Module 2 : Transmission Lines

Lecture 10 : Transmisssion Line Calculations Using Smith Chart

Objectives

In this course you will learn the following

- What is a constant VSWR circle on the Γ - plane?

- Properties of constant VSWR circles.

- Calculations of load reflection coefficient.

- Calculation of reflection co-efficient at a distance from the load.

- Calculation of transformed impedance at a distance from the load.

- VSWR on the line.

- Location of voltage maximum or minimum.

- Identifying the type of load.

Recap

In this course you have learnt the following

- What is a constant VSWR circle on the Γ - plane?

- Properties of constant VSWR circles.

- Calculations of load reflection coefficient.

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Constant VSWR Circles

■ The reflection coefficient at any location on the line is

$$\Gamma(l) = \Gamma_L e^{-j2\beta l}$$

■ Let Γ_L be written in polar form as

$$\Gamma_L = |\Gamma_L| e^{j\theta_L}$$
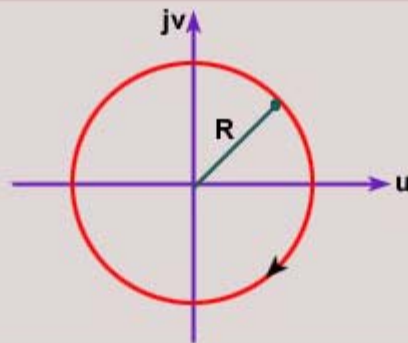
Then we get,

$$\Gamma(l) \equiv R e^{j\theta} = |\Gamma_L| e^{j(\theta_L - 2\beta l)}$$

Where,

$$R \equiv |\Gamma(l)| = |\Gamma_L|$$

and

$$\theta \equiv \theta_L - 2\beta l$$

■ As we move along the line towards the generator l increases and consequently, θ becomes more negative. However, R remains constant.
The $\Gamma(l)$ point moves on a circle with centre (0,0) and radiurs R in the complex Γ - plane.

■ If we move towards the generator, l is positive and the point moves clockwise on the circle. For movement away from the generator, the point moves anti-clockwise on the circle.

■ For a circle, $\mathrm{R} = |\Gamma_L| = \text{constant}$. Since VSWR $\rho = \dfrac{\left(1 + |\Gamma_L|\right)}{\left(1 - |\Gamma_L|\right)}$, ρ is constant for the circle.

That is, all points on the circle have same VSWR, ρ . This circle therefore is called the CONSTANT VSWR CIRCLE.

Properties of Constant VSWR Circles

■ The constant VSWR circles are shown below
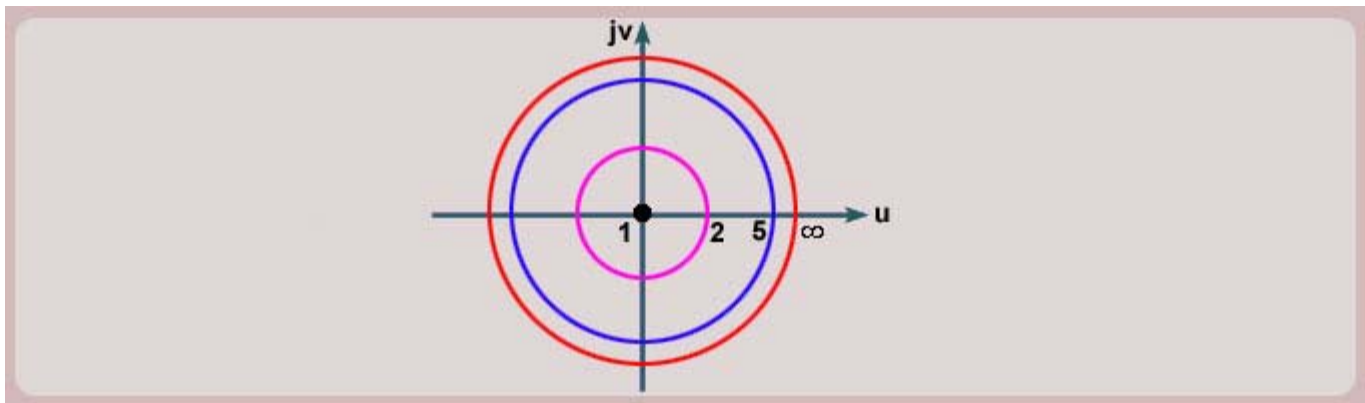
We can make following observations about the constant VSWR circles:

