FX Puzzles Replication

UIP and Backus-Smith puzzles

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1 Backus-Smith puzzle

1.1 Definition

The international risk-sharing condition that relative consumption across countries should be strongly positively correlated with the real exchange rate (implying high relative consumption in periods of low relative prices) is sharply violated in the data, with a mildly negative correlation and a markedly lower volatility of relative consumption.

1.2 Methodology

The original Backus-Smith (1993) paper tests whether the following condition holds under the assumption of complete markets:

$$\gamma \Delta(c_t - c_t^*) = \Delta q_t \tag{1}$$

where:

- γ is the relative risk aversion parameter.
- c_t is the log of domestic consumption at time t.
- c_t^* is the log of foreign consumption at time t.
- q_t is the log of the real exchange rate at time t.

Using the special case of the constant relative risk aversion utility function in log form (i.e. $\gamma \to 1$), the equation becomes: $\Delta(c_t - c_t^*) = \Delta q_t$.

Backus-Smith (1993) use the GDP deflator to calculate the real FX rate. Here, I calculate it as follows:

$$RER = \frac{E \times P^*}{P} \tag{2}$$

where:

- E is the nominal exchange rate (the price of foreign currency in terms of domestic currency, e.g., USD/EUR),
- P^* is the foreign price level, here the GDP deflator of the foreign country,
- P is the domestic price level, here the GDP deflator of the domestic country.

1.3 Data

- **Consumption and GDP deflators**: Quarterly, seasonally adjusted, total private consumption expenditure and deflator, in constant prices of 2015, for 1990:01-2023:10. Source: OECD Quarterly National Accounts comparative tables.
- **Nominal exchange rates**: Quarterly average nominal exchange rates, 19990:01-2024:01. These are expressed in units of foreign currency per USD. Source: FRED.

1.4 Results

Below I show a number of rolling plots that replicate the findings in Backus-Smith (1993), using a selection of the U.S.-OECD country pairs the authors used in their original paper. First, I plot the time series of relative consumption ($\Delta(c_t - c_t^*)$) and the real FX (Δq_t) and their respective 4-quarter rolling standard deviation. Second, I plot the 4-quarter rolling correlation of both growth rates.

1.4.1 Euro Area 19 - United States

Note: Using German consumption as a proxy for Euro Area 19 consumption.

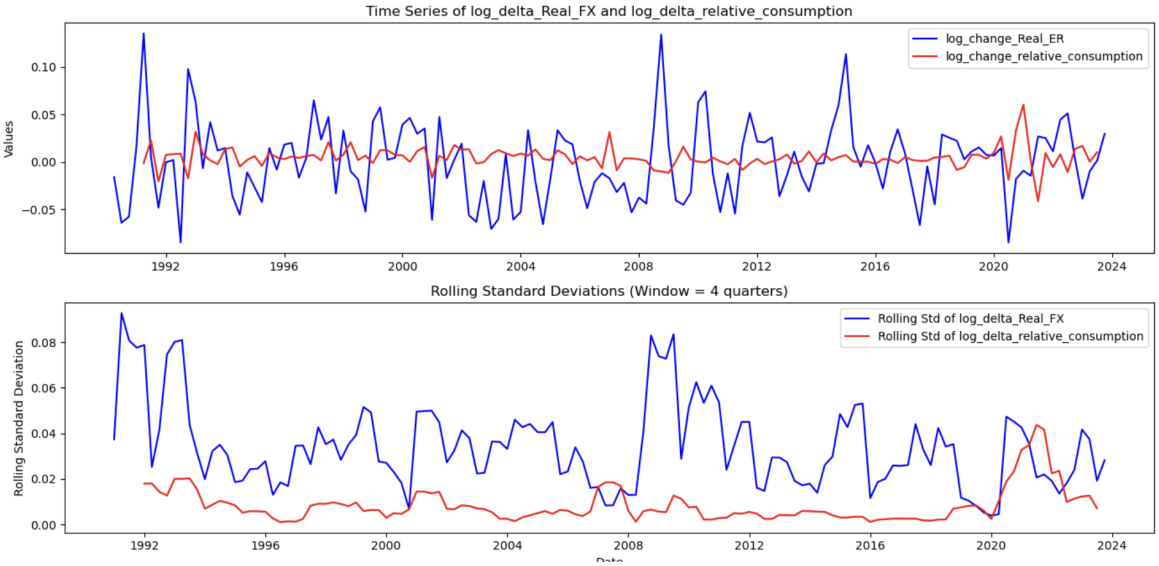

Figure 1: Euro Area 19 (Germany) - United States: Log change and volatility time series.

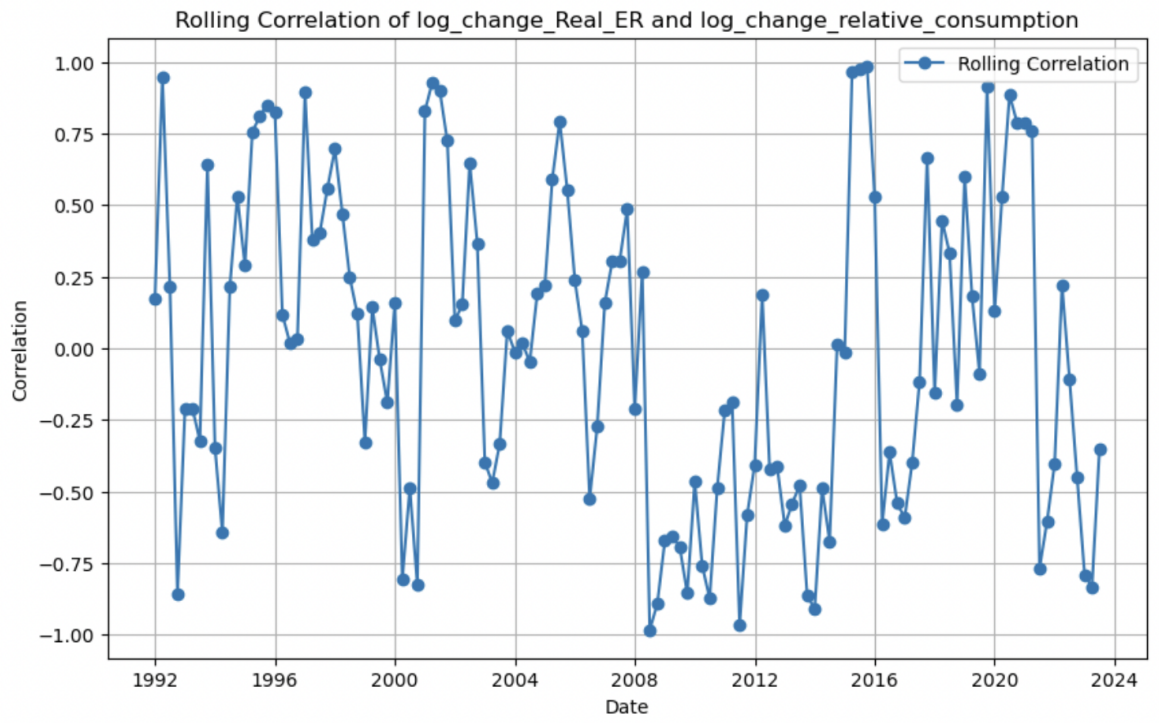

Figure 2: Euro Area 19 (Germany) - United States: Rolling correlation.

1.4.2 Canada - United States

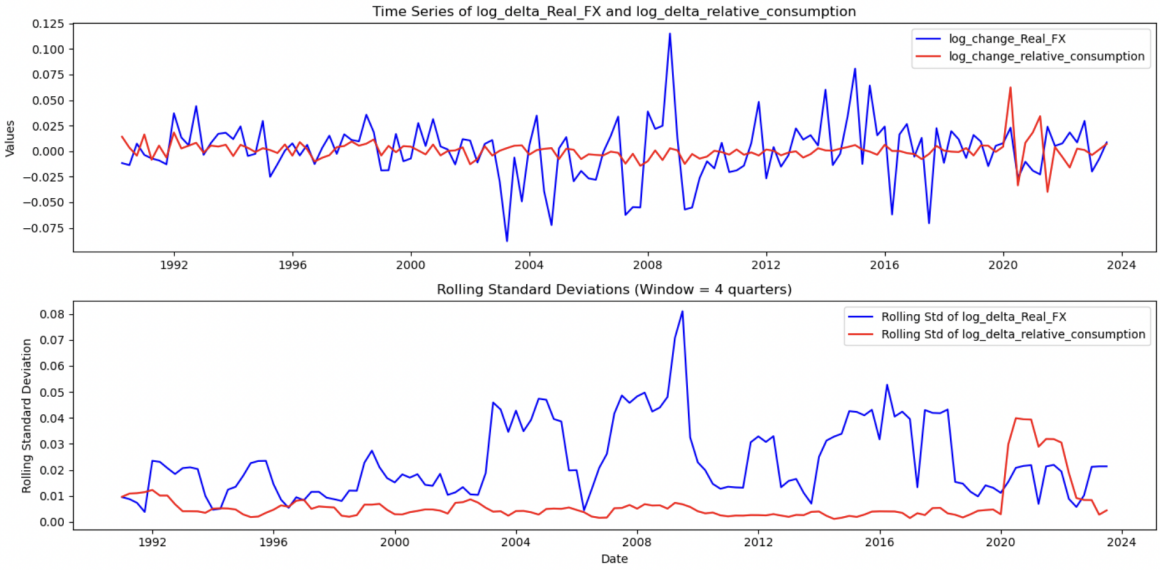

Figure 3: Canada - United States: Log change and volatility time series.

