Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Ans: Given μ = 45 & σ = 8

Since service starts 10min after the drop off, new μ = 55 (by adding μ will change not σ)

X = 60

Z= X - μ / σ = *60-55 / 8 = 0.625*

Can't meet commitment, Z > 0.625

From 1- stats.norm.cdf(60,55,8) = 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ=38 & Standard deviation σ=6 For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Given μ = 38 Years & σ = 6 Years

A. P(A>44) – Z = 44-38/6 = 1 == 15.86%
   1-stats.norm.cdf(44,38,6) --- 15.86%

   P(38<A<44) = P(A<44) – P(A<38) = 34.13 %
   stats.norm.cdf(44,38,6) – stats.norm.cdf(38,38,6)
   P(A>44) < P(38<A<44) --- So it is False

B. P(A<30) = 9.12% ---- stats.norm.cdf(30,38,6)
   No of employees = 9.12% * 400 = 36.48
   Approx. 36 so this is True

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans : X 1 & X 2 are independent Normal Random Variables such that

$X_1 \sim N(\mu, \sigma^2)$ & $X_2 \sim N(\mu, \sigma^2)$

$2X_1 \sim N(2\mu, 4\sigma^2)$ - X_1 & $2X_1$ - *both follows Normal distribution*

$X_1 + X_2 \sim N(2\mu, 2\sigma^2)$ - *follows Normal distribution*

$2X_1 - (X_1 + X_2) \sim N(2\mu, 4\sigma^2) - N(2\mu, 2\sigma^2) = N(2\mu-2\mu, 4\sigma^2+2\sigma^2) = N(0, 6\sigma^2)$

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5 - stats.norm.interval(0.99,100,20)
   E. 90.1, 109.9

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans : $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$

$1 = Rs. 45

Annual Profit in $ = $Profit_1 \sim N(5, 3^2) + Profit_2 \sim N(7, 4^2)$

= Annual Profit $\sim N(12, 25)$

Annual Profit in Rs -

$\mu = 12 \text{ X } 45 = 540$, $\sigma^2 = 45^2 * 5^2$

$\sigma = 45 * 5 = 225$

A) CI =95%, $\mu = 540$, $\sigma = 225$

stats.norm.interval(0.95,540,225)

(99.00810347848784, 980.9918965215122)

B) 5th Percentile – 5% data on left side

Z score= -1.645 ---- stats.norm.ppf(0.05)

$X = \mu + \sigma * Z = 540+(-1.645*225) = 169.88$

C) Loss = Profit <0

For $Profit_1 \sim N(5, 3^2)$ in $ – area to left of 0

$\mu = 5 \text{ X } 45 = 225$, $\sigma^2 = 45^2 * 3^2 \Rightarrow \sigma = 45 * 3 = 135$

$Profit_1 \sim N(225, 135^2)$ in Rs – area to left of 0

stats.norm.cdf(0,225,135) - AREA IS = 4.7790352272814705 %

For $Profit_2 \sim N(7, 4^2)$ in $ – area to left of 0

$\mu = 7 \text{ X } 45 = 315$, $\sigma^2 = 45^2 * 4^2 \Rightarrow \sigma = 45 * 4 = 180$

$Profit_1 \sim N(315, 180^2)$ in Rs – area to left of 0

stats.norm.cdf(0,315,180) - AREA IS = 4.005915686381709 %

Probability of making loss by Div1 > Div2