Module 5 : Plane Waves at Media Interface

Lecture 34 : Plane Wave at Dielectric Interface

Objectives

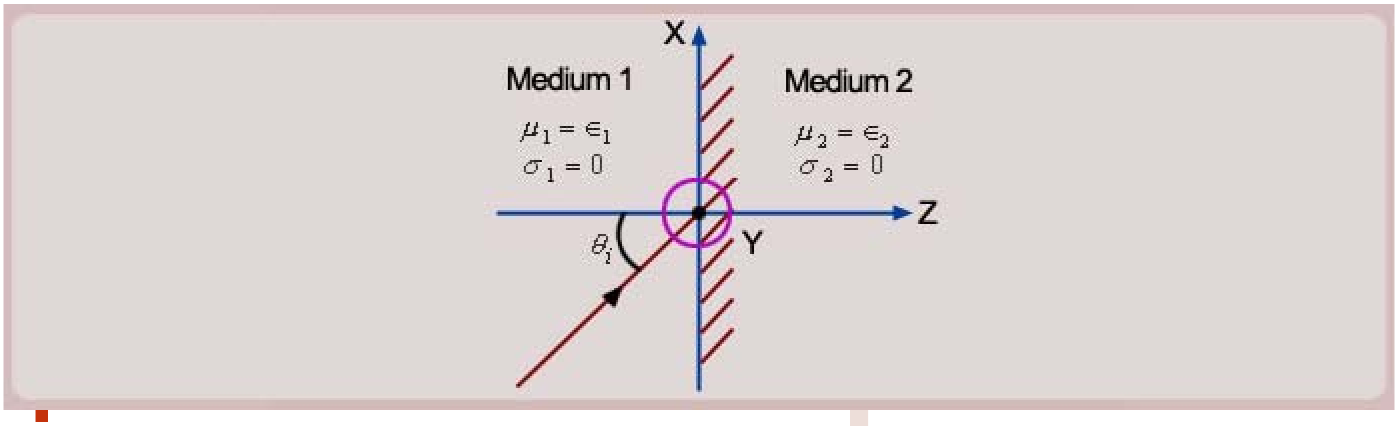
In this course you will learn the following

- Phase matching of the waves at the Interface.
- Laws of Reflection.

Phase matching of the waves at the Interface

- Let us now consider the propagation of a plane wave across media interface. Assume that the media are loss-less i.e their conductivities are zero.

- Let us orient the co-ordinate system as shown in the fig below



- A line perpendicular to media interface is called normal to the interface (z-axis in this case).
- Let the wave be incident from medium 1 such that the wave vector lies in the xz - plane making an an angle θ_i with respect to the interface normal.
- The plane containing the interface normal and the wave vector is called the 'Plane of Incidence' (in this case the xz-plane).
- The angle θ_i is called the angle of incidence.

- For this wave, we have

$$\phi_x = \frac{\pi}{2} - \theta_i, \quad \phi_y = \frac{\pi}{2}, \quad \phi_z = \theta_i$$

- We can then write the field (electric or magnetic) for this wave as

$$\begin{aligned} \mathbf{F}_i &= \mathbf{F}_{i0} e^{-j\mathbf{k}_1 \cdot \mathbf{r}} \\ &= \mathbf{F}_{i0} e^{-j\beta_1 (x\cos\phi_x + y\cos\phi_y + z\cos\phi_z)} \end{aligned}$$

- The suffix i indicates the incident field.
- $\mathbf{F_{io}}$ is a constant vector and β_1 is the phase constant of the wave in medium 1, $\beta_1 = \omega\sqrt{\mu_1\epsilon_1}$. Substituting for ϕ_x, ϕ_y, ϕ_z, we get

$$\begin{aligned} \mathbf{F_i} &= \mathbf{F_{i0}} e^{-j\beta_1 (x\cos(\pi/2-\theta_i) + y\cos(\pi/2) + z\cos\theta_i)} \\ &= \mathbf{F_{i0}} e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)} \end{aligned}$$

- The incident wave will create a phase variation at the interface which is $\beta_1 \, x \sin\theta_i$ along the x- direction and no variation in y-direction.

- When this wave is incident at the interface, on the otherside of the interface similar phase variation will be induced to maintain continuity of the fields.

- It can also be shown that the continuity for both electric and magnetic field cannot be achieved without altering the fields in medium one.

We then have

(a) Combination of incident field and the induced field in medium 1.

(b) Induced field in medium 2.

- The induced fields in medium 1 are called the 'reflected fields' and the induced fields in medium 2 are called the 'Transmitted Fields'.

