A Simple Universe Argument

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Abstract

This paper argues that one can make a prediction from the hypothesis that our universe is not designed, i.e. that it has a high level of complexity of a certain kind, and that this complexity would be easily observable everywhere. However, this is not what we observe, which falsifies the hypothesis.

1 Introduction

Many people believe that there are certain laws approximating our universe's behaviour fairly well, that we can compute its age, that we can make predictions about the distant future, and so on. In a way, all of these are really surprising. In the words of Feynman (2009):

> Incidentally, the fact that there are rules at all to be checked is a kind of a miracle; that it is possible to find a rule, like the inverse square law of gravitation, is some sort of miracle. It is not understood at all, but it leads to the possibility of prediction — that means it tells you what you would expect in an experiment you have not yet done.

This paper attempts to figure out statistically why do we observe such a surprising universe and what can we reasonably believe about it.

The mathematical part of this paper is rather small, containing only a few simple properties about set cardinalities, probabilities and ordinals. I think that the non-mathematical ideas in this paper have an intuitive appeal even without the mathematical ones, but presenting them separately would make it less clear why certain conclusions can be drawn. Although, in my opinion, the non-mathematical ideas are also somewhat obvious, I did not manage yet to find anyone drawing the same conclusions in the same way.

My argument can be considered a statistical approach to Aquina's fifth way. It tries to avoid the issues that other statistical approaches attempting to show that our universe is designed (e.g. the fine-tuning argument) have.

Sections 2 presents this paper's relation to previos work. Section 3 introduces universe approximations and a few other related notions. Section 5 introduces the argument's axioms, while section 6 uses them to make predictions about our universe, then verifies if they match our observations. Section 7 presents some clarifications and possible answers to various objections to this paper. Section 9 briefly presents some mathematical background for people interested in it.

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2 The Ordered Universe Argument

The great Catholic theologian Thomas Aquinas, in his "fifth way", attempts to show God's existence from the order of the universe, i.e. that almost all bodies, almost always behave according to simple natural laws. One can find a good exposition of this argument in "The Argument from Design"(R. G. Swinburne, 1968), but let us look at a few ideas which are interesting in the context of this paper.

There are two types of order one may consider for showing God's existence, the spatial order and the temporal order. The former is the order that can be seen in (a part of) the universe at a given moment in time, e.g. that planets, living bodies, and other things are ordered. The latter can be seen in the behaviour of things, including the laws of nature, and it's the one that will be used in this paper.

While it adresses many possible objections to Aquinas' fifth way, "The Argument from Design" leaves one of them open: this argument is based on an analogy between the order of world and the order produced by people, which limits its strength. I think that this paper's argument, although it builds on the same foundation, needs only a fairly weak analogy (see Section 6.3).

The following quote from R. G. Swinburne (1968), made when addressing Hume's objection that the order which can be observed in this universe is just an accident, makes a nice introduction for the argument described in this paper:

> But if we say that it is chance that in 1960 matter is behaving in a regular way, our claim becomes less and less plausible as we find that in 1961 and 1962 and so on it continues to behave in a regular way. An appeal to chance to account for order becomes less and less plausible the greater the order.

3 Possible Universes and Their Descriptions

If our universe is designed, then it's likely to be the way it is because its Designer wanted it to have certain properties. In order to understand why our universe works the way it does, one would need to understand the intent of its Designer. While that is interesting in itself, I will not try to pursue it here, except for a few limited ideas.

For most of the remainder of this paper, let us consider the other case. Let us assume the hypothesis that our universe is not designed and let us try to make a prediction based on it. How would a non-designed universe look like? Would it be similar to our universe? Maybe an infinity of universes exist and ours is just one of many, or maybe our universe is the only one that exists. Even if ours is the only one, one could easily imagine that it worked in a different way, e.g. maybe some constant like the speed of light would be different, or maybe gravity would work differently.

There are people who claim that all logically possible universes exist, either because they think that it simply makes sense, or because they want to give a good account of modality, or for other reasons. If that's the case, it seems, at first sight, that making predictions about non-designed universes is rather hard. However, this paper argues that there are certain things that can be said regardless of how many universes exist, whether it is one, or all logically possbile ones, or anything in between.

A possible universe could have exactly the same fundamental laws as ours, but with matter organized differently. It could have similar laws, but with different universal constants. It could have different fundamental particles (or fields, or whatever the basic building blocks of our universe are, assuming that there are any). Or it could be completely different, i.e. different in all possible ways.

It could be that our logic and reasoning are universal instruments, but it could also be that some of these possible universes are beyond what our reasoning can grasp and others have properties for which our logic is flawed. Even if that's the case, let us see if we can say anything about the possible universes that we could understand and could model in some way.

In the following, the **possible universes** term will denote only the logically possible universes which we could model (with a few more constraints that will be added below). However, the word "possible" is ambiguous, so when the distinction between logical possibility and actual possibility is important, the **conceivable universes** term may be used instead.

This notion of model is not precise enough. Let us restrict the possible universes term even more, to the possible universes that could be modelled mathematically, even if that may leave out some of them. This may seem too restrictive, especially since this paper only needs universes which can be approximated by mathematical models. We are going to relax this when talking about approximations, but, for now, let us consider only universes which are modellable with sets of axioms that are at most countable.

Let us restrict the universes we are considering even further, to universes that have something remotely resembling time and space, for which "the state of the universe at a given moment in time", or something close, makes sense, and which can plausibly contain intelligent beings. Any such universe is, for the purpose of this paper, a conceivable universe.

To keep the exposition simple, in the following I will use "the state of the universe at a given moment in time", but one should replace it with one's favourite alternative concept, e.g. with "the past of a hyperplane whose points only have spacelike intervals between them".

Let us define a **universe description** to be a consistent mathematical theory that has a set of axioms which is at most countable and which allows making predictions about the future state of the universe given its state at a certain moment in time. A **universe region description** is something similar, but only for a given space-time region of a universe, with extra axioms to take into account the border state when predicting. In the best case, for a deterministic universe, there might exist a description which allows one to correctly predict the entire future state given the state at any moment in time, but a universe description as defined here does not have to predict everything and, even when it predicts something, it does not have to always be correct.

"Predicting", as used above, would normally mean that one starts from the theory and does some formal inferences and computations, having the prediction as the result. However, as Calude, Meyerstein, and Salomaa, 2012 shows, there are many things that can't be proven this way. We don't know if the state of the universe at a given moment in time is one of those things, although we could restrict our descriptions to ones where this is possible, at least up to a reasonable level. Regardless, let us use a different meaning: a theory predicts something if that something is true in all models of that theory.

