Exercise 1:

Consider a switch with a single output line. Calls arrive according to interarrival times exponentially distributed with mean rate α. Each call has a length with exponential distribution and mean rate γ. The switch has no waiting list: if an arriving call finds a busy output line, it is blocked and lost. It is requested to model this line and find the call blocking probability P_B. What is the maximum input load in Erlangs to have a blocking probability less that 1%?

Solution:

We observe that the queuing model is of the M/M type, but with only one place in the system – the place for the served request. Thus the model is M/M/1/1 type queuing model with a two state Markov chain.

Let $\rho = \alpha/\gamma$ denote the input traffic intensity. We can state the cut equilibrium condition and the normalization condition to find the state probability distributions P_0 and P_1.

$$\alpha P_0 = \gamma P_1, \qquad \text{and} \qquad P_0 + P_1 = 1,$$

giving us $$P_0 = \gamma/(\alpha+\gamma) \qquad \text{and} \qquad P_1 = \alpha / (\alpha+\gamma).$$

Now, we see that a new call arriving in to the system is blocked and lost if the system is in state 1, i.e. the probability of blocking is same as the probability of being in state 1.

Therefore, $P_B = P_1 = \alpha / (\alpha+\gamma) = \rho / (1+\rho)$.

For the blocking probability to be less than 1%, we require that

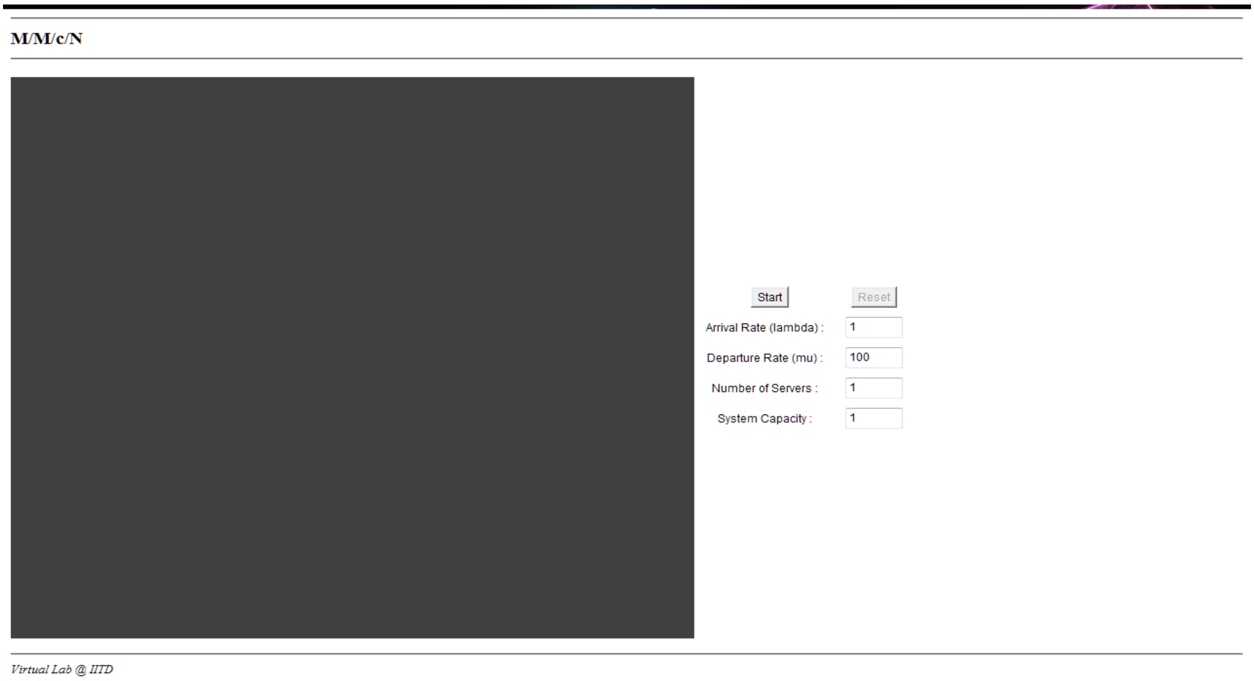
$$\rho / (1+\rho) \leq 0.01$$
$$\Rightarrow \quad \rho \leq 0.01/(1 - 0.01)$$

Hence, this is the maximum input load in Erlangs, so which the blocking probability is less than 1%.

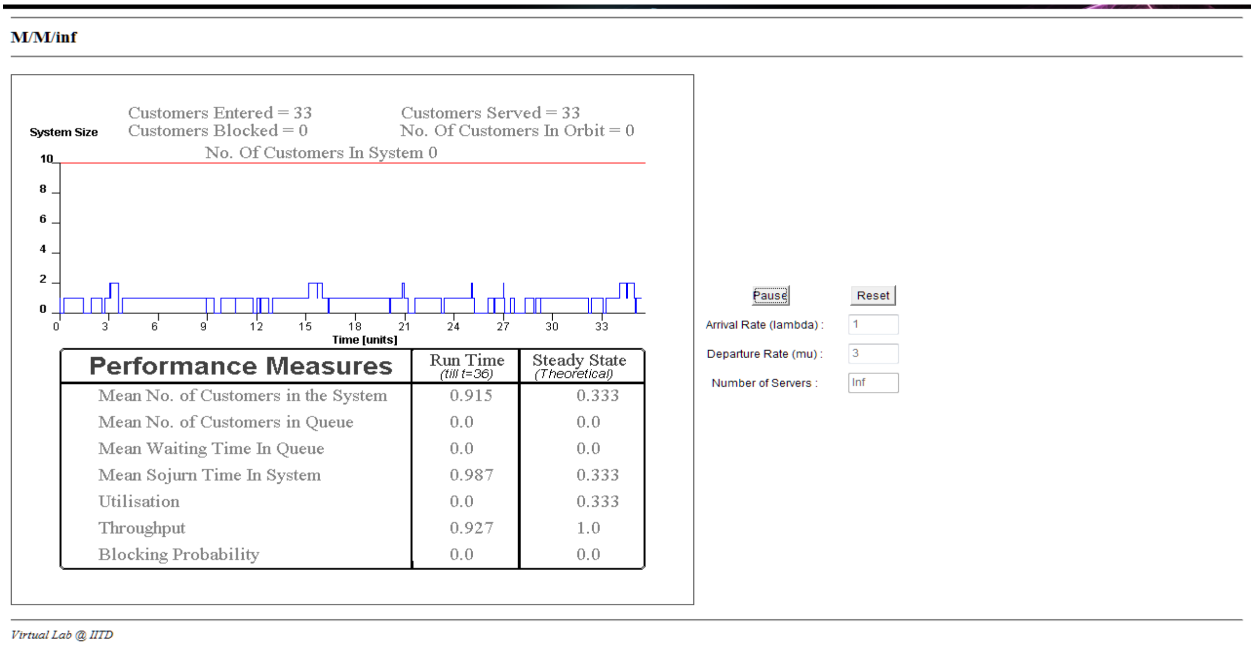
Simulation:

For simulation of the traffic generator, perform the following steps:

- → Open the page where the simulation is to be performed.
- → Next feed the data as shown. Put lambda (λ) = 1, mu (μ) = 100, 1 server and system capacity =1



- → Click Start. The applet will now generate a sample path for the queue.



- → We see that the steady state data obtained from the applet matches beautifully with the theoretically calculated data.