Module 2 : Transmission Lines

Lecture 12 : Applications of Transmission Lines

Objectives

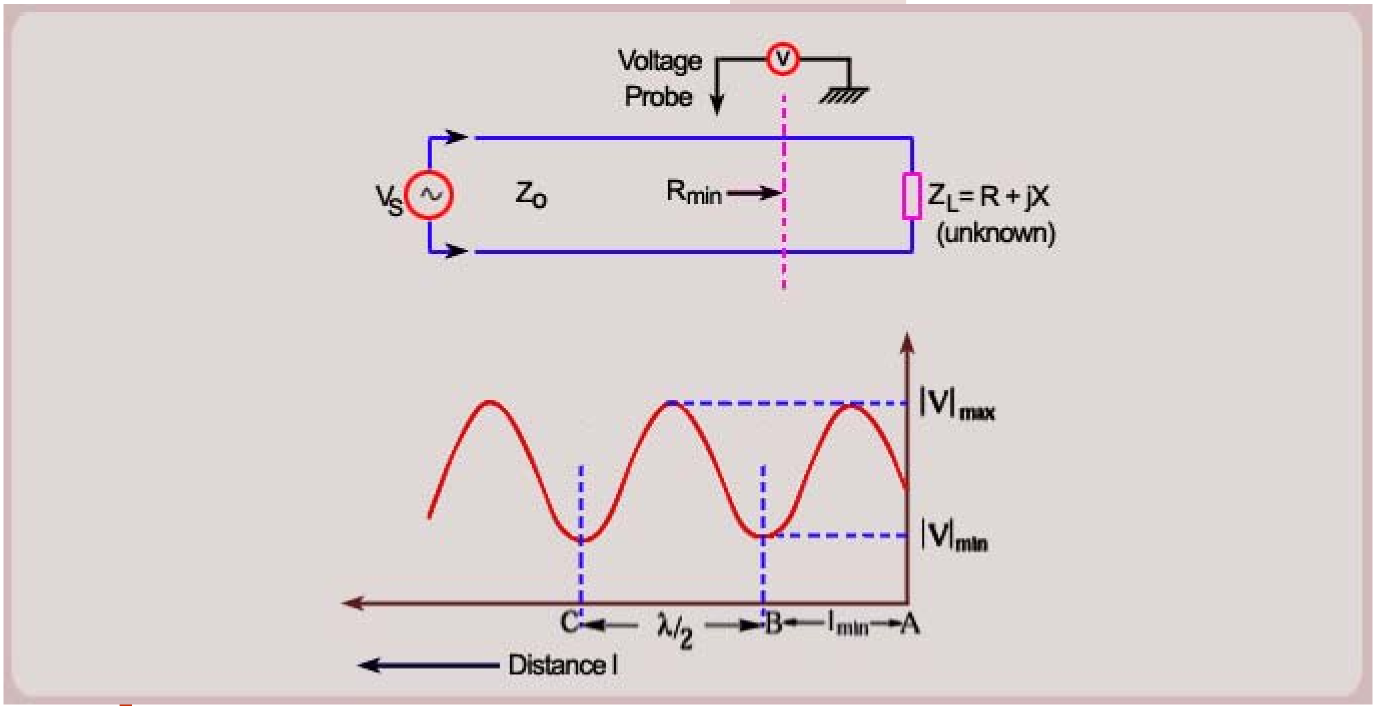
In this course you will learn the following

- Various applications of transmission lines.
- How to measure complex impedance at high frequencies where phase measurement is unreliable.
- How and why to use sections of transmission line as reactive elements in the high frequency circuits.
- Use of Smith chart and to design transmission line sections for realizing reactive impedances.

Measurement of Unknown Impedance

- The unknown impedance which is to be measured is connected at the end of the transmission line as shown in Figure below. The transmission line is excited with a source of desired frequency ω. From the standing wave pattern, three quantities, namely the maximum voltage $|V|_{max}$, minimum voltage $|V|_{min}$, and the distance of the voltage minimum from the load is measured. The ratio of $|V|_{max}$ and $|V|_{min}$ gives the VSWR on the line.