■ All the circles have same center, the origin of the complex Γ-plane.

■ The origin in the Γ-plane represents $|\Gamma_L| = 0$ or $\rho = 1$. As we move radially outwards the $|\Gamma_L|$ and hence ρ increases monotonically and for the outermost (unity) circle, $|\Gamma_L| = 1$ and $\rho = \infty$.

■ The origin corresponds to the condition $|\Gamma_L| = 0$, i.e., no reflection on the line. This point represents the best matching of the load as there is no reflected power on the line. For the outer most circle $|\Gamma_L| = 1$, and we get the worst impedance matching as the entire power is reflected on the transmission line. We can therefore make a general statement that closer is the point to the origin of the Γ-plane, (i.e., the center of the Smith chart) better is the impedance matching.

■ As we have defined earlier, the $+l$ indicates a distance towards the generator. As l becomes more positive, θ decreases and the point moves clockwise on the constant VSWR circle. If we move away from the generator, l becomes negative and then the point on the circle moves in the anticlockwise direction.

Smith chart is a very useful tool for solving transmission line problems. A variety of calculations can be carried out using the Smith chart without getting into complex computations.
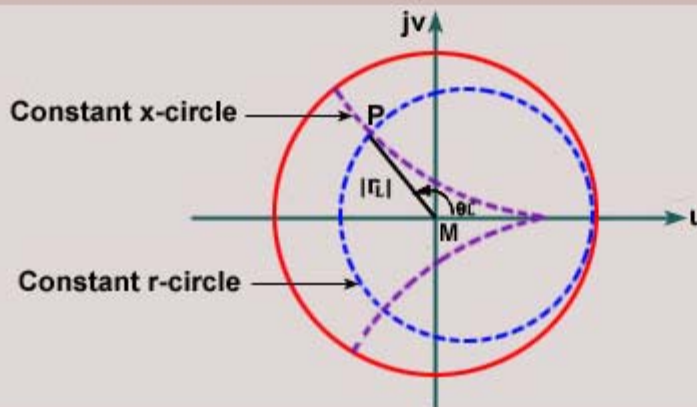
(A) Calculation of Load Reflection Co-efficient

■ Let us find the reflection coefficient for a load impedance $R + jX$. First normalize the impedance with the characteristic impedance Z_0 to get

$$r + jx \equiv \frac{R + jX}{Z_0}$$

■ Identify the constant resistance and the constant reactance circles corresponding to r and x respectively. Intersection of the two circles marks the load impedance $r + jx$ on the Smith chart as point P (see Figure)

■ Measure the radial distance of P from the centre of the chart M. This is the magnitude of the load reflection coefficient $|\Gamma_L|$.

■ The angle which the radius vector MP makes with the $+u$-axis is the phase of the load reflection coefficient θ_L.

