UKF Measurement Update with IMU Yaw Sensor

Overview

In the Unscented Kalman Filter (UKF), both the prediction and measurement updates rely on propagating sigma points through nonlinear functions. For the measurement update step, this involves computing a predicted measurement from each sigma point, even when the actual sensor provides a direct scalar reading like yaw from an IMU.

Measurement Function

Assume the robot's state is represented as:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

where x, y represent position and θ is the heading (yaw). The IMU gives a direct measurement of the heading:

$$z_t = \text{yaw at time } t = \theta_t$$

To incorporate this in the UKF, we define a measurement function:

$$h(\mathbf{x}) = \theta$$

Even though the sensor provides a direct scalar value, the UKF still needs to simulate how this measurement varies due to uncertainty in the state.

UKF Measurement Update Steps

Given:

- Predicted state mean: $\bar{\mu}_t$
- Predicted state covariance: $\bar{\Sigma}_t$
- Actual sensor measurement: z_t

1. Generate Sigma Points

Generate $2n + 1$ sigma points χ_i from the predicted state distribution $\mathcal{N}(\bar{\mu}_t, \bar{\Sigma}_t)$.

2. Propagate Sigma Points through Measurement Function

Each sigma point χ_i is passed through the measurement function:

$$\mathbf{z}_i = h(\chi_i) = \theta_i$$

This yields a set of predicted measurement sigma points $\mathbf{z}_i$.

3. Compute Predicted Measurement Mean

$$\bar{z}_t = \sum_{i=0}^{2n} w_i^{(m)} \mathbf{z}_i$$

4. Compute Innovation Covariance

$$S_t = \sum_{i=0}^{2n} w_i^{(c)} (\mathbf{z}_i - \bar{z}_t)(\mathbf{z}_i - \bar{z}_t)^T + R_t$$

where R_t is the measurement noise covariance.

5. Compute Cross-Covariance

$$C_t = \sum_{i=0}^{2n} w_i^{(c)} (\chi_i - \bar{\mu}_t)(\mathbf{z}_i - \bar{z}_t)^T$$

6. Compute Kalman Gain

$$K_t = C_t S_t^{-1}$$

7. Update State and Covariance

$$\mu_t = \bar{\mu}_t + K_t(z_t - \bar{z}_t)$$
$$\Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^T$$

Conclusion

Although the sensor measurement (e.g., IMU yaw) is a scalar, UKF requires a measurement function $h(x)$ to propagate uncertainty via sigma points. This allows the filter to capture nonlinearities in how state uncertainty translates into measurement uncertainty, even for seemingly simple sensors.