Module 2 : Transmission Lines

Lecture 9 : Graphical Approach for Transmission Analysis

Objectives

In this course you will learn the following

- Impedance transformation from the complex impedance plane to the complex reflection coefficitent plane.
- Constant resistance and constant reactance circles on complex - Γ plane.
- Simth chart - Orthogonal impedance coordinate system on complex - Γ plane.
- Location of various impedances on the Smith chart.

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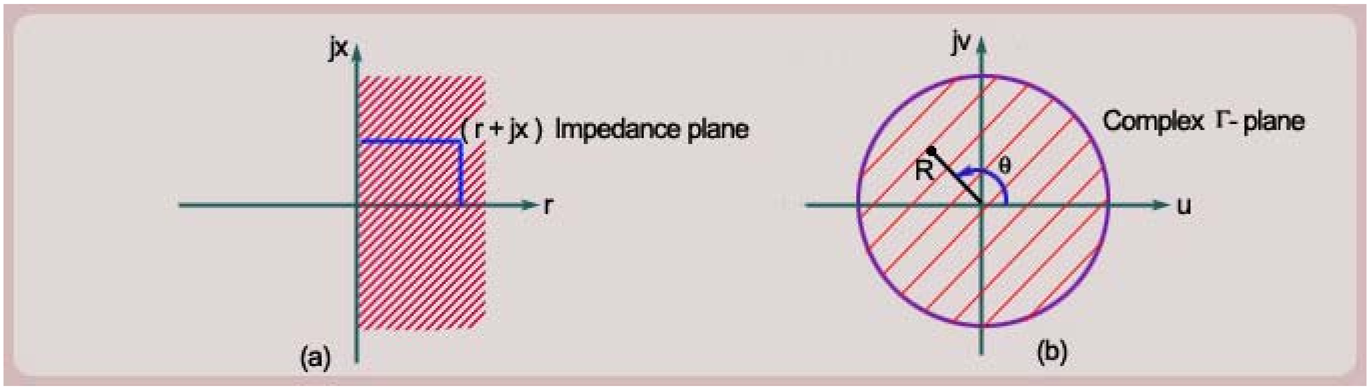
Lecture 9 : Graphical Approach for Transmission Analysis

The graphical representation given in the following mainly describes the impedance/admittance characteritics of a transmission line.

Complex Impedance (Z) & Reflection co-efficient (Γ) planes



- Let us define the normalized impedance

$$\overline{Z} = \frac{Z}{Z_0} \equiv r + jx$$

- For passive loads

$$r \geq 0 \text{ and } \infty \geq x \geq -\infty$$

- A passive load can be denoted by a point in the right half of the complex Z-plane as shown in Fig(a)
- The complex Reflection Coefficient is

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\overline{Z} - 1}{\overline{Z} + 1}$$
$$= \frac{r + jx - 1}{r + jx + 1} = \frac{(r - 1) + jx}{(r + 1) + jx}$$

- The complex Γ can be expressed in cartesian and polar form as

$$\Gamma \equiv u + jv \equiv Re^{j\theta}$$

- Since for passive loads $|\Gamma| = R \leq 1$, the reflection coefficient can be denoted by a point with the unity circle in the complex Γ- plane, as shown in Fig (b). 'R' denotes the magnitude of the reflection coefficient and θ denotes the phase of the reflection coefficient.
- Since there is one-to-one mapping between $\overline{Z}$ to Γ, the entire right half Z-plane is mapped on to the region within the unity circle in the Γ-plane.