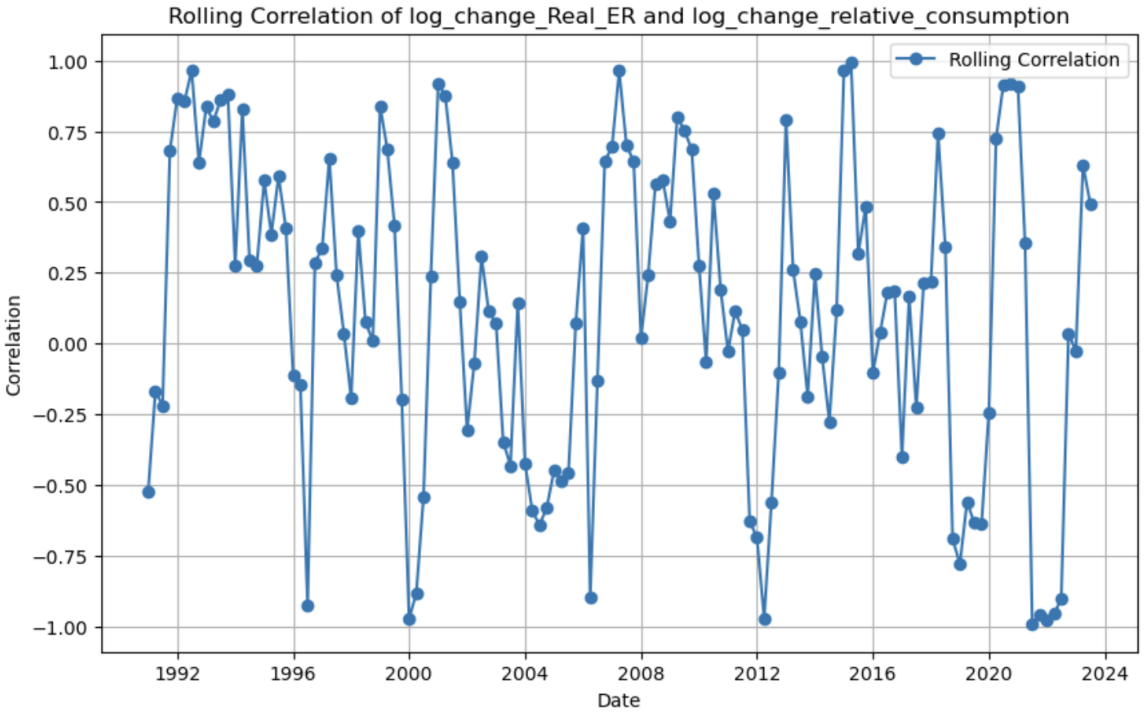

Figure 4: Canada - United States: Rolling correlation.

1.4.3 Australia - United States

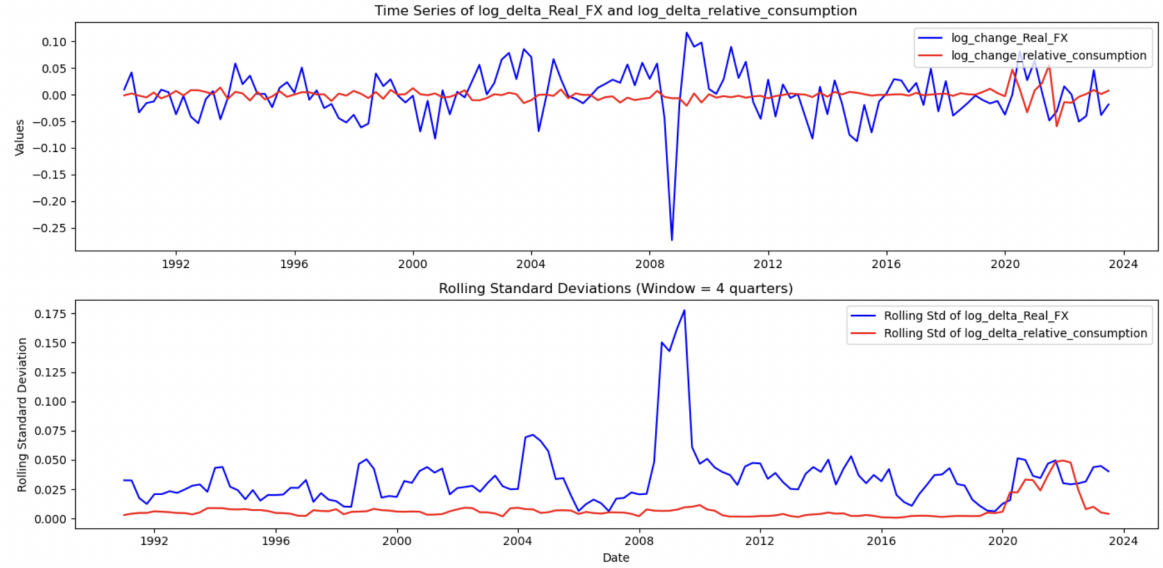

Figure 5: Australia - United States: Log change and volatility time series.

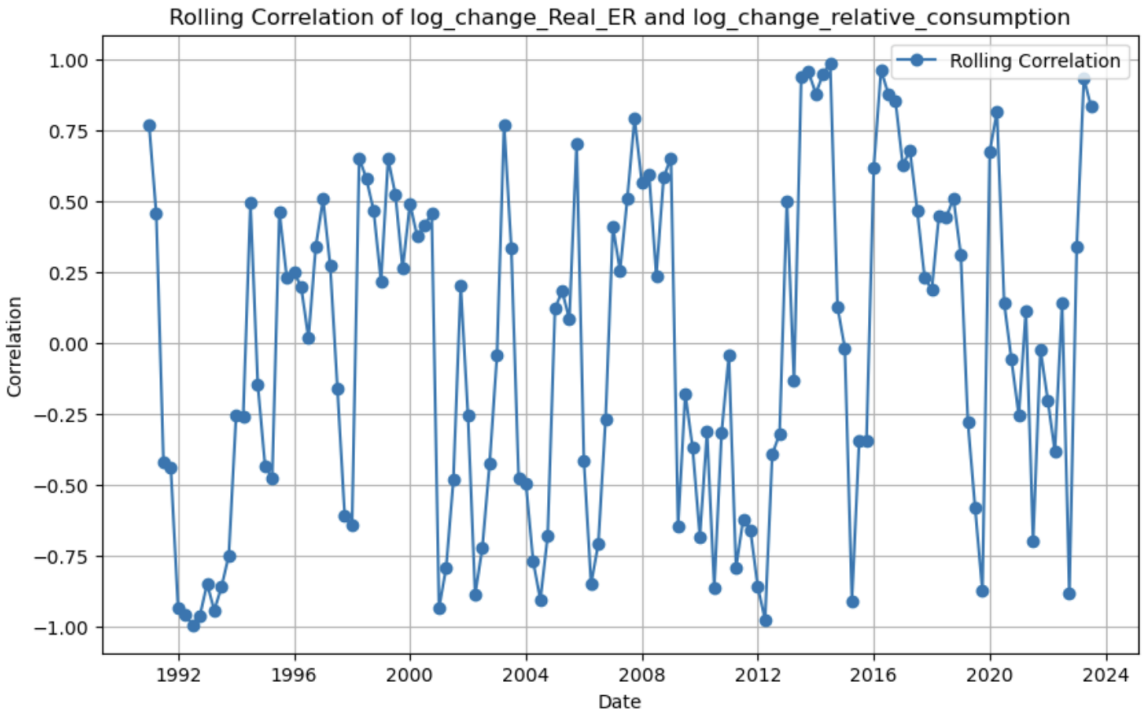

Figure 6: Australia - United States: Rolling correlation.

1.4.4 Japan - United States

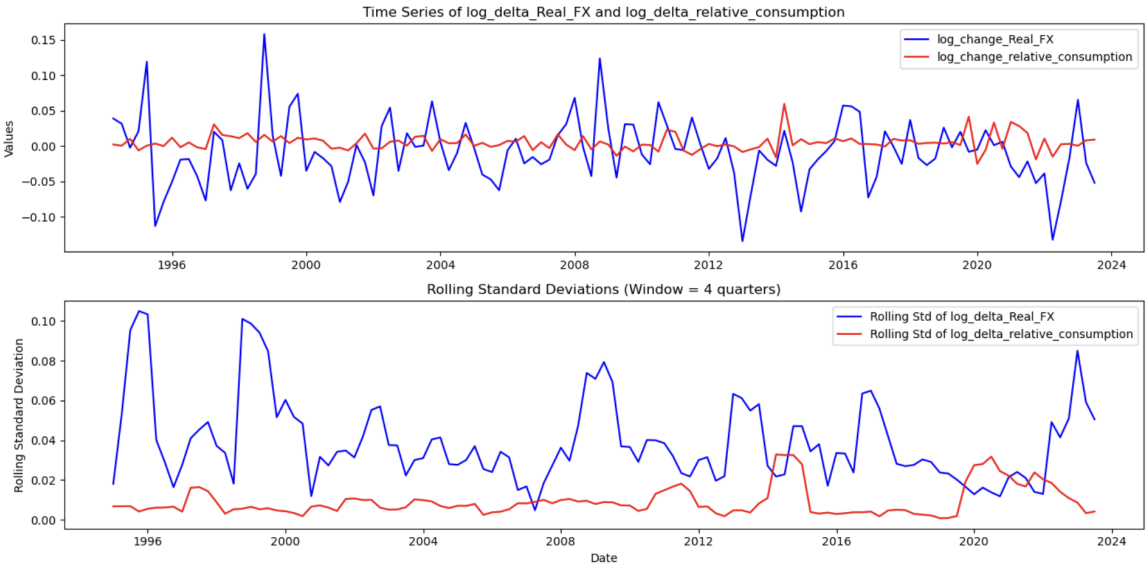

Figure 7: Japan - United States: Log change and volatility time series.

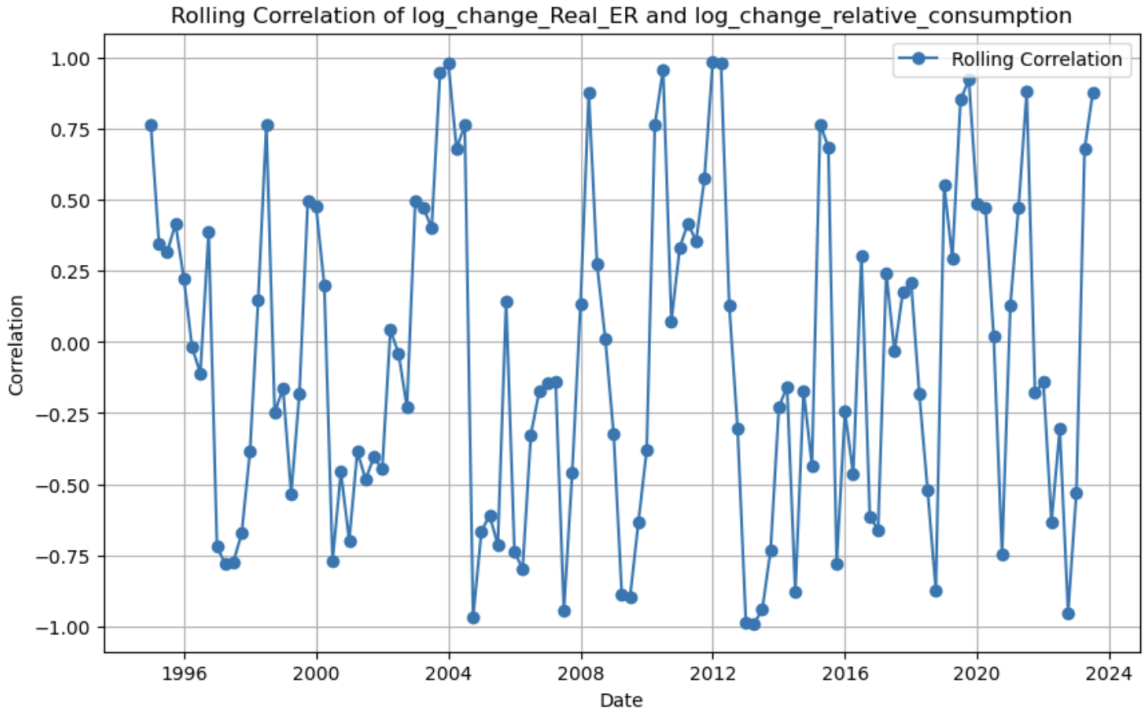

Figure 8: Japan - United States: Rolling correlation.

