Module 2 : Transmission Lines

Lecture 11 : Transmission Line Analysis in terms of Admittance

Objectives

In this course you will learn the following

- Admittance Transformation on Transmission Line.
- Admittance Smith Chart.

Admittance Transformation on Transmission Line

- For parallel connections of transmission lines, the analysis is simpler if we deal with admittances rather than impedances. We therefore develope Admittance transformation relations for a transmission line.
- To start with, we define the characteristic admittance Y_0 which is the reciprocal of the characteristic impedance Z_0, i.e.,

$$Y_0 = \frac{1}{Z_0}$$

Also $Y(l) = 1 / Z(l)$.

$$\Rightarrow Y(l) = \frac{1}{Z_0}\left\{\frac{Z_0 \cos\beta l + jZ_L \sin\beta l}{Z_L \cos\beta l + jZ_0 \sin\beta l}\right\}$$

$$= Y_0 \left\{\frac{(1/Y_0)\cos\beta l + j(1/Y_L)\sin\beta l}{(1/Y_2)\cos\beta l + j(1/Y_0)\sin\beta l}\right\}$$

- The admittance at location 'l' therefore is

$$Y(l) = Y_0\left[\frac{Y_L \cos\beta l + jY_0 \sin\beta l}{Y_0 \cos\beta l + jY_L \sin\beta l}\right]$$

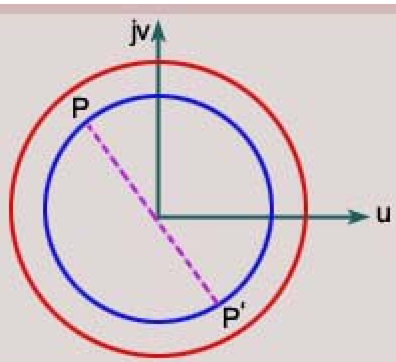
- This relation is identical to that for the impedance transformation.

Admittance Smith Chart

- Let us define the characteristic admittance as $Y_0 = \frac{1}{Z_0}$.
- Normalization of every admittance is done with the characteristic admittance Y_0 of the transmission line. An admittance $Y = G + jB$ when normalized with Y_0 is noted by
- $\bar{Y} = g + jb = \frac{G}{Y_0} + j\frac{B}{Y_0}$

The reflection coefficient is

- $$\Gamma = \left(\frac{Z - Z_0}{Z + Z_0}\right) = \frac{1/Y - 1/Y_0}{1/Y + 1/Y_0} = \frac{1 - \overline{Y}}{1 + \overline{Y}}$$



That is

- $$\Gamma = -\frac{\bar{Y} - 1}{\bar{Y} + 1} = \frac{\bar{Y} - 1}{\bar{Y} + 1}\angle\pi$$
- Now if we take normalized impedance $\bar{Z}$ equal to $\bar{Y}$ i.e., $r = g$ and $x = b$, we get Γ for $\bar{Z}$ which is 180^o out of phase with respect to the Γ for $\bar{Y}$. That means, for same numerical values, if the normalized load is impedance we get some point P on the Γ plane and if the load is admittance we get point P' which is diagonally opposite to P on the Γ-plane (see Figure). P' is obtained by rotating P by 180^o around the origin of the Γ plane.
- This is true for every $\bar{Z}$ and $\bar{Y}$ and consequently all constant resistance and constant reactance circles when rotated by 180^o around the origin of the Γ-plane give corresponding constant conductance (constant-g) and constant susceptance (constant-b) circles respectively.

