Master of Science Course in Mechanical Engineering
Politecnico di Milano - Bovisa Campus

CONTROL OF MECHANICAL SYSTEMS

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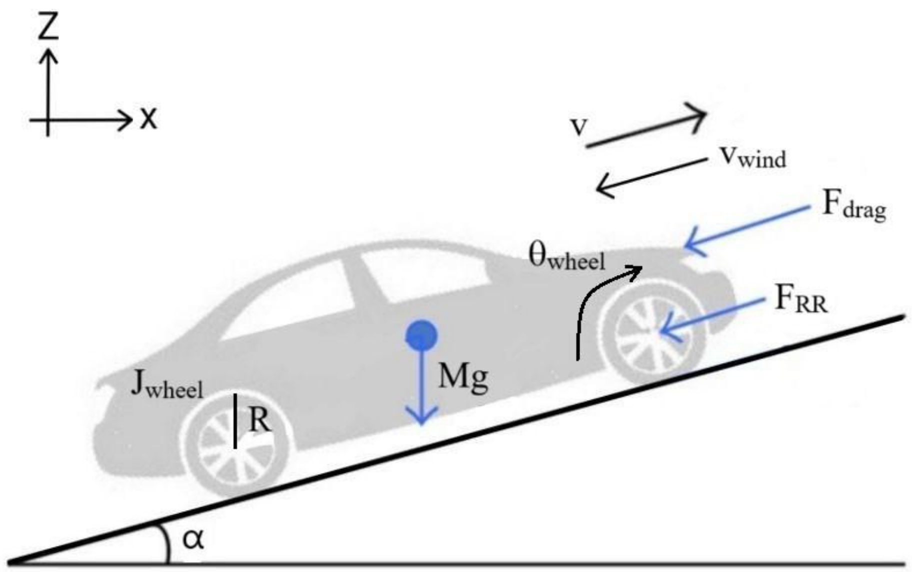
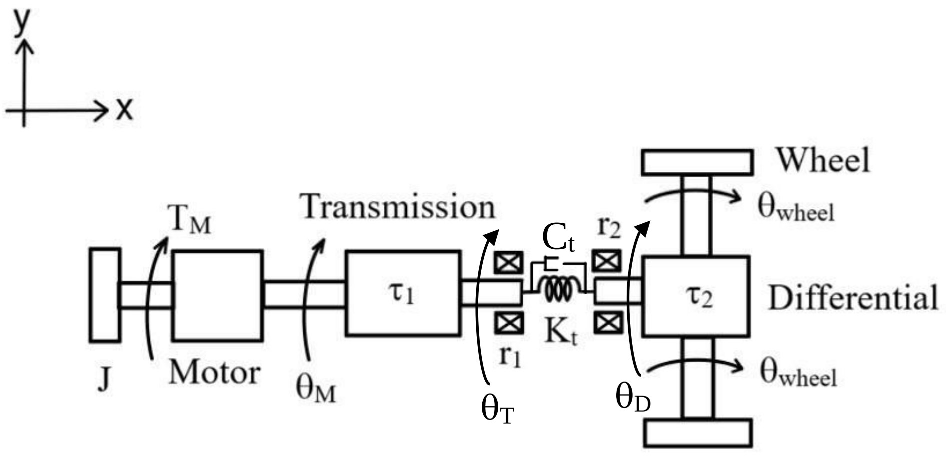
CRUISE CONTROL

The figure below shows the longitudinal dynamics of a vehicle. Aim of the exercise is to implement a regulator to **control the speed of the vehicle v by means of a torque T_M**.

The forces acting on the vehicle system are:

- The control torque T_M
- the drag force F_{drag}
- the rolling resistance F_{RR}
- the damping of the drive shaft ($\dot{\theta}_T r_1$ and $\dot{\theta}_D r_2$)
- the inertia at rotation of the tires ($4J_{wheel}\ddot{\theta}_{wheel}$)
- the inertia at rotation of the motor ($J_{motor}\ddot{\theta}_M$)
- the inertia at motion of the vehicle ($M\dot{v}$)
- the gravitational force (Mg)





PART 1

The drag force can be calculated according to the following formula:

$$F_{drag} = \frac{1}{2}\rho_{air}A_{front}C_x v_{relative}^2$$

Assuming that $v > v_{wind}$ and that the direction of these vectors is the one depicted in the picture, it holds that:

$$v_{relative} = v + v_{wind}$$

The rolling resistance force can be expressed as:

$$F_{RR} = C_{RR}Mgcos(\alpha)(1 + k_{RR}v)$$

System data

vehicle mass	M	1500	[kg]
Moment of inertia of the wheel	J_{wheel}	1	[kgm^2]
Moment of inertia of the motor	J_M	0.05	[kgm^2]
Slope of the road	α	5	[°]
wheel radius	R	0.35	[m]
Damping coefficient 1	r_1	0.005	[Nms/rad]
Damping coefficient 1	r_2	0.005	[Nms/rad]
Torsional stiffness of the drive shaft	k_t	∞	[Nm/rad]
Transmission ratio	τ_1	3	-
Differential ratio	τ_2	1	-
Air density	ρ_{air}	1.225	[kg/m^3]
Front surface of the vehicle	A_{front}	2.2	[m^2]
Drag coefficient	C_x	0.3	-
Rolling coefficients	C_{RR}	0.01	-
Rolling coefficients	k_{RR}	0.0002	[s/m]

