**Module 3 : Maxwell's Equations**

**Lecture 23 : Maxwell's equations in Differential and Integral form**

**Objectives**

**In this course you will learn the following**

- Maxwell's Equations
- Maxwell's equation for Static fields

## Maxwell's Equations

| Differential form | Integral form |
|---|---|
| $\nabla \cdot \mathrm{D} = \rho$ | $\oint_s \mathrm{D} \cdot \mathrm{da} = \iiint \rho dv$ |
| $\nabla \cdot \mathrm{B} = 0$ | $\oint_s \mathrm{B} \cdot \mathrm{da} = 0$ |
| $\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t}$ | $\oint \mathrm{E} \cdot \mathrm{dl} = -\iint \frac{\partial \mathrm{B}}{\partial t} \cdot \mathrm{da}$ |
| $\nabla \times \mathrm{H} = \mathrm{J} + \frac{\partial \mathrm{D}}{\partial t}$ | $\oint \mathrm{H} \cdot \mathrm{dl} = \iint \left(\mathrm{J} + \frac{\partial \mathrm{D}}{\partial t}\right) \cdot \mathrm{da}$ |

- The differential form of Maxwell's equations are point relations i.e., they establish relationship between fields and sources at any point is space. The differential form involves derivatives of the fields with respect to space and time.
- Since $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$, the derivative form is suitable only in those situations where medium and fields are continuous in space and the space derivatives exist. At media interfaces where the medium properties change abruptly, the differential form of Maxwell's equations can not be used. However, in this situation the integral form of Maxwell's equations can be used.

## Maxwell equation when applied to boundary

| Dielectric-dielectric interface | Dielectric-conductor interface |
|---|---|
| $E_{t1} = E_{t2}$ | $E_{t1} = 0$ |
| $D_{n1} = D_{n2}$ | $D_{n1} = \rho_s$ |
| $H_{t1} = H_{t2}$ | $H_{t1} = J_s$ |
| $B_{n1} = B_{n2}$ | $B_{n1} = 0$ |

In the above table,

- Suffix '$t$' represents Tangential compotent
- Suffix '$t$' represents Normal compotent
- The Row suffix '$s$' represent Surface charged density.

## Maxwell's equation for Static fields

- $$\nabla \cdot \mathbf{D} = \rho$$
  $$\nabla \cdot \mathbf{B} = 0$$
  $$\nabla \times \mathbf{E} = 0$$
  $$\nabla \times \mathbf{H} = \mathbf{J}$$

- We can make an important observation at this point and that is, the static electric fields are always conservative fields $(\nabla \times \mathbf{E} = 0)$. The time varying fields on the other hand are always non conservative because $\mathbf{E}$ and $\mathbf{H}$ both coexist in a medium and their time derivatives are non zero.

**Recap**

**In this course you have learnt the following**

- Maxwell's Equations
- Maxwell's equation for Static fields

**Congratulations! You have finished Module 3. To view the next Module select it from left hand side of the page.**