Transformation from Z to Γ

- Let us transform the points from the $\bar{Z}$ - plane to Γ - plane.

$$\bar{Z} = \frac{1+\Gamma}{1-\Gamma}$$

$$\Rightarrow \qquad r + jx = \frac{1+(u+jv)}{1-(u+jv)}$$

- Separating real and imaginary parts, we get

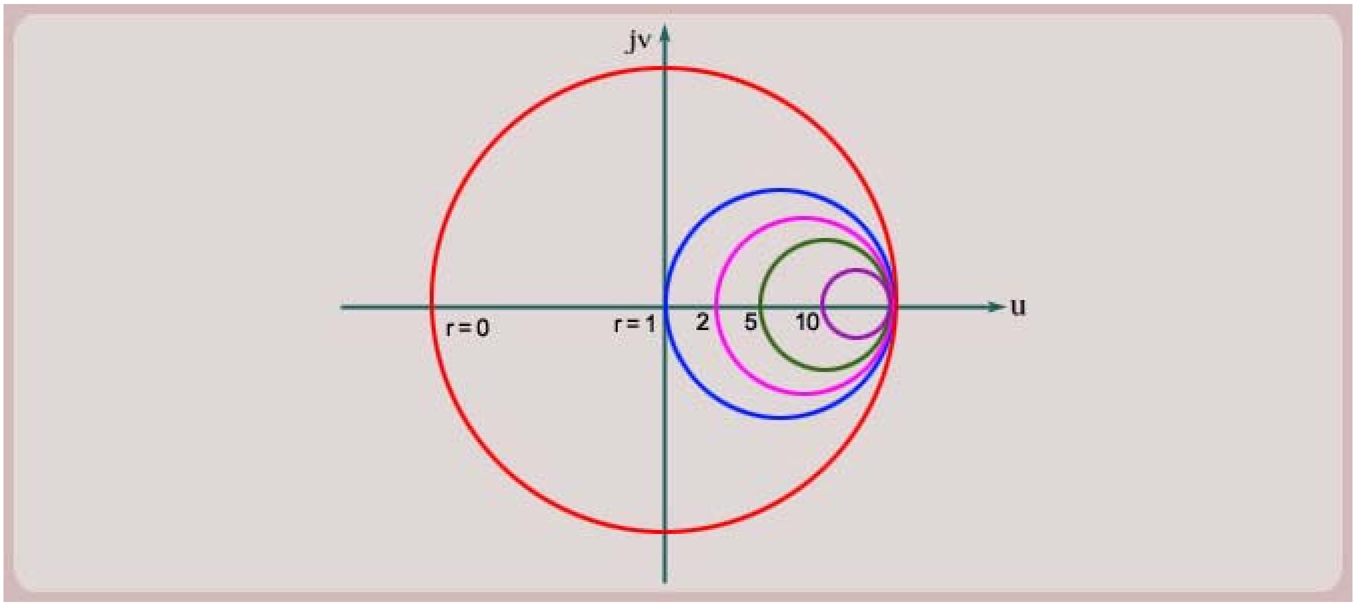
$$u^2 - 2\left(\frac{r}{r+1}\right)u + v^2 + \left(\frac{r-1}{r+1}\right) = 0 \qquad \text{--------- (2.5)}$$

$$u^2 + v^2 - 2u - \left(\frac{2}{x}\right)v + 1 = 0 \qquad \text{--------- (2.6)}$$

Equations (2.5) and (2.6) are the equations of circles. Equation (2.5) represents constant resistance circles and equation (2.6) represent constant reactance circles.

Constant Resistance Circles

- The constant resistance circles have their centres at ($\frac{r}{r+1}$, 0) and radii ($\frac{1}{r+1}$). Figure below shows the constant resistance circles for different values of r ranging between 0 and ∞.



- We can note following things about the constant resistance circles.