Note that usually the data available for making predictions is dependent on who is making the prediction. As an example, if we assume that all predictions are about things that can be perceived, directly or indirectly, then each kind of intelligent beings (e.g. humans) will make predictions about the universe as seen through their senses. If a universe contains multiple kinds of intelligent beings, with different kinds of sense organs, then that universe may have descriptions which are very different. Of course, things that are not observable directly can sometimes be mapped to things that are observable, but this may not be always true.

In order to handle this dependence on who observes the universe in a reasonable way, in the reminder of this paper we will work with universes that contain intelligent beings, and all predictions will be relative to what these intelligent beings could observe. If there are multiple kinds of intelligent beings in the universe, whenever we are talking about its description we will assume that we picked one such kind. I.e. although we will talk about universes and their descriptions, we'll actually mean (universe, intelligent-being-kind) pairs and their corresponding descriptions.

Next, let us try to specify how good an univese description should be. First, predictions speak about the future, but expecting to predict everything until the end of the universe (if any) may not be reasonable. We may want to fix an amount of time Δt, focusing on predictions about things that are at most Δt in the future. Second, we should't expect to be able to describe everything with full precision, so we may want to have a precision $\eta > 0$ for all the values that are predicted. Third, we shouldn't expect predictions to always be correct, so we should require that they are true with probability $p > 0$. The exact meaning of "true with probability" here is left open, except that we require p to be the same for each location where predictions are being made. Of course, we may add other similar constraints if needed.

As an example, we could ask that, out of all predictions that we can make at a given spacetime location, a fraction of p turn out to be true. If we are making statistical predictions, then the observed outcomes would be consistent with a fraction p of all sentences representing statistical predictions being correct.

Then let us say that an **approximate universe description** with a **level of approximation** $L = (\eta > 0,\ p > 0 \text{ and } \Delta t > 0)$ is a universe description which allows approximating the future state of the universe with a precision η, with a probability $p > 0$ for a prediction to be correct and for a limited amount of time Δt.

There is a distinction that we should make. When predicting (say) weather we can't make long-term precise predictions, and this happens because weather is chaotic, that is, a small difference in the start state can create large differences over time. This would happen even if the universe would be deterministic and we would know the laws of the universe perfectly, as long as we don't know the full current state of the universe. However, high precision predictions may be possible for a deterministic universe if the full state is taken into account and, as we mentioned, we assume that we know the full state of the universe at a moment in time when making predictions.

For a given universe or region of a universe, given a level of approximation, we will pick a canonical description in the following way: Let S be the set of descriptions which approximate the universe with the given level of aproximation. If S contains at least one finite description, then we pick the shortest such description as "the canonical description", breaking ties by using the lexicographic order. Otherwise, we simply say that the universe (region) has an infinite description, and we will abuse the terminology a bit by picking the entire set S as the canonical description (we could also pick a random description from the set). If the level of approximation is obvious from the context, we will call this canonical description **the universe's description** or the **universe region's description**.

One could also use a well-ordering on the real numbers to choose the lowest description as the universe's description, but that would complicate things without any benefit.

4 Options for Our Universe

The reminder of this paper will analyze what we can reasonably believe about the following issues:

- Our universe is designed or not.
- Our universe has a finite or infinite description.
- Option 1: There is a meta-β universe for each countable ordinal β such that our universe is the meta-0 one and the meta-β universe includes, directly or not, all meta-α universes with $\alpha < \beta$. Option 2: there is an ordinal β, possibly 1, where this stops being the case.
- The set of possible descriptions for a finite chunk of space-time that are also compatible with life has at least the cardinal of the set of real numbers, $\mathbb{R}$, or a smaller one.

5 Axioms

Throughout this paper we will implicitly use only separated probability measures, i.e. they can measure singletons (single-element sets).

Below we will use two terms, "generic" and "peculiar", which are defined precisely in section 9.1. Informally, an object is peculiar if it satisfies a peculiar predicate, and a peculiar predicate is one that has a zero probability for any continuous probability distribution. As an example, "has a finite number of digits" is a peculiar predicate over real numbers, and any real number with a finite number of digits, like 12.5, is peculiar. An object or predicate is generic if it is not peculiar.

5.1 Observing Events

Axiom 1. *If P is a probability over the set of real numbers (or a set with the same cardinal)*[1]*, we observe a real number x, and $P(x) = 0$, then x is generic.*

[1]Readers should keep in mind that, in most cases throughout this paper, what is being said about the set of real numbers is similarly valid for any set with the same cardinality.

Note that here, and in all the axioms in this paper, it is not required that P is a probability over the Borel algebra of $\mathbb{R}$, although that is, in many cases, implicitly assumed when talking about probabilities over $\mathbb{R}$.

The set of events for which $P(y)$ is greater than 0 is at most countable (see section 9.1), and, if we remove them from $\mathbb{R}$, what remains will still have the same cardinality. On this later set, the probability of all peculiar events taken together is 0, so there is no chance of us observing one. In other words, the probability of all generic events is 1, so we can be sure that we observed a generic event.

Of course, the (logical) possibility of observing a peculiar event still exists, but, practically, we will not observe it as long as the set of our observations is at most countable.

Axiom 2. *If we observe a specific real number x, when we could have observed any real number, and there is no probability distribution that could describe how x was chosen, then x is generic.*

Note that this axiom does not say that we do not know that probability distribution, it says that there is no such probability distribution. Anyone believing that this cannot happen should treat the cases where this axiom applies as invalid.

Also note that this cannot happen when using subjective probabilities.

If there is nothing that could favor peculiar numbers over generic ones, it's absurd to think that we could have observed an element of such a tiny set among something infinitely larger. Also, the similar axiom for probabilities above suggests that this is the only reasonable assumption in this case.

5.2 $\mathbb{R}^4$ Universe

The universes we are interested in are universes that can be modelled on top of a space with α dimensions, $\mathbb{R}^\alpha$, where α is finite, or something close enough to that.

In order to not define what "close enough" means, we will use the following axiom, which is true for any finite dimensional space based on the set of real numbers, but it also works on spaces based on, say, the set of rational numbers, and on many finite spaces that are similar enough.

Let us define a **generalized rational number** as being either a rational number, or one of $-\infty$ and $+\infty$. Let us denote by "cuboid" a corner-based shape in the n-dimensional space (we could take it to mean "hyperrectangle", "hypercube", or any similar shape).

Axiom 3. *The set of (generalized) cuboids using the same dimensions as our space-time and whose corners' coordinates are generalized rational numbers (i.e. they belong to $\mathbb{Q} \cup \{-\infty, +\infty\}$), is countable and covers our universe.*

Note that any spacetime based on real numbers, i.e. included in $\mathbb{R}^\alpha$, will be included in the generalized cuboid having its corners at plus or minus infinity.

