Part I. Implement a decision tree algorithm and make predictions.

In [ ]:
```
import numpy as np
```

In [ ]:
```
class TreeNode:
    """ Node class in the decision tree. """
    def __init__(self, T):
        self.type = 'leaf' # Type of current node. Could be 'leaf' or 'branch' (at default: 'leaf').
        self.left = None   # Left branch of the tree (for leaf node, it is None).
        self.right = None  # Right branch of the tree (for leaf node, it is None).
        self.dataset = T   # Dataset of current node, which is a tuple (X, Y).
                           # X is the feature array and Y is the label vector.

    def set_as_leaf(self, common_class):
        """ Set current node as leaf node. """
        self.type = 'leaf'
        self.left = None
        self.right = None
        self.common_class = common_class

    def set_as_branch(self, left_node, right_node, split_rule):
        """ Set current node as branch node. """
        self.type = 'branch'
        self.left = left_node
        self.right = right_node
        # split_rule should be a tuple (j, t).
        #   When x_j <= t, it goes to left branch.
        #   When x_j > t, it goes to right branch.
        self.split_rule = split_rule
```

In [ ]:
```
# Prepare for dataset.
def get_dataset():
    X = np.array(
        [[1.0, 2.0],
         [2.0, 2.0],
         [3.0, 2.0],
         [2.0, 3.0],
         [1.0, 3.0]
        ])
    Y = np.array(
        [1,
         1,
         0,
         0,
         0])
    T = (X, Y) # The dataset T is a tuple of feature array X and label vector Y.
    return T

T = get_dataset()
```

In this part, you are required to implement the decision tree algorithm shown in the problem description of Q2 in HW4:

The **4 steps** are marked in comments of the following code. Please fill in the missing blanks (e.g. "...") in the TODOs:

In [ ]:
```
# Initialization.
root_node = TreeNode(T)
```

In [ ]:
```
# Procedure for current node.
def build_decision_tree_procedure(node_cur, depth=0):
    # Step 1. Check if all data points in T_cur are in the same class
    #         - If it is true, set current node as a *leaf node* to predict the common class in T_cur,
    #           and then terminate current procedure.
    #         - If it is false, continue the procedure.

    T_cur = node_cur.dataset
    X_cur, Y_cur = T_cur  # Get current feature array X_cur and label vector Y_cur.
    if (Y_cur == 1).all():
        print('    ' * depth + '+-> leaf node (predict 1).')
        print('    ' * depth + '      Gini: {:.3f}'.format(Gini(T_cur)))
        print('    ' * depth + '      samples: {}'.format(len(X_cur)))
        node_cur.set_as_leaf(1)
        return
    elif (Y_cur == 0).all():
        print('    ' * depth + '+-> leaf node (predict 0).')
        print('    ' * depth + '      Gini: {:.3f}'.format(Gini(T_cur)))
        print('    ' * depth + '      samples: {}'.format(len(X_cur)))
        node_cur.set_as_leaf(0)
        return
```

```
    # Step 2. Traverse all possible splitting rules.
    #         - We will traverse the rules over all feature dimensions j in {0, 1} and
    #           thresholds t in X_cur[:, j] (i.e. all x_j in current feature array X_cur).
    all_rules = []

    #### TODO 1 STARTS ###
    # Please traverse the rules over all feature dimensions j in {0, 1} and
    #   thresholds t in X_cur[:, j] (i.e. all x_j in current feature array X_cur),
    #   and save all rules in all_rules variable.
    # The all_rules variable should be a list of tuples such as [(0, 1.0), (0, 2.0), ... ]

    for j in (0,1):
        for t in X_cur[:,j]:
            all_rules.append((j,t))
    #### TODO 1 ENDS ###

    # print('All rules:', all_rules)   # Code for debugging.

    # Step 3. Decide the best splitting rule.
    best_rule = (_, _)
    best_weighted_sum = 1.0
    for (j, t) in all_rules:

        #### TODO 2 STARTS ###
        # For each splitting rule (j, t), we use it to split the dataset T_cur into T1 and T2.
        # Hint: You may refer to Step 4 to understand how to set inds1, X1, Y1, len_T1 and inds2, X2, Y2, len_T2.

        # - Create subset T1.
        inds1 = [x for x in range(len(T_cur[0])) if T_cur[0][x][j] <= t]                 # Indices vector for those data points with
        X1 = [T_cur[0][i] for i in inds1]                    # Feature array with inds1 in X_cur.
        Y1 = [T_cur[1][i] for i in inds1]                    # Label vector with inds1 in Y_cur.
        T1 = (X1, Y1)               # Subset T1 contains feature array and label vector.
        len_T1 = len(X1)                # Size of subset T1.
        # - Create subset T2.
        inds2 = [x for x in range(len(T_cur[0])) if T_cur[0][x][j] > t]                  # Indices vector for those data points with
        X2 = [T_cur[0][i] for i in inds2]                    # Feature array with inds2 in X_cur.
        Y2 = [T_cur[1][i] for i in inds2]                    # Label vector with inds2 in Y_cur.
        T2 = (X2, Y2)               # Subset T2 contains feature array and label vector.
        len_T2 = len(X2)                # Size of subset T2.
        #### TODO 2 ENDS ###

        # Calculate weighted sum and try to find the best one.
        weighted_sum = (len_T1*Gini(T1) + len_T2*Gini(T2)) / (len_T1 + len_T2)
        # print('Rule:', (j, t), 'len_T1, len_T2:', len_T1, len_T2, 'weighted_sum:', weighted_sum)  # Code for debugging.
        if weighted_sum < best_weighted_sum:

            #### TODO 3 STARTS ####
            # Update the best rule and best weighted sum with current ones.

            best_rule = (j,t)
            best_weighted_sum = weighted_sum
            #### TODO 3 ENDS ####

    # Step 4. - We split the dataset T_cur into two subsets best_T1, best_T2 following
    #               the best splitting rule (best_j, best_t).
    #         - Then we set current node as a *branch* node and create child nodes with
    #               the subsets best_T1, best_T2 respectively.
    #         - For each child node, start from *Step 1* again recursively.

    best_j, best_t = best_rule
    # - Create subset best_T1 and corresponding child node.
    best_inds1 = X_cur[:,best_j] <= best_t
    best_X1 = X_cur[best_inds1]
    best_Y1 = Y_cur[best_inds1]
    best_T1 = (best_X1, best_Y1)
    node1 = TreeNode(best_T1)
    # - Create subset best_T2 and corresponding child node.
    best_inds2 = X_cur[:,best_j] > best_t
    best_X2 = X_cur[best_inds2]
    best_Y2 = Y_cur[best_inds2]
    best_T2 = (best_X2, best_Y2)
    node2 = TreeNode(best_T2)
    # - Set current node as branch node and create child nodes.
    node_cur.set_as_branch(left_node=node1, right_node=node2, split_rule=best_rule)
    print('    ' * depth + '+-> branch node')
    print('    ' * depth + '      Gini: {:.3f}'.format(Gini(T_cur)))
    print('    ' * depth + '      samples: {}'.format(len(X_cur)))
    # - For each child node, start from Step 1 again recursively.
    print('    ' * (depth + 1) + '|-> left branch: x_{} <= {} (with {} data point(s)).'.format(best_j, best_t, len(best_X1)))
    build_decision_tree_procedure(node1, depth+1) # Note: The depth is only used for logging.
    print('    ' * (depth + 1) + '|-> right branch: x_{} > {} (with {} data point(s)).'.format(best_j, best_t, len(best_X2)))
    build_decision_tree_procedure(node2, depth+1)

def Gini(Ti):
    """ Calculate the Gini index given dataset Ti. """
```

```
    Xi, Yi = Ti         # Get the feature array Xi and label vector Yi.
    if len(Yi) == 0:    # If the dataset Ti is empty, it simply returns 0.
        return 0

    #### TODO 4 STARTS ####
    # Implement the Gini index function.
    P_Y1 = len([x for x in Yi if x == 1]) / len(Yi)         # Estimate probability P(Y=1) in Yi
    P_Y0 = len([x for x in Yi if x == 0]) / len(Yi)         # Estimate probability P(Y=0) in Yi
    Gini_Ti = 1 - (P_Y1 ** 2) - (P_Y0 ** 2)      # Calculate Gini index: Gini_Ti = 1 - P(Y=1)^2 - P(Y=0)^2
    #### TODO 4 ENDS ####
    return Gini_Ti
```

After you finish the above code blank filling, you can use the following code to build the decision tree. The following code also shows the structure of the tree.

