**Module 2 : Transmission Lines**

**Lecture 8 : Power Transfer through a Transmission Line**

**Objectives**

**In this course you will learn the following**

- Power delivered to a complex load connected to a generator through a section of a line.
- Complex power at any location on the line.
- How to obtain the amplitude of the forward travelling wave?

## Power delivered to the load obtained using Circuit concept

- Consider a loss-less transmission line with characteristic impedance $Z_0$. Let the line be terminated in a complex load impedance $Z = R + jX \neq Z_0$. Since the load impedance is not equal to the characteristic impedance, there is reflection on the line, and the voltage and the current on the line can be given as

$$V(l) = V^+ e^{j\beta l}\left\{1 + \Gamma_L e^{-j2\beta l}\right\}$$
$$I(l) = \frac{V^+}{Z_0} e^{j\beta l}\left\{1 - \Gamma_L e^{-j2\beta l}\right\}$$

- Since the reference point $l = 0$ is at the load end, the power delivered to the load is

$$\begin{aligned} P_L &= \frac{1}{2} Re(VI^*) \text{ at } l = 0 \\ &= \frac{1}{2} Re\left\{[V^+(1 + \Gamma_L)]\left[\frac{V^+}{Z_0}(1 - \Gamma_L)\right]^*\right\} \\ &= \frac{1}{2} Re\left\{\frac{|V^+|^2}{Z_0}\left[1 - |\Gamma_L|^2 + (\Gamma_L - \Gamma_L^*)\right]\right\} \end{aligned}$$

- Since the difference of any complex number and its conjugate is in the purely imaginary part, $(\Gamma_L - \Gamma_L^*)$ is a purely imaginary quantity. Therefore the power delivered to the load is

$$P_L = \frac{|V^+|^2}{2Z_0}\left\{1 - |\Gamma_L|^2\right\}$$

## Power delivered to the load using Wave concept

- The power delivered to the load can also be calculated using a different approach and that is, the power given to the load is the difference of the power carrried by the incident wave towards the load and the power carried away by the reflected wave.

  Since the travelling waves always see the characteristic impedance, the incident and reflected powers $P_{inc}$ and $P_{ref}$ respectively are,

$$P_{inc} = \frac{1}{2}Re\{V^+(I^+)^*\} = \frac{1}{2}Re\{V^+(\frac{V^+}{Z_0})^*\}$$

$$= \frac{|V^+|^2}{2Z_0} \quad \text{Note: } Z_0 \text{ is real for a loss-less line}$$

$$P_{ref} = \frac{|\Gamma_L V^+|^2}{2Z_0}$$

$$= \frac{|V^+|^2}{2Z_0}|\Gamma_L|^2$$

We therefore get

$$P_L = P_{inc} - P_{ref}$$

$$P_L = \frac{|V^+|^2}{2Z_0}\{1 - |\Gamma_L|^2\}$$

## Complex Power at any point on the line

- The complex power at any point on the line is

$$P(l) = \frac{1}{2}\{V(l)[I(l)]^*\}$$

Substituting for voltage and current at location '$l$' ,

$$P(l) = \frac{1}{2}\left\{V^+ e^{j\beta l}\left(1+\Gamma_L e^{-j2\beta l}\right)\left[\frac{V^+}{Z_0} e^{j\beta l}\left(1-\Gamma_L e^{-j2\beta l}\right)\right]^*\right\}$$

$$= \frac{|V^+|^2}{2Z_0}\left\{1-|\Gamma_L|^2 + \mathrm{Im}\left[\Gamma_L e^{-j2\beta l}\right]\right\}$$

$$\Rightarrow \mathrm{Re}\{P(l)\} = \frac{|V^+|^2}{2Z_0}\left\{1-|\Gamma_L|^2\right\} = P_L$$

$$\mathrm{Im}\{P(l)\} = \frac{|V^+|^2}{2Z_0}\mathrm{Im}\left[\Gamma_L e^{-j2\beta l}\right]$$

