**Module 5 : Plane Waves at Media Interface**

**Lecture 37 : Total Internal Reflection**

## Objectives

**In this course you will learn the following**

- Condition for Total Internal Reflection.

## Condition for Total Internal Reflection

- From Snell's law we have seen that $\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$ .
- Therefore if $\frac{\beta_1}{\beta_2}\sin\theta_i \geq 1$ , then $\sin\theta_t > 1$ and consequently the angle of transmission does not exist.
- That means if a wave is launched at an angle which satisfies the above condition there is no transmitted wave.
- The incidence angle for which the angle of transmission $\theta_t$ is $90^0$ is called the critical angle $\theta_c$ . We therefore get

$$\sin\theta_c = \frac{\beta_2}{\beta_1}$$

- For angle of incidence greater than the critical angle the reflection coefficient for the two polarization can be written as

$$\Gamma_{\parallel} = \frac{\eta_1\cos\theta_i - j\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}{\eta_1\cos\theta_i + j\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}$$

$$\Gamma_{\perp} = \frac{\eta_2\cos\theta_i - j\eta_1\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}{\eta_2\cos\theta_i + j\eta_1\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}$$

the quantity inside the square root sign is positive in this case.

- We note that now the magnitude of a reflection coefficient is unity and the reflection coefficients have become complex i.e when $\theta > \theta_c, |\Gamma_\perp| = 1$ and $|\Gamma_\parallel| = 1$, but $\Gamma_\parallel$ and $\Gamma_\perp$ are complex.
- The phase angle's of the two reflection coefficients respectively are

$$\phi_\parallel = -2\tan^{-1}\left(\frac{\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}{\eta_1\cos\theta_i}\right)$$

$$\phi_\perp = -2\tan^{-1}\left(\frac{\eta_1\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}{\eta_2\cos\theta_i}\right)$$

- Since the magnitude of reflection coefficient are unity the entire power incident on the dielectric interface is reflected back into medium 1. This phenomena is called therefore called **TOTAL INTERNAL REFLECTION.**
- We can note the following important points about **Total Internal Reflection** :

**(1)** Since $\sin\theta_t$ has to be $\leq 1$ the Total Internal Reflection can take place only if

$$\mu_2\epsilon_2 \leq \mu_1\epsilon_1$$

i.e., medium 2 is rarer compared to medium 1.

**(2)** For dielectric media a special case of this is an ideal dielectric interface, for which $\mu_1 = \mu_2 = \mu_0$,

$$\epsilon_1 = \epsilon_0\epsilon_{r1} = \epsilon_0 n_1^2$$

$$\epsilon_2 = \epsilon_0\epsilon_{r2} = \epsilon_0 n_2^2$$

Where $n_1$ and $n_2$ are refractive indices of the two dielectric media. The above condition then reduces to the well known relation

$$n_2 \leq n_1$$

**(3)** The wave undergoes a phase change at a Total Internal Reflection. The phase change depends upon the media parameters as well as the angle of incidence.

**(4)** For a given angle of incidence, waves with two polarization, parallel and perpendicular, undergo different phase change.

**(5)** Although the magnitude of reflection coefficient is unity for total internal reflection the transmission coefficient is not zero. That means although there is no wave propogation in medium 2 there are electric and magnetic fields in medium 2. Since there is no wave propogation in medium 2 these fields do not constitute any power flow. These fields are called ' **Evanescent Fields**'.

**(6)** The **Evanescent Fields** exponentially deals away from the interface. At critical angle's, fields are constant in medium 2 and as the angle increases beyond the critical angle the fields decay more rapidly. Nevertheless there will always be fields in medium 2 even in case of Total Internal Reflection.

## Problem

**Recap**

**In this course you have learnt the following**

- Condition for Total Internal Reflection.