Module 2 : Transmission Lines

Lecture 13 : Application of Transmission Lines continues

Objectives

In this course you will learn the following

- What is the resonant section of a transmission line?
- Frequency response of a resonant section of a line.
- Input impedance of a resonant section of a line.
- Voltage and current on a resonant section of a line.

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Transmision Lines as Resonant Circuits

- If the length of a short or open circuited line is exact multiple of $\lambda/4$, the imput impedance of the line is zero or ∞. Let us plot the input impedance as a function of frequency `f', for a given length of transmission l and a given termination (short circuit or open circuit).

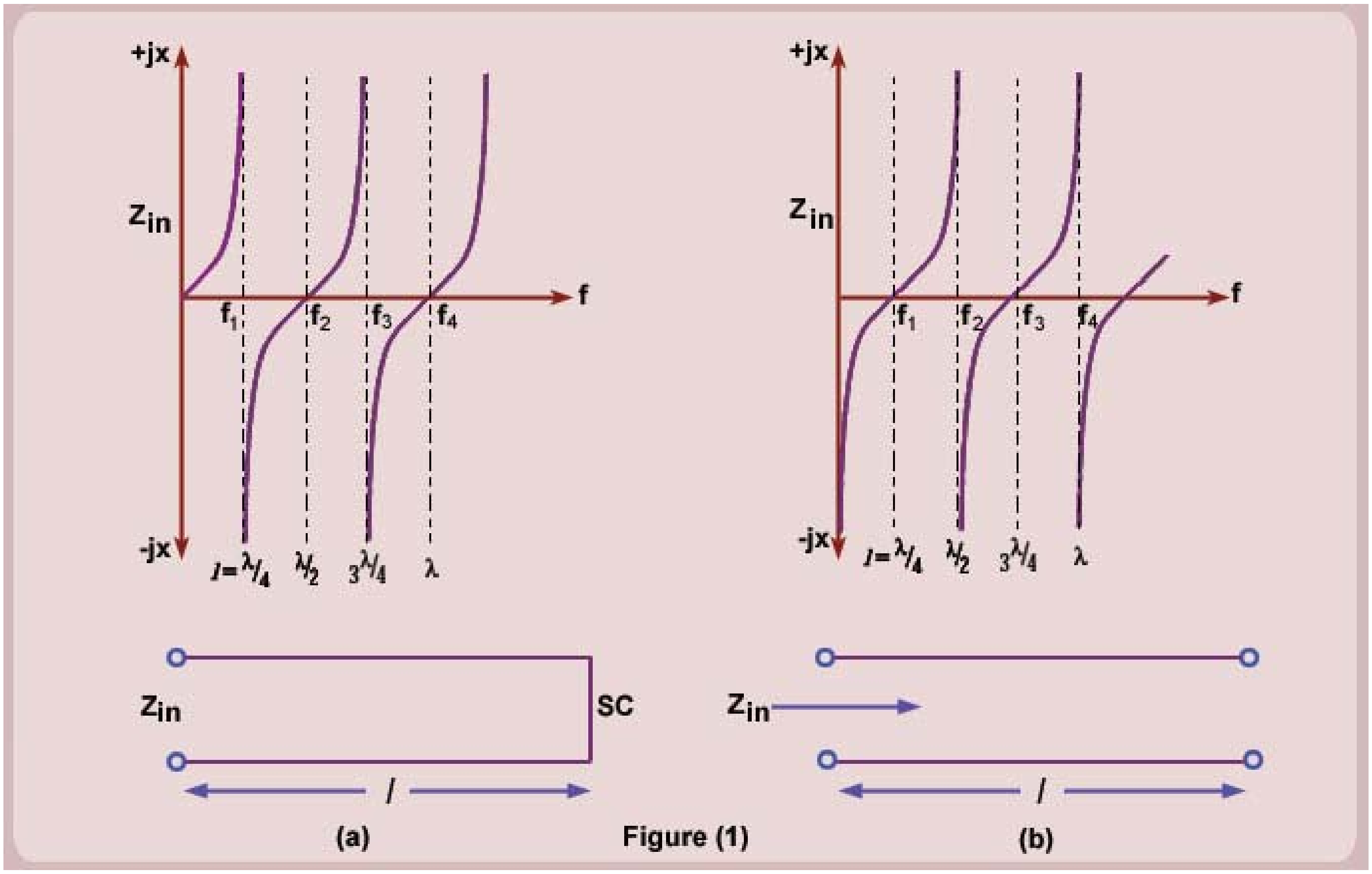


Figure (1)

- Figure shows the variation of reactance as a function of frequency for open and short circuited sections of a transmission line. It is clear that around frequencies f_1, f_2, f_3, f_4.., for which the length l is an integer multilple of $\lambda/4$, the impedance variation is identical to an L-C resonant circuit. In the vicinity of these frequencies the line can be used as a LC - resonant circuit.

