**Module 2 : Transmission Lines**

**Lecture 6 : Loss Less Transmission Line**

**Objectives**

**In this course you will learn the following**

- What is a loss-less transmission line?
- Variation of voltage and current on a loss less line.
- Standing waves on a loss-less line.
- Voltage standing wave ratio (VSWR) and its relation to the voltage reflection co-efficient.
- Importance of VSWR and its values for various impedances.
- Concept of return-loss (RL). Return loss a measure of reflection on the line.

## Analysis of Loss Less Transmission Line

- In any electrical circuit the power loss is due to ohmic elements. A loss less transmission line therefore implies $R = 0$ and $G = 0$. For a loss less transmission line hence we get

  Propagation constant :

$$\gamma = \sqrt{j\omega L j\omega C} = j\omega\sqrt{LC} \quad = \text{Purely imaginary}$$

  That is, $\alpha \equiv 0$ and $\beta = \omega\sqrt{LC}$ .

- The charateristic impedance

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} \;=\; \sqrt{\frac{L}{C}} \quad = \text{Purely real}$$

- The reflection coefficient at any point on the line is

$$\Gamma(l) = \Gamma_L e^{-j2\beta l} = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j2\beta l}$$

- The voltage and current expressions become

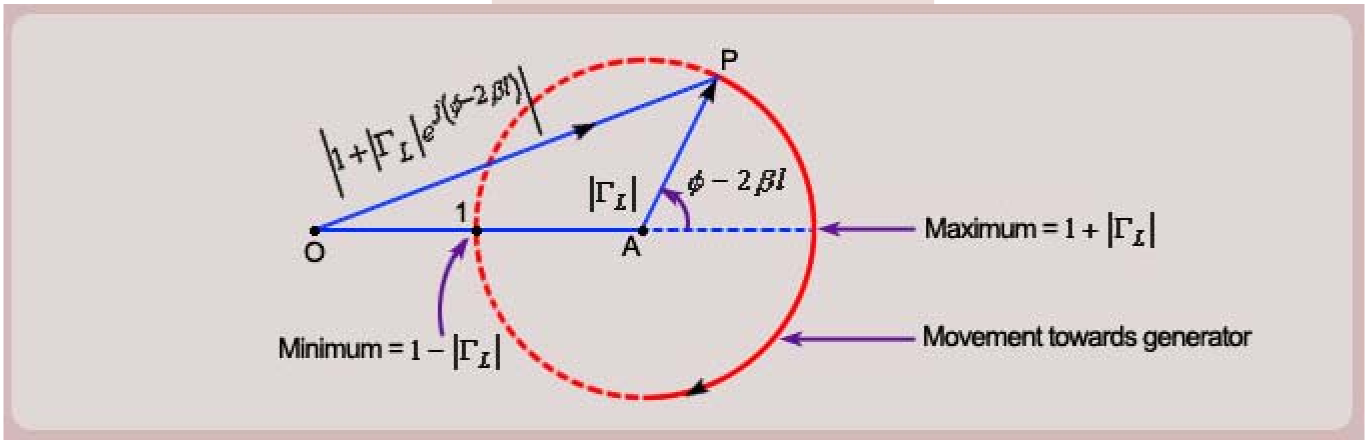
$$V(l) = V^+ e^{j\beta l}\left\{1 + \Gamma_L e^{-j2\beta l}\right\}$$
$$I(l) = \frac{V^+}{Z_0} e^{j\beta l}\left\{1 - \Gamma_L e^{-j2\beta l}\right\}$$

- Let the reflection coefficient at the load end be written in the amplitude and phase form as