$$\rho = \frac{|V|_{max}}{|V|_{min}}$$



- We know that at point B on the transmission line where the voltage is minimum, the impedance is real and its value is $R_{min} = Z_0/\rho$. The impedance R_{min} is nothing but the transformed value of the load impedance Z_L. We can therefore obtain the unknown impedance Z_L by transforming back R_{min} from point B to point A. Let the distance of the voltage minimum from the load be l_{min}. Since the transformation from B to A is away from the generator, the distance BA is negative. The unknown impedance therefore is

$$Z_L \equiv R + jX = Z_0\left[\frac{R_{min}\cos\beta(-l_{min}) + jZ_0\sin\beta(-l_{min})}{Z_0\cos\beta(-l_{min}) + jR_{min}\sin\beta(-l_{min})}\right]$$

- Substituting for $R_{min} = Z_0/\rho$, we get

$$Z_L \equiv R + jX = Z_0\left[\frac{\frac{Z_0}{\rho}\cos\beta l_{min} - jZ_0\sin\beta l_{min}}{Z_0\cos\beta l_{min} - j\frac{Z_0}{\rho}\sin\beta l_{min}}\right]$$

$$= Z_0\left[\frac{1 - j\rho\tan\beta l_{min}}{\rho - j\tan\beta l_{min}}\right]$$

- Separating real and imaginary parts we get

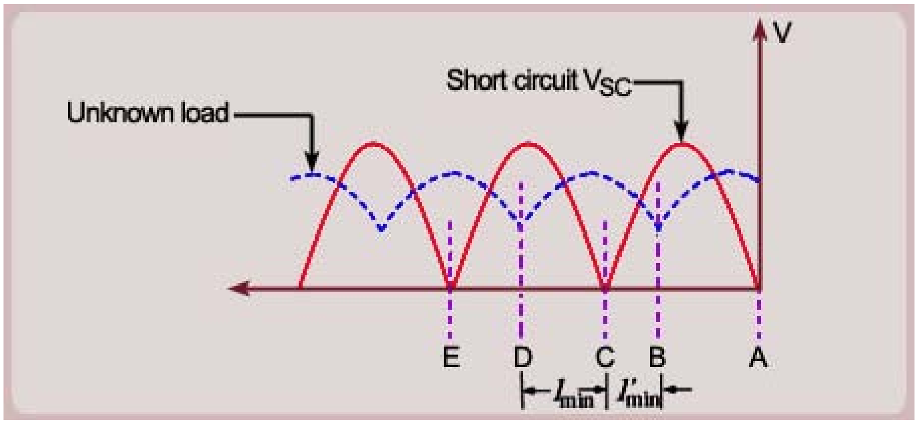
$$R = \frac{\rho(1 + \tan^2 \beta l_{min})}{\rho^2 + \tan^2 \beta l_{min}}$$

$$X = \frac{(1 - \rho^2)\tan \beta l_{min}}{\rho^2 + \tan^2 \beta l_{min}}$$

Measurement of Unknown Impedance (Practical Consideration)

- While practically implementing the above scheme one would also notice that invariably the location of unknown impedance Z_L is not precisely defined. As a result the measurement of l_{min} may have some error which in turn will result into an error in the load impedance.

- To overcome this problem the measurement is carried out in two steps. First, the standing wave pattern is obtained with the unknown load as explained above. Now replace the unknown impedance by an ideal short-circuit and obtain the standing wave pattern again. The two standing wave patterns are shown as below



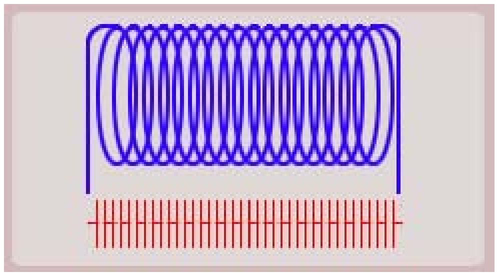
- At the short circuit point (which is also the location of the unknown impedance) the voltage V_{sc} is zero. The voltage is also zero at points which are multiple of $\lambda/2$ away from it. i.e., at point C, E etc. The points C, E etc represent impedance conditions identical to that at A, that is, the impedance at C or E is equal to the unknown impedance. The unknown impedance therefore can be obtained by transforming impedance Z_0/ρ at B or D to point C. If impedance is transformed from D to C the distance l_{min} is negative, whereas if the transformation is made from B to C the distance l'_{min} is positive. The unknown impedance therefore can be evaluated as

$$Z_L = R + jX = Z_0\left[\frac{1 - j\rho\tan\beta l'_{min}}{\rho - j\tan\beta l'_{min}}\right] = Z_0\left[\frac{1 + j\rho\tan\beta l_{min}}{\rho + j\tan\beta l_{min}}\right]$$

- One can note here that $l_{min} + l'_{min} = \lambda/2$. In the impedance calculation either of l_{min} or l'_{min} can be used. As long as the sign of the distance is taken correctly it does not matter which of the minima is taken for impedance transformation.