1.4.5 United Kingdom - United States

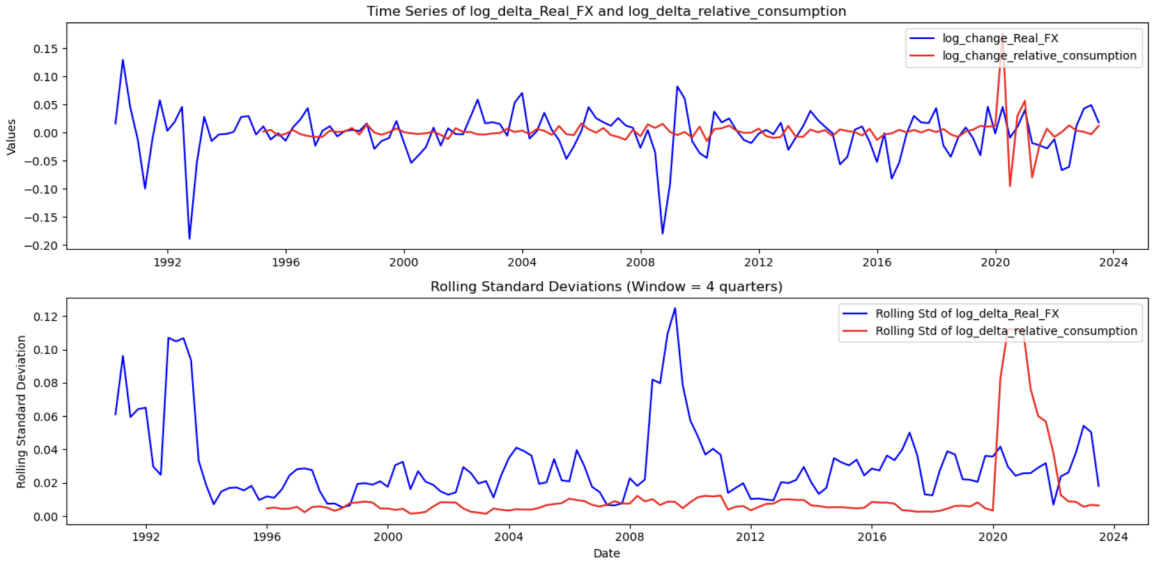

Figure 9: United Kingdom - United States: Log change and volatility time series.

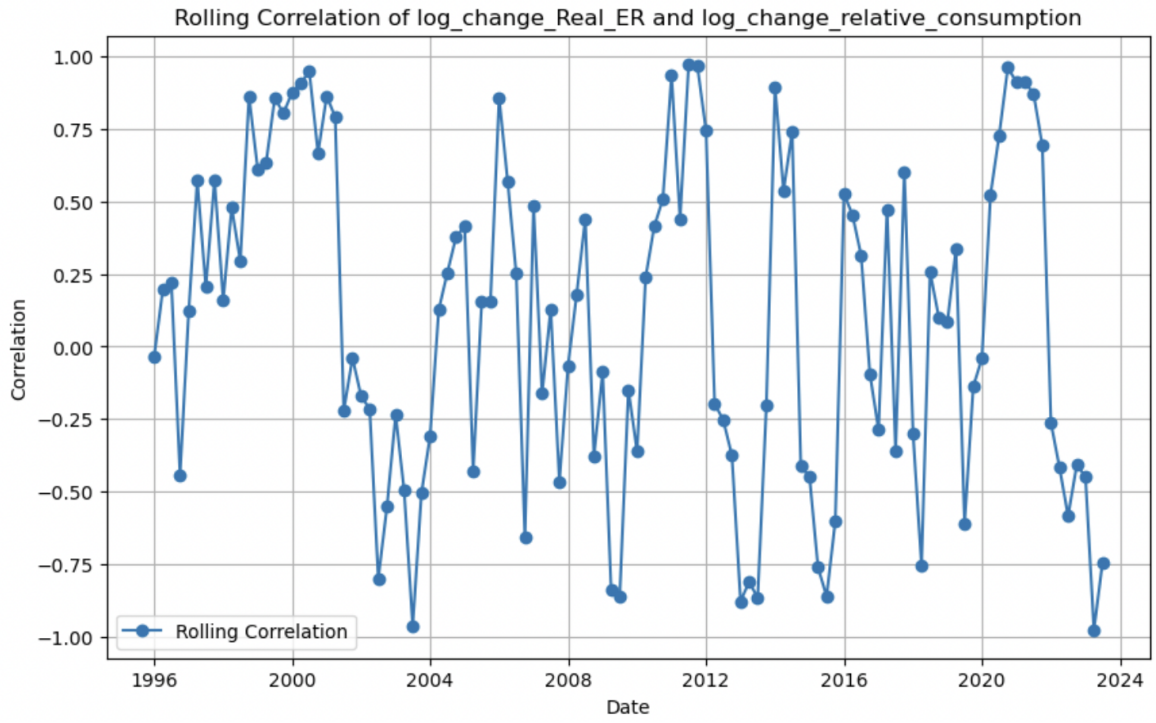

Figure 10: United Kingdom - United States: Rolling correlation.

1.5 Conclusion

The results I found are in line with the original paper by Backus and Smith (1993) and therefore reinforce the existence of the Backus-Smith or consumption-real FX puzzle.

First, for every currency pair, I observe that relative consumption is markedly less volatile than the real FX. In addition, find that the 4-quarter rolling correlation is also consistently very volatile, indicating there is no systematic cross-correlation between the two growth rates. This result was robust to different time windows (6-quarters, 8-quarters and 12-quarters) which I tested but did not include here

2 UIP/Forward premium puzzle

2.1 Definition

Cross-currency interest rate differentials are not balanced out by expected depreciations and instead predict exchange rate appreciations (albeit with a very low R^2), resulting in positive expected returns on currency carry trades.

2.2 Methodology

Recall the uncovered interest parity (UIP) regression,

$$s_{t+1} - s_t = \alpha_0 + \alpha_1(f_{t+1} - s_t) + u_{t+1}, \tag{3}$$

Where:

- s_t is the log of the exchange rate at time$t$.

- f_{t+1} is the log of time $t+1$ forward exchange rate contract for delivery at time $t+1$.
- u_{t+1} is the regression error.

In practice, the most common way in which testing the validity of UIP has been implemented is by way of the Fama regression (Fama, 1984), where the forward premium is treated as being equivalent to the interest differential:

$$s_{t+h} - s_t = \alpha_0 + \alpha_1(i_{h,t} - i^*_{h,t}) + u_{t+h} \tag{4}$$

where:

- $i_{h,t}$ is the domestic interest rate of maturity h at time t.
- $i^*_{h,t}$ is the foreign interest rate of maturity h at time t.

Notwithstanding the possible effect of Jensen inequality terms, rational expectations and risk neutrality implies that $\alpha_1 = 1$ and $\alpha_0 = 0$, the linear expression of the UIP.

Note - Given in some cases the interest differential yields a negative number, I add a constant value to the data ($\log(Y + a)$) prior to applying the log transformation, such that $\min(Y + a)$ is a very small positive number (like 0.001).

2.3 Data

- **Nominal exchange rate**: Monthly average nominal exchange rates, 19990:01-2024:01. These are expressed in units of foreign currency per USD. Source: FRED.
- **3-month interest rate**: Monthly 3-month maturity government bond spot yield, 2004:09-2024:06 for euro area (source: ECB). Monthly 3-month Treasury Bill, bank bill or interbank rate, 1990:01-2024:05, for United States, Canada, Australia (source: FRED).

2.4 Results

Below I include a number of plots to illustrate the UIP puzzle. For consistency, I use the same OECD-U.S. currency pairs as in Section 1 for the Backus-Smith puzzle.

First, I show a scatter plot of the 3-month ex post depreciation against the 3-month interest rate differential for each currency pair. Second, I show the estimated coefficients for the intercept and the interest rate differential regressor from the Fama regression, as shown in equation 4.

In order to illustrate the changing dynamics of the regression over time, as documented in the literature, for all currency pairs I split these plots into three time periods: pre-Great Financial Crisis (GFC) (from September 2004 to December 2007), GFC (from December 2007 to June 2009) and post-GFC (from June 2019 to December 2019).

