Module 3 : Maxwell's Equations

Lecture 22 : Basic Laws of Electromagnetics (contd.)

Objectives

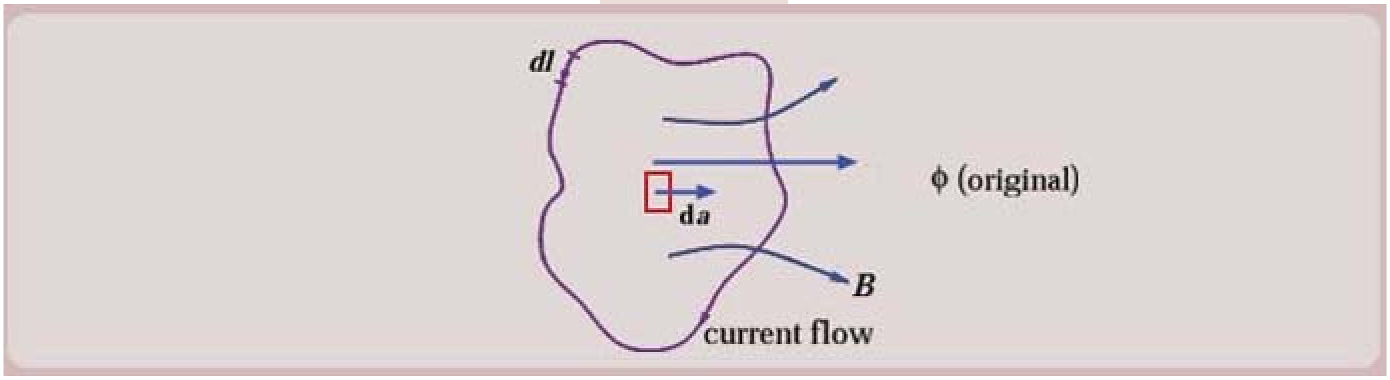
In this course you will learn the following

- Ampere's Circuit Law
- Faraday's Law of Electromagnetic Induction

Ampere's Circuit Law

- The Ampere's circuit law states that the total magnetomotive force along a closed loop is equal to the net current enclosed by the loop. The magnetomotive force is nothing but the line integral of the tangential component of the magnetic field around the loop. Writing mathematically the Ampere's circuit law is

$$\oint \mathbf{H} \cdot \mathbf{dl} = I$$



- Consider now a loop as shown in Fig. The distribution of current passing through the loop may not be uniform. Also the current may not be flowing perpendicular to the plane of the loop. It is therefore appropriate to define vector current density through the loop. The total current enclosed by the loop then is

$$I = \iint_S J.\, da$$

- Now Applying the Stokes theorem , we get

$$\oint H.dl = \iint_A (\nabla \times H).\, da = \iint_S J.\, da$$

$$\Rightarrow \iint_A \{\nabla \times H - J\}.\, da = 0$$

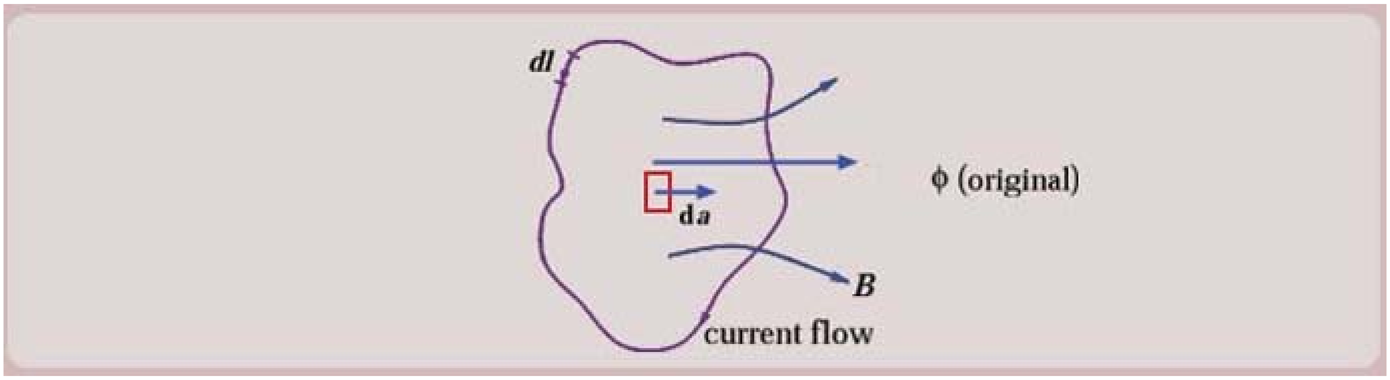
$$\nabla \times \mathrm{H} = \mathrm{J}$$

Faraday's Law of Electromagnetic Induction

- According to Faraday's law, the net electromotive force (EMF) in a closed loop is equal to the rate of change of magnetic flux enclosed by the loop (Φ). Now, the current due to induced EMF will produce a magnetic field. Therefore there will be magnetic field induced by the current which will be enclosed by the loop. There are two possibilities :

(i) The magnetic field due to the induced current is in same direction as the original field

(ii) The magnetic field due to the induced current is in opposite direction to the original field. In the first case, the two magnetic fields will add to enhance the net flux enclosed by the loop. This will increase the current and hence the magnetic flux enclosed. The process will be regenerative as there is no stabilizing element in this process. In the second case on the other hand, the induced magnetic field opposes the original magnetic and therefore the original magnetic field feels a opposition from the induced current. This process is a more appropriate physical process. We therefore find that the EMF in the loop is produced in such a way that the magnetic field due to induced current is in opposite direction to that of the original field. This is called the Lenz's law.



- Consider a loop as showm in Fig . Now to satisfy the Lenz's law the current direction in the loop must be opposite to the direction of the length segment $\mathbf{dl}$.
- The length segment $\mathbf{dl}$ and the area segment $\mathbf{da}$ are chosen by the right hand rule. Mathematically the Faraday's law can be written as

$$V = \oint \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial \Phi}{\partial t}$$

where V is the net EMF around the loop, and the negative sign is due to the Lenz's law.

- Now if the loop has magnetic flux density $\mathbf{B}$, the total flux enclosed by the loop is

$$\Phi = \iint_A \mathbf{B} \cdot \mathbf{da}$$

- The Faraday's law then becomes

$$\oint \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_A \mathbf{B} \cdot \mathbf{da}$$

From the above Fig. we can note that the magnetic flux enclosed by the loop can be varied in time in three ways

- Keeping loop area stationary and varying the magnetic flux density with time,
- Keeping magnetic flux density static but moving the loop,
- Moving the loop in a time varying magnetic field
- Since here we are interested in time varying fields we consider the first case only, that is the area of the loop is stationary

and the magnetic flux is time varying. In this case we can take the time derivative inside the integral giving

$$\oint \mathbf{E} \cdot \mathbf{dl} = -\iint_A \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{da}$$

- Applying the Stoke's theorem we get

$$\begin{aligned} & \oint \mathbf{E} \cdot \mathbf{dl} = \iint_A (\nabla \times \mathbf{E}) \cdot \mathbf{da} = -\iint_A \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{da} \\ \Rightarrow \quad & \iint_A \left\{ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} \right\} \cdot \mathbf{da} = 0 \end{aligned}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- This is Faraday's law of electromagnetic induction in differential form.

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Recap

In this course you have learnt the following

- Ampere's Circuit Law
- Faraday's Law of Electromagnetic Induction