Master of Science Course in Mechanical Engineering
Politecnico di Milano - Bovisa Campus

CONTROL OF MECHANICAL SYSTEMS

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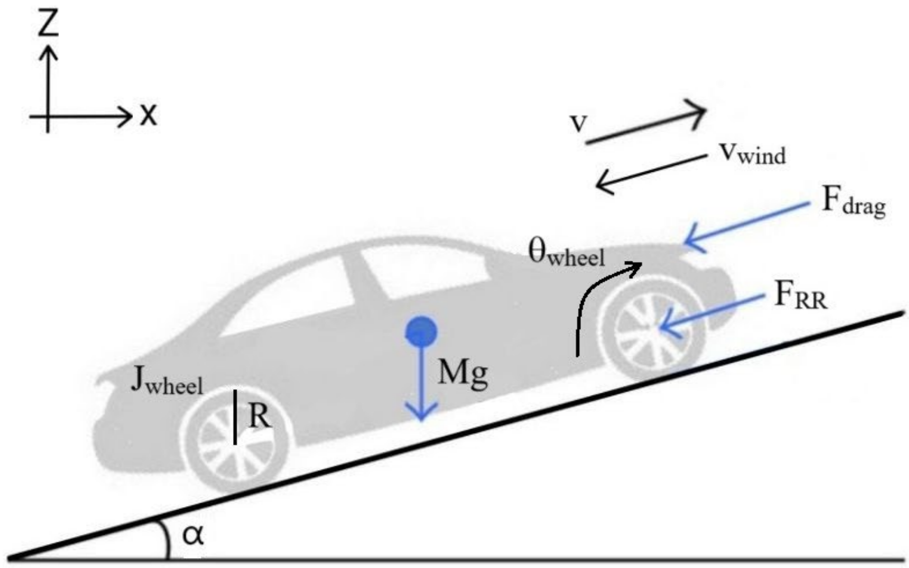
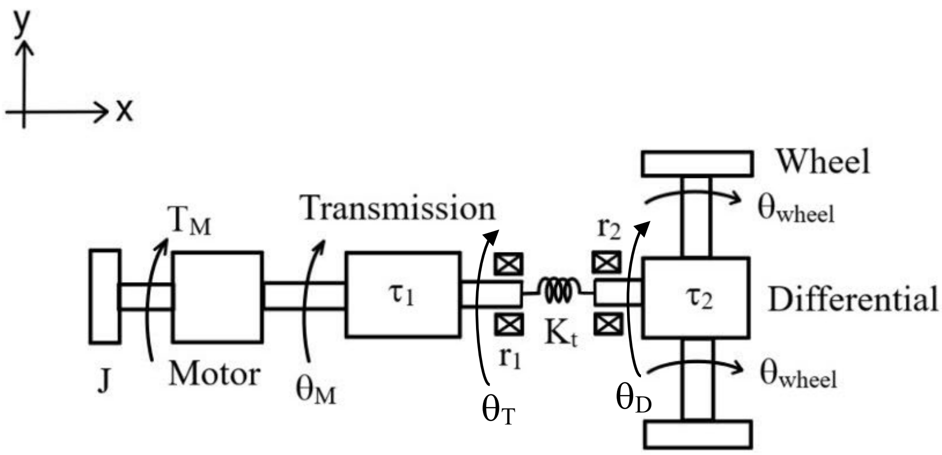
CRUISE CONTROL

The figure below shows the longitudinal dynamics of a vehicle. Aim of the exercise is to implement a regulator to **control the speed of the vehicle v by means of a torque T_M**.

The forces acting on the vehicle system are:

- The control torque T_M
- the drag force F_{drag}
- the rolling resistance F_{RR}
- the damping of the drive shaft ($\dot{\theta}_T r_1$ *and* $\dot{\theta}_D r_2$)
- the inertia at rotation of the tires ($4J_{wheel}\ddot{\theta}_{wheel}$)
- the inertia at rotation of the motor ($J_{motor}\ddot{\theta}_M$)
- the inertia at motion of the vehicle ($M\dot{v}$)
- the gravitational force (Mg)