2.4.1 Euro area - United States

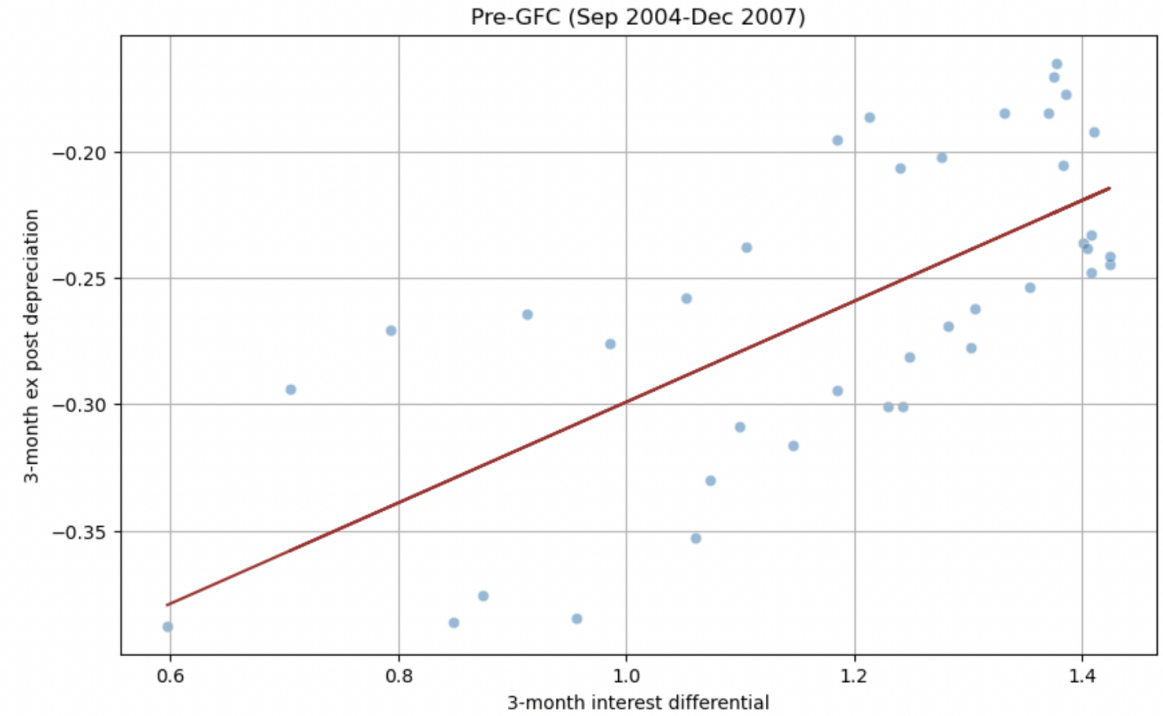

Figure 11: Euro area - United States: 3-month ex post depreciation vs 3-month interest differential

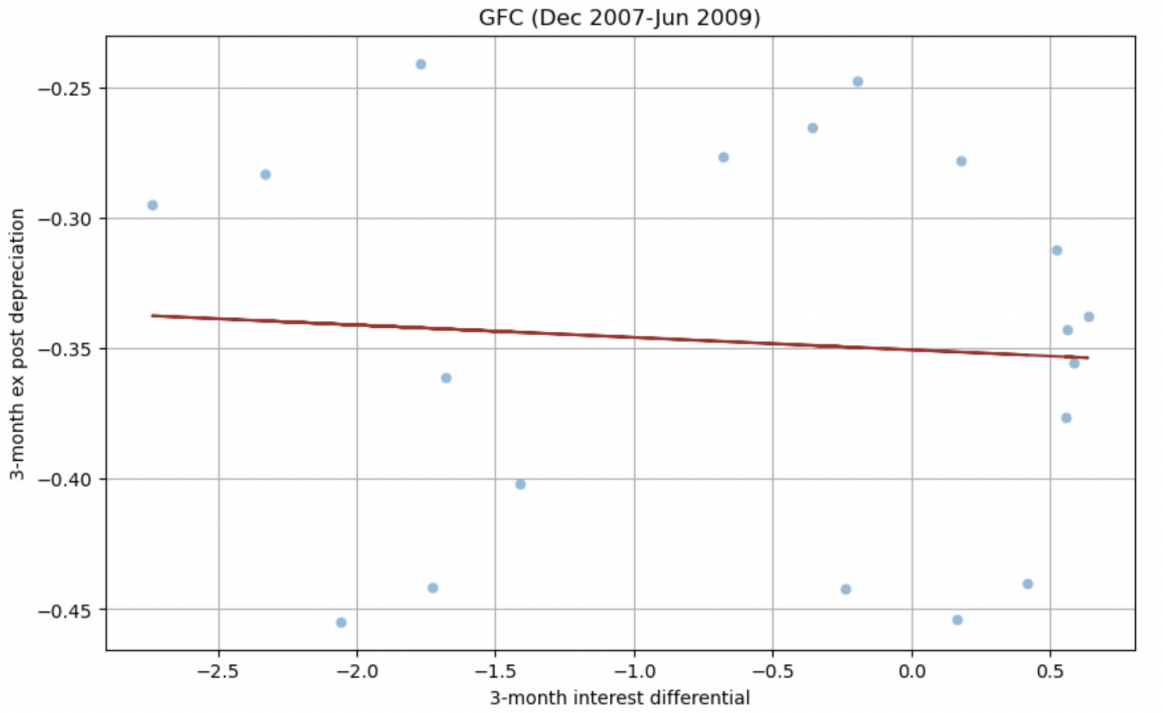

Figure 12: Euro area - United States: 3-month ex post depreciation vs 3-month interest differential

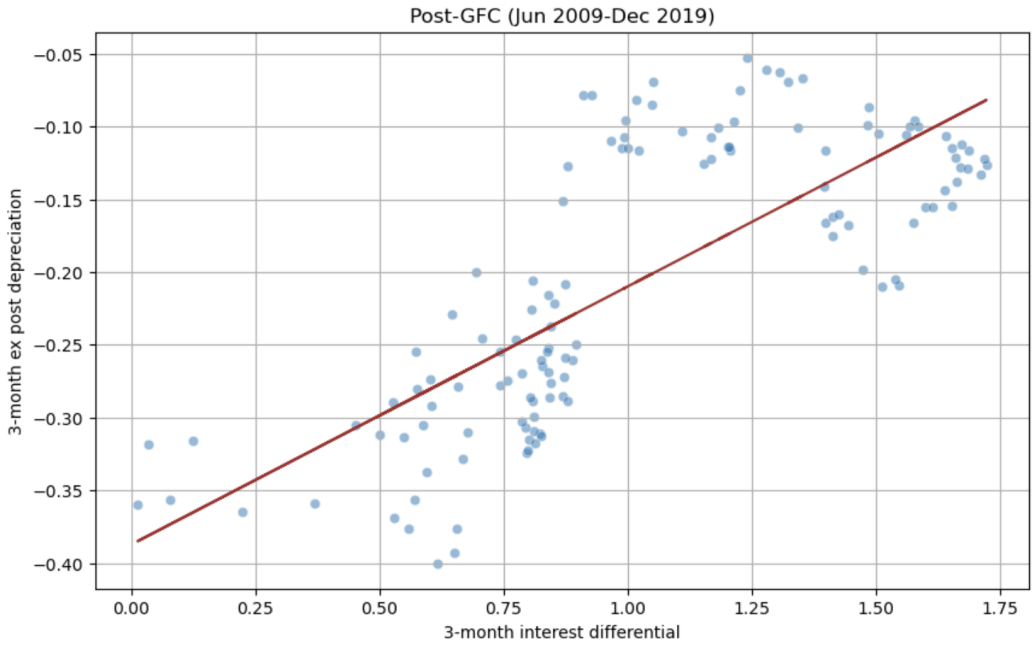

Figure 13: Euro area - United States: 3-month ex post depreciation vs 3-month interest differential

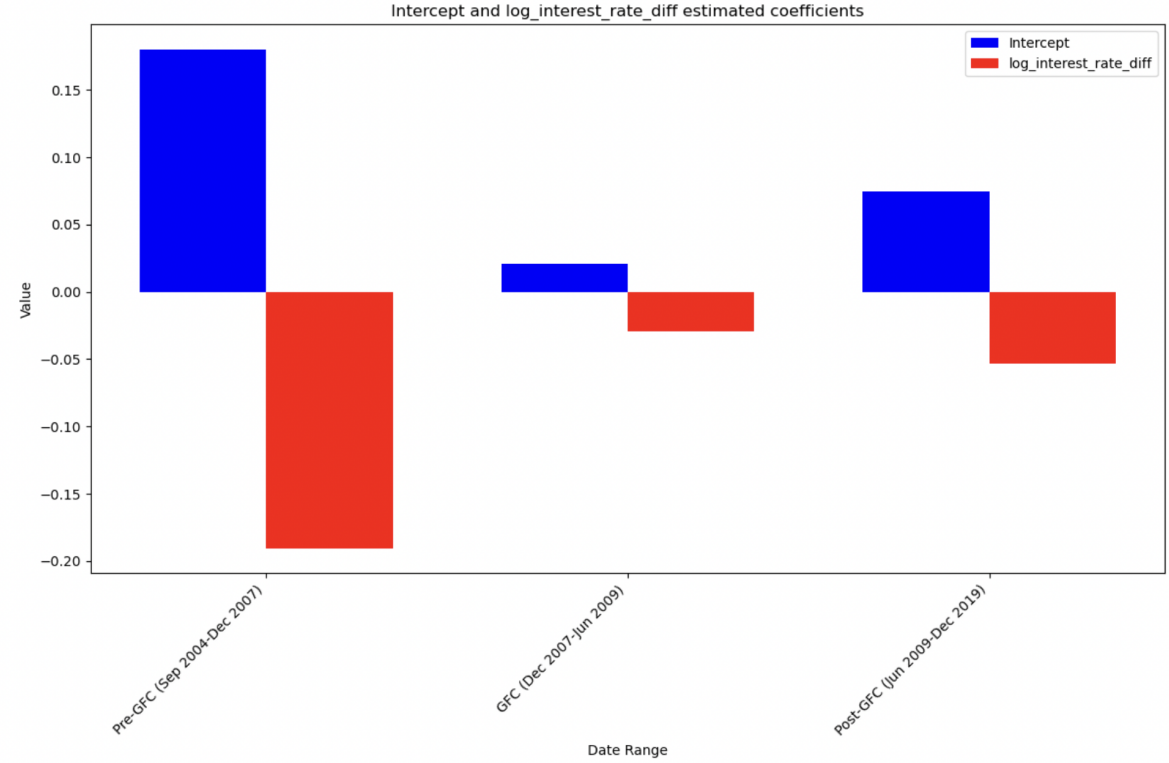

Figure 14: Euro area - United States: Fama regression estimated coefficients

Note: Results for the GFC (Dec 2007-Jun 2009) period are statistically insignificant at a 10% significance level.