Frequency response of Resonant Circuit

- The impedance characteristics of a series and a parallel resonant circuit are shown in the figure below

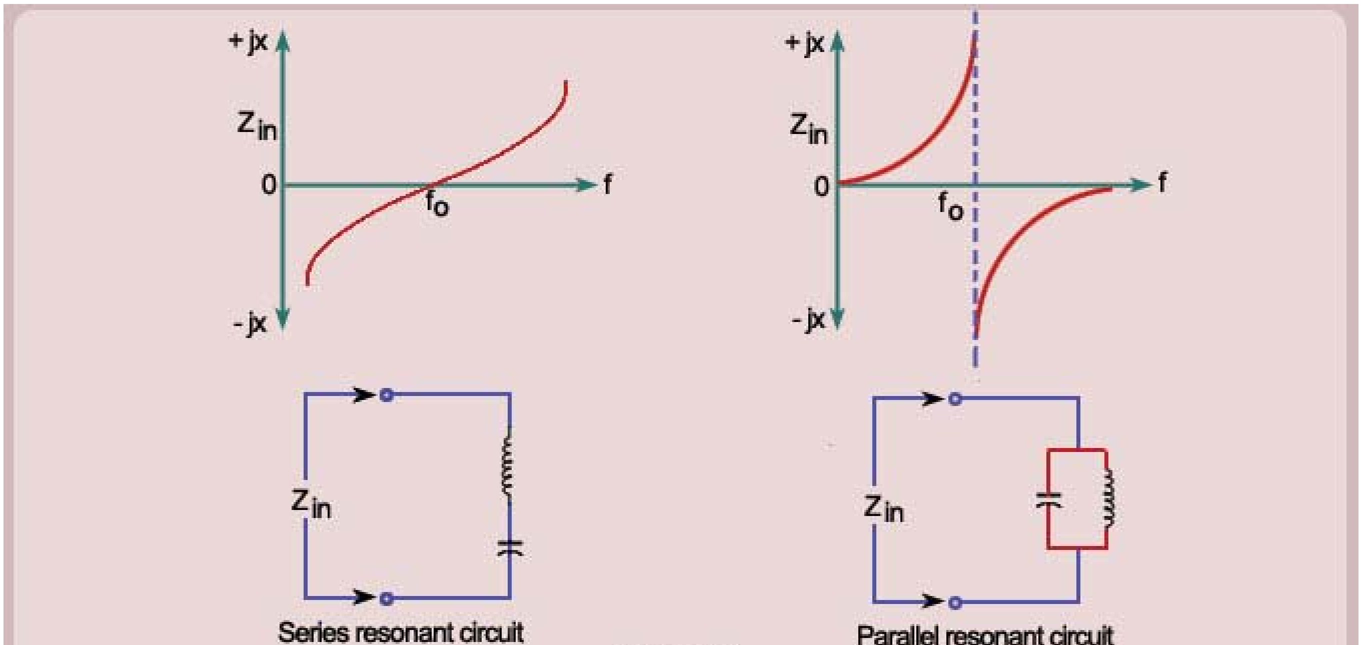


Figure (2)

- Comparing Figure (1) with Figure (2), one can observe that a short circuited line behaves like a parallel resonant circuit around frequencies f_1 and f_3, whereas around f_2 and f_4 its behaviour is like a series resonant circuit.
- In general a short circuited section of a line is equivalent to a parallel resonant circuit. If $l = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$ (odd multiples of $\lambda/4$)
- Similarly, the line is equivalent to a series resonant circuit. If $l = \lambda/2, \lambda, 3\lambda/2, \ldots$ (even multiples of $\lambda/4$)
- A converse is true for an open circuited section of a line i.e., if the length of the line is equal to odd multiples of $\lambda/4$, the line behaves like a series resonant circuit, and if the length of the line is equal to even multiple of $\lambda/4$, the line behaves like a parallel resonant circuit.

