Module 7 : Antenna

Lecture 47 : Introduction

Objectives

In this course you will learn the following

- What is an Antenna?
- When do structures radiate?
- Magnetic vector potential
- Lorentz gauge condition
- Green's function
- Spherical wave

Introduction

- An is transducer which converts electrical signals into electromagnetic waves and vice versa. So, if an antenna is excited with a voltage/current it generates electromagnetic waves, and if placed in front of an electromagnetic wave, it extracts power from the wave and delivers to the load connected to it.

- The phenomenon of electromagnetic radiation is related to the acceleration of electric charges. An accelerated charge corresponds to the time-varying current (a steady flow of charge gives the DC current and the AC current requires acceleration of charges).

- In principle, every time-varying current can give EM radiation no matter how small the frequency of the current is.

- An antenna however is a structure which generates EM radiation with high efficiency. Also it will be seen subsequently that the antennas do not generate EM waves uniformly in all direction. Every antenna preference for certain directions and no preference fo other directions.

- Antenna design therefore focuses on two issues

(1) How to get highest possible radiation efficiency from an antenna

(2) How to design antenna structure to achieve desired spatial distribution of the EM waves

- Let us investigate the characteristics of basic antennas.

Theory

Potential Functions

- Here we solve the Maxwell's equations with electrical sources to obtain the electromagnetic fields. The four Maxwell's equations with sources are given as

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathrm{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathrm{J} + \epsilon \frac{\partial \mathbf{H}}{\partial t}$$

- The solution of the Maxwell's equation for electric and magnetic fields is rather difficult. Instead, one can define potential functions related to the fields and find their solution.

Eqn. is identically satisfied by a vector A defined as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\Rightarrow \quad \mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A}$$

Substituting in the curl equation, we get

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0$$

- This eqn can be identically satisfied if we define the quantity inside the bracket as the gradient of some function as

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V$$

- The vector A is called the magnetic vector potential and the quantity V is called the electric scalar potential.

Laurentz Gauge Condition and Wave Equation

- If we substitute for B and E in the remaining two Maxwell's equations we get

$$\nabla \cdot \left(-\frac{\partial \mathbf{A}}{\partial t} - \nabla V \right) = \frac{\rho}{\epsilon}$$

$$\nabla \cdot \left(-\frac{\partial \mathbf{A}}{\partial t} - \nabla^2 V \right) = -\frac{\rho}{\epsilon}$$

$$\text{and } \frac{1}{\mu} \nabla \times \nabla \times \mathbf{A} = \mathbf{J} + \epsilon \frac{\partial}{\partial t} \left(-\frac{\partial \mathbf{A}}{\partial t} - \nabla V \right)$$

  The two equations are coupled equation for V and A.

- The equations can be decoupled using what is called the **Laurentz gauge** condition given as

$$\nabla \cdot \mathbf{A} = -\mu \epsilon \frac{\partial V}{\partial t}$$

- The decoupled equations for the electric scalar and magnetic vector potential are

$$\nabla^2 \mathbf{A} - \mu \epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$$

$$\text{and } \nabla^2 \mathbf{V} - \mu \epsilon \frac{\partial^2 \mathbf{V}}{\partial t^2} = -\frac{\rho}{\epsilon}$$

- Two things can be verified from these equations:

(1) For the non-time varying (static) case the equations reduce to well known Poisson's equation.

(2) For the source free case the equation reduce to the wave equation discussed earlier.

- Since the magnetic vector potential and the electric scalar potential are related through the Laurentz gauge condition, solving the wave equation for one of them is adequate.

- Generally we find the solution for the magnetic vector potential.

- For sinusoidal variation of the current and potential with angular frequency ω, the equation becomes

$$\nabla^2 \mathbf{A} + \omega^2 \mu \epsilon \mathbf{A} = -\mu \mathbf{J}$$

- Noting that the quantity $\omega \sqrt{\mu \varepsilon} = \beta$, the phase constant of the wave, the wave equation becomes

$$\nabla^2 \mathbf{A} + \beta^2 \mathbf{A} = -\mu \mathbf{J}$$

- The equation is solved by using the **Green's function technique.**

Green's Function Technique

- The Green's function is a solution of the differential equation with the driving term replaced by the δ -function. The Greens function , G therefore satisfies the equation

$$\nabla^2\mathbf{G} + \beta^2\mathbf{G} = \delta(space)$$

- The Green's function essentially is the spatial impulse response of the system described by the wave equation.

