Exercise 2:

We consider a buffer that receives messages to be sent. The transmission is made by means of two modern lines that operate at the same speed. We know that:

- The message arrival time is Poisson with mean rate λ.
- The message transmission time is exponentially distributed with mean value E[X].

We are requested to determine:

- The traffic intensity in Erlangs that is offered to the buffer.
- The mean number of messages in the buffer.
- The mean delay for a message from its arrival to the buffer till it is completely transmitted.
- Could the buffer support an input traffic characterised by λ=10 msg/sec and E[X]=2s?

Solution:

We can model the system as a M/M/2 queue with arrival rate λ and completion rate $\mu = 1/E[X]$. The transmission queue is stable as long as the ergodicity condition is satisfied, i.e. $\lambda/2\mu < 1$ Erlang, implying that the limiting input load, $\rho = \lambda/\mu$ that the system can handle is 2 Erlangs. The system is described by a Markov chain of the number of the number of messages in the system as the random variable, as shown. As long as we have stability, we can solve the system by the means of cut equilibrium conditions.

The various cut equations are:

Cut 1:	$\lambda P_0 = \mu P_1$	=>	$P_1 = \rho P_0$
Cut 2:	$\lambda P_1 = 2\mu P_2$	=>	$P_2 = (\rho^2/2)P_0$
Cut 3:	$\lambda P_2 = 2\mu P_3$	=>	$P_3 = (\rho^3/2^2)P_0$

Generalizing,

Cut n:	$\lambda P_{n-1} = \mu P_n$	=>	$P_n = 2\,(\rho/2)^n\,P_0$

We can use the normalization condition to evaluate P_0.

$$P_0 = \frac{1}{1 + \sum P_n/P_0} = \frac{1}{1 + 2\left(\sum\left(\frac{\rho}{2}\right) - 1\right)} = \frac{2 - \rho}{2 + \rho}$$

Since $P_0 > 0$, we have that $\rho < 2$ Erlangs.

We can now evaluate the mean number of messages in the system by the means of the first derivative of the PGF.

We get the mean number of messages in the system $N = 4\rho / (4 - \rho^2)$.

The mean delay can be now calculated using Little's theorem as $T = N / \lambda$.

$T = 4\rho/ (4- \rho^2)\ \lambda$

As for the last question, we need to evaluate the input traffic intensity load ρ, which as per the given data is 20 Erlangs, which exceeds the maximum allowed load (2 Erlangs) for system stability.

Simulation:

We can confirm our results with a simulation, following the procedure below:

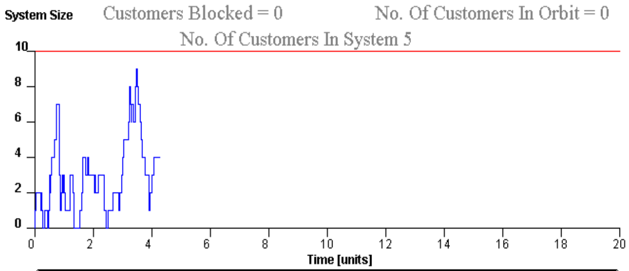
- → Open the simulation page.
- → Put the values of the parameters, λ = 9 msg/sec, and μ = 5sec, and 2 servers and start the simulation.

→ The applet now produces a simulation.

M/M/c

Customers Entered = 44 | Customers Served = 39
Customers Blocked = 0 | No. Of Customers In Orbit = 0

No. Of Customers In System 5



Performance Measures	Run Time *(till t=5)*	Steady State *(Theoretical)*
Mean No. of Customers in the System	2.927	9.473
Mean No. of Customers in Queue	1.276	7.673
Mean Waiting Time In Queue	0.124	0.852
Mean Sojurn Time In System	0.285	1.052
Utilisation	0.825	0.9
Throughput	9.094	9.0
Blocking Probability	0.0	0.0

Pause Reset

Arrival Rate (lambda) : 9

Departure Rate (mu) : 5

Number of Servers : 2

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