Module 7 : Antenna

Lecture 51 : Antenna Arrays

Objectives

**In this course you will learn the following**

- What is an antenna array?

- Why antenna arrays are required?

- Uniform linear array.

- End fire and broadside array.

- Direction main beam, directions of nulls and sidelobes.

- Amplitudes of side lobes.

- Half Power Beam Width (HPBW).

- Directivity of a uniform array.

- Grating lobe and condition for avoiding grating lobe.

Introduction

- In the last module we discussed the principles of antenna and the linear dipole antennas.

- The half-wavelength dipole antenna is the most commonly used antenna as it has good terminal characteristics. However, the antenna has a very broad radiation pattern and consequently low directivity.

- There are many applications where we need highly directive antennas and in general a much better control of the radiation pattern. Also while doing this we wish to keep the antenna terminal characteristics more or less unaffected.

- Antenna Arrays precisely is the mechanism by which we can realize complex radiation patterns without significantly altering the antenna impedance.

- Here we first discuss the principle of linear arrays. The principle can be extended to the planar and volume arrays in a rather straight forward manner.

- The problem of antenna arrays can be divided into two categories:
  - (1) **Array Analysis:** Here we investigate the radiation patterns for a given antenna array configuration.
  - **(2)** **Array Synthesis:** Here we design the array configurations to achieve a desired radiation pattern.

- The array analysis problem is rather an academic exercise whereas the array synthesis is an engineering design problem. For a good design of antenna arrays, the basics understanding of array analysis is essential.

Uniform Linear Array

■ An antenna array consists of identical antenna elements with identical orientation distributed in space. The individual antennas radiate and their radiation is coherently added in space to form the antenna beam.

■ For a linear array, the antennas are placed along a line called the Axis of the array. The antenna elements in general could have arbitrary spacing between them and could be excited with different complex currents. However, here we analyze first the uniform array.

■ **In a uniform array the antennas are equi-spaced and are excited with uniform current with constant progressive phase shift** (phase shift between adjacent antenna elements) as shown in Fig.
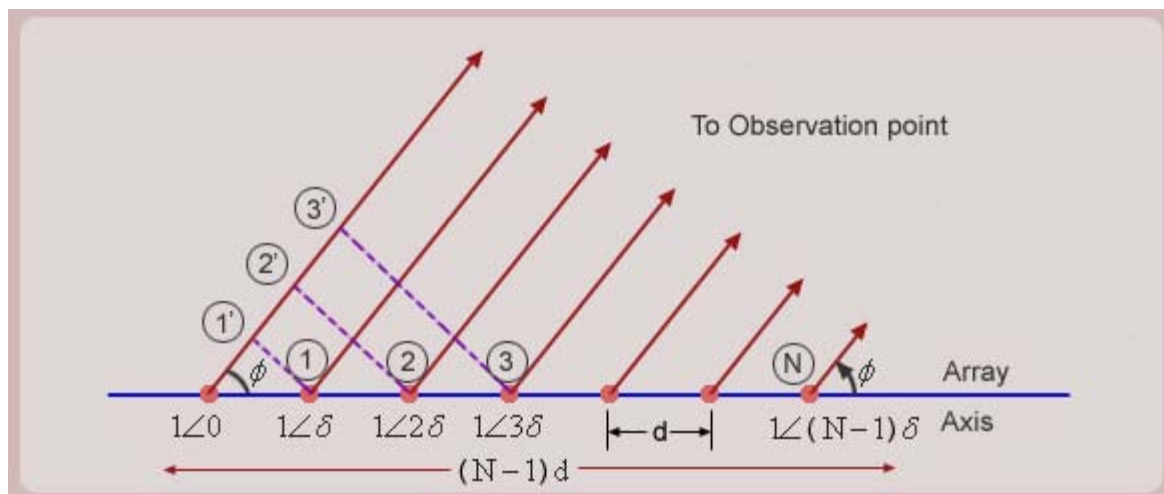
■ Let the array have N elements and let the antennas be isotropic (this condition will be relaxed later). All the antennas are excited with equal amplitude currents. Let us define the following for the array.

■ **Inter-element spacing, d** : This the spacing between any two adjacent elements of the array.