Note

The Smith chart should be placed in such a way that the most clusterred portion of the chart lies on the right side. The horizontal line towards right then indicates the real $+u$ - axis.
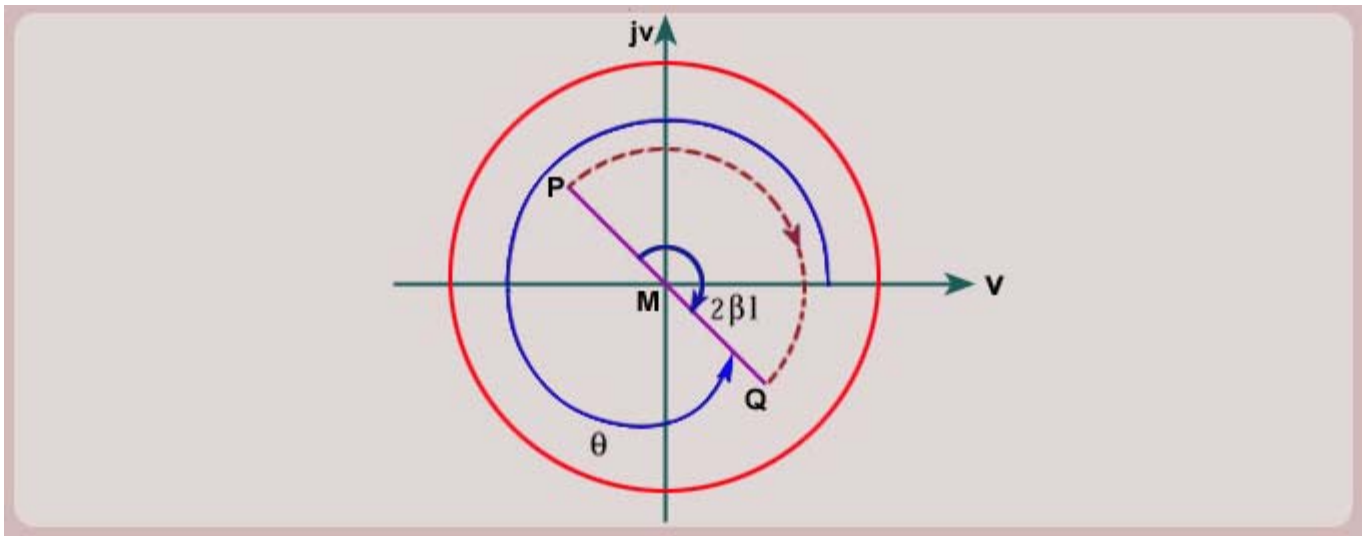
(B) Calculation of Reflection Co-efficient at a Distance from the Load

- Let the transmission line be terminated in a load impedance Z_L. Let us find the reflection coefficient at a distance l from the load using the Smith chart.

- First, mark the load impedance as described in (A)

- Draw the constant VSWR circle passing through P.

- Rotate the radius vector MP by an angle $2\beta l$ in the clockwise direction to get to point Q (See Fig). Radial distance MQ gives the magnitude of the reflection coefficient, $|\Gamma(l)|$. Angle which the radius vector MQ makes with the $+u$-axis gives the phase of $\Gamma(l)$, θ.

(C) Calculation of Transformed Impedance at a Distance from the Load

- Suppose we have to obtain the transformed impedance at a distance l from the load. Carry out the steps in (a) and (b) to get to point Q. Now instead of measuring reflection coefficient, identify the constant resistance and the constant reactance circles passing through Q. They provide the transformed normalized resistance and reactance $r(l)$ and $x(l)$ respectively. Multiplying by Z_0 we get the transformed impedance $Z(l) = Z_0(r(l) + jx(l))$.

- The same procedure is used for transforming impedance from any point on the line to any other point.