**Note**

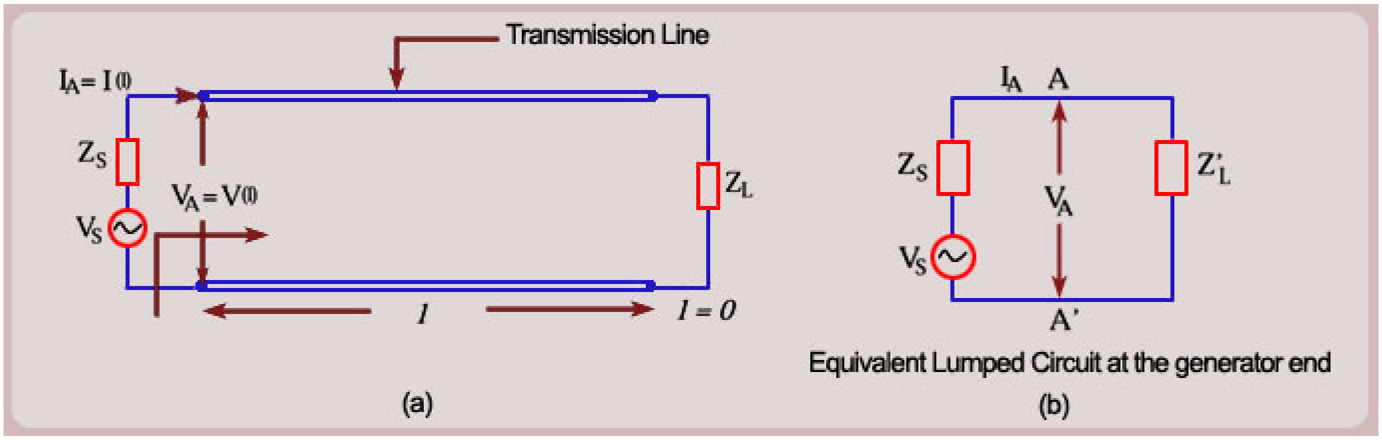
**(i)** **The $Re\left(P(l)\right) = P_L$ i.e., the power loss at any point on the line is same as that at the load. This makes sense because since the line is lossless, any loss of power is only in the load impedance.**

**(ii)** **The imaginary power which is related to the energy stored in the reactive fields is a function of length. This is due to the fact that for mismatched lines we have loads $\Gamma_L \neq 0$, and hence there is voltage and current variations on the line due to standing waves. The capacitive and inductive energies are different at different locations.**

## Evaluation of Arbitrary Constant $V^+$

**For Impedance calculations the knowledge of $V^+$ is not needed. However for power calculation we need to know $V^+$**

- We can obtain $V^+$ by transforming the load impedance to the generator end of the line and then applying lumpled circuit analysis.




(a)

Equivalent Lumped Circuit at the generator end

(b)

- The transformed impedance at the generator end is

$$Z'_L = Z_0 \left[ \frac{Z_L \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_L \sin \beta l} \right] \equiv R' + jX' \text{ (say)}$$

- From circuit (b) the voltage and current at AA' are

$$I_A = \frac{V_s}{Z_S + Z'_L}$$

$$V_A = Z'_s I_s = \frac{Z'_L V_s}{Z_S + Z'_L}$$

- From Fig(a) the voltage and the current at the generator end are

$$V(l) = V^+ e^{j\beta l} \{1 + \Gamma_L e^{-j2\beta l}\}$$

$$I(l) = \frac{V^+}{Z_0} e^{j\beta l} \{1 - \Gamma_L e^{-j2\beta l}\}$$

- Equating the two voltages we get

$$V^+ e^{j\beta l} \{1 + \Gamma_L e^{-j2\beta l}\} = \frac{Z'_L V_s}{Z_S + Z'_L}$$

$$V^+ = \frac{Z'_L V_s e^{-j\beta l}}{(Z_s + Z'_L)(1 + \Gamma_L e^{-j2\beta l})}$$

- Since the line is lossless, the power supplied to the transformed impedance $Z'_L$ is same as that supplied to the load $Z_L$ .

$$P_L = P = \frac{1}{2}\mathrm{Re}(V_A I^*{}_A)$$

$$= \frac{1}{2} R' \left| \frac{V_s}{Z'_L + Z_s} \right|^2$$

**Recap**

**In this course you have learnt the following**

- Power delivered to a complex load connected to a generator through a section of a line.
- Complex power at any location on the line.
- How to obtain the amplitude of the forward travelling wave?