■ **Progressive phase-shift, δ** : This is the phase shift between currents on any two adjacent antenna elements of the array.

■ The field due an antenna is proportional to its current. Also for a far away point, the fields due to individual antennas have equal amplitude but different phases.

■ The phase of the field has two components:
   (1) The phase due to the phase of the excitation current.
   (2) The phase due to propagation. If the observation point P is in a direction which makes an angle ϕ with the array axis, the propagation phase difference between radiation from two adjacent elements is $\beta d \cos\phi$.

Uniform Linear Array (contd.)

■ The total phase difference, φ , between the fields due to adjacent elements is algebraic sum of the current and the propagation phase difference.

$$\varphi = \beta d \cos \phi + \delta$$

■ Without losing generality, let us assume that the electric field due to individual antennas has unit amplitude at the observation point P. Also let the first element be the reference element. Then by definition the phase of the field due to antenna 1 is zero.

■ The total field at the observation point is

$$E = e^{j0} + e^{j\varphi} + e^{j2\varphi} + \ldots\ldots + e^{j(N-1)\varphi}$$

$$E = \left\{ 1 + e^{j\varphi} + e^{j2\varphi} + \ldots\ldots + e^{j(N-1)\varphi} \right\}$$

■ The RHS of the equation is a geometric series with summation given by

$$E = \frac{1 - e^{jN\varphi}}{1 - e^{j\varphi}}$$

■ After some algebraic manipulation, we get the electric field at the observation point as

$$|E| = \left| \frac{\sin\left(\dfrac{N\varphi}{2}\right)}{\sin\left(\dfrac{\varphi}{2}\right)} \right|$$

■ The maximum electric field is obtained when all the terms in the series add in phase (i.e. for $\varphi = 0$). The maximum filed therefore is N.

■ The expression gives the variation of field as a function of the direction, ϕ, and hence is the radiation pattern of the antenna array.

■ The radiation pattern is generally normalized with respect to the maximum value N to get the ' **Array Factor** ' as

$$AF \equiv \frac{1}{N} \frac{\sin\left(\dfrac{N\varphi}{2}\right)}{\sin\left(\dfrac{\varphi}{2}\right)}$$

This is the general expression for the radiation pattern of a uniform array.

■ A typical radiation pattern is shown in Fig.

Radiation pattern as the cartesian plot   Radiation pattern as the polar plot
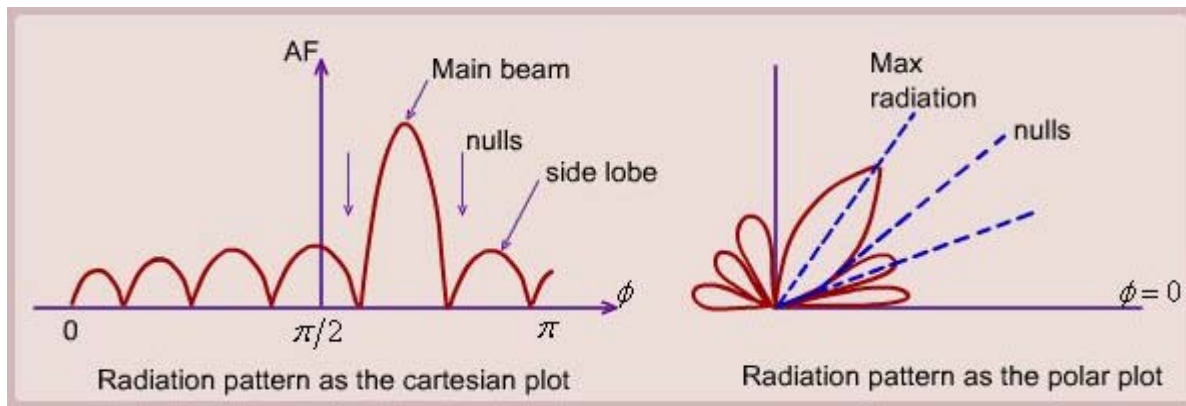
- The range of the angle ϕ is from 0 to π, and the 3-D radiation pattern is the figure of revolution of the Array Factor around the axis of the array.

- From the general array factor we can study the Direction of Maximum Radiation.