- The most appropriate coordinate system for analyzing EM radiation is the spherical coordinate system.

- The Green's function for the δ -function located at the origin of the coordinate system is obtained as

$$G = -\frac{e^{-j\beta r}}{4\pi r}$$

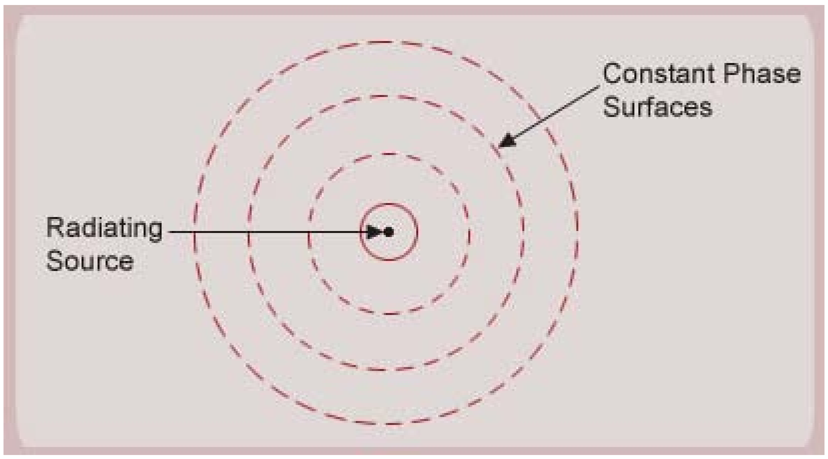
- It can be noted that

(1) The expression represents a traveling wave in r direction.

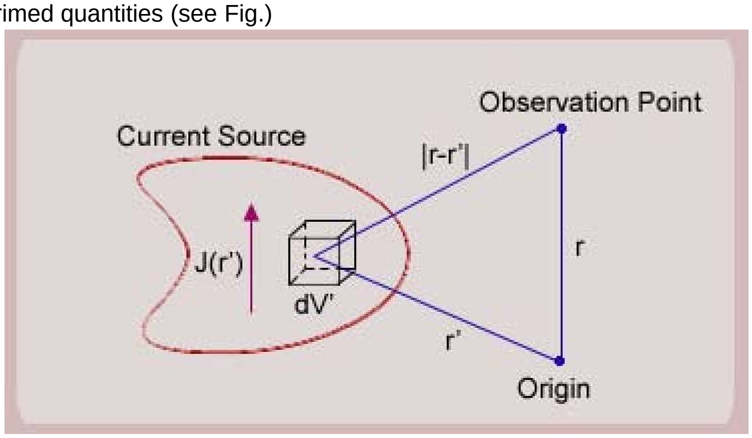
(2) The amplitude of the Greens function is inversely proportional to r.

(3) The constant phase surfaces for the traveling wave are spheres.

(4) The phenomenon therefore represents a outward traveling spherical wave as shown in the following Fig.



- Let now the current be distributed over a volume denoted by primed quantities and let the location of the observation point be denoted by the un-primed quantities (see Fig.)



- The total magnetic vector potential due to the current distribution is then given as

$$\mathbf{A} = \int_v \mu \mathbf{J}(r') \frac{e^{-j\beta|r-r'|}}{4\pi|r-r'|} dv'$$

- Note that the integral is a convolution of the spatial impulse response (the Green's function) and the driving source function $-\mu \mathrm{J}$.

- So the antenna analysis problem reduces to finding the vector potential from the current distribution on an antenna. Once the vector potential is known, the electric and magnetic fields, and subsequently the power radiated by the antenna can be obtained in a rather straight forward manner.

- Obtaining current distribution on an antenna structure is a rather complex task an is beyond the scope of this course. Here therefore we will assume a current distribution and then proceed to find the fields radiated by it.

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Recap

In this course you have learnt the following

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