Module 5 : Plane Waves at Media Interface

Lecture 35 : Reflection & Refraction from Dielectric Interface

Objectives

In this course you will learn the following

- Reflection and Refraction from Dielectric Interface.
- Reflection and Refraction with Perpendicular Polarization.

Reflection and Refraction from Dielectric Interface

- After applying the law of reflection and Snell's law the problem of wave propagation across the dielectric interface reduces to finding the amplitudes of the fields of three waves at the interface.
- Once the fields are known at the interface the fields in region 1 can be obtained by super position of F_i and F_r . Whereas in medium 2 the fields are F_t .
- For arbitarty polarization handling waves across the media interface is quite complicated.
- The problem is elegantly handled by decomposing the fields into its components i.e one in the plane of incidence and other perpendicular to it.
- We therefore analyze two cases:

(1) **Electric field in the plane of incidence called Parallel Polarization.**

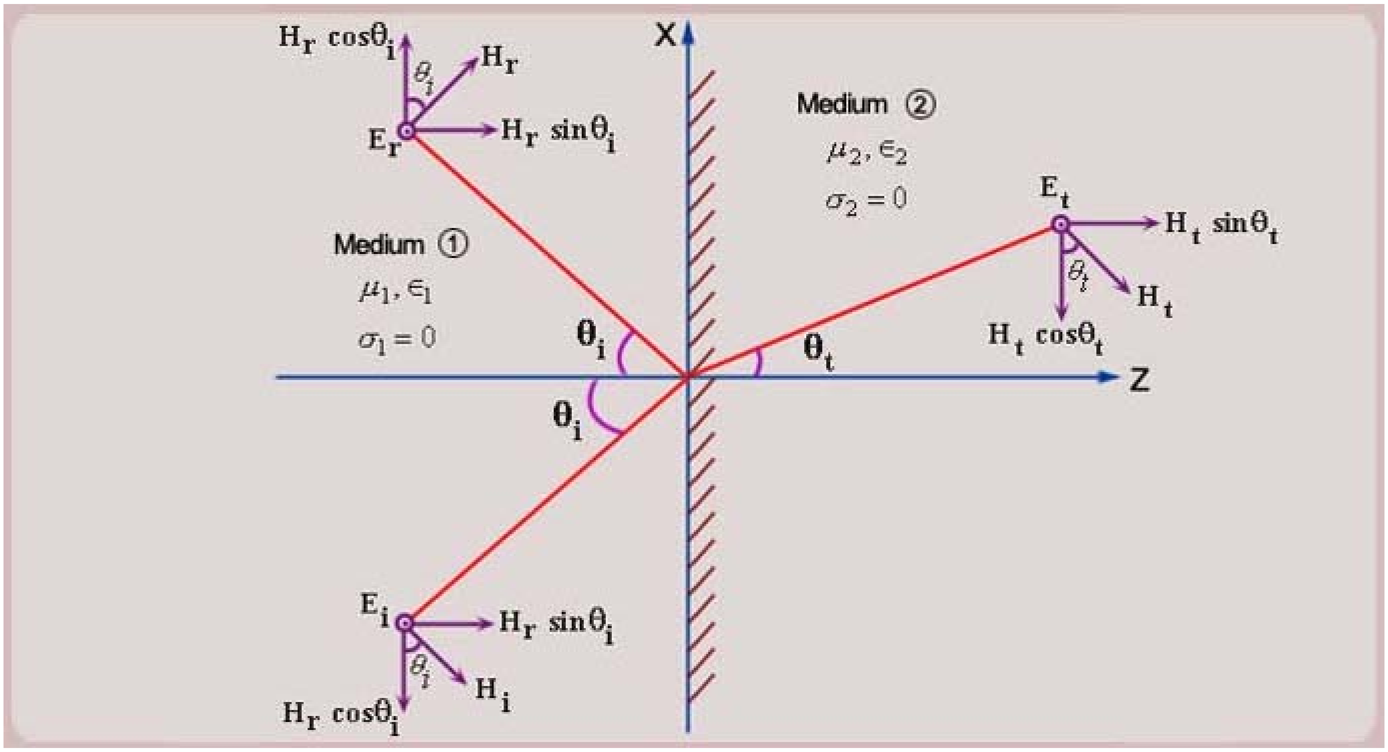
(2) **Electric field perpendicular to the plane of incidence called Perpendicular Polarization.**