- However if the distance `l' is away from the generator, it should be treated negative and hence the rotation of the radius vector must be by $2\beta l$ in the anticlockwise direction.
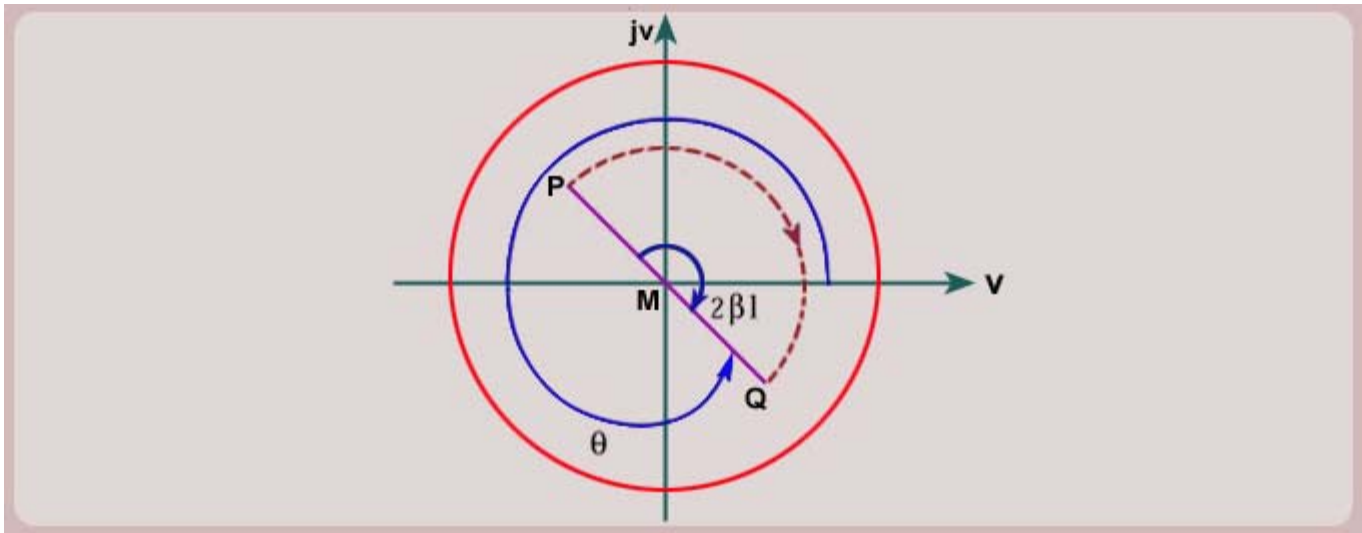
(D) VSWR on the line

- As we have seen earlier if the load impedance Z_L is not equal to Z_0 there is a standing wave on the transmission line. We also know that the maximum impedance seen on the line $R_{max} = Z_0\rho$. This means that the maximum normalized impedance $r_{max} = R_{max}/Z_0$ measured on the line is nothing but the VSWR, ρ. The task of finding ρ is then very simple. Find the normalized impedance (r_{max}) corresponding to point T. Then,

$$\rho = r_{max}$$
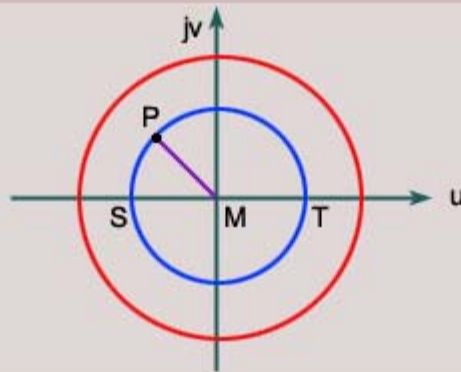
- Mark the impedance on the Smith chart (P).

- Draw the circle with center as the centre of the Smith chart (M) and radius PM.

- Mark point T where the circle intersects the $x = 0$ line ($+u$ axis) on the right hand side of the Smith chart.

- Read r_{max} corresponding to point T.