(a) **The circles always have centres on the real Γ-axis (u -axis).**

(b) **All circles pass through the point (1,0) in the complex Γ plane.**

(c) **For $r = 0$, the centerof the circle lies at the origin of the Γ plane and it shifts to the right as r increases.**

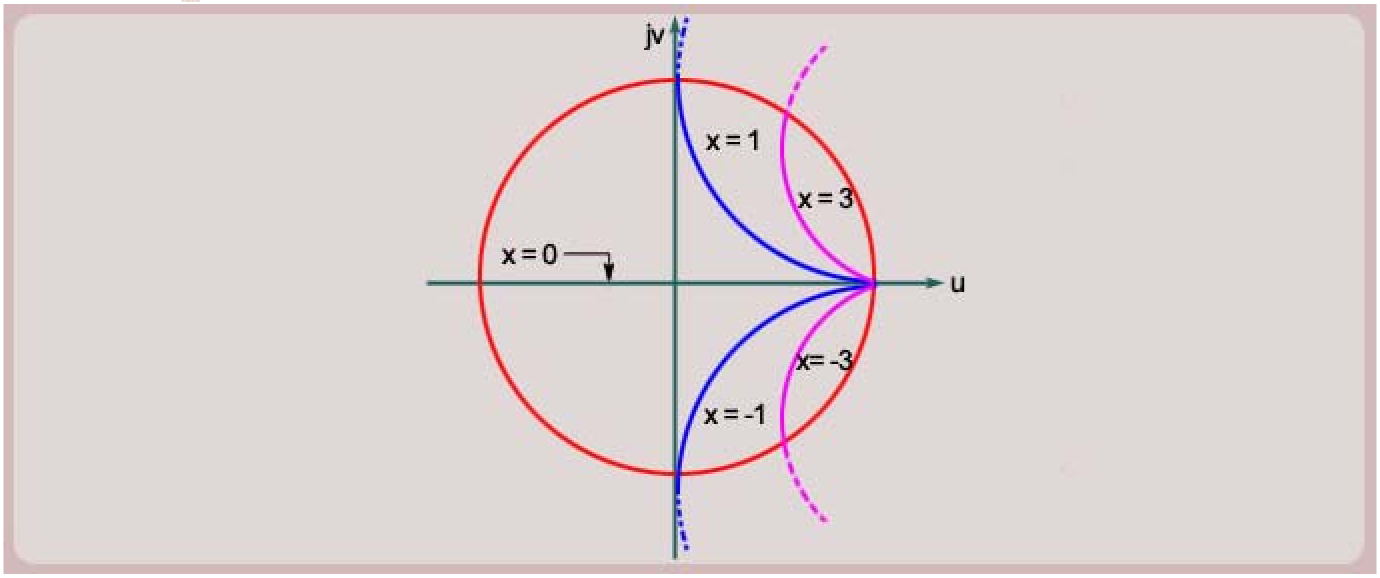
(d) **As r increases the radius of the circle goes on reducing and for $r \to \infty$ the radius approaches zero, i.e., the circle reduces to a point.**

(e) **The outermost circle with center (0,0) and radius unity, corresponds to $r = 0$ or in other words represents purely reactive impedances.**

(f) **The right most point on the unity circle, $(1,0)$ represents $r = 0$ as well as $r = \infty$.**

Constant Reactance Circles

- The constant reactance circles have their centers at $(1, \frac{1}{x})$ and radii $(\frac{1}{x})$. The centres for these circles lie on a vertical line passing through point (1,0) in the Γ-plane. The constant reactance circles are shown in figure below for different values of x



Note again that only those portions of the circles are of significance which lie within the unity circle in the Γ-plane. The curves shown dotted portion do not correspond to any passive load impedance.

We can note following things about the constant reactance circles:

(a) **These circles have their centers on a vertical line passing through point $(1, 0)$.**

(b) **For positive x the center lies above the real Γ-axis and for negative x, the center lies below the real Γ-axis.**

(c) **For $x = 0$ the center is at $(1, \pm\infty)$ and radius is ∞.This circle therefore represents a straight line.**

(d) **As the magnitude of the reactance increases the center moves towards the real Γ-axis and it lies on the real Γ-axis at (1,0) for $x = \pm\infty$.**

(e) **As the magnitude of the reactance increases, the radius of the circle, $(\frac{1}{x})$,decreases and it approaches zero as $x \to \pm\infty$.**

