Module 2 : Transmission Lines

Lecture 5 : Standing Waves on Transmission Line & Impedance Transformation

Objectives

In this course you will learn the following

- Formation of Voltage and Current standing waves on a transmission line.
- Partial and full standing waves.
- How backward wave is developed?
- What is voltage reflection coefficient?
- Relation of the voltage reflection coefficient to the load impedance.
- Impedance transformation on a transmission line.

How standing waves are formed on a line?

- The voltage and current on the line are superposition of the two waves travelling in the opposite directions.

$$V(l) = V^{+}e^{\gamma l} + V^{-}e^{-\gamma l}$$
$$I(l) = \frac{V^{+}}{Z_0}e^{\gamma l} - \frac{V^{-}}{Z_0}e^{-\gamma l}$$

Where l is the distance measured from the load towards the generator $\left(l \equiv -x\right)$

- The result is a 'Standing Wave'. Ofcourse in general it is a partial standing wave since the amplitudes of the two travelling waves may not be equal.

- Figure shows the voltage standing wave on the line. We may note how the nature of the wave changes from 'travelling' to 'standing' when we vary V^{+} and V^{-} . (Try different values for V^{-}/V^{+})

- When , $V^{-} = 0$, there is no backward wave and therefore the net wave is the 'Forward Travelling Wave'.

- On the other hand when $V^{-} = V^{+}$, the wave will be fully standing wave.

Origin of Backward Wave

- In our discussion, the generator is connected to the left end of the line. So a voltage travelling wave moving away (the forward wave) from the generator is understandable. However, one would wonder about the origin of the backward wave. There is no energy source at the right end of the line.

- The only possibility then is, that the forward wave reaches the right end of the line and does not find correct conditions for transfering the full power to the load impedance. The part of the energy then gets reflected from the load which results into the 'Backward Wave'.

- The strength of the backward wave then should be related to the load impedance with which the line is terminated.

- Since the forward wave carrys energy towards the load, we call this wave as the 'Incident Wave'. The backward wave which carrys reflected energy from the load is called the 'Reflected Wave'. We therefore have

$$\text{Incident Wave} \;:\; V^{+} e^{-\gamma x} \equiv V^{+} e^{\gamma l}$$

$$\text{Reflected Wave} : V^{-} e^{\gamma x} \equiv V^{-} e^{-\gamma l}$$

Voltage Reflection Co-efficient and its Relation to Load Impedance

- As a measure of reflected energy we define a quantity called ' Voltage Reflection Coefficient ' as

$$\Gamma(l) = \frac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}} = \frac{V^-}{V^+} e^{-2\gamma l}$$

- Impedance seen at any distance 'l' from the load in terms of the ' Reflection Coefficient ' then is

$$Z(l) \equiv \frac{V(l)}{I(l)} = Z_0 \frac{1 + \Gamma(l)}{1 - \Gamma(l)}$$

- Inverting the relation we get the reflection coefficient at any point on the line which is at a distance 'l' from the load is

$$\Gamma(l) = \frac{Z(l) - Z_0}{Z(l) + Z_0}$$

- Now at $l = 0$, the impedance $Z(l) = \text{Load impedance } Z_L$. Therefore the reflection coefficient at the load end of the line is

$$\Gamma_L = \frac{V^-}{V^+} = \frac{\text{Reflected voltage at the load-end}}{\text{Incident voltage at the load-end}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Interesting to Note

- **The transmission line provides a medium of impedance Z_0 for the energy flow. Any departure from Z_0 creates an impedance step. This impedance step disrupts the smooth flow of energy and the part of the energy is reflected. Larger the impedance step more is the reflected energy and higher the reflection coefficient.**

Impedance at any Point on the Line

- Impedance at a distance 'l' from the load is

$$Z(l) = \frac{V(l)}{I(l)} = Z_0 \left\{ \frac{e^{\gamma l} + \left(\frac{V^-}{V^+}\right) e^{-\gamma l}}{e^{\gamma l} - \left(\frac{V^-}{V^+}\right) e^{-\gamma l}} \right\}$$ and

$$\frac{V^-}{V^+} = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

We therefore get,

$$Z(l) = Z_0 \left[\frac{Z_L(e^{\gamma l} + e^{-\gamma l}) + Z_0(e^{\gamma l} - e^{-\gamma l})}{Z_L(e^{\gamma l} - e^{-\gamma l}) + Z_0(e^{\gamma l} + e^{-\gamma l})} \right]$$

Rearranging terms and noting that $\frac{e^{\gamma l} + e^{-\gamma l}}{2} = cosh\ \gamma l$ and $\frac{e^{\gamma l} - e^{-\gamma l}}{2} = sinh\ \gamma l$, we get

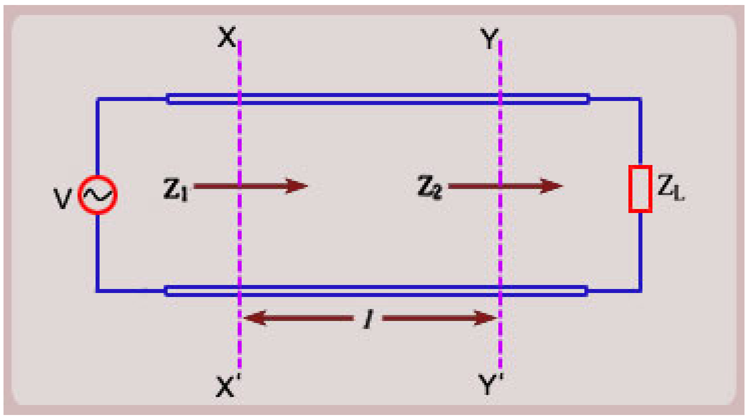
$$Z(l) = Z_0 \left[\frac{Z_L \cosh\gamma l + Z_0 \sinh\gamma l}{Z_L \sinh\gamma l + Z_0 \cosh\gamma l} \right]$$

Important

- **Impedance measured at line is not same as Z_L and is location dependent.**
- **Impedance seen by the generator for a given load impedance varies a function of the line length and consequently the power supplied by the generator becomes a function of line length.**
- **Just changing the connecting wires the circuit performance will change.**

General Impedance Transformation

- The impedance at any point of line is a transformed version of the load impedance.
- Infact there is nothing special about the load impedance. The impedance transformation can be between any two locations on the line. It should be remembered however, that the sign convention for the distance on the line must be correctly taken.
- **If the length is measured towards the generator it is taken positive.**
- **If the length is measured away from the generator, it is taken negative.**



- In the figure if we go from X to Y, 'l' is negative and if we go from Y to X, 'l' is positive.
- If the impedance at YY' is Z_2 , its transformed version at XX' will be Z_1 given by

$$Z_1 = Z_0 \left[\frac{Z_2 \cosh \gamma l + Z_0 \sinh \gamma l}{Z_2 \sinh \gamma l + Z_0 \cosh \gamma l} \right] \text{-------- (2.3)}$$

- Inverting the relation we get,

$$Z_2 = Z_0 \left[\frac{Z_1 \cosh \gamma l - Z_0 \sinh \gamma l}{-Z_1 \sinh \gamma l + Z_0 \cosh \gamma l} \right] \text{-------- (2.4)}$$

- It can be noted that the above two expressions 2.3 and 2.4 are same expression with Z_1 and Z_2 interchanged and 'l' replaced by '$-l$'

Conclusion

Expression 2.3 is the general impedance transformation relation which can be used for transforming impedance on one location on the line to the other. If the impedance is transformed to a point towards the generator, 'l' is positive, and if it is transformed to a point away from the generator, 'l' is negative.

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Recap

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