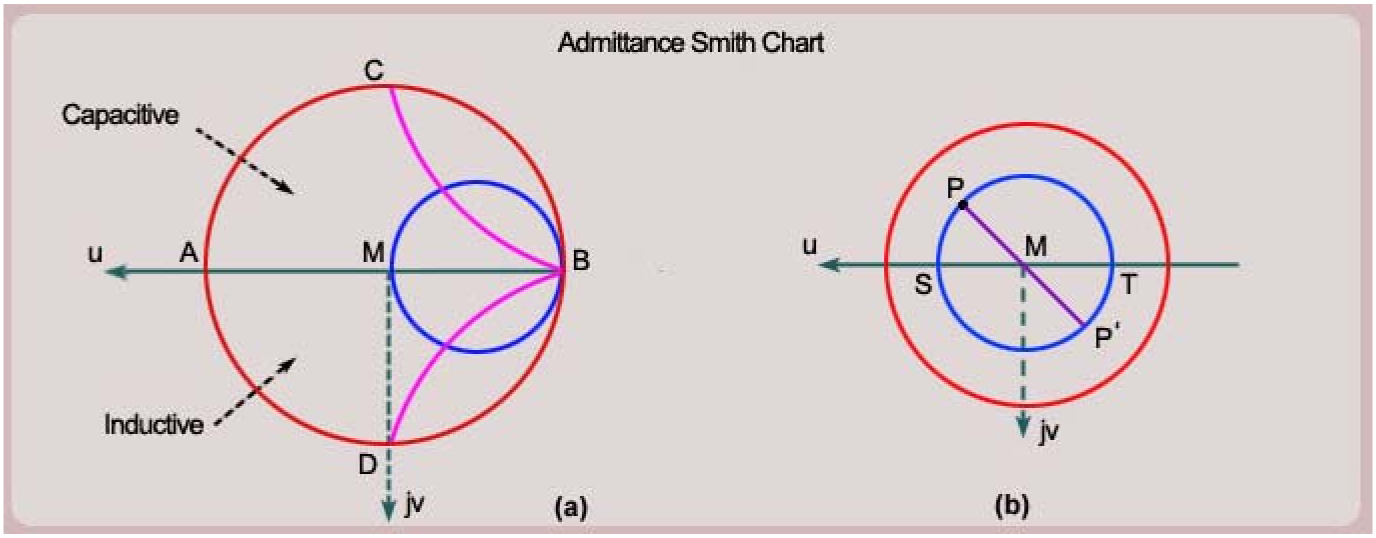
Admittance Smith Chart (contd.)

- **The** Admittance Smith Chart **therefore appears as in the following : Figure**

- The admittance Smith chart therefore is obtained by rotating the impedance Smith chart by 180^o and replacing r by g and x by b. Since it is just a matter of rotation, there is no need to have separate Smith charts for impedance and admittance.

- Generally we keep the Smith chart fixed and rotate the co-ordinate axis of the complex Γ - plane by 180^0 if the chart is used for admittance calculation.

Admittance Smith Chart (contd.)



Following points should be kept in mind while making their use of the Smith chart for transmission line calculations.

(1) While calculating phase of the reflection coefficient from the admittance Smith chart the phase must be measured from the rotated u-axis.

(2) Although the r and x can be interchanged with g and b respectively and a point (r, x) and (g, b) will have the same spatial location on the Smith chart for $r = g$ and $x = b$, physical conditions corresponding to the two will not be identical. Let us take some specific examples.

- Upper half of the Smith chart with $+jx$ represents inductive loads where as $+jb$ represents capacitive loads.

- Point A in Figure (a) is $r = 0, x = 0$ as well as $g = 0, b = 0$. But $r = 0, x = 0$, represents short circuit load hereas, $g = 0, b = 0$, represents an open circuit load. The point A therefore represents the short circuit in the impedance chart whereas it represents the open circuit in the admittance chart.

- Similarly point B in Figure (a) represents the open circuit for the impedance chart but in admittance chart it represents the short circuit.

- In Figure (b), point T corresponds to the voltage maximum if the chart is the impedance chart, and a voltage minimum if the chart is the admittance chart. The opposite is true for point S. Now since the voltage maximum coincides with the current minimum and vice-versa, the point T in admittance Smith chart represents the location of the current maximum and point S represents location of the current minimum. So we find that the voltage standing wave pattern and the impedance have the same relationship as the current standing wave pattern and the admittance.

- As we have seen, the reflection coefficients for same normalized impedance and admittance values are 180^o out of phase. Therefore any normalized impedance can be converted to normalized admittance and vice-versa by taking a diagonally opposite point on the constant VSWR circle. In Figure (b), P' gives normalized admittance corresponding to the normalized impedance at P. We can therefore switch between admittance and impedance Smith charts freely without any additional computation.

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Recap

In this course you have learnt the following

- Admittance Transformation on Transmission Line.

- Admittance Smith Chart.