The axiom above does not require the space-time to include the cuboids or their corners. If, say, our universe is based on the set of integers, i.e. it would be included in $\mathbb{Z}^\alpha$, we could consider it as being included in $\mathbb{Q}^\alpha$, where we could check if it is included in one of the cuboids mentioned above.

The following definition could also be written as an axiom.

Definition 1. *A part of our universe is **finite** if it can be covered with a finite cuboid, i.e. one for which all corner coordinates are rational numbers.*

There are a lot of possible definitions for "finite" which are not included here. While this paper could probably be extended to also handle many of these, most likely there is no point in doing so. As an example, in $\mathbb{R}^3$ one can define it as "having a finite volume", which would mean that there are finite things that do not fit in finite cuboids. In order to handle this, when defining a level of approximation, we could allow ignoring what happens in a small part of the region we model.

5.3 Neighborhood Modelling

Axiom 4. *There is a large compact time-space region of our universe which includes our solar system, and there is a level of approximation L such that:*

1. *Any cuboid included in that region has a finite approximate description for the level L (i.e. we can make non-trivial approximate predictions in all such cuboids).*
2. *A description for one of the cuboids also works for all other cuboids with the same size in the given time-space region (i.e. the space region is isotropic enough).*

Many people assume, implicitly or explicitly, that this is true, and, even more, that a cuboid's description works for the entire universe. This is especially visible when, e.g., claiming that the universe is around 14 billion years old, that the sun will, in some distant future, become a white dwarf, or that standard-candle supernovae are not ilussions. In order to believe this, one must assume that our universe has a finite approximate description or, at least, that our solar system/galaxy/observable part of the universe has such a description.

5.4 Logically Possible Universes

Axiom 5. *For any level of approximation L above a certain minimum level (see below) there is a set D_L of universe descriptions such that the following are true:*

1. *D_L has the same cardinality as the set of real numbers, $\mathbb{R}$.*
2. *For all descriptions d in D_L there is at least one conceivable universe U_d which*
   (a) *has a time-space or something similar enough;*
   (b) *can plausibly contain intelligent beings that use mathematics;*
   (c) *for the intelligent beings mentioned above and for the level of approximation L, d is U_d's description.*
3. *If d and d' are descriptions from D_L, then d does not work for $U_{d'}$, the universe corresponding to d'.*
4. *D_L contains a description for our universe.*

The same is true for universe regions, except that D_L may have a lower cardinality.

The minimum level for which this is true is left unspecified, but we should include some common-sense restrictions, e.g. the minimum length we would need to measure is not below Planck's length. All levels of approximation used below will be above this minimum level, even if this is not mentioned explicitly.

This axioms states that, for a given level of approximation, there is a large set of conceivable universes, which in some narrow respects are similar to ours, but which are, in general, wildly different. Also, our universe belongs to this set.

To see why that is reasonable, let us first note that, most likely, we have an approximate description for the observable part of our universe given by classical mechanics, maybe with some additions. Alternately, one could use a description based on, say, quantum field theory.

It may be that there is a description (possibly different from the one above), that works for our entire universe. If not, some parts of the following would need to be adjusted to include this possibility, but the argument should stay essentially the same.

Next, for almost any countable axiom system that still has an n-dimensional real space, $\mathbb{R}^n$, as a base, one could imagine an alternate universe which, in the present, is exactly like ours inside (say) Mars' orbit, but what is what is outside of Mars' orbit is described by that axiom set.

Some of these axiom sets would describe laws of nature which still allow life to exist inside Mars' orbit, are similar enough to ours to allow us to observe what happens outside of this orbit, but different enough that we would notice (e.g. gravity could work differently, depending on the region of space in which one travels).

For any approximation level L, and any region that is not trivial for L (the meaning of "trivial" is left open, but, as an example, if we use an approximation level that does not distinguish things smaller than a size l, then the region must be significantly larger than l), there are multiple possible descriptions that are distinct for L, i.e. there is no possible region where both descriptions would be valid within the level L (see 7.4). By splitting an infinite timespace into disjoint regions defined by a set of finite rational coordinates (e.g. we could split it into cubes whose edges have length 1 and whose corners are integers) and taking all possible ways of assigning descriptions to these regions we get a set of universe descriptions with infinitely countable axioms which we will denote by D_L. Since the set of regions is countable, and we can assign at least two descriptions to each region, we get a set of assignments with the same cardinality as $\mathbb{R}$ (also see 9.3).

This does not change if we identify a region that can sustain intelligent life, whose description is fixed, and we require that the other regions' descriptions are compatible with life in the fixed region.

This means that the set of descriptions mentioned in the axiom, D_L, has the cardinality of $\mathbb{R}$. See section 7.3 for another take on this issue.

Also see, e.g., (Manson & Thrush, 2003), which suggests that something similar might be happening in our universe.

6 Valid Options for Our Universe

This section will try to develop the axioms above in order to find out what is reasonable to believe about the issues presented in section 4.

We will focus mostly on what happens when our universe is not designed since in this case it is easier to make predictions about our universe and to falsify them, but we will also take a look at created universes.

R. G. Swinburne (2003, Section "Why a world with human bodies is unlikely if there is no God") comes sort of close to the argument presented here, but while R. G. Swinburne argues that human bodies are unlikely, I am arguing that, in the context of all possible universes that could have human-like beings, our universe is extremely unlikely unless a Designer intended it, the most obvious reason for that being that the Designer wanted to design for human beings. Since the existence of human bodies is not directly related to the subject of this paper, I will not discuss that section more than it is strictly needed.

Note that R. G. Swinburne says that individual sets of laws have non-zero probability while I'm claiming that their probability is 0. It seems to me that R. G. Swinburne implicitly assumes that such a set has a finite number of laws, while I am explicitly removing that constraint, so both can be right within their contexts.

6.1 Peculiar Descriptions and Meta-universes

Let us assume denote our universe's description by $OURD$. Let us take D_L be the set of descriptions from axiom 5, with $OURD$ belonging to D_L. Let us also consider that, perhaps, our universe is contained in a meta-universe, which is, perhaps, contained in a meta-meta-universe, and so on. Since these meta-universes included in other meta-universes resemble ordinals, we will label them with ordinal numbers, starting with 0 for the first meta-universe.

For the purpose of this paper, a meta-universe is something that contains our universe and which influences, one way or another, which universes could exist and in which quantity. This influence will be represented as a probability distribution over universe descriptions.

In the following we will ignore that there could be meta-universe without a corresponding probability distribution, since they are handled by axiom 2.

As an example, our universe could be one of the many universes in a meta-universe that would allow only the existence of universes having the same laws as ours, but with different fundamental constants. The probability distribution mentioned above would be zero for all plausible universes with different laws, and for the universes that have the same set of laws, it would be equivalent to a probability distribution over the fundamental constants.

