CentraleSupélec

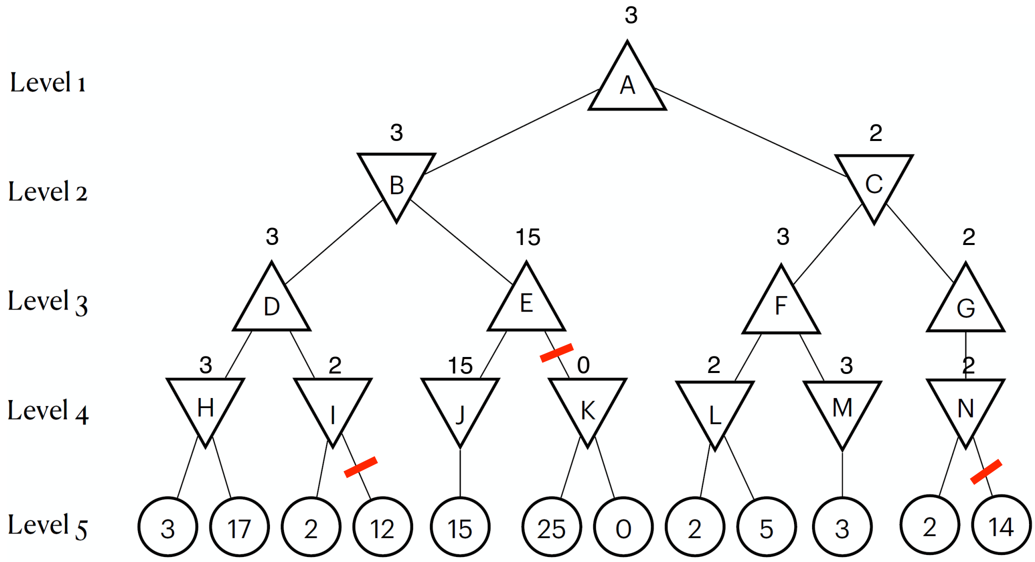
2nd Year
Elective Course

2022 – 2023

Artificial Intelligence – Tutorial Session 3
TS 3: Multiagent

1 Minimax - Expectimax

Consider the following game tree, where the triangles pointing down are minimizers, the triangles pointing up are maximizers, and the round leaf nodes are terminal states:



(a) What is the value of the root node if we use the minimax algorithm?

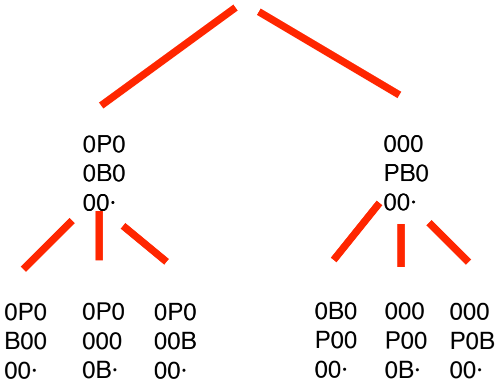
(b) What is the value of the root node if we use the alpha-beta pruning algorithm? What nodes will be pruned?

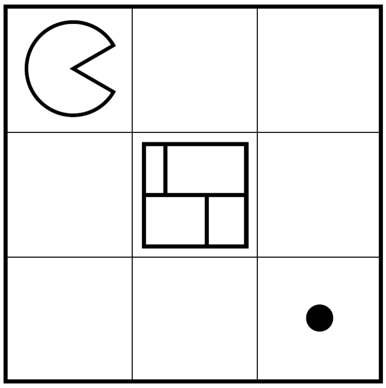
(c) Replace the minimizers by chance node and run the expectimax algorithm. What is the new value of the root node?

2 Surrealist Pacman

In the game of Surrealist Pacman, Pacman plays against a moving wall. On Pacman's turn, Pacman must move in one of the four cardinal directions, and must move into an unoccupied square. On the wall's turn, the wall must move in one of the four cardinal directions, and must move into an unoccupied square. The wall cannot move into a dot-containing square. Staying still is not allowed by either player. Pacman's score is always equal to the number of dots he has eaten.