- For each case we obtain two parameters namely

$$\boxed{\begin{array}{lll} \textbf{Reflection Coefficient} & : & \Gamma = \dfrac{\mathbf{E}_r}{\mathbf{E}_i} \\[2ex] \textbf{Transmission Coefficient} & : & \tau = \dfrac{\mathbf{E}_t}{\mathbf{E}_i} \end{array}}$$

Reflection and Refraction with Perpendicular Polarization

- Without losing generality let us assume incidence electric field $\mathbf{E_i}$ is pointing out of the plane of the paper as shown in the figure



- The direction of magnetic field can be chosen appropiately to give the poynting vector in the direction of the wave.
- From the Laws of Reflection and Refraction the wave vectors of the reflected and transmitted waves also lie in the plane of incidence.
- The electric field of the incident wave is tangential to the interface. The continuity of the tangential components of the electric field therefore demand that the electric field for transmitted and reflected wave are perpendicular to the wave of the paper. Let us therefore assume that $\mathbf{E_r}$ and $\mathbf{E_t}$ are also oriented in y -direction.
- The magnetic field direction can be obtained from the poynting vector for the transmitted and reflected waves. The direction of the magnetic field for the three waves are therefore as shown in the figure. We therefore have the electric fields for the three waves

$$\text{Incident wave} \quad : \quad \mathbf{E_i} = \mathbf{E_{i0}} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\text{Reflected wave} \quad : \quad \mathbf{E_r} = \mathbf{E_{r0}} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$\text{Transmitted wave} \quad : \quad \mathbf{E_t} = \mathbf{E_{t0}} e^{-j\beta_2(x\sin\theta_t - z\cos\theta_t)}$$

- Where $\beta_1 = \omega\sqrt{\mu_1\epsilon_1}$, $\beta_2 = \omega\sqrt{\mu_2\epsilon_2}$ and $\mathbf{E_{i0}}$, $\mathbf{E_{r0}}$, and $\mathbf{E_{t0}}$ are the vector amplitudes of the three waves. For all the fields, a time variation of $e^{j\omega t}$ is implicitly assumed. And Magnetic fields as

$$|\mathbf{H_i}| = \frac{|\mathbf{E_i}|}{\eta_1}$$

$$|\mathbf{H_r}| = \frac{|\mathbf{E_r}|}{\eta_1}$$

$$|\mathbf{H_t}| = \frac{|\mathbf{E_t}|}{\eta_2}$$

- Where $\eta_1 = \sqrt{\mu_1/\epsilon_1}$ and $\eta_2 = \sqrt{\mu_2/\epsilon_2}$ are the intrinsic impedances of medium 1 and medium 2 respectively.
- Since there are no surface currents, applying continuity of the tangential components of the electric and magnetic fields at the interface (i.e $z = 0$), we get

$$\mathbf{E_i} + \mathbf{E_r} = \mathbf{E_t}$$
$$|\mathbf{H_i}|\cos\theta_i - |\mathbf{H_r}|\cos\theta_i = |\mathbf{H_t}|\cos\theta_t$$

- Solving above equations we obtain the reflection and transmission coefficients as

$$\text{Reflection coeff} \quad : \Gamma_\perp = \frac{|\mathbf{E_{r0}}|}{|\mathbf{E_{i0}}|} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\text{Transmission coeff} \quad : \tau_\perp = \frac{|\mathbf{E_{t0}}|}{|\mathbf{E_{i0}}|} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Note

(1) **The reflection and transmission coefficients are real i.e the phase differences between incident, transmission and reflected wave are either 0 or π .**

(2) **The magnitutude of the reflection coefficient $\Gamma_\perp$ is always less than unity.**

(3) **The transmission coefficient could be greater or less than unity. However, a transmission coefficient greater than**
unity does not mean higher transmitted power compared to the incident wave.

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Recap

In this course you have learnt the following

- Reflection and Refraction from Dielectric Interface.
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