Alternately, the probability over universe descriptions could be a subjective one, i.e. it can measure what we believe about what can exist. The text below is usually written with the objective meaning in mind, but most conclusions should also work for subjective probabilities.

There is a difference between what universes exist, and in which proportion, and what universes could exist, which may seem important. However, for this argument, we will ignore this distinction, focusing on only on the probability distribution (or the lack of) that describes how the descriptions for the existing universes were generated.

As an example, let us take a look at what happens if only one universe exists. If there is absolutely no reason for it existing and the other ones not existing, then no meta-universe which includes it exists. If, somehow, this universe is contained in a meta-universe which enforces that only one universe exists, the probability distribution will assign 1 to this universe's description, and 0 to everything else. If other universes could exists, but, somehow, they don't, we will get a probability distribution specifying what could exist, and we would be talking about a meta-universe in which only one universe was generated.

As mentioned at the beginning of this section, we are associating meta-universes with ordinals. Let us consider the case when these universe ordinals stop at some point, i.e. when there are ordinals which do not correspond to any meta-universe. If the smallest ordinal not associated with a universe is countable, let us denote it by α. If the smallest such ordinal is not countable, or if there is a universe for each ordinal, then let α be the lowest uncountable ordinal. α will be an upper limit for any ordinal that we will consider in the following.

If $\alpha = 0$, then there is no meta-universe containing ours, so there is no probability distribution over universe descriptions, which means that $OURD$ is generic (axiom 2).

If $\alpha > 0$ and the probability distribution over universe descriptions given by the first meta-universe is continuous, then the probability of all peculiar descriptions is 0, so the only reasonable conclusion is that our universe has a generic approximate description (axiom 1).

This result should probably be good enough, and if you agree with this, please skip to section 6.2.

However, one could also wonder what happens when this probability distribution is not continuous. If the discontinuities are generic, then we can easily show that, with probability 1, our universe's description is generic.

However, if they are not generic, then we can ask ourselves whether we have a probability description for these discontinuities. Since the meta-universe containing our universe may be contained in a meta-meta-universe, maybe the meta-meta-universe can provide such a distribution. If the meta-meta-universe probability distribution is continuous, then the discontinuities are generic. If not, then we have to ask ourselves if a meta-meta-meta-universe can give us more information.

If the meta-meta-meta-universe contains a probability distribution that tells us what is the probability that a given number is a discontinuity for both the meta-universe and the meta-meta-universe, then, if this distribution is continuous (or there is no such distribution), the discontinuities are generic. If not, we can go to the next meta level.

Let β be ordinal where this process stops (if any such ordinal exists) i.e. let β the least ordinal lower than α for which one of the following happens: either there is no meta-universe corresponding to it (and no probability distribution), or the probability for our universe's description is 0. Note that β may be infinite.

If β exists, let us note that we can consider that we made a countable number of observations on various probability distributions, one for each ordinal/meta-universe, and that each time we observed our universe's description. Since one of them was made on a probability distribution where either there is no probability distribution, or the probability of $OURD$ is 0, then $OURD$ is generic (axioms 2 and 1).

If such a $\beta < \alpha$ does not exist, then for any ordinal $\delta < \alpha$ there is a probability distributions which has $OURD$ as a discontinuity.

If α is countable, then, similar to the above reasoning, $OURD$ is generic.

This means that, if our universe is not designed, we have two options that might be reasonable:

- our universe's description, $OURD$, is generic
- $OURD$ is peculiar, α is uncountable and, for all $\delta < \alpha$, the corresponding probability distribution exists and, according to it, $OURD$'s probability is greater than 0.

In other words, in order to claim that $OURD$ is peculiar, one needs to postulate the existence of an uncountable chain of meta-universes, all of them favouring a peculiar $OURD$, which, by default, is prohibitively unlikely for any of them. But that's not all, since, although the current argument does not work anymore when the chain of meta-universes becomes uncountable, intuitively the peculiarness problem still remains: why would it suddenly become reasonable to make only peculiar observations if we make enough of them? A few, maybe, but all of them? Normally, when we make observations with a continuous probability distribution, which is the default, we expect to observe only generic elements as long as we make a countable number of observation, and only when the observation set becomes uncountable we expect to, perhaps, also observe some peculiar elements.

From now on, I will assume that the possible objection in the preceding paragraph is unreasonable, which means that, practically speaking, either $OURD$ is generic or our universe is designed.

Since we don't know the limits of our universe, and how similar the entire universe is to the part we can observe, let us consider next what happens for universe regions.

6.2 Peculiar Descriptions for Universe Regions

Let us consider all generalized cuboids whose corners' coordinates are rational numbers or $+\infty$ or $-\infty$. From axiom 3, their set is countable.

Let us examine whet happens when the set of descriptions for a cuboid that are compatible with life in that cuboid (and, perhaps, in the universe around it) has the same cardinal as $\mathbb{R}$: we can apply the same argument as in section 6.1. The main difference would be the way we assign ordinals: the region would correspond to ordinal 0, our universe would correspond to 1, the meta-universe would correspond to 2, and so on. I.e. we would treat the region as it would be an universe and our universe as being its meta-universe.

Then, since the set of all the generalized cuboids mentioned above is countable, all of their descriptions should be generic.

However, we usually believe that we can have finite approximate descriptions for the observable part of our universe, or, at least, for a large part of if (axiom 4). This means that the only options that have a chance of being plausible are that our universe is designed, or that the set of descriptions compatible with life for a finite cuboid has a cardinality smaller than $\mathbb{R}$.

Let us focus on the latter case above. Note that, from section 6.1, our universe has a generic approximate description. But we assumed that the region around us has a finite approximate description. If we were to extend it to the entire universe, we would find a peculiar description for our universe, which is a contradiction. This means that there is at least one region in our universe which has a different description.

Let A be the set of possible approximate descriptions for a cuboid of size, say, $1\ second \times meter^3$. As argued in the paragraph above, unless our approximation level is extremely coarse, A will have multiple elements. We will assume that we are working with a reasonable approximation level. For any possible description a in the set A, let $P(a)$ be the probability of encountering a cuboid with a as its description in our universe.

If A is finite, we should, by default, pick the uniform probability distribution on A, assigning equal probabilities to all elements of A. However, A can have an infinite cardinal, so we have to consider more general probabilities.

In any case, since, as mentioned above, A has multiple elements, we can't reasonably expect to have one element with probability 1. Then let $p_1 < 1$ be the probability of the description that we use for the cuboids around us. The probability of observing n non-overlapping cuboids with this description without observing any other description is[2] p_1^n.

