Advanced Dynamics of Mechanical System

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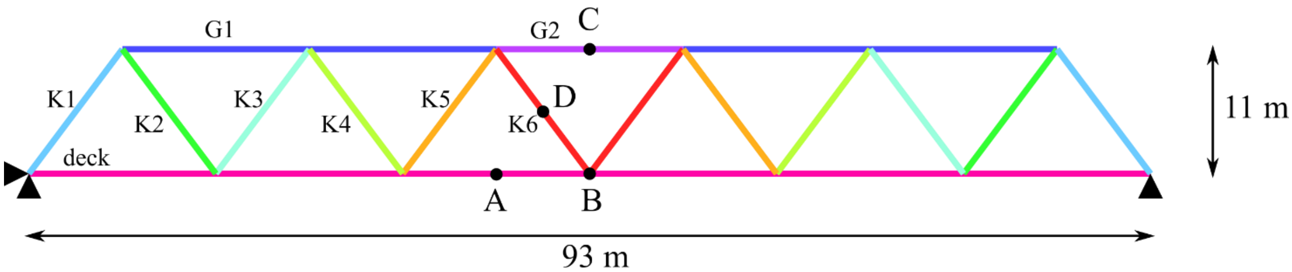
Assignment 2

In-plane Finite Element Model of a Truss Bridge



	Colour	m [kg/m]	EA [N]	EJ [Nm^2]
Deck Element	Pink	2.36E+03	1.59E+10	1.36E+09
Diag member K1	Light blue	4.42E+02	1.18E+10	2.80E+08
Diag member K2	Green	2.56E+02	6.85E+09	1.36E+08
Diag member K3	Cyan	2.75E+02	7.35E+09	1.51E+08
Diag member K4	Light green	1.75E+02	4.68E+09	4.48E+07
Diag member K5	Orange	1.45E+02	3.89E+09	6.13E+07
Diag member K6	Red	94.2	2.52E+09	1.63E+07
Upper chord G1	Blue	5.34E+02	1.43E+10	1.21E+09
Upper chord G2	Purple	5.93E+02	1.59E+10	1.36E+09

For the structure reported in the figure above, whose elements properties are reported in the table, work out the following items and prepare a short report summarizing the solution obtained.

1. Define a FE model of the structure in the 0 – 7 Hz frequency range (check that for each element k, $\frac{\omega_{1,k}}{\Omega_{max}} \geq 1.5$).
2. Compute the structure's natural frequencies and vibration modes up to the 6th one. Plot the mode shapes with the indication of the associated natural frequencies.

3. Compute the damping matrix of the system according to the proportional damping assumption: $[C] = \alpha[M] + \beta[K]$, where α and β have to be fixed such that they result in the following damping ratios for the first two vibration modes: ξ_1 = 1%, ξ_2 = 0.75%.

4. Calculate the frequency response functions which relate the input force (vertical force in node **A**) to the following outputs.

   a) Vertical displacement and vertical acceleration of nodes **A, B** and **C**. Plot the corresponding magnitude and phase diagrams.

   b) Shear force, bending moment and axial force evaluated in nodes **C** and **D**. Plot the corresponding magnitude and phase diagrams.

   c) Constraint forces. Plot the corresponding magnitude and phase diagrams.

   Assume the input force to vary in the 0 – 7 Hz frequency range and set the frequency resolution to 0.01 Hz.

5. Using the modal superposition approach and considering the structure's first three modes, calculate the frequency response functions which relate the same input force of question 4 (vertical force applied in node **A**) to the vertical displacement and vertical acceleration of nodes **A, B** and **C**. Plot the corresponding magnitude and phase diagrams superimposed to those obtained in item 4a. Point out the differences and comment the results.

6. Design a TMD to be installed on the structure so as to reduce by at least 15% the amplitude of the vertical vibration of node **A** in correspondence with the resonance peak of the first mode of vibration, for the same excitation condition of question 4. The increase of the structure mass must be limited to maximum 2% while the damping ratio of the TMD alone must not exceed 30%. Provide a plot comparing the FRF results corresponding to the structure with and without the TMD.

7. Compute the static response of the structure due to:

   a) the deck weight only;

   b) the weight of the entire structure.

   Plot the deformed shape of the structure compared to the undeformed configuration and compute the value of the maximum deflection.

8. Assuming that the bridge is being crossed by a long train, compute the critical velocities for the first two modes considering a value for the distance d of 27.5 m.