Direction of Maximum Radiation

■ The direction of maximum radiation (also called the direction of the main beam) is one of the important features of the array.

■ The maximum radiation is obtained when $\varphi = 0$. If the direction of maximum radiation is denoted by ϕ_{max} we have

$$\varphi = \beta d \cos \phi_{max} + \delta = 0$$

$$\Rightarrow \cos \phi_{max} = -\frac{\delta}{\beta d}$$

$$\Rightarrow \phi_{max} = \cos^{-1}\left(-\frac{\delta}{\beta d}\right) = \cos^{-1}\left(\frac{\delta \lambda}{2\pi d}\right)$$

■ The array phase in terms of the direction of main beam is written as

$$\varphi = \beta d \left(\cos \phi - \cos \phi_{max}\right)$$

■ Two things can be noted from the equation.

　(1)　The direction of the maximum radiation is independent of the number of elements in the array.

　(2)　The direction of the main beam can be changed from 0 to π by changing the progressive phase shift δ from $-\beta d$ to $+\beta d$.

■ An array is said to be End-fire array if the main beam is along the axis of the array. An array is said to be Broadside array if the main beam is perpendicular to the axis of the array.

■ There are two end-fire directions for an array but the broadside is a plane perpendicular to the array axis (see Fig below)
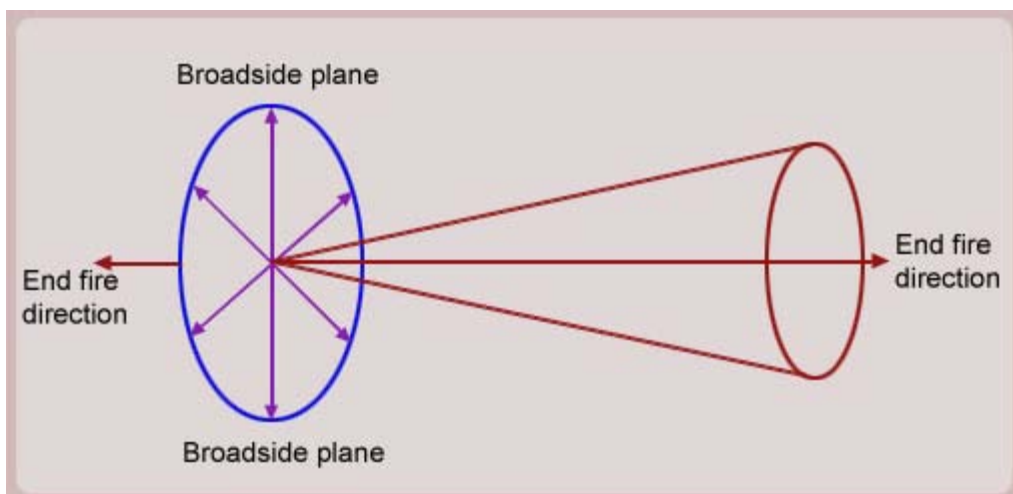
Directions of Nulls

- The nulls of the radiation patterns can be obtained by equating the array factor to zero.

- The directions of the nulls, ϕ_{null} is given as

$$\sin\left(\frac{N\varphi}{2}\right) = 0$$

$$\Rightarrow \frac{N\varphi}{2} = \pm m\pi, \quad m = 1, 2, 3\ldots$$

$$\Rightarrow \varphi = \pm\frac{2m\pi}{N}, \quad m = 1, 2, 3\ldots$$

Which can be simplified as

$$\beta d\left\{\cos\phi_{null} - \cos\phi_{max}\right\} = \pm\frac{2m\pi}{N}$$

$$\Rightarrow \cos\phi_{null} = \cos\phi_{max} \pm\frac{2m\pi}{\beta dN}$$

$$\cos\phi_{null} = \cos\phi_{max} \pm\frac{m\lambda}{dN}$$

- For finding the directions of the nulls all possible values of $\pm m$ have to be tested.

Directions of Side-lobes

- Local maximum in the radiation pattern is called the side lobe.

- There is one side lobe between two adjacent nulls except the main beam.

- Whenever the numerator of the AF is maximum, there is a side-lobe in the radiation pattern.

- Generally, the directions of the side-lobes are taken approximately half way between the two adjacent nulls.