Even if p_1 is very close to 1, p_1^n converges quickly to 0. As an example, if $p_1 = 0.9$ then observing $n = 70$ consecutive cuboids with the same description is enough to make p_1^n go below one to one thousand odds, $n = 140$ is enough to go below one to one million, $n = 210$ goes below one to one billion.

Then, if we compare the non-design hypothesis with another one non-zero probability, almost always the consistency of a small spacetime region around us is enough to make the non-design hypothesis unlikely enough to disregard[3].

Next, let us see hat happens if we assume the design hypothesis.

6.3 Design Probability

Let us examine the hypothesis that our universe is designed. Since, in this case, the way our world works would based on the designer's intent, it is no longer obvious that, say, continuous probability distributions should be the default. On the other hand, do we actually have better options?

How large is the the probability of having a large consistent time-space region when a designer is involved? Can we repeat the previous argument to draw the same conclusion, that the probability of observing our universe is vanishingly small?

We can separate this probability in two: the probability that a designer would want rational beings[4], denoted by p_r, and the probability that the uni-

[2] This assumes independence between the cuboids, but, given that our universe's description is a generic one, chosen among all possible ones without a Creator biasing it, this is a reasonable assumption.

[3] Let ND be the hypothesis that our universe is not designed, D be the hypothesis that it is designed, $OURD$ be our universe's description, our be the description for the region of space around us. Note that $P(our) \geq P(OURD)$ since our occurs in any universe with $OURD$, but may also occur in others. Also, assuming that $OURD$ has a non-zero probability in the design case, then $P(our) \geq P(OURD) = P(OURD|D) \cdot P(D) > 0$. From Bayes' rule, $P(D|our) = \frac{P(our|D) \cdot P(D)}{P(our)} > 0$. Similarly, $P(ND|our) = \frac{P(our|ND) \cdot P(ND)}{P(our)} = \frac{P(ND) \cdot p_1^n}{P(our)}$. To compare the two we have to compare $P(our|D) \cdot P(D)$ with $P(ND) \cdot p_1^n$.

[4] A designer could want a consistent universe region with a finite description without wanting intelligent beings, but this possibility is not analyzed in this paper

verse region containing those intelligent beings is consistent given that it was designed for them.

I think it's safe to assume that the latter probability is positive, and, probably, fairly high. See as an example this quote from R. G. Swinburne (2003), which argues that, if God exists, there is a fairly good chance that humans can understand their universe:

> So, in order to have significant freedom and responsibility, humans need at any time to be situated in a "space" in which there is a region of basic control and perception, and a wider region into which we can extend our perception and control by learning which of our basic actions and perceptions have which more distant effects and causes when we are stationary, and by learning which of our basic actions cause movement into which part of the wider region. If we are to learn which of our basic actions done where have which more distant effects (including which ones move us into which parts of the wider region), and which distant events will have which basically perceptible effects, the spatial world must be governed by laws of nature. For only if there are such regularities will there be recipes for changing things and recipes for extending knowledge that creatures can learn and utilize. So humans need a spatial location in a law governed universe in which to exercise their capacities, and so there is an argument from our being thus situated to God.

With that in mind, let us compare the probability that we observe consistency under the "design for intelligent beings", "design without intelligent beings in mind" and "no design" hypotheses.

If the probability that a designer would want rational beings, p_r, is 0, then we can't differentiate the design and non-design cases, and the the probability of observing our world's consistency is the same for both.

However, if p_r is non-zero, then the the probability of observing our world's consistency becomes larger in the design case, since p_r is providing some ammount of support for that hypothesis.

Oversimplifying a bit, and assuming that the probability of having consistency is large if the universe was designed for intelligent beings, the ammount of support would be roughly equal to the ratio between p_r and the probability of observing our world's consistency in the non-design case.

Since observing consistency without design has an extremely small probability, having almost any non-zero probability for "a designer of worlds would want rational beings" would provide overhelming support for the design hypothesis. As an example, a probability of 1 in one billion billion would easily be large enough.

From a natural theology point of view, one may argue for a higher design probability through analogy with our own intents: we would be interested in intelligent beings, it seems to us that intelligent beings would be interested in other intelligent beings, so maybe the Designer would be interested in creating other intelligent beings.

How much is this analogy worth? It's hard to tell, but, on the other hand, our options are:

- Our universe is not designed, and the probability of observing the consistency that we see around us is vanishingly small.
- Our universe is designed, but the above analogy does not work and peculiar descriptions still have a zero probability. Again, the probability of observing the consistency that we see around us would be vanishingly small.
- The probability that the above analogy works is not vanishingly small, and we are observing the consistency that we would expect from a universe designed for intelligent beings.

For completeness, the first two options are valid only under the additional assumption mentioned earlier: for a given approximation level, in a universe that has intelligent life, the set of possible approximate descriptions for a finite cuboid must have a lower cardinality than $\mathbb{R}$.

It seems to me that the only reasonable option is that the analogy is not as bad as some people may think it is.

7 Objections and Clarifications

This section includes various possbile objections to this argument. Since the fine-tuning argument addresses the same problem, and it's also using a probabilistic argument (though in a completely different way), some of the objections below are similar to the fine-tuning ones, and it may be helpful to compare the two approaches.

For the fine-tuning argument see, e.g., (Friederich, 2018). For objections to the fine-tuning argument that are relevant here, see, e.g., (Manson & Thrush, 2003; Manson, 2009; McGrew, McGrew, & Vestrup, 2001; Narveson, 2003; Sober, 2009). For possible answers see, e.g. Leslie (2003), R. G. Swinburne (2003), Monton (2006), Kotzen (2012).

7.1 Observation Selection Effect and Multiple Universes

In this paper, we only look at universes that contain intelligent life, and that restricts the set of possible universe descriptions. Even more, we don't see a universe as it is, instead we see it through the eyes of the intelligent beings inhabiting it. It can be argued that, in a non-created universe, beings might be intelligent only if their intelligence is useful to them. But this likely means that those beings live in a timespace region which seems consistent from their point of view, so maybe it's not that unlikely to see consistency around us.

However, let us look more carefully at how much consistency we would expect. There are possible universes with consistent regions in which intelligent life can exist, and whose consistency ends abruptly at some random time. There are possible universes whose consistent regions are strictly the size needed for allowing intelligent life, and there are possible universes with large consistent regions.

Let us assume that our existence means that some consistency is required. Is there any non-required consistency around us, consistency which is due to chance? To be more precise, how large is the time-space region whose consistency is required? Well, perhaps at very distant times in the past, the consistent region included the entire observable universe, but, right now, there is no reason to require full consisency outside of Earth's orbit. Even more, since non-consistency only means observable non-consistency, and does not require something wildly different, it's likely that we don't even need full consistency inside of Earth's orbit.

However, as far as we know, our solar system is consistent, our galaxy is fairly consistent, and distant galaxies are also fairly consistent.