PART 1

The drag force can be calculated according to the following formula:

$$F_{drag} = \frac{1}{2}\rho_{air}A_{front}C_x v_{relative}^2$$

Assuming that $v > v_{wind}$ and that the direction of these vectors is the one depicted in the picture, it holds that:

$$v_{relative} = v + v_{wind}$$

The rolling resistance force can be expressed as:

$$F_{RR} = C_{RR}Mg\cos(\alpha)(1 + k_{RR}v)$$

System data

vehicle mass	M	1500	[kg]
Moment of inertia of the wheel	J_{wheel}	1	[kgm^2]
Moment of inertia of the motor	J_M	0.05	[kgm^2]
Slope of the road	α	5	[°]
wheel radius	R	0.35	[m]
Damping coefficient 1	r_1	0.005	[Nms/rad]
Damping coefficient 1	r_2	0.005	[Nms/rad]
Torsional stiffness of the drive shaft	k_t	∞	[Nm/rad]
Transmission ratio	τ_1	3	-
Differential ratio	τ_2	1	-
Air density	ρ_{air}	1.225	[kg/m^3]
Front surface of the vehicle	A_{front}	2.2	[m^2]
Drag coefficient	C_x	0.3	-
Rolling coefficients	C_{RR}	0.01	-
Rolling coefficients	k_{RR}	0.0002	[s/m]

Consider the system without the actuator, **under the assumption of a non-deformable drive shaft ($k_t = \infty$),** the system has 1 d.o.f.: the speed of the vehicle v. Assuming small oscillations around the steady state condition: $v_0 = 15\frac{m}{s}$ *and* $v_{wind,0} = 3.5\frac{m}{s}$:

1. Write the linearized equation of motion of the system putting in evidence the generalized mass (m*), the generalized stiffness (k*) and the generalized damping (c*)

2. Calculate the torque $T_{M,0}$ at steady state condition.

3. Write the expression of the transfer functions
$$G(s) = \frac{v}{T_M} \; and \; D(s) = \frac{v}{v_{wind}}$$
4. Calculate the poles of the transfer function G(s) by using MATLAB.
5. Draw the Bode diagrams of the transfer function G(s) by using MATLAB.

PART 2

Let's suppose that the torque T_M (t) is governed by a PI regulator having transfer function R(s).

6. Check the stability of the feed-back control system considering the different cases that may arise based on the regulator time constant T_i (MatLab commands *margin*, *nyquist*, *rlocus*).

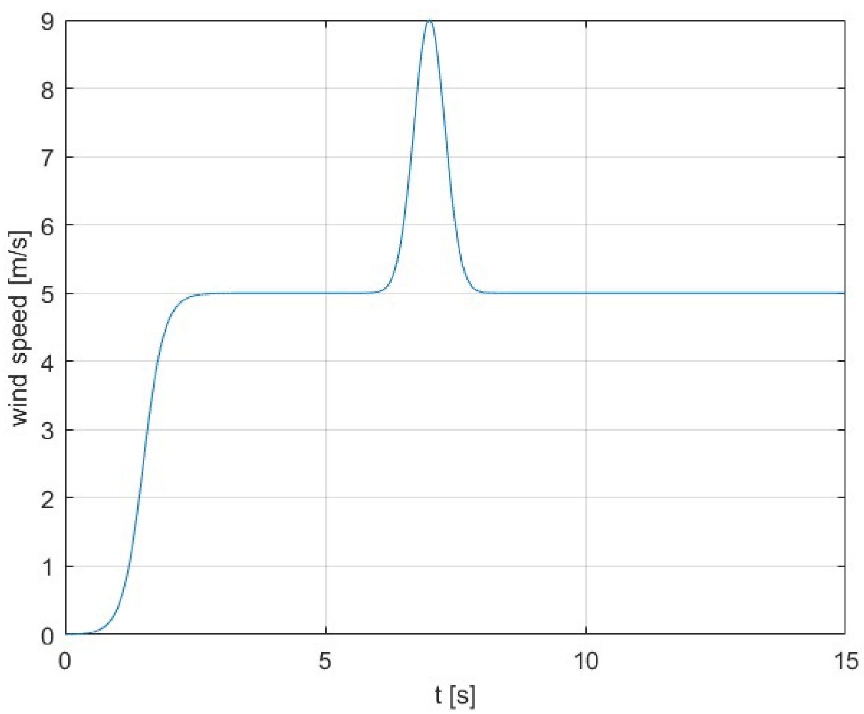
After selecting the time constants of the PI regulator:

7. draw the Bode diagrams of the open loop transfer function H(s) = R(s)G(s), draw the Bode diagrams of the closed loop transfer functions

$$L_D(s) = \frac{v}{v_{wind}}$$

8. analyse the performances of the feedback control system (MatLab commands *bandwidth*, *step*) for different values of the proportional gain k_p.

9. assuming that the reference time history for the wind force is the one shown in the figure below, check the performances of the feed-back control system in the time domain by using Simulink.



$$Speed_{wind}(t) = \frac{5}{1+e^{-5(t-1.5)}} + 4e^{-\frac{(t-7)^2}{2\cdot 0.3^2}}$$