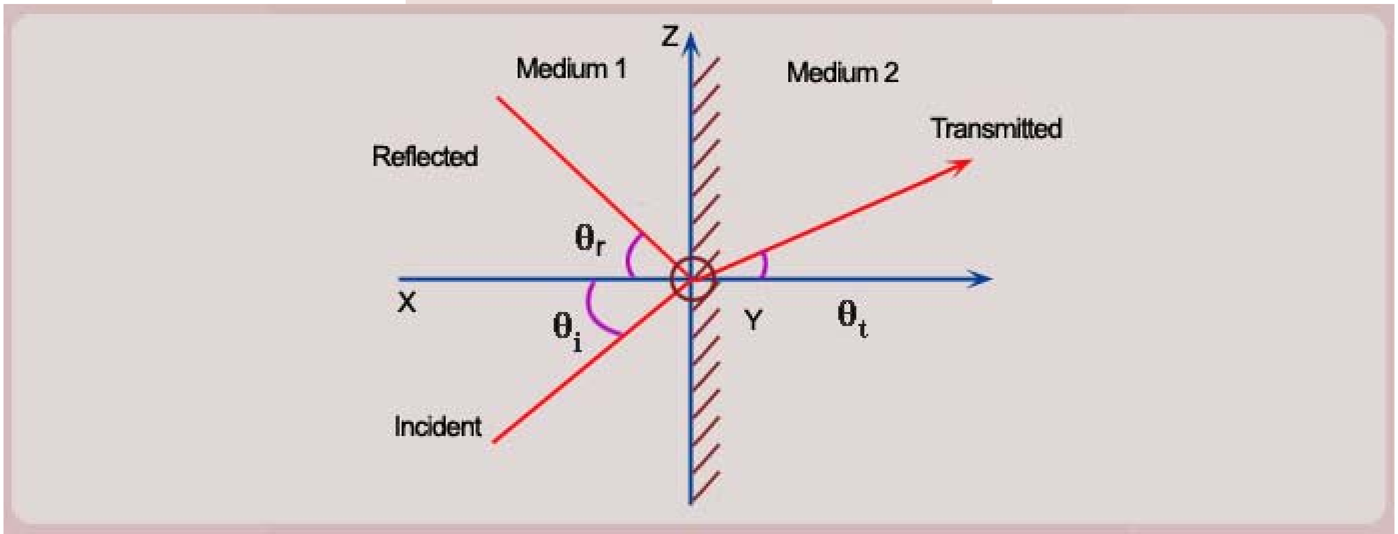
- The induced fields constitute waves in both the media going away from the interface.

Laws of Reflection

- Since the phase is constant in y-direction the reflected and transmitted wave have wave vectors in the xz-plane i.e the plane of incidence. We can conclude that the incident reflected and transmitted wave vectors lie in the same plane. This is the first law of reflection.
- If we assume that the reflected wave or reflected wave vector makes an angle θ_r with respect to the interface normal and the transmitted wave vector makes an angle θ_t with respect to the interface normal as shown in the figure, we can write reflected and transmitted fields as

$$\begin{aligned}\mathbf{F_r} &= \mathbf{F_{r0}} e^{-j\beta_1\{x\cos(\pi/2-\theta_r)+y\cos\pi/2+z\cos(\pi-\theta_r)\}} \\ &= \mathbf{F_{r0}} e^{-j\beta_1\{x\sin\theta_r - z\cos\theta_r\}}\end{aligned}$$

$$\begin{aligned}\mathbf{F_t} &= \mathbf{F_{t0}} e^{-j\beta_2\{x\cos(\pi/2-\theta_t)+y\cos\pi/2+z\cos\theta_t\}} \\ &= \mathbf{F_{t0}} e^{-j\beta_2\{x\sin\theta_t + z\cos\theta_t\}}\end{aligned}$$



- At the interface i.e $z=0$ continuity of the fields demands

$$(\mathbf{F_{i0}})_{tan} e^{-j\beta_1 x\sin\theta_i} + (\mathbf{F_{r0}})_{tan} e^{-j\beta_1 x\sin\theta_r} = (\mathbf{F_{t0}})_{tan} e^{-j\beta_2 x\sin\theta_t}$$

- Since this condition has to be true for every value of x and y , we get

$$\begin{aligned} & \beta_1 x\sin\theta_i = \beta_1 x\sin\theta_r = \beta_2 x\sin\theta_t \\ \Rightarrow \quad & \sin\theta_i = \sin\theta_r \\ \Rightarrow \quad & \theta_i = \theta_r \end{aligned}$$

This is the second Law of Reflection i.e "The Angle of Reflection = The Angle of Incidence"

Law of Refraction

- From the above equation we also get

$$\begin{aligned} & \beta_1 \sin\theta_i = \beta_2 \sin\theta_t \\ \Rightarrow \quad & \sqrt{\mu_1\epsilon_1}\sin\theta_i = \sqrt{\mu_2\epsilon_2}\sin\theta_t \end{aligned}$$

- This is known as "Snell's Law of Refraction".
- For ideal dielectrics $\mu_1 = \mu_2 = \mu_0$ (free space permeability), $\epsilon_1 = \epsilon_0\epsilon_{r1}$ and $\epsilon_2 = \epsilon_0\epsilon_{r2}$ where ϵ_{r1} and

ϵ_{r2} are the dielectric constants of the two media. The above equation can be written as

$$
\begin{aligned}
& \sqrt{\mu_0 \epsilon_0 \epsilon_{r1}}\, \sin\theta_i = \sqrt{\mu_0 \epsilon_0 \epsilon_{r2}}\, \sin\theta_t \\
\Rightarrow \quad & \sqrt{\epsilon_{r1}}\, \sin\theta_i = \sqrt{\epsilon_{r2}}\, \sin\theta_t \\
\Rightarrow \quad & n_1 \sin\theta_i = n_2 \sin\theta_t
\end{aligned}
$$

- This is the Snell's law for ideal dielectric media.

Reflection & Refraction for Dielectric interface

- $\mu r1$ and $\mu r2$ are relative permeabllities of the two media.
- $\epsilon r1$ and $\epsilon r2$ are the relative permittivities of the two media.
- Angle of incidence can be varies using scroll.

Recap

In this course you have learnt the following

- Phase matching of the waves at the Interface.
- Laws of Reflection.