

Module 6 : Wave Guides

Lecture 44 : Transverse Electric Mode in Rectangular Waveguide

Objective

In this course you will learn the following

- Cutt-off Frequecy of TE and TM mode.

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- The analysis of TE mode in a rectangular waveguide can be carried out on the line similar to that of the TM mode.

- For TE mode we have

$$E_z = 0 \quad \text{and} \quad H_z \neq 0 \quad \text{-----} (6.57)$$

- The wave equation is solved for H_z in this case.

- In the case of TM mode the wave equation was solved for E_z which was tangential to all the four walls of the waveguides. We therefore had boundary conditions on E_z .

- In the TE case however the independent component H_z is tangential two the walls of the waveguide which do not impose any boundary conditions on H_z . The tangential component of magnetic field is balanced by the appropriate surface currents on the walls of the waveguides.

- The analysis procedure for TE mode therefore is slightly different than that of the TM mode. We have seen in the case of parallel plane waveguide that the tangential component of the magnetic field is maximum at the waveguide walls. Also in Cartesian co-ordinate system the solution to the wave equation are sinusoidal in nature.

- One can note that for $x = 0$, $x = a$ (vertical walls) and for $y = 0$, $y = b$ (horizontal walls) the tangential component of magnetic field is maximum.

- Substituting for H_z from

$$H_z = D \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z} \quad \text{-----} (6.58)$$

and $E_z = 0$ in 6.31, 6.32, 6.33 and 6.34, we get the transverse field components as

$$E_x = \frac{+j\omega\mu}{h^2} D \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z} \quad \text{-----} (6.59)$$

$$E_y = \frac{-j\omega\mu}{h^2} \left(\frac{m\pi}{a}\right) D \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z} \quad \text{-----} (6.60)$$

$$H_x = \frac{+j\beta}{h^2} \left(\frac{m\pi}{a}\right) D \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z} \quad \text{-----} (6.61)$$

$$H_y = \frac{j\beta}{h^2} \left(\frac{n\pi}{b}\right) D \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z} \quad \text{-----} (6.62)$$

- In this case also we get

$$h^2 = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \text{-----} (6.63)$$

- Following observations can be made regarding the TE mode :

- (1) The fields for the TE modes have similar behaviour to the fields of the TM modes i.e they exist in the form of discrete

pattern, they have sinusoidal variations in x and y directions, indices m and n represent number of half cycles of the field amplitudes in x and y direction respectively and so on.

(2) Unlike TM mode both indices m and n need not be non-zero for the existence of the TE mode. However, if both the indices are zero, the magnetic field is independent of space and therefore cannot exist. In other words, TE_{00} mode cannot exist but TE_{m0} and TE_{0n} modes can exist.

(3) The lowest order mode for the TE case therefore would be TE_{10} and TE_{01} .

Cutt-off Frequency of TE and TM mode

For both TE_{mn} and TM_{mn} modes the phase constant is given by

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad \text{----- (6.64)}$$

For the mode to be travelling β has to be a real quantity. If β becomes imaginary then the fields no more remain travelling but become exponentially decaying

The frequency at which β changes from real to imaginary is called the cut-off frequency of the mode. At cut-off frequency therefore $\beta = 0$ giving

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right]^{1/2} \quad \text{----- (6.65)}$$

$$\Rightarrow f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad \text{----- (6.66)}$$

The cut-off frequencies for lowest TM and TE modes i.e. TM_{11} , TE_{10} & TE_{01} can be obtained from eqn. 6.70 as

$$f_{cTM_{11}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} \quad \text{----- (6.67)}$$

$$f_{cTE_{10}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \frac{\pi}{a} = \frac{1}{2a\sqrt{\mu\epsilon}} \quad \text{----- (6.68)}$$

$$\text{and } f_{cTE_{01}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \frac{\pi}{b} = \frac{1}{2b\sqrt{\mu\epsilon}} \quad \text{----- (6.69)}$$

Since by definition we have $a > b$ we get the frequencies as

$$f_{cTE_{10}} < f_{cTE_{01}} < f_{cTM_{11}} \quad \text{----- (6.70)}$$

We can make an important observation that if at all the electro magnetic energy travels on a rectangular waveguide its frequency has to be more than the lowest cut-off frequency i.e. $f_{cTE_{10}}$.

As the order of the mode increases the cut-off frequency also increases i.e. with increasing frequency there is possibility of existence of higher order mode.

The very first mode that propagates on the rectangular waveguide is TE_{10} mode and therefore this mode is called the dominant mode of the rectangular waveguide. The cut-off frequency for the dominant mode is

$$\lambda = \frac{c}{f_{cTE_{10}}} = 2a \quad \text{----- (6.71)}$$

The equation suggest that for propagation of an electro magnetic wave inside a rectangular waveguide the width of a waveguide should be greater than half the wave length of the wave.

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Recap

In this course you have learnt the following

- Cutt-off Frequey of TE and TM mode.