2.4.2 Canada - United States

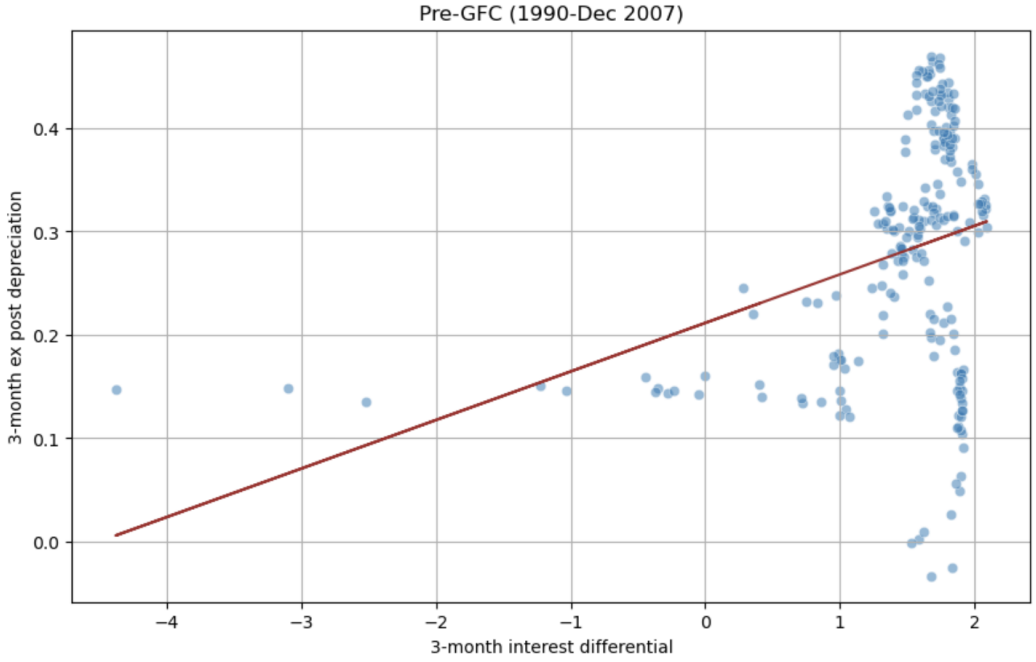

Figure 15: Canada - United States: 3-month ex post depreciation vs 3-month interest differential

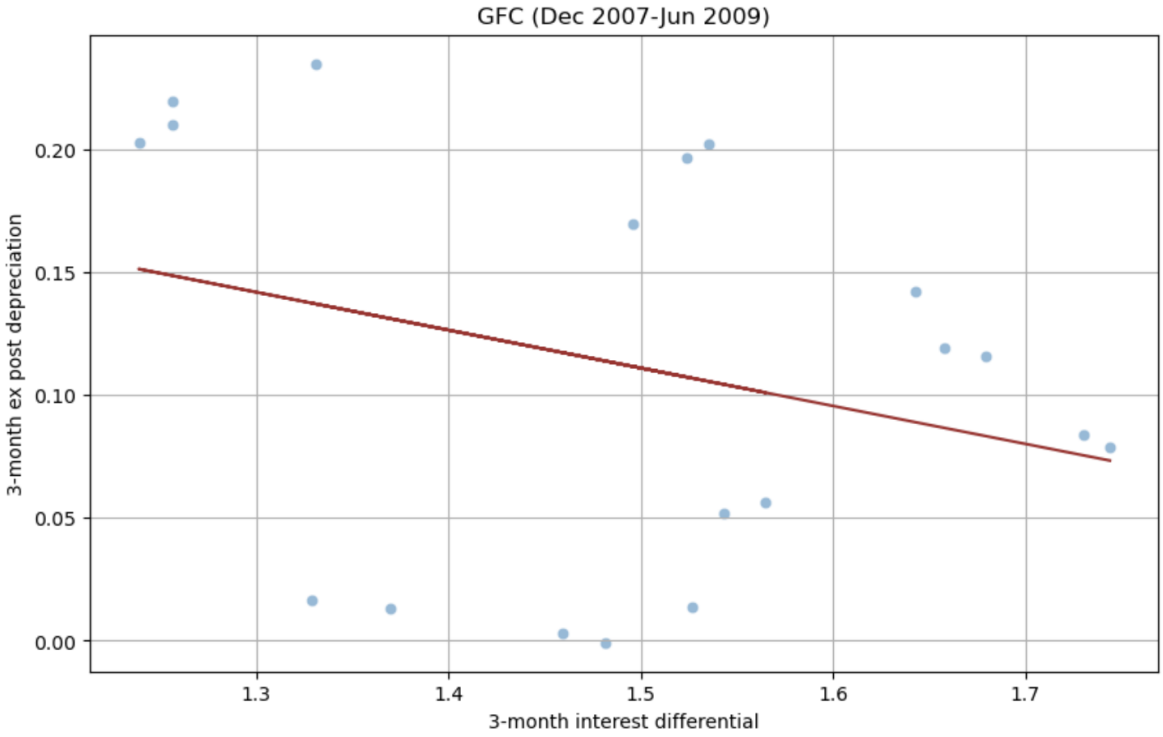

Figure 16: Canada - United States: 3-month ex post depreciation vs 3-month interest differential

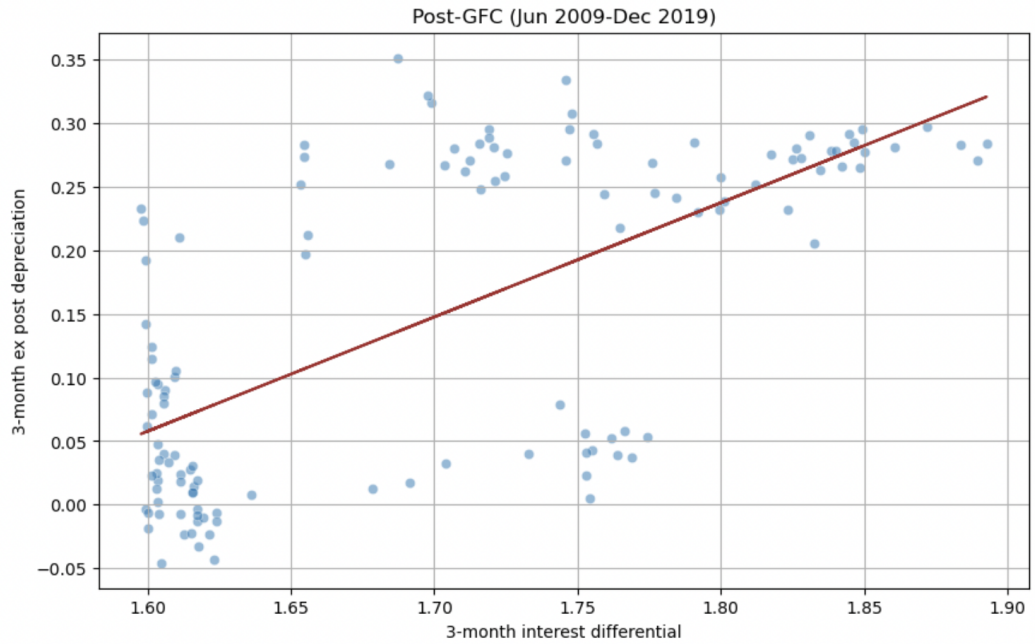

Figure 17: Canada - United States: 3-month ex post depreciation vs 3-month interest differential

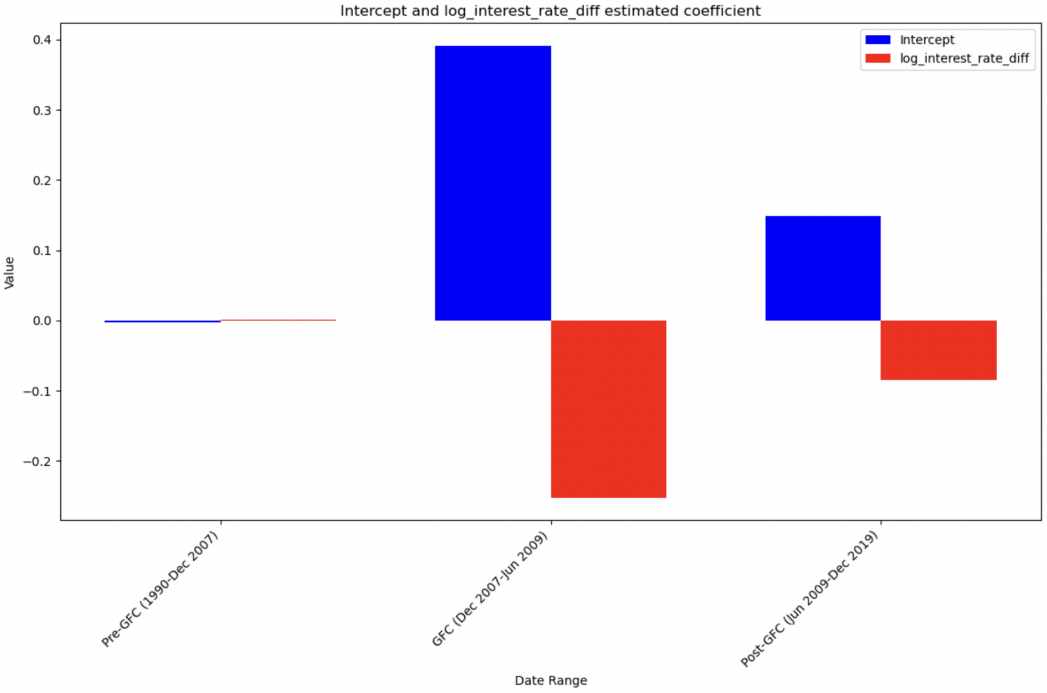

Figure 18: Canada - United States: Fama regression estimated cof

Note: Results for the pre-GFC (Jan 1990-Dec 2007) period are statistically insignificant at a 10% significance level.