Consider the system without the actuator, **under the assumption of a non-deformable drive shaft ($k_t = \infty$)**, the system has 1 d.o.f.: the speed of the vehicle v. Assuming small oscillations around the steady state condition: $v_0 = 15\frac{m}{s}$ *and* $v_{wind,0} = 3.5\frac{m}{s}$:

1. Write the linearized equation of motion of the system putting in evidence the generalized mass (m*), the generalized stiffness (k*) and the generalized damping (c*)

2. Calculate the torque $T_{M,0}$ at steady state condition.

3. Write the expression of the transfer functions (*where Tm represents the torque around steady state condition*)

$$G(s) = \frac{v}{T_M} \; and \; D(s) = \frac{v}{v_{wind}}$$

4. Calculate the poles of the transfer function G(s) by using MATLAB.
5. Draw the Bode diagrams of the transfer function G(s) by using MATLAB.

PART 2

Let's suppose that the torque T_M (t) is governed by a PI regulator having transfer function R(s).

6. Check the stability of the feed-back control system considering the different cases that may arise based on the regulator time constant T_i (MatLab commands *margin*, *nyquist*, *rlocus*).

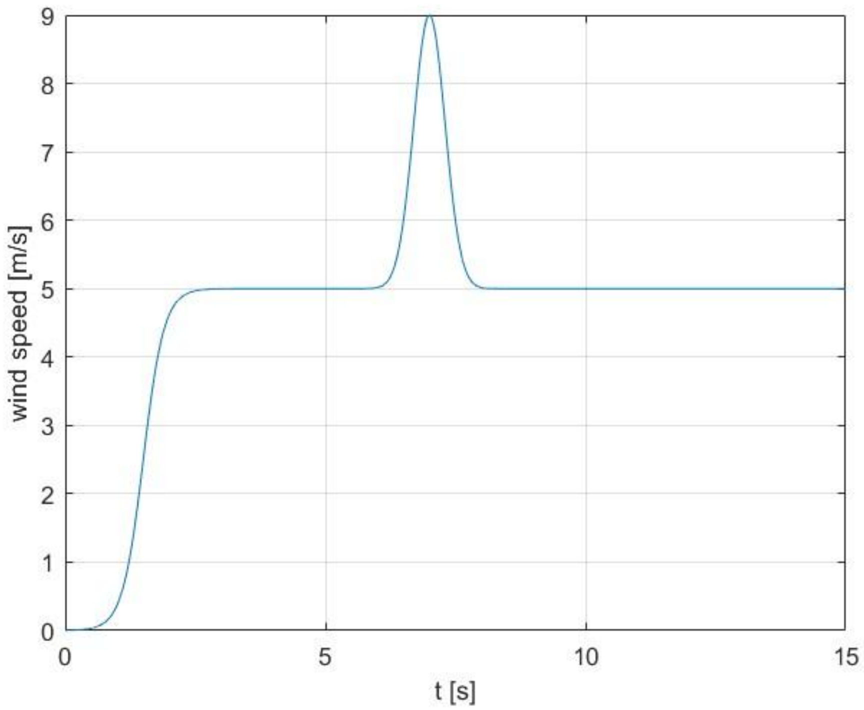
After selecting the time constants of the PI regulator:

7. draw the Bode diagrams of the open loop transfer function H(s) = R(s)G(s)
8. draw the Bode diagrams of the closed loop transfer function

$$L_D(s) = \frac{v}{v_{wind}}$$

9. analyse the performances of the feedback control system (MatLab commands *bandwidth*, *step*) for different values of the proportional gain k_p *(you could assume that v_{ref} = 1 and consider L_{vref} to better analyze the time performance).*

10. assuming that time history for the wind force is the one shown in the figure below, check the performances of the feed-back control system in the time domain by using Simulink.



$$Speed_{wind}(t) = \frac{5}{1 + e^{-5(t-1.5)}} + 4e^{-\frac{(t-7)^2}{2 \cdot 0.3^2}}$$

PART 3

Let's suppose that the control torque T_M is provided by a **permanent magnet DC motor**.

Permanent magnet DC motor data.

Armature resistance	R_a	0.05	[Ω]
Armature inductance	L_a	0.00015	[H]
	K_ϕ	0.8	[Nm/A]

Under the assumption of a non-deformable drive shaft ($k_t = \infty$), the system has 1 d.o.f.: the speed of the vehicle v. Assuming small oscillations around the steady state condition: $v_0 = 15\frac{m}{s}$ *and* $v_{wind,0} = 3.5\frac{m}{s}$:

1. Write the linearized equations of motion of the electro-mechanical system, considering the speed of the vehicle v as the independent coordinate.
2. Calculate the voltage $V_{a,0}$ at steady state condition.
3. Write the expression of the transfer function G(s) between the armature input voltage V_a (consider V_a as the difference from steady state condition) and the speed v.
4. calculate the poles of the transfer function G(s) by using MatLab.
5. draw the Bode diagrams of the transfer function G(s) by using MatLab.