The first game begins in the configuration shown below. Pacman moves first.



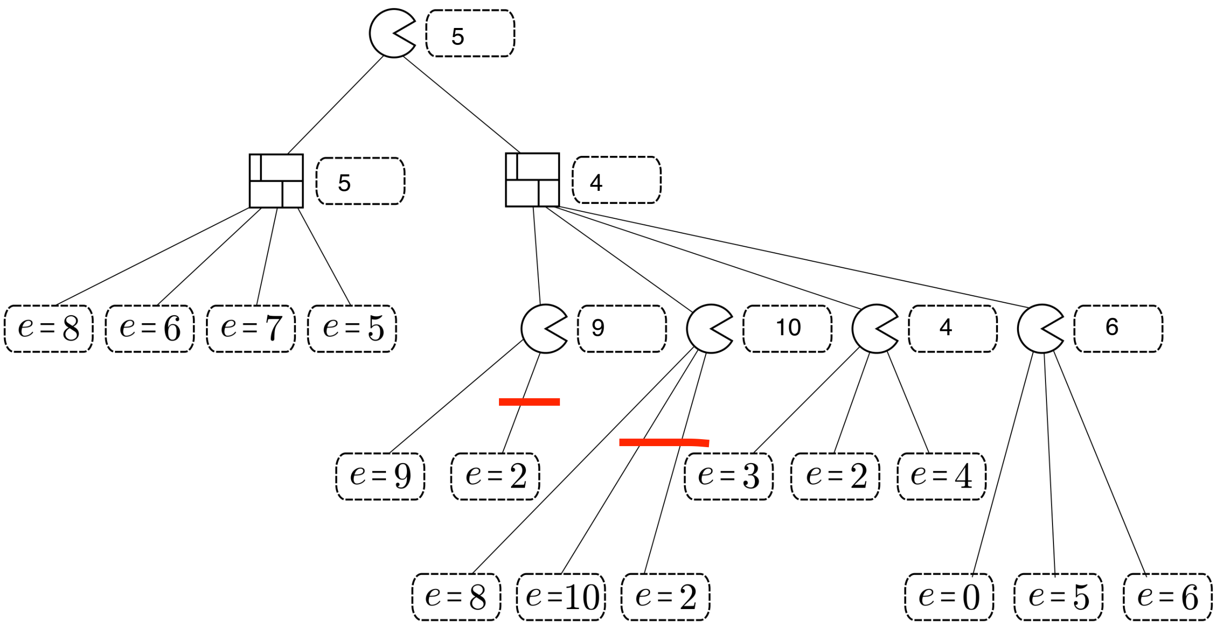


(a) Draw a game tree with one move for each player. Draw only the legal moves.

(b) According to the depth-limited game tree you drew above what is the value of the game? Use Pacman's score as your evaluation function. No dot eaten thus -1

(c) If we were to consider a game tree with ten moves for each player (rather than just one), what would be the value of the game as computed by minimax? Possible to eat a dot thus 10-10=0

A second game is played on a more complicated board. A partial game tree is drawn, and leaf nodes have been scored using an (unknown) evaluation function e.

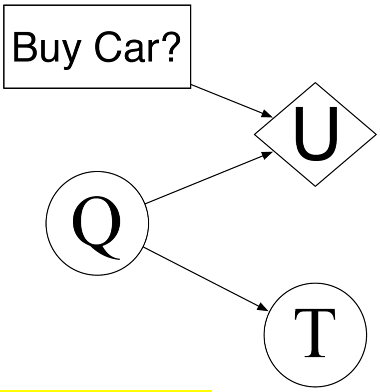


(d) In the dashed boxes, fill in the values of all internal nodes using the minimax algorithm.

