

Assignment: Weighted Least Squares Fitting of Silver Grade vs. Zinc Grade

Introduction

In this assignment you will fit a dataset of **silver grade vs. zinc grade** using a **straight line**, a **second-order polynomial**, and an **exponential function**. The zinc grade is treated as the independent variable and the silver grade as the dependent variable.

Data

The Zinc and silver grades with associated error in silver grade is provided below

Zinc grade	Silver grade	Error in silver grade
0.8	1.0	0.3
1.5	2.4	0.4
2.1	4.1	0.5
3.0	6.9	0.6
3.8	9.3	0.7
4.6	12.2	0.9
5.5	16.0	1.0

Part A — Normal Equations Approach for Weighted Least Squares (L2)

Let the weights be defined as

$$w_i = \frac{1}{\sigma_i^2},$$

where σ_i is the error in the silver grade measurement.

A1) Weighted Straight Line Fit

Fit the straight line model

$$y = a + bx$$

using the weighted normal equations.

- Report the fitted parameters a and b .
- Compute and report the RMS error of the residuals.
- Compute and report the coefficient of determination R^2 .

A2) Weighted Quadratic Fit

Fit the second-order polynomial model

$$y = a + bx + cx^2$$

using the weighted normal equations.

- Report the fitted parameters a , b , and c .
- Compute and report the RMS error of the residuals.
- Compute and report the coefficient of determination R^2 .

A3) Plot and Model Comparison

Create a plot that includes:

- the data points with error bars in silver grade,
- the fitted straight line,
- the fitted quadratic curve.

Answer the following questions:

1. Which model provides the better fit based on RMS error and R^2 ?
2. Are there regions where either model appears invalid or physically unrealistic?