- $VSWR = r_{max}$

(E) Location of Voltage Maximum or Minimum

■ At the location of voltage maximum the impedance is maximum ($R_{\max}$), and at the location of voltage minimum the impedance is minimum ($R_{\min}$). Hence point T indicates the location of voltage maximum and point S indicates the location of voltage minimum in figure below
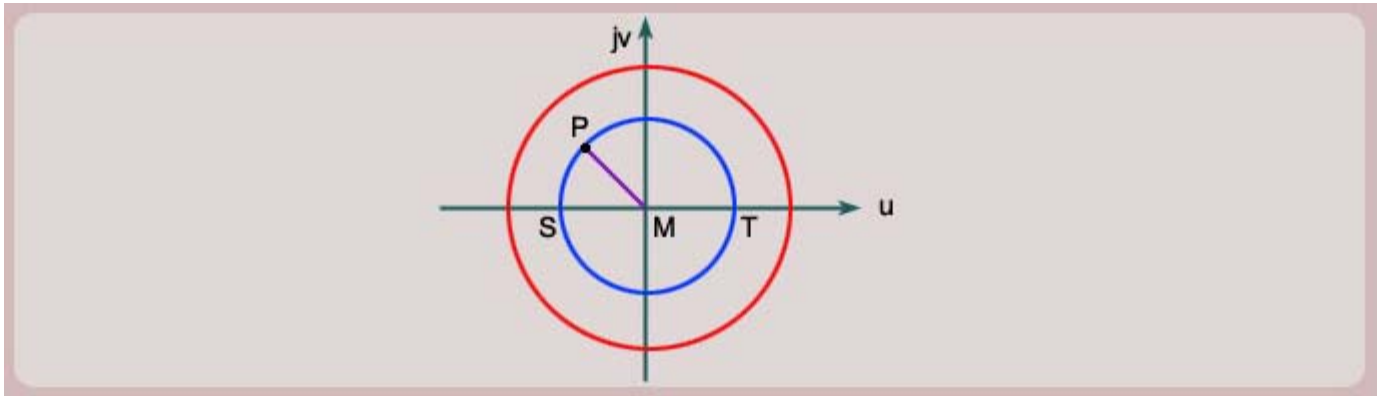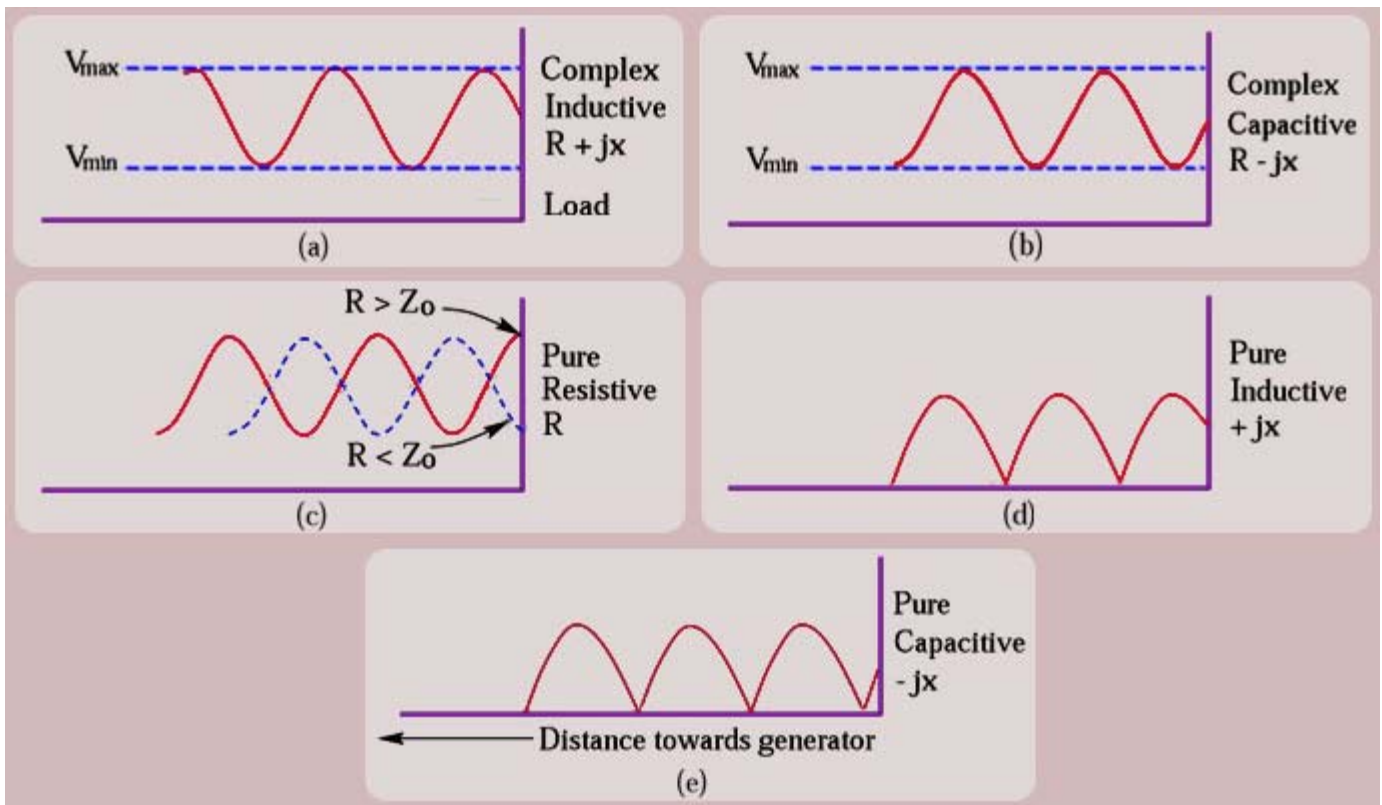