This paper argues that, since we do observe much more consistency than we would expect, design is the right explanation.

However, there is a possible objection related to this: if multiple universes exist, perhaps all possible ones, there will be some beings living in the implausibly consistent ones.

While this is correct, the probability of an intelligent being living in a fully consistent universe is still 0. In virtually all universes the nonhomogeneity of the universe would be easily observable, meaning that, for the relatively few intelligent beings living in the other universes, as long as the design hypothesis has some plausibility, it would be unreasonable to think that their universe is not designed (assuming that the argument presented in this paper is correct).

Also, observing a large consistent region of space is still very unlikely among all existing universes.

7.2 Unknown Designer's Intent

From a natural theology point of view, one can't know what the Designer wanted (e.g. one can't know that a universe designer would want to create a universe having life) (Sober, 2009; Narveson, 2003), so, by default, any argument showing that the probability of our universe is small if it's not designed would also show that the probability is small even if it is designed. To fix this, one would need an independent way to show that the Designer wanted the universe to have life (Sober, 2008). In order for this paper's argument to work, one would actually need a weaker requirement, i.e. it would be enough to have a non-zero probability to the hypothesis that the Designer wanted the universe to have intelligent life.

This is an interesting objection for this approach. If we can't assign a non-zero probability to this hypothesis, then this argument can't explain that the universe around us seems to be consistent. Instead, it would just point out how extremely odd it is to observe it. This would force us to take it as a brute fact that the universe has a finite description (or we could just live in a bubble of consistency whose probability becomes inimaginably smaller with each passing second, which would be fairly similar). In this case we would be in the right setting for the argument presented in R. Swinburne, 2004, which uses this consistency as an argument for God's existence.

Returning to the objection, let us note that the design and non-design cases have some differences.

One difference is that, for an intelligent Designer/Creator, what is being created corresponds, one way or another, to a purpose. To be consistent with the approach used in this paper, let us assume that the set of possible purpose has the same cardinality as the set of real numbers, $\mathbb{R}$. We should expect a number of universes with distinct descriptions for each purpose, but we should not expect that each possible purpose is an actual purpose. Given that intelligence and design both imply choice, I think that we should assume that, for each actual purpose, there were many similar purposes that were discarded when the actual one was choosen.

So then we could ask ourselves two things: how large is the set of actual purposes, and whether one of them involves creating intelligent beings.

I think it's hard to estimate the likelyhood of any of these, but, for the purpose of this paper, it's probably good enough to show that, by default, we should assign a non-zero probability to both.

First, let us note that, for real numbers, the similarity of two numbers is usually linked to the distance between them (i.e. it's the inverse of the distance). So, if we are given a non-zero distance (i.e. a non-infinite similarity) between real numbers, and we want to pick a subset of real numbers such that no two numbers are too similar, then that subset is at most countable.

It is not obvious that similarity between purposes works in the same way as similarity between real numbers, but I think that the above shows that there's a non-zero chance that if no two actual purposes are too similar, then their set is at most countable.

Given a countable set of options, then we can (and should) assign by default a non-zero probability to all options.

Second, let us take a look at the probability that a Designer would want to create intelligent beings.

If the Designer's purposes have a structure similar to the one used for descriptions, then we would expect to have a countable number of basic purposes that can be expressed in words (similar to the axioms used for descriptions), and each actual purpose would contain a subset of these basic purposes. This may be too restrictive, but it may be a decent approximation.

In this case, what matters is the probability of the "intelligent beings" basic purpose to show up in one of the chosen purposes. But, since we have a countable number of basic purposes, for each of them we should assign a non-zero probability that it is part of a chosen purpose.

So, if we have a non-zero probability that the set of purposes is countable, a non-zero probability that one of them involves creating intelligent beings, and each of the purposes in the set has a default non-zero probability, then the overall probability of having a universe created for intelligent beings is non-zero.

If the Designer intends to create intelligent beings, then the Designer will create a universe for them. As argued in R. Swinburne, 2004, it's very likely that this universe is, indeed, consistent on a large scale. But if there is a non-zero probability of observing consistency around us, then there is a non-zero probability of observing an universe like ours.

7.3 Few Universes Exist

Another possible objection is: we used a meta-universe definition that's too restrictive. What would happen if, say, we had a meta-universe that, instead of providing a probability distribution over universes, simply restricts the possible universe descriptions to a set with cardinality less than $\mathbb{R}$, preventing us from applying axiom 1?

Note that the current argument works with conceivable universes, so the issue is not which universes exist, but which could exist.

As a parenthesis, let us note that using conceivable universes makes sense. If we are trying to make a prediction about how a non-designed universe would look like, we would have to imagine that we are outside any universe, and we are about to observe one that is (say) non-designed. What could we say about it? Not much, maybe that it makes logical sense (i.e. that it can be modelled mathematically), maybe it has something close to time and space, but, besides that, anything is possible. I.e. we could say that it can be any of the conceivable universes, or, perhaps, that all conceivable universes are valid candidates.

That being said, the first possible answer to this section's issue is that the main argument of this paper can be reused here, perhaps by slightly changing this paper's axioms. If we assume the continuum hypothesis, the set of descriptions mentioned above can be a countable one or a finite one, both of which are peculiar, so we can safely assume that this is not the case. Without the continuum hypothesis, the cardinality of the set of descriptions can also be somewhere between countable and the cardinality of the real numbers. If so, then, by default, we should still use continuous probability distributions over the set of descriptions. If that does not seem good enough, then the set of discontinuities would still be at most countable and, following the same line of reasoning as the main argument, the discontinuities should be generic. Peculiar non-continuous events would require some explanation, which can be provided by a Designer.

This means that, if we replace the set of real numbers by a set whose cardinality is above countable, but at most that of the real numbers in axioms 1, 2 and 5, the argument should still work.

The second possible answer is that, if there is no Designer, we should actually expect the set of possible descriptions for the meta-universe to have the same cardinal as $\mathbb{R}$, and that's true even if we ask these descriptions to be "different enough", i.e. different even though we work with approximations. To see why, let us consider the following argument:

The set of all conceivable descriptions has the same cardinality as the set of real numbers, $\mathbb{R}$. This means that the set of all sets of conceivable descriptions has the same cardinality as the power set of $\mathbb{R}$, i.e. $2^{\mathbb{R}}$.

A claim about what is possible specifies a particular set of descriptions, and any set of descriptions can be such a claim, which means that the set of possible claims has the same cardinal as the power set of $\mathbb{R}$, $2^{\mathbb{R}}$. The cardinal numbers remain the same even if we take into account only claims according to which our universe is possible.

Unlike in the main topic discussed in this paper, this time it's clear that only one of these claims can be true.

