Module 3 : Maxwell's Equations

Lecture 21 : Basic Laws of Electromagnetics

Objectives

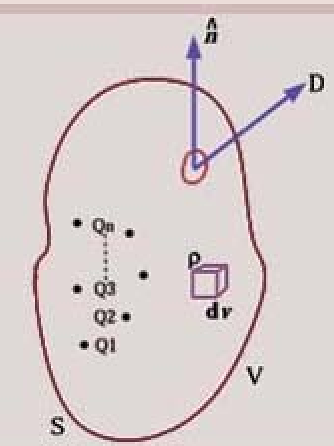
In this course you will learn the following

- Gauss's Law
- Gauss's Law for Magnetic Flux Density

Gauss's Law

- The Gauss's law states that the total outward electric displacement through any closed surface surrounding charges is equal to the total charge enclosed.
- Let us now consider a closed surface S surrounding the charges Q_1, Q_2, Q_3 as shown in Fig. Then through an incremental area $\mathbf{da}$ on the surface, the outward electric displacement will be $\mathbf{D}\cdot\mathbf{da}$. The total electric displacement can be obtained by integrating over the surface S. Then according to the Gauss's law

$$\oint_S \mathbf{D}\cdot\mathbf{da} = Q_1 + Q_2 + \ldots + Q_n$$



- Instead of discrete charges Q_1, Q_2, ... Q_n if there is a continious distribution of charge inside the closed surface, we have to find total charge by integrating over the enclosed volume. The distributed charges can be correctly represented by a charge density ρ which in general is a function of space. The ρ is the charge per unit volume having units $Coulomb/m^3$. The charge in a small volume dV will be ρdV. The total charge enclosed by the volume can be obtained by integrating over the volume V. For distributed charges then becomes

$$\oint_S \mathbf{D}\cdot\mathbf{da} = \iiint_V \rho dv$$

- This is the Gauss law in the integral form
- The equivalent differential form can be obtained by applying the Divergence theorem to the above eqn.

$$\oint_S \mathbf{D}\cdot\mathbf{da} = \iiint_V (\nabla\cdot\mathbf{D}) dv$$

- Substituting and bringing all the terms on the left hand side of the equality sign we get

$$\iiint_V \{(\nabla\cdot\mathbf{D}) - \rho\} dv = 0$$

$$\nabla\cdot\mathbf{D} = \rho$$

- This is the differential form of the Gauss's law.

Gauss's Law for Magnetic Flux Density

- The total magnetic flux coming out of a closed surface is equal to the total magnetic charges (poles) inside the surface. However there are no isolated magnetic monopoles. The magnetic poles are always found in pair with opposite polarity. As a result, there are always equal number of north and south poles inside any closed surface making net magnetic charges identically zero inside a volume. The total outward magnetic flux from any closed surface therefore must identically be equal to zero. Writing mathematically,

$$\oint_S \mathbf{B} \cdot \mathbf{da} = 0$$

- Where **B** is the magnetic flux density. Applying the Divergence theorem we get

$$\iiint_V (\nabla \cdot \mathbf{B}) dv = 0$$

or

$$\nabla \cdot \mathbf{B} = 0$$

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Recap

In this course you have learnt the following

- Gauss's Law
- Gauss's Law for Magnetic Flux Density