Let's think about controlling the speed v of the wheels by means of the torque $T_M(t)$ governed by a PID regulator acting on the armature voltage $V_a(t)$ and having transfer function R(s).

6. Check the stability of the feed-back control system considering the different cases that may arise on the basis of the regulator constants T_i and T_D (MatLab commands *margin*, *nyquist*, *rlocus*).

After selecting the time constants of the PID regulator

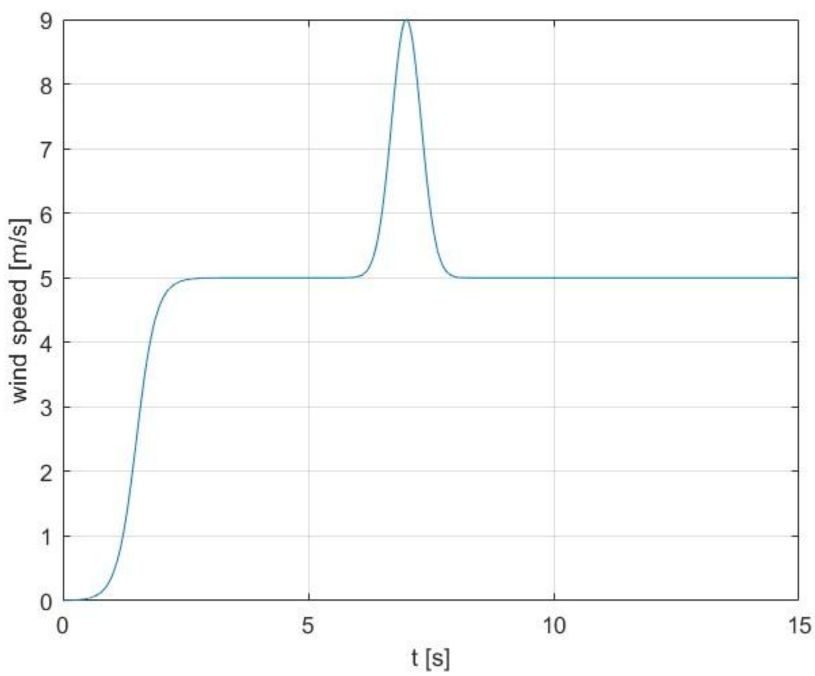
7. draw the Bode diagrams and the polar plot of the loop transfer function H(s) = R(s)G(s)
8. draw the Bode diagrams of the closed loop transfer function

$$(s) = \frac{v}{v_{wind}}$$

9. analyse the performances of the feedback control system (MatLab commands *bandwidth*, *step*) for different values of the proportional gain k_p *(you could assume that v_{ref} = 1 and consider L_{vref} to better analyze the time performance).*

After selecting the proportional and the derivative gains of the PID regulator

10. assuming that the reference time history for the wind force is the one shown in the figure below, check the performances of the feed-back control system in the time domain by using Simulink.



$$Speed_{wind}(t) = \frac{5}{1+e^{-5(t-1.5)}} + 4e^{-\frac{(t-7)^2}{2\cdot 0.3^2}}$$

PART 4

Under the assumption of a deformable drive shaft ($k_t = \frac{400Nm}{rad}, c_t = \frac{4Nms}{rad}$), the system has 2 d.o.f.: the displacement of the vehicle x and the angular displacement of the motor θ_M. **(Consider the system without the actuator)** Assuming small oscillations around the steady state condition: $v_0 = 15\frac{m}{s}$ *and* $v_{wind,0} = 3.5\frac{m}{s}$:

1. write the linearized equations of motion of the mechanical system
2. write the expressions of the transfer functions
    - $G_{11}(s) = \frac{\theta_M(s)}{T_M(s)}$
    - $G_{21}(s) = \frac{v(s)}{T_M(s)}$
3. plot the Bode diagrams of the transfer functions using MatLab.

Let's think about controlling the speed v by means of the torque $T_M(t)$ and let's assume that the torque T_M (t) is governed by a PID regulator having transfer function R(s).

4. Check the stability of the feed-back control system considering the different cases that may arise on the basis of the regulator constant T_D and T_i (MatLab commands *margin*, *nyquist*, *rlocus*).

5. Evaluate the performances of the feed-back control system against the wind force (*pay attention to the TF you need*) considering the different cases that may arise on the basis of the regulator constants T_i, T_D and kp ((*you could assume that* $v_{ref} = 1$ *and consider* L_{vref} *to better analyze the time performance*, MatLab commands *step*, *bandwidth*, *dcgain*).