(e) Cross off any nodes that are not evaluated when using alpha-beta pruning (assuming the standard left-to-right traversal of the tree).

3 Decision Networks and VPI

A used car buyer can decide to carry out various tests with various costs (e.g., kick the tires, take the car to a qualified mechanic) and then, depending on the outcome of the tests, decide which car to buy. We will assume that the buyer is deciding whether to buy car c and that there is time to carry out at most one test which costs 50€ and which can help to figure out the quality of the car. A car can be in good shape (of good quality $Q = +q$) or in bad shape (of bad quality $Q = \neg q$), and the test might help to indicate what shape the car is in. There are only two outcomes for the test T : pass (T = pass) or fail (T = fail). Car c costs 1500€, and its market value is 2000€ if it is in good shape; if not, 700€ in repairs will be needed to make it in good shape. The buyer's estimate is that c has 70% chance of being in good shape. The Decision Network is shown below.



E(buy) = 0,7*(2000-1500) + 0,3*(2000-1500-700) = 290€

(a) Calculate the expected net gain from buying car c, given no test.

(b) Tests can be described by the probability that the car will pass or fail the test given that the car is in good or bad shape. We have the following information:

P(+q | pass) = P(pass | +q) * P(+q) / P(pass)
= 0,9 * 0,7 / 0,69 = 0,91

P(pass) = P(pass | +q) * P(+q) + P(pass | -q) * P(-q)
= 0,9*0,7 + 0,2*0,3 = 0,69
P(fail) = 1 - P(pass) = 0,31

$$P(T = \text{pass} \mid Q = +q) = 0.9$$
$$P(T = \text{pass} \mid Q = \neg q) = 0.2$$

P(+q | fail) = 0,2 * 0,3 / 0,31 = 0,22
P(-q | pass) = 0,2 * 0,7 / 0,69 = 0,2
P(-q | fail) = 0,77

Calculate the probability that the car will pass (or fail) its test, and then the probability that it is in good (or bad) shape given each possible test outcome.

E(buy | pass) = 437 et E(buy | fail) = -46

(c) Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

(d) Calculate the value of (perfect) information of the test. Should the buyer pay for a test?

VPI(T) = P(pass)*E(buy | pass) + P(fail)*E(buy | fail) - E(buy) = 11,53
Le test est à 50€ donc on perd de l'argent à le faire

4 VPI

You are the latest contestant on Monty Hall's game show, which has undergone a few changes over the years. In the game, there are n closed doors: behind one door is a car ($U(\text{car}) = 1000$), while the other $n-1$ doors each have a goat behind them ($U(\text{goat}) = 10$). You are permitted to open exactly one door and claim the prize behind it.

You begin by choosing a door uniformly at random.

(a) What is your expected utility? E(gain) = (1000*1 + 10*(n-1))/n = 10 + 990/n

(b) After you choose a door but before you open it, Monty offers to open k other doors, each of which are guaranteed to have a goat behind it. E(new gain) = (1000*1 + 10*(n-k-1))/(n-k) = 10 + 990/(n-k) on change car plus grand

If you accept this offer, should you keep your original choice of a door, or switch to a new door?

(c) What is the value of the information that Monty is offering you?

(d) Monty is changing is offer! VPI = 990(1/(n-k) - 1/n) = 990*k / n(n-k)

After you choose your initial door, you are given the offer to choose any other door and open this second door. If you do, after you see what is inside the other door, you may switch your initial choice (to the newly opened door) or keep your initial choice.

What is the value of this new offer? VPI = 990/n

(e) Monty is generalizing his offer: you can pay d^3€ to open d doors as in the previous part. (Assume that $U(x€) = x$.) You may now switch your choice to any of the open doors (or keep your initial choice). What is the largest value of d for which it would be rational to accept the offer?

E(a) = 990 d/n - d^3, on pait au maximum quand on ne gagne plus rien donc
d = sqrt(900/n)