■ To find distance of these points from the load, measure the angle between the load point P and point T and between point P and point S respectively in the clockwise direction from P to T.The angle PMT in the clockwise direction when divided by 2β gives the distance of the voltage maximum from the load, $l_{\max}$. Similarly one can obtain distance of voltage minimum, $l_{\min}$ by measuring angle PMS in clockwise direction.

■ Alternatively one can make use of the fact that the voltage maximum and minimum are seperated by a distance of $\lambda / 4$. That is $l_{\min} = l_{\max} \pm \lambda/4$

Identifying the Type of Load
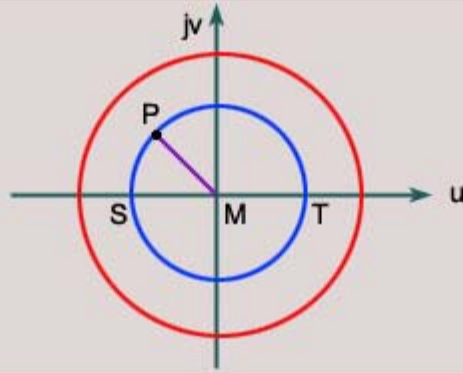
■ If the load is inductive, point P lies in the upper half of the Smith chart. Then while moving clockwise on the constant-VSWR circle, we first meet point T and then we meet poing S. In other words, for inductive loads, voltage maximum is closer to the load point than the voltage minimum. Exactly opposite occurs for capacitive loads i.e., the voltage minimum is closer to the load than the maximum.

■ We can therefore quickly identify the load looking at the standing wave pattern.

■ If the pattern is like the one given in Figure (a), that is the voltage drops towards the load, the load is inductive.

■ Similarly if the pattern is like that in Figure (b), that is, the voltage rises towards load, the load is capacitive.

■ If the voltage is maximum or minimum at the load, the load impedance is purely resistive Figure (c).

■ For the purely reactive loads the point P will lie on the outermost circle making $\left|\Gamma_L\right| = 1$ and $V_{min} = 0$.

The pattern for pure inductive loads will be like that in Figure (d) and that for the pure capacative loads will be like that in Figure (e).

(a) — Complex Inductive $R + jx$ Load, V_{max}, V_{min}

(b) — Complex Capacitive $R - jx$, V_{max}, V_{min}

(c) — $R > Z_0$, Pure Resistive R, $R < Z_0$

(d) — Pure Inductive $+ jx$

(e) — Pure Capacitive $- jx$, Distance towards generator

Note

For identifying the load, observe two things on the standing wave pattern

(1) Minimum Voltage

(2) Location of Voltage minimum/maximum