2.4.3 Australia - United States

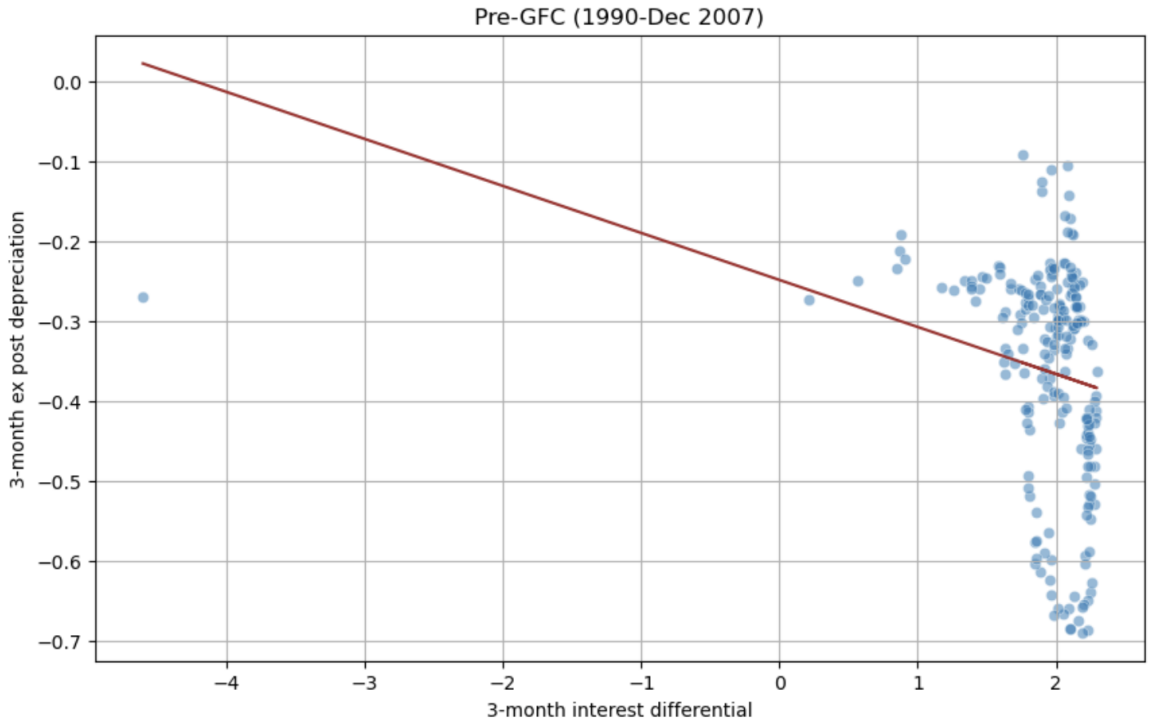

Figure 19: Australia - United States: 3-month ex post depreciation vs 3-month interest differential

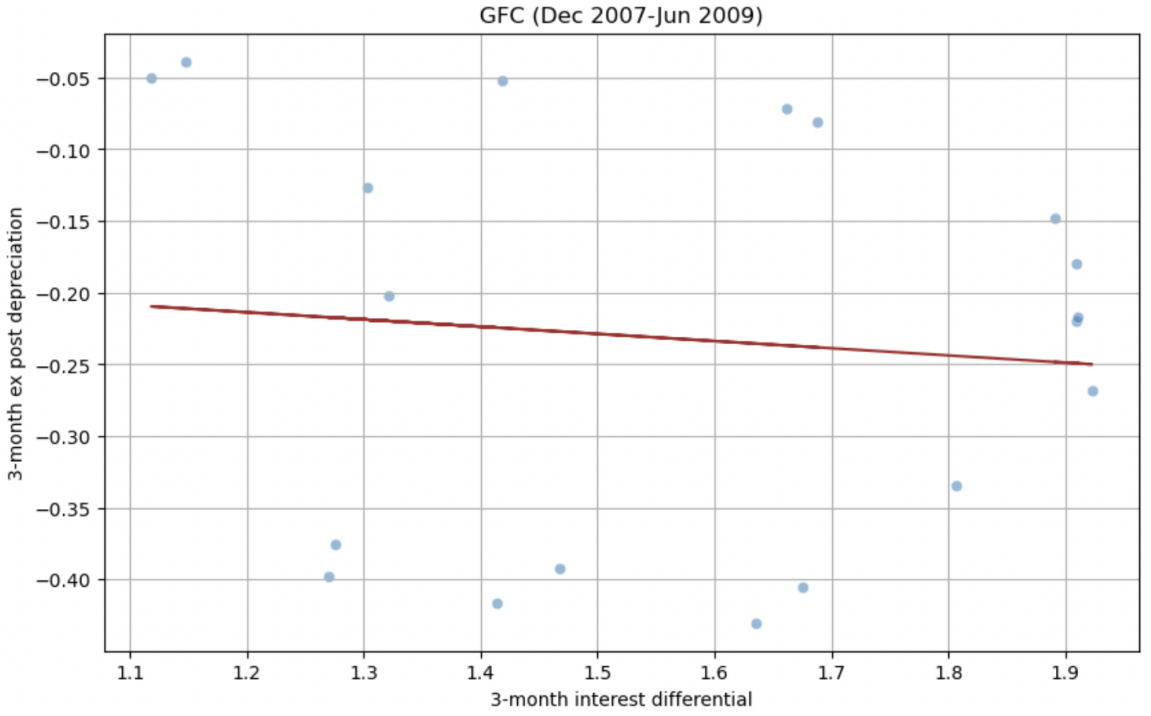

Figure 20: Australia - United States: 3-month ex post depreciation vs 3-month interest differential

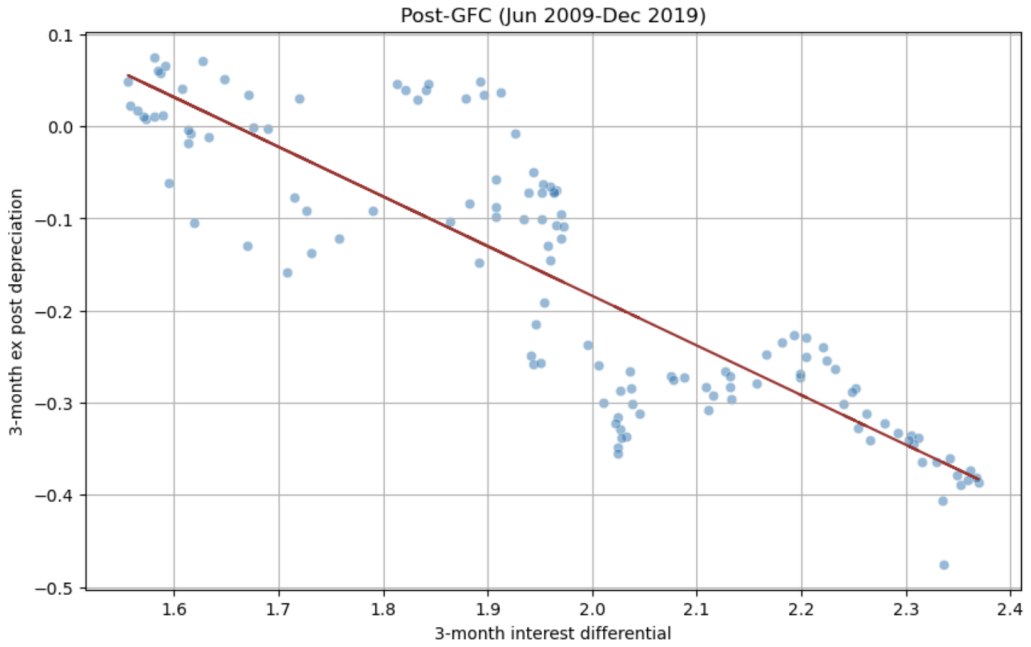

Figure 21: Australia - United States: 3-month ex post depreciation vs 3-month interest differential

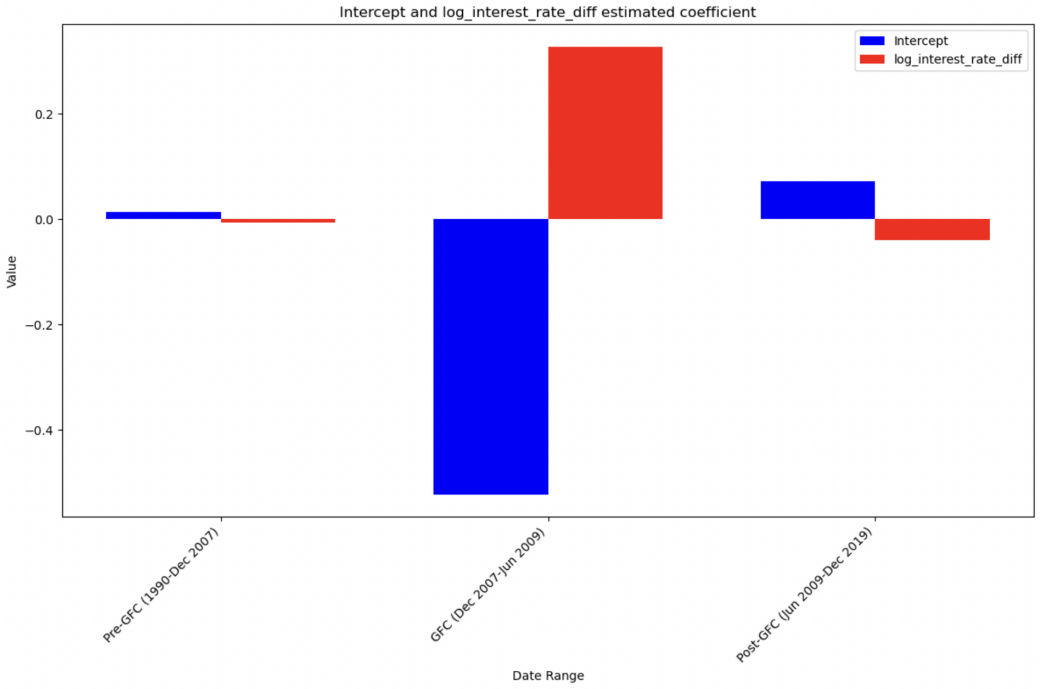

Figure 22: Australia - United States: Fama regression estimated coefficients

Note: Results for the pre-GFC (Jan 1990-Dec 2007) period are statistically insignificant at a 10% significance level.