Input Impedance of Resonant Line

- Input impedance of a resonant lossless line is either 0 or ∞ . However, in practice, the lines have finite loss. This loss has to be included in the calculations while analysing the resonant lines. The complex propagation constant γ has to be used in impedance calculations of a resonant line.
- The input impedance of a short or open circuited line having propagation constant γ can be written as

$$Z_{sc} = Z_0 \tanh \gamma l \qquad \text{for short circuit load}$$

$$Z_{oc} = Z_0 \coth \gamma l \qquad \text{for open circuit load}$$

- Note that although γ has been taken complex for a low-loss transmission line, Z_0 is almost real. Substituting for $\gamma = \alpha + j\beta$, we get

$$Z_{sc} = Z_0 \tanh(\alpha + j\beta)l = Z_0 \left[\frac{\tanh \alpha l + \tanh(j\beta)l}{1 + \tanh \alpha l \tanh(j\beta)l} \right]$$

- For a low-loss line, taking $\alpha l << 1$, we have $\tanh \alpha l \approx \alpha l$. Also $\tanh(j\beta)l = j \tan \beta l$. Hence we get

$$Z_{sc} \approx Z_0 \left[\frac{\alpha l + j \tan \beta l}{1 + j\alpha l \tan \beta l} \right]$$

- Similarly for an open circuited line we get

$$Z_{oc} \approx Z_0 \left[\frac{1 + j\alpha l \tan \beta l}{\alpha l + j \tan \beta l} \right]$$

- For resonant lines, l is integer multiples of $\lambda/4$ i.e., $\beta l (= 2\pi l/\lambda)$ is integer multiples of $\pi/2$. If we take l odd multiples of $\lambda/4$, $\tan \beta l = \infty$, and we get

$$Z_{sc} \approx \frac{Z_0}{\alpha l}$$

$$Z_{oc} \approx Z_0 \alpha l$$

- On the other hand if we take l even multiples of $\lambda/4$, $\tan \beta l = 0$, giving

$$Z_{sc} \approx Z_0 \alpha l$$

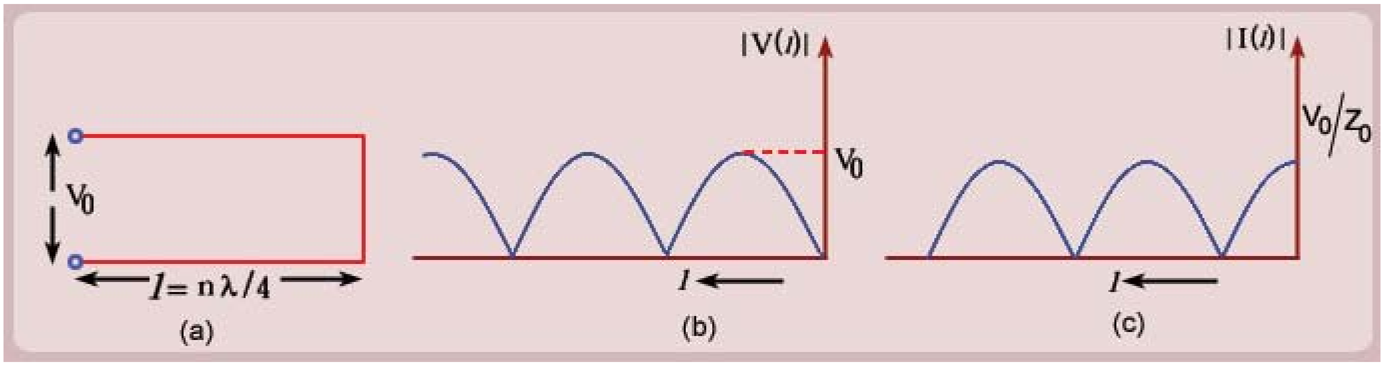
$$Z_{oc} \approx \frac{Z_0}{\alpha l}$$

Conclusion

- A parallel resonant section of a line has an impedance $Z_0/\alpha l$ and a series resonant section has an impedance $Z_0 \alpha l$.
- One can cross-check the result with that of an ideal loss-less line. In the absence of any loss the parallel resonant circuit shows infinite impedance and a series resonant circuit shows zero impedance at the resonance.

Voltage & Current on a Resonant Section of a line

- Consider a short circuited section of a line having length equal to odd multiples of $\lambda/4$. This line is equivalent to a parallel resonant circuit. Let the line be applied with a voltage V_0 between its input terminals as shown in Figure(a).



- The voltage and current standing wave patterns on the line are shown in Figure(b,c).

- The voltage is zero at the short-circuit-end of the line and is maximum at the input end of the line. similarly, the current is maximum at the short-circuit end and minimum at the input end of the line.
- The maximum value of the voltage on the line is V_0 and maximum value of current is V_0/Z_0 . For a short-circuited line the voltage and current on the line are given as

$$V(l) = |V_0 \sin \beta l|$$
$$I(l) = |\frac{V_0}{Z_0} \cos \beta l|$$

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Recap

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