$$\Gamma_L = |\Gamma_L| e^{j\phi}$$

  then we have

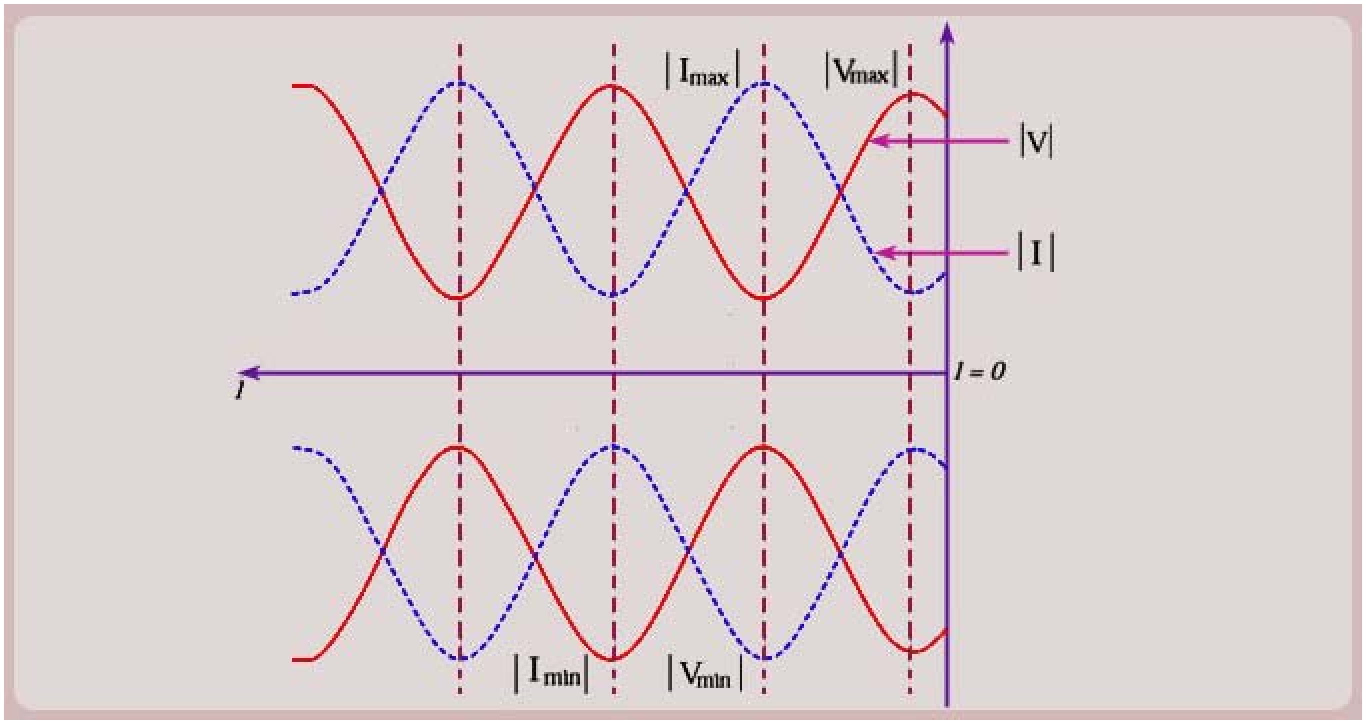
$$V(l) = V^+ e^{j\beta l}\left\{1 + |\Gamma_L| e^{j(\phi - 2\beta l)}\right\}$$
$$I(l) = \frac{V^+}{Z_0} e^{j\beta l}\left\{1 - |\Gamma_L| e^{j(\phi - 2\beta l)}\right\}$$




- As we move towards the generator the phase $(\phi - 2\beta l)$ becomes more negative and point P rotates clockwise on the dotted circle. The radius of the circle is $|\Gamma_L|$ . Length of the vector OP gives the magnitude of the quantity $(1 + |\Gamma_L| e^{j(\phi - 2\beta l)})$

## Spatial Variation of Current & Voltage

- The previous equations indicate that the amplitudes of the voltage and current vary as a function of distance on the line.




- Wherever $\phi - 2\beta l = 0$ or even multiple of $\pi$, the quantity in the brackets is maximum $(1+|\Gamma_L|)$ in the voltage expression, and minimum $(1-|\Gamma_L|)$ in the current expression. That is wherever the voltage amplitude is maximum, the current amplitude is minimum.

- Similarly wherever $\phi - 2\beta l = \text{odd multiple of } \pi$ , the voltage is minimum and the current is maximum

**Note**

**The voltage and current variation at every point on the line is $e^{j\omega t}$ only.**

- The distance between two adjacent voltage maxima (or minima) or two adjacent current maxima (or minima) corresponds to

$$2\beta l = 2\pi$$
$$\Rightarrow 2.\frac{2\pi}{\lambda}.l = 2\pi$$
$$l = \frac{\lambda}{2}$$

- The distance between adjacent voltage and current maxima or minima corresponds to

$$2\beta l = \pi$$
$$\Rightarrow l = \frac{\lambda}{4}$$

- We then say that the voltage and current are in space quardrature, i.e, when voltage is maximum the current is minimum and vice versa.

## Voltage Standing Wave Ratio

- The maximum and minimum peak voltages measured on the line are

$$|V|_{\max} = |V^{+}|\left(1 + |\Gamma_L|\right)$$
$$|V|_{\min} = |V^{+}|\left(1 - |\Gamma_L|\right)$$

- Let us define a quantity called ' Voltage Standing Wave Ratio (VSWR) ' as

$$\rho = \frac{|V|_{\max}}{|V|_{\min}}$$

- Substituting for $|V|_{\max}$ and $|V|_{\min}$ we get

$$\rho = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$
$$\text{or } |\Gamma_L| = \frac{\rho - 1}{\rho + 1}$$

- The VSWR is a measure of the reflection on the line. Higher the value of VSWR, higher is $|\Gamma_L|$ i.e., higher is the reflection and is lesser the power transfer to the load.

- Since $0 \le |\Gamma_L| \le 1$ , we get

$$1 \le \rho \le \infty$$

  VSWR of 1 corresponds to the $|\Gamma_L| = 0$ . That is the best situation.

- Ideally for a perfect match VSWR = 1. However, generally a $\text{VSWR} \le 2$ is considered acceptable in all experimental works.

## Return Loss & Reflection Co-efficient

- The return loss is defined as

Return loss (RL) = -20 log $|\Gamma_L|$ **dB**

- The return loss indicates the factor by which the reflected signal is down compared to the incident signal.
- For perfect match $|\Gamma_L| = 0$ and the return loss is $\infty$, whereas for the worst case of $|\Gamma_L| = 1$ the return loss is 0 dB
- Higher the return loss better is the match.

For acceptable value of VSWR = 2,

$$|\Gamma_L| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} = \frac{2 - 1}{2 + 1} = \frac{1}{3}$$

$$\Rightarrow \text{Return Loss RL} = -20 \log(1/3)$$

$$= 9.54$$

**The return loss should be higher than 9.54**

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**Recap**

**In this course you have learnt the following**

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