In [ ]:
```
# Build the decision tree.
build_decision_tree_procedure(root_node)

# If your code is correct, you should output:
#
# +-> branch node
#       Gini: 0.480
#       samples: 5
#     |-> left branch: x_1 <= 2.0 (with 3 data point(s)).
#     +-> branch node
#           Gini: 0.444
#           samples: 3
#     .....
#
# You can also use the sklearn results to validate your decision tree
# (the threshold could be slightly different but the structure of the tree should be the same).
```

```
+-> branch node
      Gini: 0.480
      samples: 5
    |-> left branch: x_1 <= 2.0 (with 3 data point(s)).
    +-> branch node
          Gini: 0.444
          samples: 3
        |-> left branch: x_0 <= 2.0 (with 2 data point(s)).
        +-> leaf node (predict 1).
              Gini: 0.000
              samples: 2
        |-> right branch: x_0 > 2.0 (with 1 data point(s)).
        +-> leaf node (predict 0).
              Gini: 0.000
              samples: 1
    |-> right branch: x_1 > 2.0 (with 2 data point(s)).
    +-> leaf node (predict 0).
          Gini: 0.000
          samples: 2
```

With the obtained decision tree, you can predict the class of new feature vectors:

In [ ]:
```
def decision_tree_predict(node_cur, x):
    if node_cur.type == 'leaf':
        return node_cur.common_class
    else:
        j, t = node_cur.split_rule
        if x[j] <= t:
            return decision_tree_predict(node_cur.left, x)
        else:
            return decision_tree_predict(node_cur.right, x)
```

In [ ]:
```
for x in [(2,1), (3,1), (3,3)]:
    y_pred = decision_tree_predict(root_node, x)
    print('Prediction of {} is {}'.format(x, y_pred))
```

```
Prediction of (2, 1) is 1
Prediction of (3, 1) is 0
Prediction of (3, 3) is 0
```

Part II. Use Scikit-learn to build the tree and make predictions.

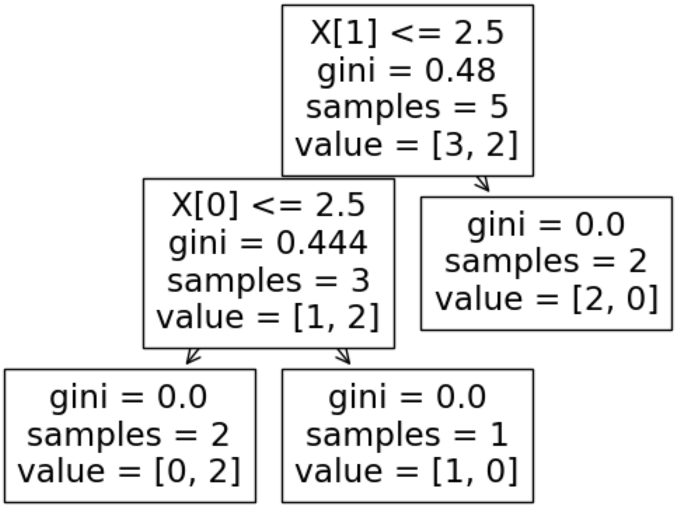
The following code uses Scikit-learn to build the decision tree. You can use it to check if your previous implementation is correct or not.

In [ ]:
```
# Ref: https://scikit-learn.org/stable/modules/tree.html#tree-algorithms-id3-c4-5-c5-0-and-cart
from sklearn import tree
X, Y = T
clf = tree.DecisionTreeClassifier()
clf = clf.fit(X, Y)
```

The following code illustrates the obtained decision tree. It should have same structure and similar rules compared with the tree in your own implementation.

```
In [ ]: # Plotting the tree.
        tree.plot_tree(clf)
```

```
Out[ ]: [Text(0.6, 0.8333333333333334, 'X[1] <= 2.5\ngini = 0.48\nsamples = 5\nvalue = [3, 2]'),
         Text(0.4, 0.5, 'X[0] <= 2.5\ngini = 0.444\nsamples = 3\nvalue = [1, 2]'),
         Text(0.2, 0.16666666666666666, 'gini = 0.0\nsamples = 2\nvalue = [0, 2]'),
         Text(0.6, 0.16666666666666666, 'gini = 0.0\nsamples = 1\nvalue = [1, 0]'),
         Text(0.8, 0.5, 'gini = 0.0\nsamples = 2\nvalue = [2, 0]')]
```



The following code makes the predictions using the obtained decision tree. It should have identical results as the ones for your own implementaion.

```
In [ ]: # Predict the class.
        for x in [(2,1), (3,1), (3,3)]:
            y_pred = clf.predict(np.array([x]))[0]
            print('Prediction of {} is {}'.format(x, y_pred))
```

```
Prediction of (2, 1) is 1
Prediction of (3, 1) is 0
Prediction of (3, 3) is 0
```