We first represent the system by 1 dof : v. We need to write every rotational speed as a function of v:

$$\omega_M = \tau_1 \omega_T \; ; \; \omega_T = \omega_D \; ; \; \omega_D = \tau_2 \omega_W \; ; \; \omega_W = \frac{v}{R} \;\Rightarrow\; \omega_T = \omega_D = \tau_2 \frac{v}{R} \; ; \; \omega_M = \tau_1 \tau_2 \frac{v}{R}$$

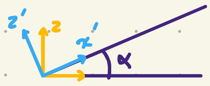
We then can compute the Lagrange equation for $v = \dot{x}$ in order to find the EoM:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{x}}\right) - \frac{\partial E_c}{\partial x} + \frac{\partial D}{\partial \dot{x}} + \frac{\partial V}{\partial x} = Q = \frac{\delta W}{\delta x}$$

- $E_c = \frac{1}{2} J_M \omega_M^2 + 2 J_W \omega_W^2 + \frac{1}{2} M v^2 + \frac{1}{2} \cancel{J_T}^{\,0} \omega_T^2 = \frac{1}{2}\left(J_M \left(\tau_1 \tau_2 \frac{v}{R}\right)^2 + 4 J_W \left(\frac{v}{R}\right)^2 + M v^2\right)$

$$= \frac{1}{2} \underbrace{\left(J_M \left(\frac{\tau_1 \tau_2}{R}\right)^2 + 4 \frac{J_W}{R^2} + M\right)}_{m^*} v^2 = \frac{1}{2} m^* v^2$$

- $D = \frac{1}{2}(r_1 + r_2)\, \dot{\theta}_T^2 = \frac{1}{2} \underbrace{(r_1 + r_2)\left(\frac{\tau_2}{R}\right)^2}_{r^*} v^2 = \frac{1}{2} r^* v^2$
- $V_R = 0$



- $\delta W = \vec{T_M} \cdot \delta \vec{\theta_M} + \vec{F_{DRAG}} \cdot \delta \vec{x}' + \vec{F_{ROLL}} \cdot \delta \vec{x}' - Mg \sin(\alpha) \vec{x}' \cdot \delta \vec{x}'$ and $\delta \vec{\theta_M} = \tau_1 \tau_2 \frac{\delta \vec{x}}{R} + \cancel{\vec{z}}^{\,\vec{0}}$

$$\Rightarrow Q = \frac{\delta W}{\delta x} = T_M \frac{\tau_1 \tau_2}{R} - F_D - F_R - Mg \sin(\alpha)$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{x}}\right) = \frac{d}{dt}\left(m^* \dot{x}\right) = m^* \ddot{x} \; ; \; \frac{\partial E_c}{\partial x} = 0 \; ; \; \frac{\partial D}{\partial \dot{x}} = r^* \dot{x} \; ; \; \frac{\partial V}{\partial x} = 0$$

$\Rightarrow$ EoM: $m^* \ddot{x} + r^* \dot{x} = T_M \frac{\tau_1 \tau_2}{R} - F_D - F_R - Mg \sin(\alpha)$

- $F_R = C_{RR} M g (1 + K_{RR} v)$

we consider that wind is horizontal

- $F_D = \frac{1}{2} \rho_a C_x A v_{rel}^2 \; ; \; v_{rel}^2 = (v - v_{WIND})^2 \Rightarrow F_D(v, v_W) = \frac{1}{2} \rho_a C_x A (v - v_W)^2$ $\quad v^2 - 2 v v_W + v_W^2$

$\longrightarrow$ linearization: $F_D(v, v_W) \simeq F_D(v = v_0, v_W = v_{W0}) + \left.\frac{\partial F_D}{\partial v}\right|_{v_0, v_{W0}} \cdot (v - v_0) + \left.\frac{\partial F_D}{\partial v_W}\right|_{v_0, v_{W0}} \cdot (v_W - v_{W0})$

- By writing $v = v_0 + \delta v \; ; \; v_W = v_{W0} + \delta v_W \;\longrightarrow\; \dot{v} = \delta \dot{v}$:

$$F_D(\delta v, \delta v_W) \simeq \frac{1}{2} \rho_a C_x A \left((v_0 - v_{W0})^2 + 2(v_0 - v_{W0})(\delta v - \delta v_W)\right)$$

$$\Rightarrow m^* \delta \dot{v} + r^*(v_0 + \delta v) = (T_{M0} + T_c) \frac{\tau_1 \tau_2}{R} - \frac{1}{2} \rho_a C_x A \left((v_0 - v_{W0})^2 + 2(v_0 - v_{W0})(\delta v - \delta v_W)\right) - C_{RR} M g (1 + K_{RR}(v_0 + \delta v)) - Mg \sin\alpha$$

$$\Leftrightarrow m^* \delta \dot{v} + \delta v \left(r^* + C_{RR} K_{RR} M g + \rho_a C_x A (v_0 - v_{W0})\right) = (T_{M0} + T_c) \frac{\tau_1 \tau_2}{R} - \frac{1}{2} \rho_a C_x A \left((v_0 - v_{W0})^2 - 2(v_0 - v_{W0}) \delta v_W\right) - C_{RR} M g (1 + K_{RR} v_0) - Mg \sin\alpha - r^* v_0$$

We can compute the steady-state torque T_{M0} by having $\delta v = 0$ and $\delta v_W = 0$:

$$T_{M0} \frac{\tau_1 \tau_2}{R} - \frac{1}{2} \rho_a C_x A (v_0 - v_{W0})^2 - C_{RR} M g (1 + K_{RR} v_0) - Mg \sin\alpha - r^* v_0 = 0$$

$$\Leftrightarrow T_{M0} = \frac{R}{\tau_1 \tau_2} \left(\frac{1}{2} \rho_a C_x A (v_0 - v_{W0})^2 + M g \left(C_{RR}(1 + K_{RR} v_0) + \sin\alpha\right) + r^* v_0\right)$$

We can then compute numerically this torque for $v_0 = 15\, m \cdot s^{-1}$ and $v_{W0} = 3{,}5\, m \cdot s^{-1}$:

$T_{M0} = 173{,}2237$ or $183{,}1278$ N.m depending on the direction of the wind.

$$m^* \delta \dot{v} + \delta v \overbrace{\left(r^* + C_{RR} K_{RR} M g + \rho_a C_x A (v_0 - v_{W0})\right)}^{\tilde{r}} = T_c \frac{\tau_1 \tau_2}{R} + \rho_a C_x A (v_0 - v_{W0}) \delta v_W$$

$$G = \frac{1}{m^* s^2 + \tilde{r} s} \frac{\tau_1 \tau_2}{R} \quad ??$$