Module 2 : Transmission Lines

Lecture 4 : Physical Interpretation of Voltage & Current Solution

Objectives

In this course you will learn the following

- Demonstration of wave motion.
- Forward and backward travelling waves.
- Interpretation of the propagation constant γ .
- Attenuation and phase constants and their units.
- Definition of wavelength and its relation to the phase constant.
- Characteristic impedance of the transmission line.
- Relation between voltage and current for forward and backward travelling waves.

Forward Travelling Wave

- Combining now the space-time we get what is called the 'Wave Motion'. See Figure. The voltage pattern appears travelling from left to right.

- The first term of the voltage solution $V^+ e^{-\alpha x} e^{j\omega t - j\beta x}$ represents a voltage travelling wave in $+x$ direction (left to right), and $\left| V^+ \right|$ gives the amplitude of the wave at $x = 0$. We call this wave, the ' Forward Travelling Wave '

Backward Travelling Wave

- Similarly, the second term in the voltage solution $V^- e^{\alpha x} e^{j\omega t + j\beta x}$ gives a travelling wave but travelling in negative' x' direction (right to left) as shown in Figure $|V^-|$ gives the amplitude of the wave at $x = 0$. This wave we call the 'Backward Travelling Wave' .

Current Travelling Waves

- The first term of the current solution $I^+ e^{-\alpha x} e^{j\omega t - j\beta x}$ represents a current travelling wave in $+x$ direction (left to right), and $\left| I^+ \right|$ gives the amplitude of the wave at $x = 0$. We call this wave, the ' Forward Current Travelling Wave '

- Similarly, the second term in the current solution $I^- e^{\alpha x} e^{j\omega t + j\beta x}$ gives a travelling wave but travelling in negative ' x ' direction (right to left) as shown in Figure. $\left| I^- \right|$ gives the amplitude of the at $x = 0$. This wave we call the 'Backward Current Travelling Wave' .

Important Conclusion

- **The Voltage and the Current exist in the form of waves on a transmission line.**
- **In general, we can say that in a circuit, any time varying voltage and/or current always exist in the form of waves, although the wave nature may not be evoked at low frequencies where the transit time effects are negligible.**

Complex Propogation Constant

- The propagation constant γ in general is complex

$$\gamma = \alpha + j\beta$$

The wave amplitude varies as $e^{-\alpha x}$. That is α denotes the exponential decay of the wave along its direction of propagation. α therefore is called the 'Attenuation Constant' of the line . It has the unit Neper/m . For α =1 Neper/m, the wave amplitude reduces to 1/e of its initial value over a distance of 1m.

- Many times the attenuation of a wave is measured in terms of dB/m. α therefore can be given in dB/m, where

$$\boxed{\text{1 Neper/m} = \text{8.68 dB/m}}$$

Note

In voltage/current expressions, α should always be in Neper/m. Therefore if α is given in dB/m it should be converted to Neper/m before it is used in the voltage/current equations.

The wave phase has two components

- Time phase ωt
- Space phase $\pm\beta x$

The parameter β gives the phase change per unit length and hence called the 'Phase Constant' of the line. Its units are Radian/m.

Now for a wave the distance over which the phase changes by 2π is called the ' wavelength ' (λ) . Therefore the phase change per unit length

$$\boxed{\beta = \frac{2\pi}{\lambda}}$$

Characteristic Impedance of Transmission Line

- Substituting the voltage and current solutions in the differential equations, and noting that the equations must be satisfied by two waves indepentely we get,

$$\frac{V^+}{I^+} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\frac{V^-}{I^-} = -\frac{R + j\omega L}{\gamma} = -\sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

- We define a parameter called the 'Characteristic Impedance' of the line as

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

Note

- **The ratio of Forward Voltage and Current waves is always Z_0 , and the ratio of the Backward Voltage and Current waves is always $-Z_0$.**
- **The parameters γ and Z_0 completely define the voltage and current behaviour on a transmission line. These two parameters are related to R, L, G, and C, and the frequency of the signal. In transmission line analysis knowledge of γ and Z_0 is adequate and the explicit knowledge of R, L, G, C is rarely needed.**

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Recap

In this course you have learnt the following

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