2.4.4 Japan - United States

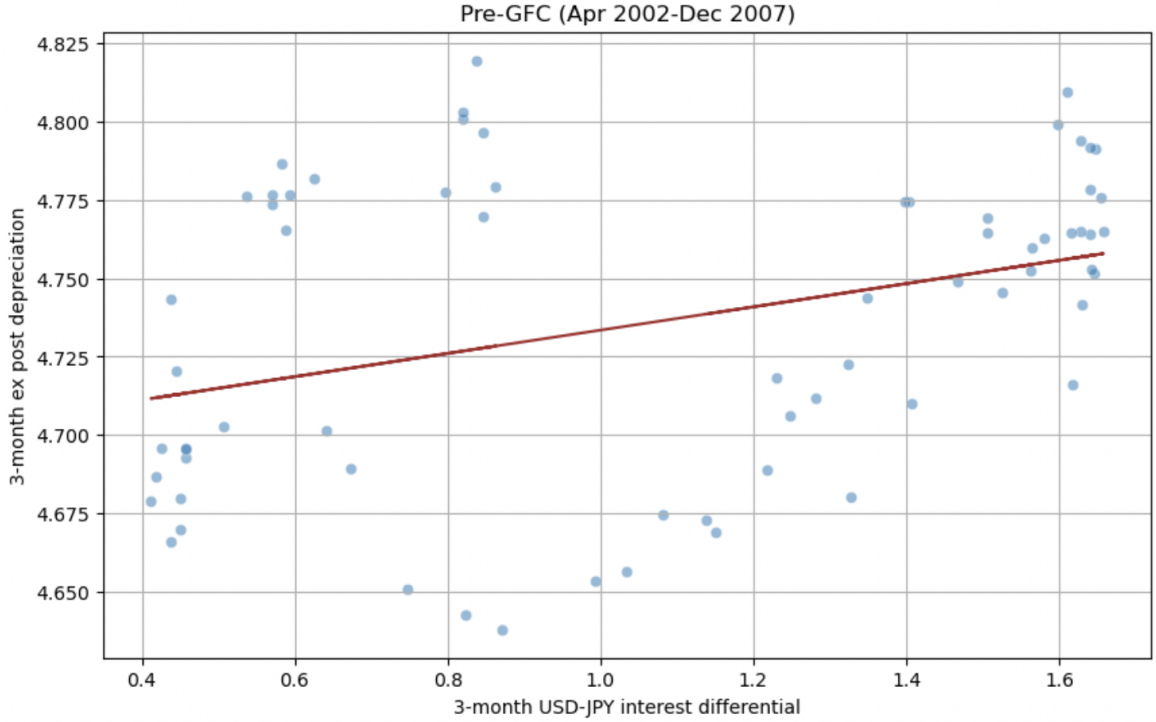

Figure 23: Japan - United States: 3-month ex post depreciation vs 3-month interest differential

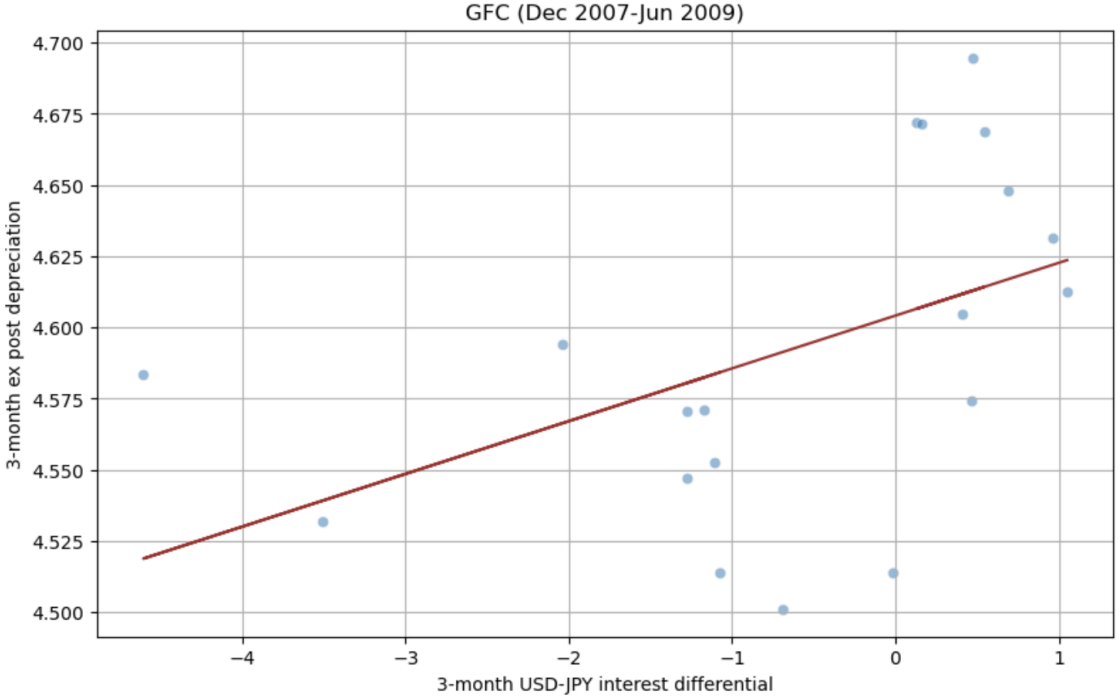

Figure 24: Japan - United States: 3-month ex post depreciation vs 3-month interest differential

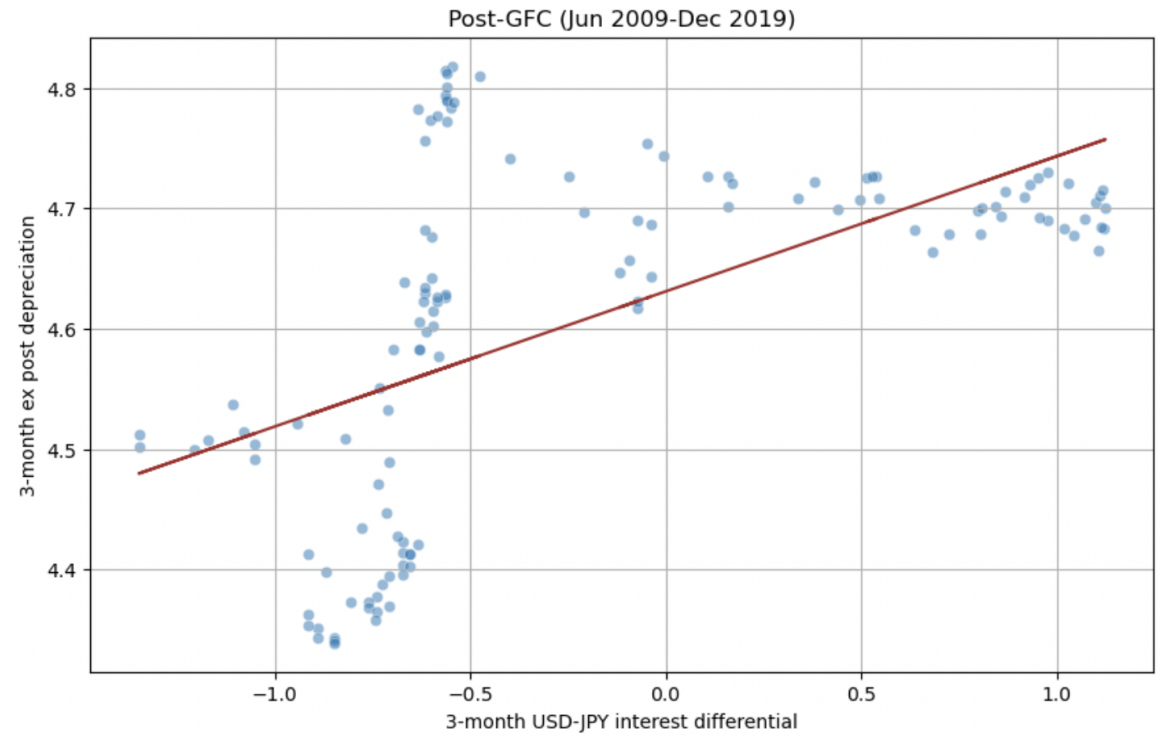

Figure 25: Japan - United States: 3-month ex post depreciation vs 3-month interest differential