- The direction of a side lobe is given as

$$\frac{N\varphi}{2} = \pm\left(m+\frac{1}{2}\right)\pi$$

$$\Rightarrow \varphi = \beta d\left\{\cos\phi_{SL} - \cos\phi_{max}\right\} = \pm\left(m+\frac{1}{2}\right)\pi, \quad m = 1, 2,\ldots$$

$$\cos\phi_{SL} = \cos\phi_{max} \pm\left(m+\frac{1}{2}\right)\frac{\lambda}{2d} \quad m = 1, 2,\ldots$$

- The amplitude of the side lobe is obtained by substituting the value of φ in the AF.

- The amplitude of the mth side lobe is

$$\frac{1}{N}\left|\frac{1}{\sin\left(\pm\left(m+\dfrac{1}{2}\right)\dfrac{\pi}{N}\right)}\right|$$

- For a large array $N \gg 1$ and the side lobe amplitude is approximately

$$\approx \frac{1}{N}\frac{1}{\left(m+\dfrac{1}{2}\right)\dfrac{\pi}{N}} = \frac{2}{(2m+1)\pi}$$

- The first, second, third side lobe amplitudes are $2/3\pi, 2/5\pi, 2/7\pi$ respectively.

- The important thing to note: **The side lobe amplitudes are independent of the array size and the direction of the main beam.**

Half Power Beam Width (HPBW)

■ For large arrays, the HPBW is approximately taken as half of the BWFN.

■ For a given array the HPBW is a function of the direction of the main beam. The HPBW is minimum for a broadside direction and maximum for the end-fire direction. The beam width monotonically increases as the main beam tilts towards the axis of the array.

■ The HPBW for the broad-side array and the end-fire array are approximately given as

$$\varphi_{BS} = \frac{\lambda}{dN} = \frac{\lambda}{\text{Length of the array}}$$

$$\varphi_{EF} = \sqrt{\frac{2\lambda}{dN}} = \sqrt{\frac{2\lambda}{\text{Length of the array}}}$$

■ The HPBW is inversely related to the array length. Larger the array narrower is the beam i.e., smaller HPBW.

Directivity

■ The directivity of the uniform array is given by

$$D = \frac{4\pi}{\iint |AF|^2 \, d\Omega}$$

■ For a large array, the integral can be approximated by the solid angle of the main beam and the directivities for the broadside and end-fire arrays are given as

$$D_{BS} = \frac{4\pi}{2\pi\phi_{BS}} = \frac{2dN}{\lambda}$$

$$D_{EF} = \frac{4\pi}{\pi\left(\frac{\phi_{EF}}{2}\right)^2} = \frac{16}{\left(\frac{2\lambda}{dN}\right)} = \frac{8dN}{\lambda}$$

■ **Important Observation:**

The HPBW of the broadside array is less than that of the end-fire array but the directivity of the end-fire array is larger than the broadside array.

Grating Lobe

■ A grating lobe or grating beam is a beam identical to the main beam but in undesired direction.

■ The power radiated by the array gets divided between the main beam and the grating lobe. The power efficiency in the direction of the main beam is consequently reduced. A grating lobe should be avoided in the radiation pattern.

■ A grating lobe appears when

$$\varphi = 2m\pi$$

Where m is an integer.

■ For broadside array the grating lobe appears when $d \geq \lambda$, and for the end-fire array it appears when $d \geq \lambda/2$. We can conclude that to avoid grating lobe in the radiation pattern for any array the inter-element spacing should be $< \lambda/2$.

Effect of Antenna radiation pattern on the array

■ The radiation pattern of the antenna elements used in the array is called the 'primary pattern'.

■ If the array consists of non-isotropic but identical elements, the effect of the primary pattern can be accounted for very easily.

■ Since the radiation due to every element is weighted by the primary pattern, the total radiation pattern of an array is the product of the primary pattern and the AF.

Radiation pattern = Primary pattern   X   Array Factor

■ While analyzing the array of non-isotropic but identical elements therefore first find the AF assuming the elements to be isotropic and then multiply the AF with the primary pattern to get the total radiation pattern.

■ Using this concept the arrays can be analyzed by dividing the array into sub-arrays.

Recap

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