Transmission Line as a Circuit Element

- At frequencies of hundreds and thousands of MHz where lumped elements are hard to realize, the use of sections of transmission line as reactive elements may be more convenient.



- The turns in the wire of the inductor have small distributed capacitors.

- As the frequency increases, the capacitance begins to play a role in the response of the circuit and beyond the resonant frequency, the capacitance predominates the response. That is the inductance coil effectively behaves like a capacitor.

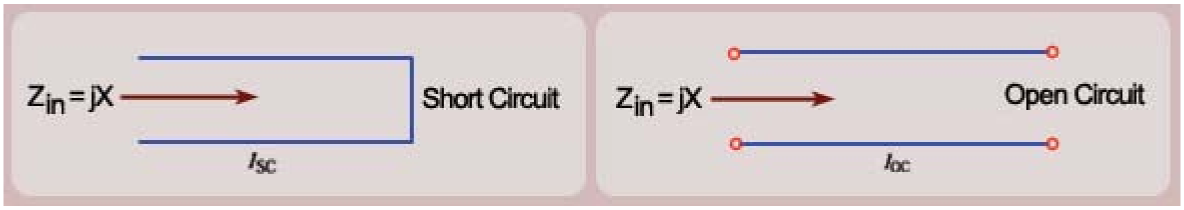
- Similarly, for a capacitance, there exist lead inductance. As the frequency increases, the lead inductance starts dominating over the capacitance and beyond the resonant frequency of the LC combination, the capacitor effectively behaves like an inductor.

- So, it is clear that at high frequencies, realization of reactive element is not that simple.

- On the other hand at high frequencies, the wavelength and the length of the transmission line section reduces and becomes more manageable.

Use of Smith Chart for calculating l_{sc} and l_{oc}

- From the impedance relation we can see that if a line of length l is terminated in a short circuit or open circuit (shown in Figure below) the input impedance of the transmission line is purely reactive.



- The input impedance of a loss-less line can be written as

$$Z_{in} = jZ_0 \tan \beta l \qquad \text{for short circuit load}$$
$$= -jZ_0 \cot \beta l \qquad \text{for open circuit load}$$

- Since the range of 'tan' and 'cot' functions is from $-\infty$ to $+\infty$, any reactance can be realized by proper choice of l. Moreover, any reactance can be realized by either open or short circuit termination. This is a very useful feature because depending upon the transmission line structure, terminating one way may be easier than other. For example, for a microstrip type line (see in later section), realizing an open circuit is easier as short circuit would require drilling a hole in the substrate.
- Now if a reactance X is to be realized in a high frequency circuit one can use a short circuited line of length l_{sc} or an open circuited line of length l_{oc} given by

$$l_{sc} = \frac{1}{\beta}\tan^{-1}\left(\frac{X}{Z_0}\right)$$
$$l_{oc} = \frac{1}{\beta}\cot^{-1}\left(\frac{-X}{Z_0}\right)$$