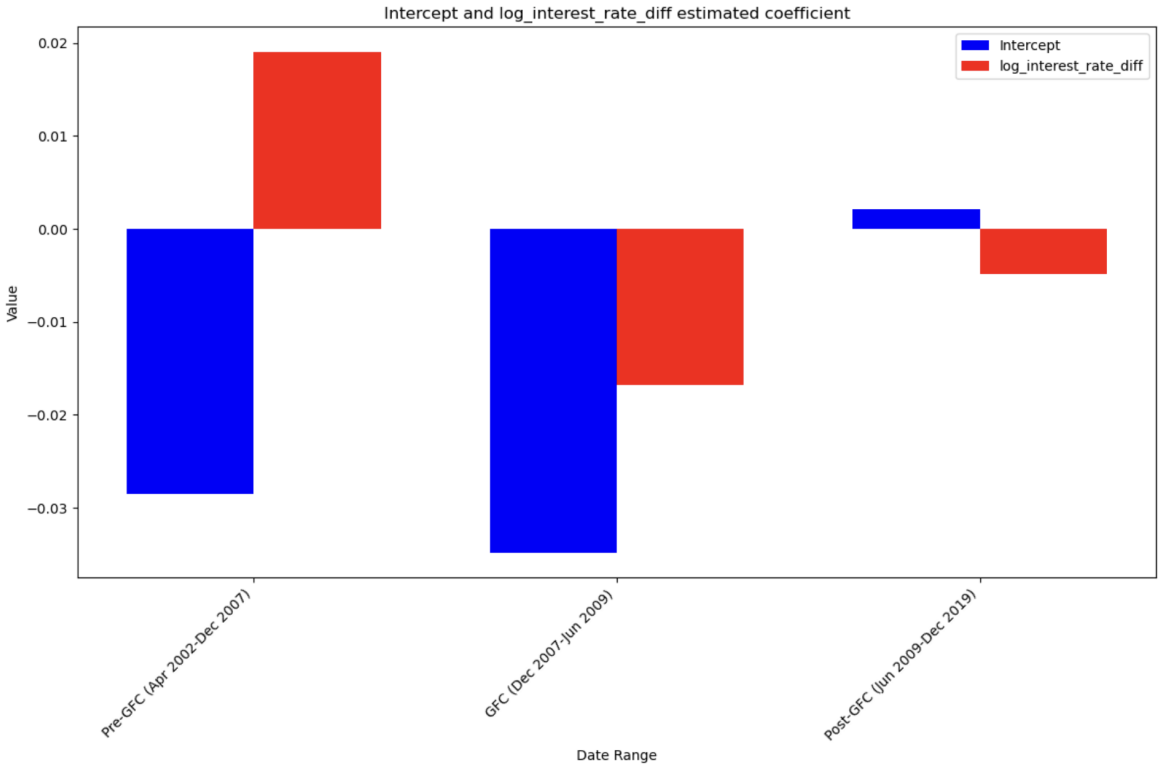

Figure 26: Japan - United States: estimated coefficients Fama regression

2.4.5 United Kingdom - United States

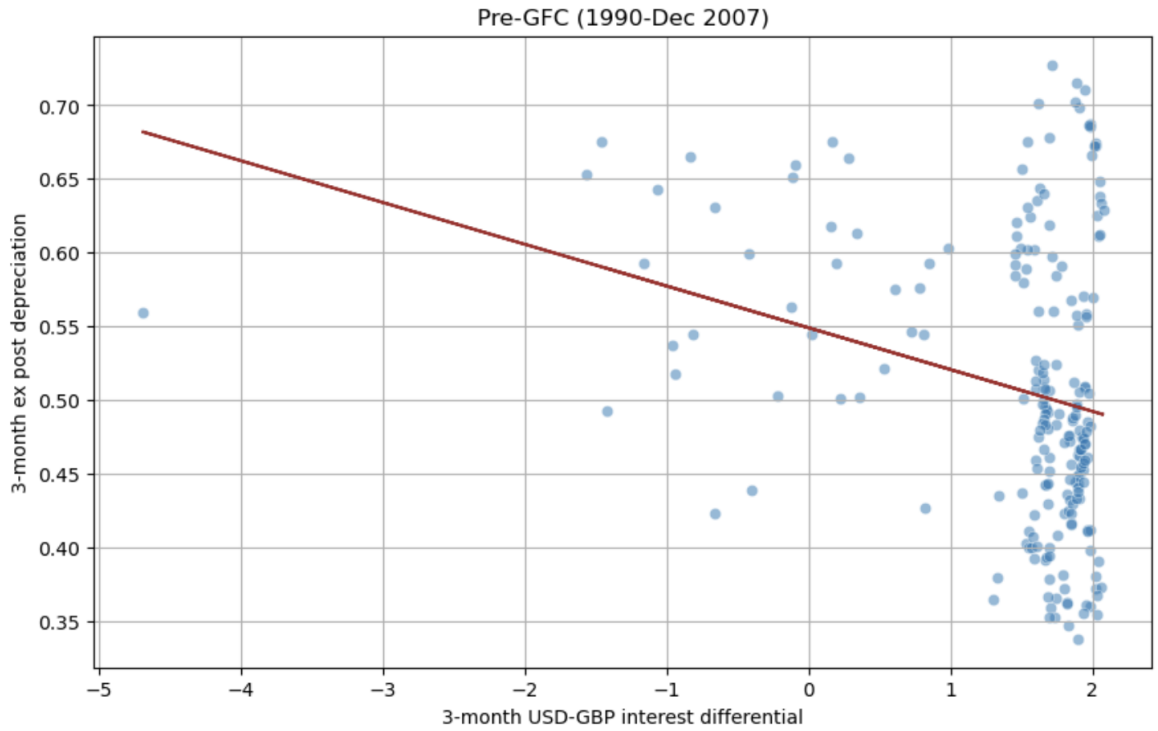

Figure 27: United Kingdom - United States: 3-month ex post depreciation vs 3-month interest differential

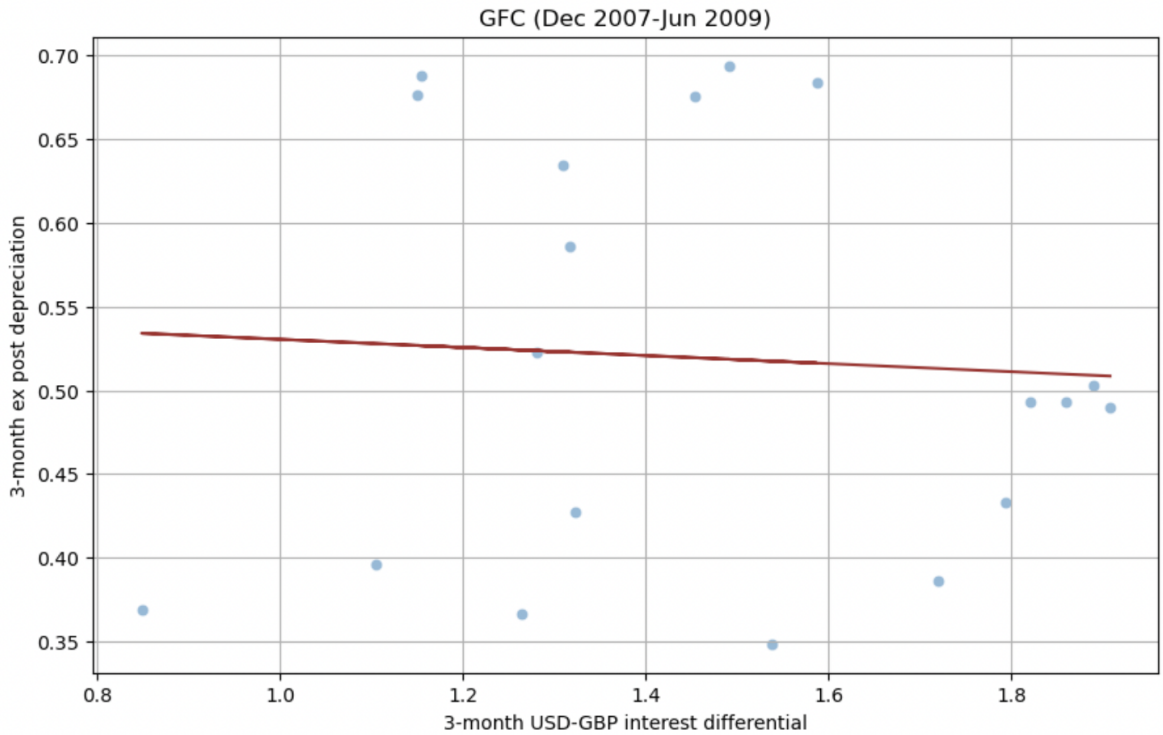

Figure 28: United Kingdom - United States: 3-month ex post depreciation vs 3-month interest differential

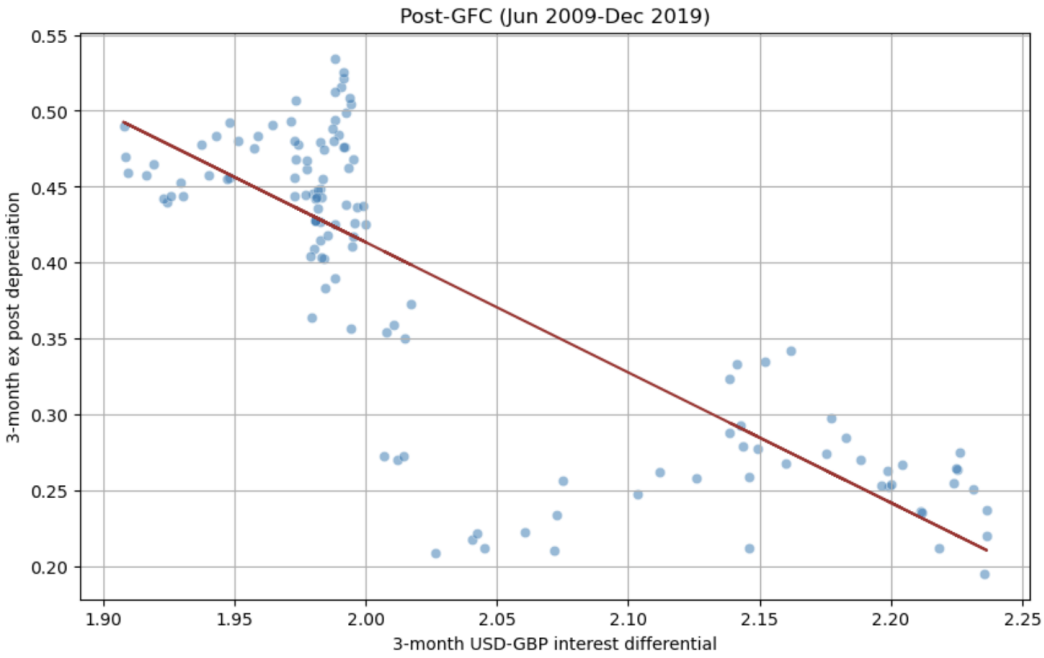

Figure 29: United Kingdom - United States: 3-month ex post depreciation vs 3-month interest differential

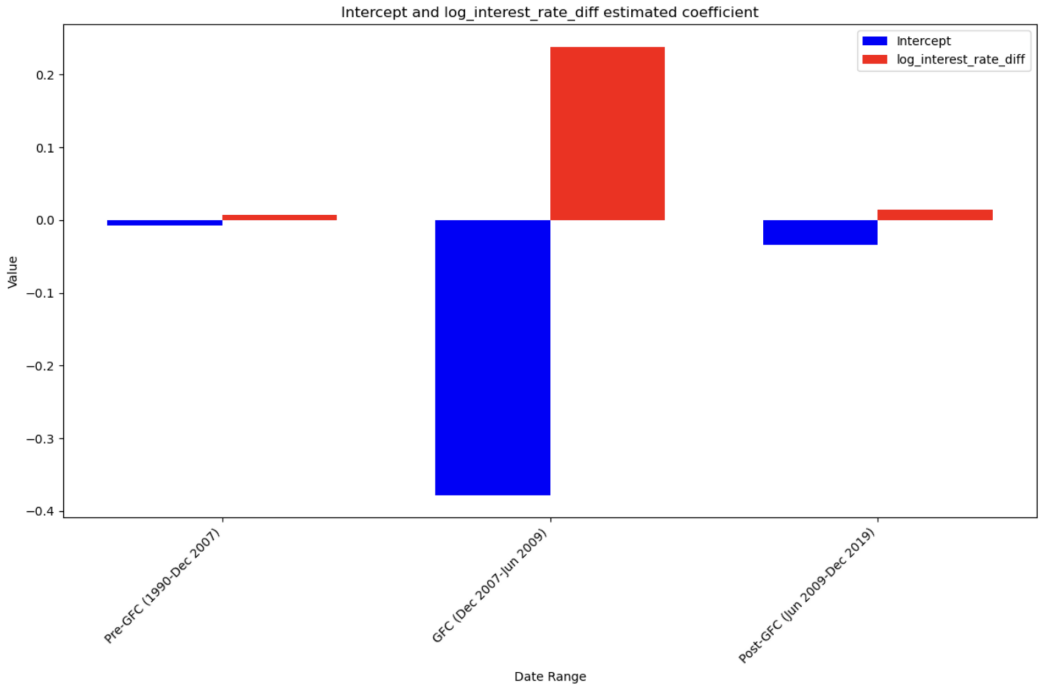

Figure 30: United Kingdom - United States: Fama regression estimated coefficients

Note: Results for the pre-GFC (Jan 1990-Dec 2007) and post-GFC (Dec 2007-Dec 2019) periods are statistically insignificant at a 10% significance level.

2.5 Conclusion

The results show that the null hypotheses that $\alpha_1 = 1$ and $\alpha_0 = 0$ in the Fama regression in equation (4) are violated, indicating UIP does not hold in the data (i.e. the UIP puzzle), as suggested by Fama (1984) in his original paper.

The estimated coefficients are generally small or negative and tend to increase in absolute magnitude during the GFC period. However, I do not find a consistent sign or size across time periods or currency pairs. Additionally, as indicated, the estimated coefficients for some time periods were found to be statistically insignificant at a 10% significance level.