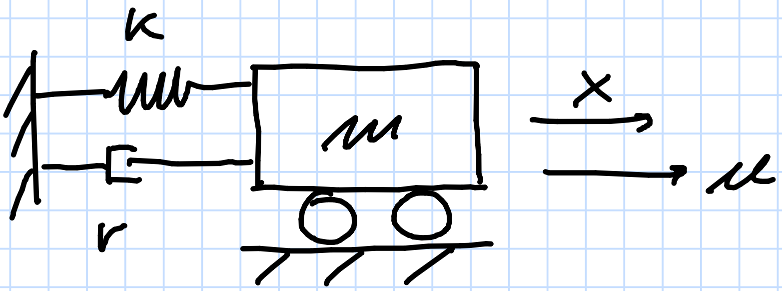


$$\begin{Bmatrix} \dot{x} \\ x \end{Bmatrix}$$

$$m\ddot{x} + r\dot{x} + kx = u$$

$$\begin{cases} \ddot{x} = -\dfrac{r}{m}\dot{x} - \dfrac{k}{m}x + \dfrac{1}{m}u \\ \dot{x} = \dot{x} \end{cases}$$

$$\begin{Bmatrix} \ddot{x} \\ \dot{x} \end{Bmatrix} = \underbrace{\begin{bmatrix} -\dfrac{r}{m} & -\dfrac{k}{m} \\ 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} \dot{x} \\ x \end{Bmatrix}}_{\underline{X}} + \underbrace{\begin{bmatrix} \dfrac{1}{m} \\ 0 \end{bmatrix}}_{B} \underbrace{u}_{\underline{u}}$$

$$\det(A - \lambda I) = 0 \Rightarrow \mathrm{eig}(A)$$

$$\Rightarrow \left(-\frac{r}{m} - \lambda\right)(-\lambda) + \frac{k}{m} = 0$$

$$\lambda^2 + \frac{r}{m}\lambda + \frac{k}{m} = 0 \qquad \lambda = \frac{-\dfrac{r}{m} \pm \sqrt{\left(\dfrac{r}{m}\right)^2 - 4\dfrac{k}{m}}}{2}$$

$$\lambda = -\frac{r}{2m} \pm \sqrt{\left(\frac{r}{2m}\right)^2 - \frac{k}{m}} =$$

$$\begin{cases} \omega_0 = \sqrt{\dfrac{k}{m}} & \text{NATURAL FREQUENCY} \\ h = \dfrac{r}{2m\omega_0} & \text{DAMPING FACTOR} \\ \alpha = h\omega_0 \end{cases}$$

$$= -\alpha \pm \omega_0 \sqrt{h^2 - 1}$$

- $h > 1$ OVERDAMPED
- $h = 1$ CRITICAL
- $h < 1$ UNDERDAMPED

$$m\ddot{x} + r\dot{x} + kx = u$$

$$mXs^2 + rXs + kX = U \Rightarrow \frac{X}{U} = \frac{1}{ms^2 + rs + k}$$

$$(i\Omega = s) \qquad = \frac{1}{-m\Omega^2 + i\Omega r + k}$$

P - CONTROL

$$m\ddot{x} + r\dot{x} + kx = K_p \left(x_{REF} - x \right)$$

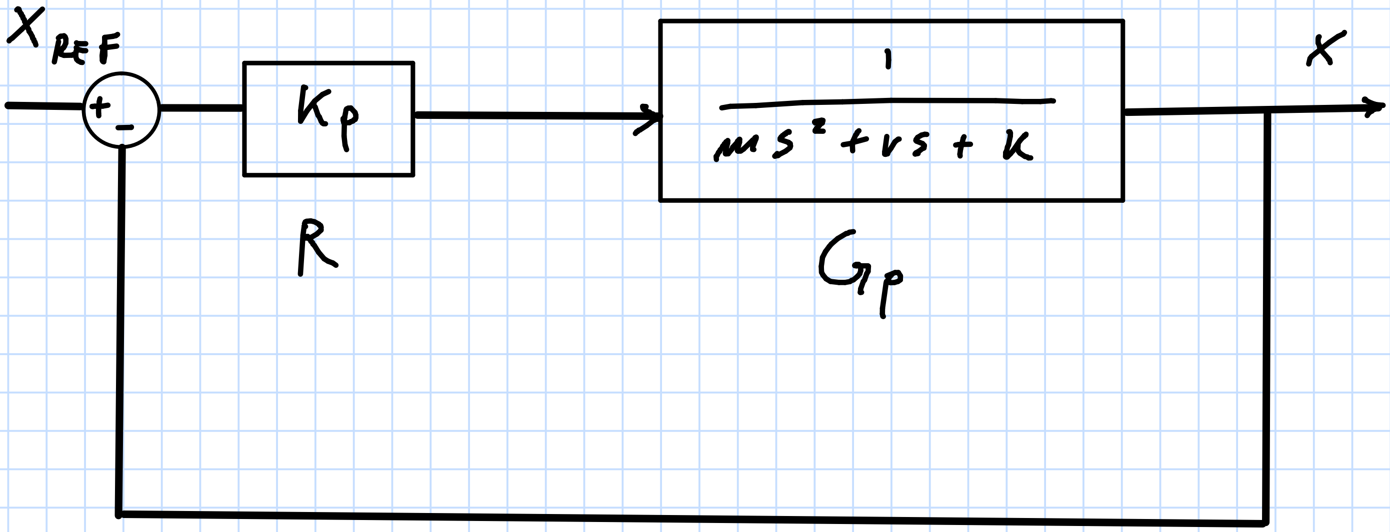
$$m\ddot{x} + r\dot{x} + \left(k + K_p \right) x = K_p x_{REF}$$

$$m\ddot{x} + r\dot{x} + k^* x = K_p x_{REF}$$

$$\lambda = -\frac{r}{2m} \pm \sqrt{\left(\frac{r}{2m}\right)^2 - \frac{k^*}{m}}$$

$$\begin{cases} \omega_c = \sqrt{\dfrac{k^*}{m}} \\ h_c = \dfrac{r}{2m\omega_c} \\ \alpha_c = h_c \omega_c \end{cases}$$

$$\frac{x}{X_{REF}} = \frac{K_p}{ms^2 + rs + \left(K + K_p \right)} \xrightarrow{ss} \frac{K_p}{K + K_p}$$


X_REF
+
-
Kp
R
1
ms² + rs + k
Gp
x