TMID MJD: A study Python Notebook: phase_analysis.ipynb
- Tempo2 Polyco Phase Error Variation Analysis Python Notebook : tmid_pred.ipynb
- Phase of vela at GMRT given the observation time Python Notebook: phase at observation time
- What is maxHA parameter? Python Notebook: HA_check.ipynb
- A basic trial for Pint polyco features Python Notebook: This is still in progress

References

[1] R. T. Edwards, G. B. Hobbs, and R. N. Manchester. tempo2, a new pulsar timing package – II. The timing model and precision estimates. *Monthly Notices of the Royal Astronomical Society*, 372(4):1549–1574, 10 2006. ISSN 0035-8711. doi: 10.1111/j.1365-2966.2006.10870.x. URL `https://doi.org/10.1111/j.1365-2966.2006.10870.x`.

[2] D. R. Lorimer and M. Kramer. *Handbook of Pulsar Astronomy*, volume 4. 2004.