Let us take all the claims in which the set of possible descriptions is less than $\mathbb{R}$. The set of these claims (let us denote it by S) has a cardinality which is less than $2^{\mathbb{R}}$. The set of claims not in S (let us denote it by T) has the same cardinality as $2^{\mathbb{R}}$. But T is infinitely larger[5] than S, which means that it's

[5] S is infinite, which means that, if we take a distinct copy of S for each element of S, and we put them together, we get a set that still has the same cardinal as S (i.e. $S \times S$ has the same cardinal as S). In order to get the same cardinal as T we need to take even more distinct copies of S, which means that T is infinitely larger than S.

How many copies? We actually need one for each element in T, and no lower cardinal would reasonable to assume that S has probability 0. If so, then, with probability 1, the set of possible descriptions has the same cardinality as $\mathbb{R}$.

We should be able to get the same results if, instead of looking at the set of all distinct descriptions, we group descriptions based on their similarity and we work with the set of these groups of descriptions. As argued in section 7.4, there are $\mathbb{R}$ conceivable descriptions which are different enough, so each of these would be in a different group, which means that there are $\mathbb{R}$ groups of conceivable descriptions. Using the same reasoning as above, with probability 1, the set of groups of conceivable descriptions which are possible according to the meta-universe rules has the same cardinality as $\mathbb{R}$. From these groups, we can extract a similar cardinal of possible descriptions, which should be different enough since we grouped them based on similarity.

7.4 Not Enough Descriptions

When working without approximations, there are $\mathbb{R}$ possible descriptions that are essentially different. However, this is less obvious when working with approximations, especially since that means we can probably have incompatible descriptions for the same universe.

Let us assume (as an axiom) that, for a given level of approximation L, we can find a finite cuboid size for which there is a finite set of (full-precision) descriptions, let us call it D_f, which cannot all be approximated with a single finite approximate description. In other words, if we have a set of cuboids whose descriptions include the D_f set, we can't find a single finite approximate description that works for all of them within the level of approximation L.

In the following, the difference between cuboid descriptions and universe descriptions is not always explicit, but can be inferred from the context.

Here is an incomplete example that shows why the above axiom is reasonable. First, let us assume that we have a set of "primary measurements" that we can do for our model, such that any other measurement that we could do can be computed from those. Then let us take two descriptions, the first saying "the value of each primary measurement is 0", and the second saying "the value of each primary measurement oscillates quickly between 0 and something large enough to be easily detected within the approximation level".

Then, let us consider a hypothetical infinite universe whose timespace is R^n. Let us divide it in cuboids of the size mentioned above and let us assign the incompatible descriptions in the set mentioned above to these cuboids in a roughly even way. The set of cuboids is countable, so there are $\mathbb{R}$ ways of assigning these descriptions.

Let us pick an assignment and let us pick a timespace point very close to the time end of a space-time cuboid boundary. Let us take that entire point's past and let us try to make a prediction based on that. First, based on just that past, we can't predict the type of the next cuboid, since there will be many possible assignments that have the same past, but in which the next cuboid will be different from the current assignment.

Still, a universe description must predict the future from the past, and the future depends only on the next cuboid type. Then, the universe description must allow predicting the next cuboid type, so it must include, implicitly or

do.

explicitly, a function predicting the next cuboid from a cuboid's past and its context (i.e. position). Now we must find out how many such functions we need in order to describe all possible assignments.

To make things simpler, let us assume that the number of dimensions, n, is 1 and that we have only two cuboid types. It should be obvious that the reminder of this example can be generalized to any number of dimensions and to any number of cuboid types greater than 2.

Having $n = 1$, means that we have a one-to-one correspondence between integers and cuboids, so let us identify a cuboid with its number. Let us also identify the two cuboid types with $\{0, 1\}$ and let us represent a universe as a function from the set of integers, $\mathbb{Z}$, to $\{0, 1\}$, identifying, for each cuboid, its type.

We can represent the information available for predicting the type of a future cuboid by a pair between the cuboid and its past, let us denote it by '(x, f)' where x is the cuboid's number, 0 or 1, and f, which represents x's past, is a function from the set of natural numbers to $\{0, 1\}$.

However, we are not trying to predict just one cuboid's future, we are trying to predict the future of all cuboids. Any way of making these predictions identifies with a function from the past of cuboids to their future, i.e. from (x, f) pairs to $\{0, 1\}$.

Let us find out how many distinct functions we need. To make things simple, let us consider only universes which, for negative integers, have only cuboids of type 0.

The past of cuboid 0 is perfectly identical in all these universes, so, in order to predict the type of the cuboid with index 0, we need two distinct prediction functions. In order to predict the types of cuboids 0 and 1 we need four distinct prediction functions. In general, in order to predict the types of all cuboids between 0 and n we need 2^{n+1} distinct prediction functions. And, in order to predict the type of all cuboids which are greater or equal to 0, we need $2^{\mathbb{N}}$, i.e. the power set of the natural numbers, i.e. $\mathbb{R}$ prediction functions.

As mentioned previously, intelligent beings may need some consistent space-time around them. Even if that's the case, fixing the cuboid type for a finite chunk of an infinite universe does not change the the cardinal for the set of functions mentioned above.

7.5 Multiple Universes Based on the Same Laws

As mentioned above, our universe could be included in a meta-universe in which, colloquially speaking, all universes have the same laws, but different fundamental constants, and no other universes are allowed. Of course, using this paper's definitions, the fundamental constants would be part of a universe's description.

When using approximate universe descriptions, since our universe seems to be based on laws that are continuous in these fundamental constants, the set constants that are "different enough" should be at most countable.

Since no other universe is allowed, and the set of allowed approximate descriptions is countable, the meta-universe's probability has to be non-zero for some of these descriptions. In other words, the meta-universe probability has some peculiar discontinuities, which, as mentioned above, is implausible.

7.6 Complicated universe

Above we have argued that we can divide the universe region around us in small identical pieces, all of which have finite approximate descriptions, and a description that works for one of them works for all of them.

Of course, in practice, we can only work with finite descriptions when modelling our universe. Fortunately, quantum mechanics, together with relativity (although they may be incompatible), seem to fit our universe almost perfectly. Even Newtonian physics seems to be pretty good. However, we can ask ourselves whether our timespace region is actually much more complex, perhaps infinitely complex, but we didn't manage to notice that yet.

This is possible, but the essential part of this paper's argument relies on the timespace around us being consistent. As long as it is conceivable that a small region of space around us can have two incompatible descriptions, then it becomes unreasonable to assign probability 1 to any description, which means that we are observing something having a really low probability.

8 Conclusion

We have the following reasonable possibilities:

1. The Designer intent analogy makes some sense, our universe is designed, and the universe is, roughly, as homogenous as we would expect it to be (at least around us).

