Module 6 : Wave Guides

Lecture 42 : Phase Velocity and Dispersion

Objectives

In this course you will learn the following

- Cut-off Frequency of a Mode.

Cut-off Frequency of a Mode

- The propagation constant of the modal fields in the z-direction is given as

$$\beta = \beta_1 \sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2} \quad \text{---------- (6.17)}$$

$$= \sqrt{\beta_1^2 - \left(\frac{m\pi}{d}\right)^2} \quad \text{---------- (6.18)}$$

- If the propagation constant β is real the mode will be at travelling mode whereas, if β becomes imaginary the wave exponentially decays in the z-direction and the fields do not represent a wave. These fields are then called '**EVANASENT FIELDS**' . The evanasent fields do not carry any power. The power is carried only by the travelling modes.

- For the travelling mode therefore we need β to be real which implies

$$\beta = real$$

$$\Rightarrow \qquad \beta_1 \geq \frac{m\pi}{d} \quad \text{---------- (6.19)}$$

Since

$$\beta_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi f}{v_1}, \quad \text{---------- (6.20)}$$

where v_1 is the velocity of the uniform plane wave in medium 1, we get

$$f \geq \frac{mv_1}{2d} \quad \text{---------- (6.21)}$$

$$\Rightarrow \qquad \lambda_1 \leq \frac{2d}{m} \quad \text{---------- (6.22)}$$

- We can note the following important things at this stage:

(1) For a given waveguide height d, the frequency has to be higher than certain threshold frequency for propagation of a particular mode. The threshold frequency is called **CUT-OFF** freuquency at the mode and given by

$$f_{cm} = \frac{mv_1}{2d} \quad \text{---------- (6.23)}$$

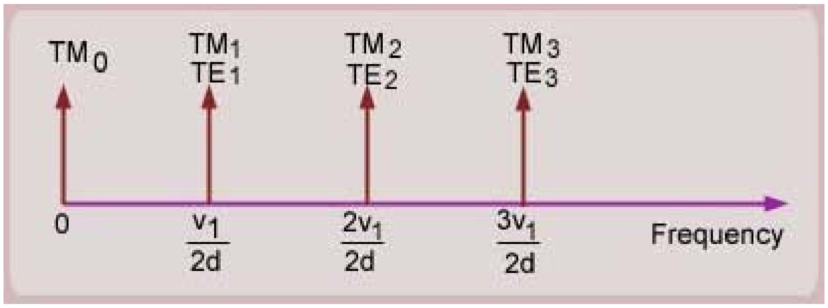
The corresponding cut-off wavelength is

$$\lambda_{cm} = \frac{2d}{m} \quad \text{---------- (6.24)}$$

(2) For a given waveguide height, d, and frequency, f, only those modes propagate for which $m \leq \frac{2df}{v_1} = \frac{2d}{\lambda}$.
This means inside a wave guide there is a possibility of only finite number of modes at a given frequency.

(3) As the mode number (m) increases and the cut-off frequency also increases meaning higher order mode get excited only at higher frequencies.

- The cut-off frequencies for different modes are shown in the following figure:



- If a mode has the cut-off frequency less than the frequency of operation the mode propagates otherwise it does not propagate.

NOTE :

- The TM_0 mode which is also the TEM mode has no cut-off frequency. This is the mode which can propagate at any frequencies starting from dc .

- The phase velocity of a mode is

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\beta_1\sqrt{1-(\frac{m\lambda_1}{2d})^2}} = \frac{v_1}{\sqrt{1-(\frac{m\lambda_1}{2d})^2}}$$

---------- (6.25)

where v_1 is the velocity of the uniform plane wave in the medium filling the region between the wave guide walls.