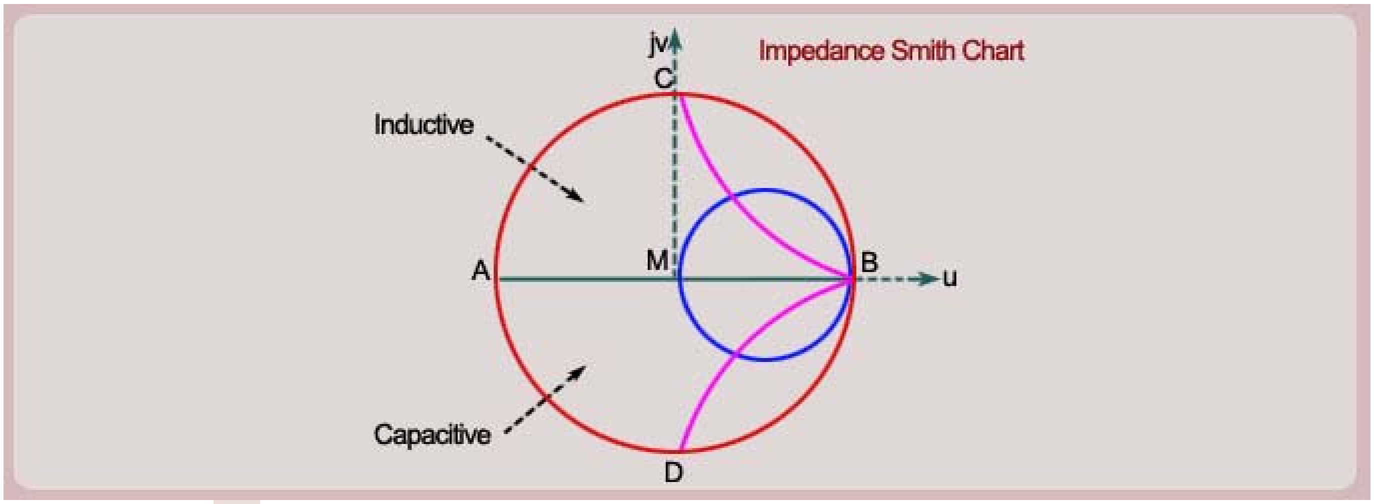
(f) **All circles pass through the point $(1, 0)$.**

(g) **The real Γ-axis (u-axis) corresponds to $x = 0$ and therefore represents real impedances, i.e., purely resistive impedances.**

(h) **The right most point on the unity circle, $(1, 0)$, corresponds to $x = 0$ as well as $x = \pm\infty$.**

The Smith Chart

- The Smith chart is a graphical figure which is obtained by superposing the constant resistance and the constant reactance circles within the unity circle in the complex Γ-plane. Since we have mapped here the impedances to the Γ-plane, let us call this Smith chart the Impedance Smith chart.



- Generally the u, v axes are not drawn on the Smith chart. However one should not forget that the Smith chart is a figure which is drawn on the complex Γ-plane with its center as origin.

- The intersection of constant resistance and constant reactance circles uniquely defines a complex load impedance on the Γ-plane.

Let us identify some special points on the Smith Chart.

(a) **The left most point A on the smith chart corresponds to $r = 0, x = 0$ and therefore represents ideal short-circuit load.**

(b) **The right most point B on the Smith chart corresponds to $r = \infty$, $x = \infty$ and therefore represents ideal open circuit load.**

(c) **The center of the Smith chart M , corresponds to $r = 1, x = 0$ and hence represents the matched load.**

(d) **Line AB represents pure resistive loads and the outermost circle passing through A and B represents pure reactive loads.**

(e) **The upper most point C represents a pure inductive load of unity reactance and the lower most point D represents a pure capacitive load of unity reactance.**

(f) **In general the upper half of the Impedance Smith Chart represents the complex inductive loads and the lower half represents the complex capacitive loads.**

A ready made Smith Chart looks as in the following : <u>Figure</u>

Recap

In this course you have learnt the following

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- Location of various impedance on the Smith chart.