2. The analogy does not make sense, we don't know whether our universe is designed or not, our universe is not homogenous, but we observe some extremely unlikely homogeneity around us.

The most reasonable option seems to be that our universe is designed.

9 Mathematical Background

This section presents the mathematical results used in this paper. They are usually presented without proofs or references, but most introductory courses on the topic of each subsection should cover them.

This section also contains precise definitions for some of the concepts that are specific to this paper (namely "generic" and "peculiar").

9.1 Probabilities

Let A be a set with the same cardinality as the set of real numbers, $\mathbb{R}$. Let F be the set of all mathematical predicates of one variable over A that can be written as a finite formula. If f is such a predicate then let A_f be the subset of A where f is true, i.e. $A_f = \{a \in A \mid f(a) \text{ is true}\}$. Given a probability distribution over A, we define $P(f) = P(A_f)$.

Let F_0 be the set of predicates in F whose probability is 0 for all continuous probability distributions over A, i.e.

$$F_0 = \{f \in F \mid P(A_f) = 0 \text{ for all continuous probability distributions over } A\}.$$

As an example, any predicate f which is true for a finite subset of elements, i.e. A_f is finite, would belong to F_0.

Let us identify by A_0 the set of elements of A for which at least one predicate of F_0 is true, i.e.

$$A_0 = \{a \in A \mid \exists f \in F_0 \text{ with } f(a) \text{ true}\}.$$

Let us also denote by F_1 and A_1 the complements of F_0 and A_0, respectively.

Let P be a continuous probability distribution.

Since F is countable, F_0 must be at most countable. Then we can compute $P(F_0)$ as the sum of the probability of F_0's elements, so obviously, $P(F_0) = 0$, which means that the probability of its complement, $P(F_1)$, is 1. From the way we defined $P(f)$, $P(F_0) = P(A_0)$, so $P(A_0)$ must be 0 and $P(A_1)$ must be 1, which justifies the indexes used for these.

Let us say that an element $a \in A_1$ is **generic** and an element $a \in A_0$ is **peculiar**. Then we could rewrite the equalities above to $P(x \text{ is generic}) = 1$ and $P(x \text{ is peculiar}) = 0$.

When the probability for a subset of A is 0, $P(E) = 0$, where $E \subset A$, it does not mean that observing an element of E is logically impossible, it means that, if we make a set of (independent) observations that is at most countable of elements of A, we have no chance at all of observing an element of E.

Let us now consider a different issue. Let P be a probability over the set of real numbers, $\mathbb{R}$ (or any infinite uncountable set). Let A be the set of real numbers with non-zero probability. We will show that A is at most countable.

Indeed, let us take all subsets of A that are at most countable. The probability of any such subset is between 0 and 1, so we can take S to be the supremum of the probabilty of these subsets, and S will also be between 0 and 1.

There is a sequence (C_n) of at most countable sets such that each C_n includes all the preceding ones, and $P(C_n)$ converges to the supremum, S. But the union of all C_n (let us call it C) is also countable, and its probability is larger or equal than the probability of any C_n, so the union's probability is exactly S.

If there would be a number x of non-zero probability that is not in the union C, then the union of x and C would still be countable, and would have a larger probability than S, which is a contradiction. This means that C, which is countable, contains all numbers with non-zero probability.

9.2 Ordinals

Ordinals are generalizations of natural numbers. Natural numbers can be defined by identifying each natural number with the set of natural numbers less than it. Then 0 is identified with the empty set $\emptyset$, 1 with $\{0\} = \{\emptyset\}$, 2 with $\{0, 1\} = \{\emptyset, \{\emptyset\}\}$, and so on.

Each of these natural numbers is an ordinal. To define the smallest ordinal that is not a natural number, denoted by ω, we will use the same rule: let ω to be the set of ordinals smaller than it, i.e the set of natural numbers, $\omega = \{0, 1, 2, \ldots\}$.

Of course, the next ordinal, called $\omega + 1$, will be the set $\{0, 1, 2, \ldots \omega\}$ and the next one, $\omega + 2$, will be $\{0, 1, 2, \ldots \omega, \omega + 1\}$. We can continue and, in the same way, define $\omega \cdot 2 = \omega + \omega$ to be the ordinal that comes after all ordinals of the form $\omega + n$ where n is a finite ordinal (i.e a natural number).

Then we can define $\omega \cdot 3$, $\omega \cdot 4$ and so on, and we can take $\omega \cdot \omega$ to be the ordinal that comes after all the ones defined by using the above rules.

Let us note that ω is countable, and that all the ordinals mentioned above that come after it are also countable. By using the same kind of reasoning as above we can produce other countable ordinals like ω^ω (from ordinals like $\omega \cdot \omega \cdot \cdots \cdot \omega$) and ϵ_0 (from $\omega^{\omega^{\cdots^{\omega}}}$).

After going through many similar processes, at some point we obtain the smallest uncountable ordinal, ω_1, which is the set of all countable ordinals.

Let us note that some ordinals, like all the finite ones except 0, and like $\omega + 1$, can be obtained from the previous one by using a succesor relation, i.e. $succesor(\alpha) = \alpha \cup \{\alpha\}$. All ordinals have a succesor, but not all are succesors, some, like ω and $\omega \cdot 2$ can be defined only as the set of all smaller ordinals. The former are called "successor ordinals", the later are called "limit ordinals". Note that 0 is a limit ordinal.

9.3 Cardinals

The set of natural numbers $\mathbb{N}$, the set of integers $\mathbb{Z}$ and the set of rational numbers $\mathbb{Q}$ are countable.

The set of real numbers $\mathbb{R}$ has a larger cardinality, the same cardinality as the powerset of a countable set. Another way to put this is that if we have a countable set, and, for each element, we can choose one of two options, and we take all ways of choosing options for the entire countable set, then we get a set with the same cardinal as the set of real numbers[6]. That is, the set of all functions

$$f : \mathbb{N} \longrightarrow \{1, 2\}$$

has the same cardinality as $\mathbb{R}$.

If we have a set with the same cardinality as the set of real numbers, and we remove a countable set, the resulting set still has the same cardinality as the set of real numbers. As an example, $\mathbb{R}$, $\mathbb{R} \setminus \mathbb{N}$, $\mathbb{R} \setminus \mathbb{Z}$ and $\mathbb{R} \setminus \mathbb{Q}$ all have the same cardinality. In general, if A and B are infinite sets and B's cardinality is lower than A, then removing B from A does not change its cardinal, i.e. $A \setminus B$ has the same cardinal as A.

If A has an infinite cardinal, then the product set $A \times A$ has the same cardinality as A.

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[6] This is also valid for more than two options, as long as the cardinality of their set is at most countable.

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