- Noting that $\frac{2d}{m} = \lambda_{cm}$ the phase velocity can be written as

$$v_p = \frac{v_1}{\sqrt{1-(\frac{\lambda_1}{\lambda_{cm}})^2}} = \frac{v_1}{\sqrt{1-(\frac{f_{cm}}{f})^2}}$$ ---------- (6.26)

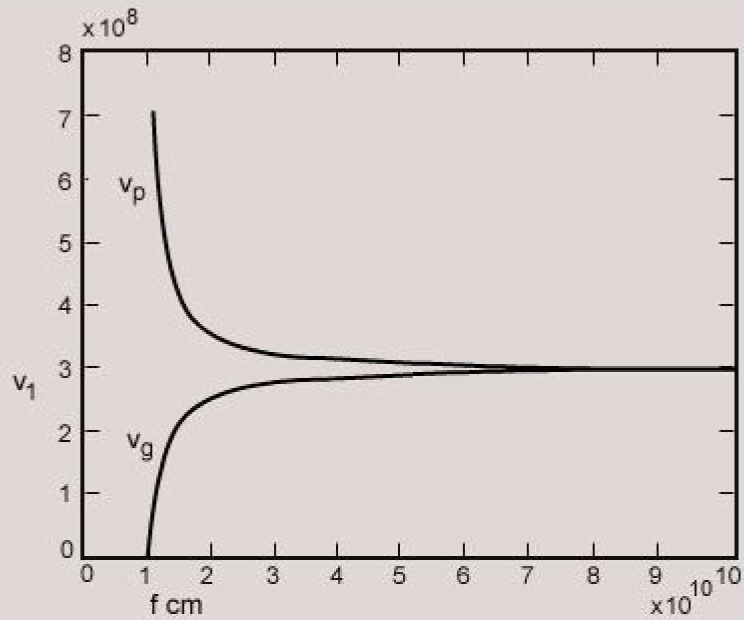
- It is clear from the expression that in general the phase velocity of a mode is a function of frequency except when the cut-off frequency is zero. This phenomena is called ' **WAVE DISPERSION**'.

- In general, one can then say that the modal propagation on a wave guide is dispersive in nature.

- The group velocity of the mode is

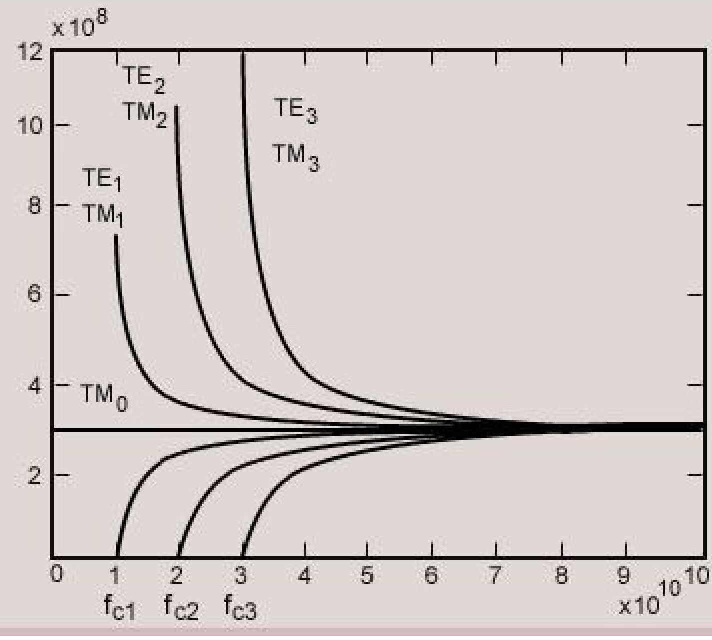
$$v_g = \frac{v_1^2}{v_p} = v_1\sqrt{1-\left(\frac{f_{cm}}{f}\right)^2}$$ ---------- (6.27)

- Typical plot for group and phase velocities is shown in figure below



- At the cut-off frequency the phase velocity approaches infinity whereas the group velocity approaches zero. That means the energy propagation seizes as the mode approaches cut-off. As the frequency increases both group and phase velocities asymptotically approach the velocity of the plane wave in the media.

- The figure below shows the typical velocity plot for different modes as a function of frequency.



Recap

In this course you have learnt the following

- Cut-off Frequency of a Mode.