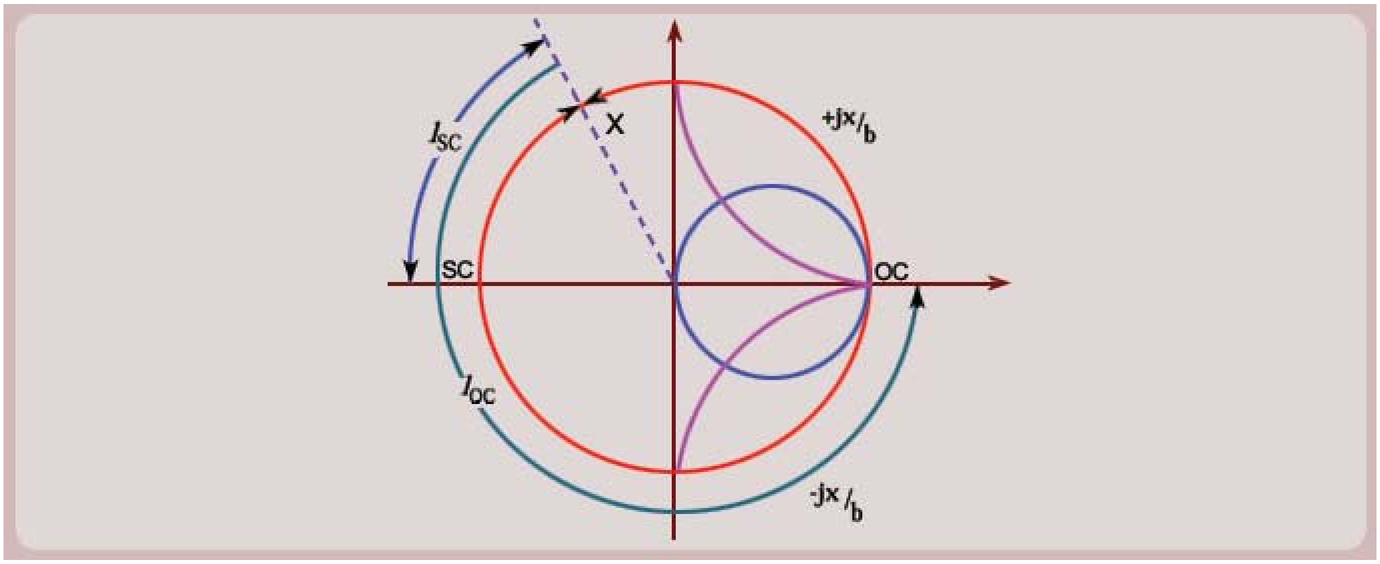
- Smith chart can be used to find l_{sc} or l_{oc} as follows:
- Choose suitable characteristics impedance of the line, Z_0
- Normalize the reactance to be realized (X) by Z_0 to give normalized reactance x .

(a) Mark the reactance jx to be realized on the Smith chart to get point 'X' in Figure.

(b) Move in anticlockwise direction from point X to the short circuit (SC) point on the Smith chart to get l_{sc} (see Figure below).

(c) Move from X in the anticlockwise direction upto open circuit (OC) to get l_{oc} as indicated in Figure.

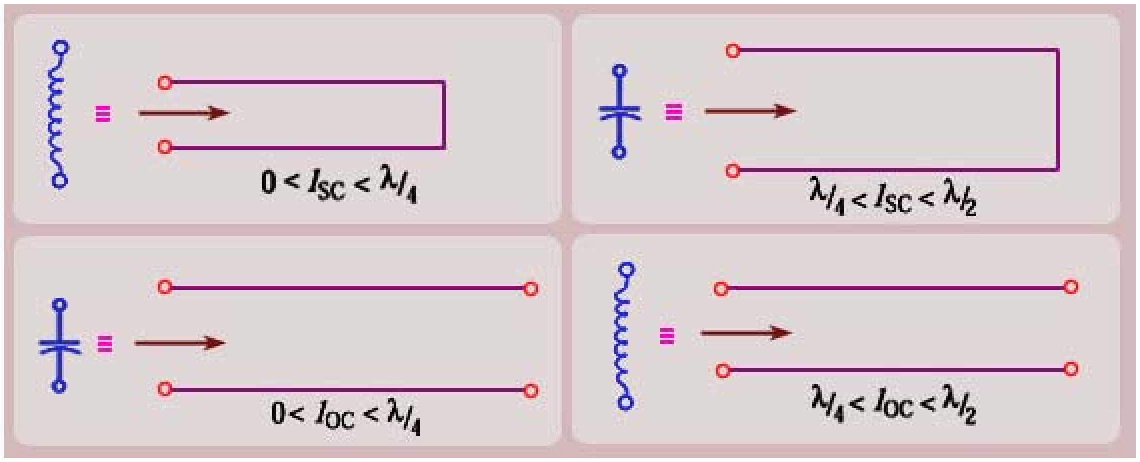
(d) Note here that instead of reactance if we had to realize a normalized susceptance b, the procedure is identical except that SC and OC points are interchanged.



l_{SC}
X
+jx/b
SC
OC
l_{OC}
-jx/b

Line length and their Equivalent Reactants

- The following figure shows the range of transmission line lengths and the corresponding reactances which can be realized at the input terminals of the line.



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Recap

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