Exercise 4:

An *internet service provider* (ISP) must design the number of access lines to a *point-of-presence* (POP), S, in order to guarantee a blocking probability lower than or equal to 2%. The following data is available:

- The served users produce a mean total arrival rate of calls in the rush hour equal to 6 calls / min.
- Each call (internet dial-up connection) has a duration modelled by an exponentially distributed variable with mean value of 3 minutes.

It is requested to derive the analytical model of the system, to express the blocking probability and to derive the value of S according to the Erlang-B table.

Solution:

The POP system can be modelled as a M/M/S/S queuing system assuming that dial-up connections occur according to a Poisson process with the mean rate $\lambda = 6$ per minute. In this model, the mean completion rate is $\mu = 1/3\ \text{min}^{-1}$. Note that such a queuing model is always stable, since it follows the Blocked Calls Cleared discipline.

Using the cut equilibrium condition, the normality condition, and the fact that a call is blocked iff the system is in state 's', we get the following result.

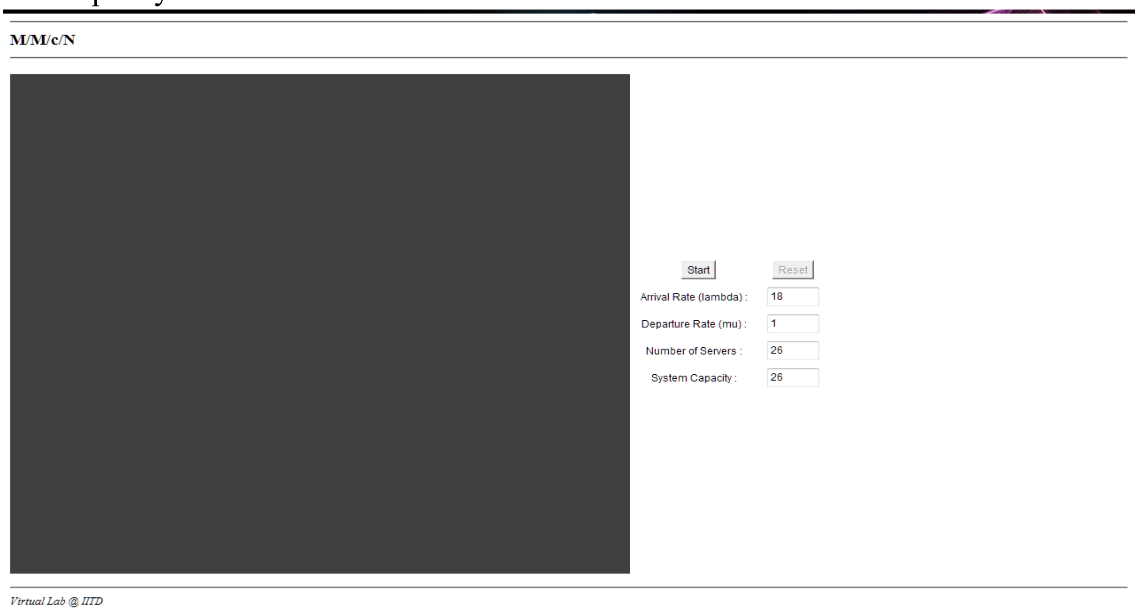
$$P_B = P_S = \rho^S/(S!*\Sigma\ (\rho^S/i!)),$$ where the summation is from i = 0 to i = S.

We need to find the value of S for which $P_B <= 0.02$ for an input traffic intensity of $\rho = \lambda/\mu$ =18 Erlangs. Using the Erlang B table, we reach the conclusion that S = 26 lines.

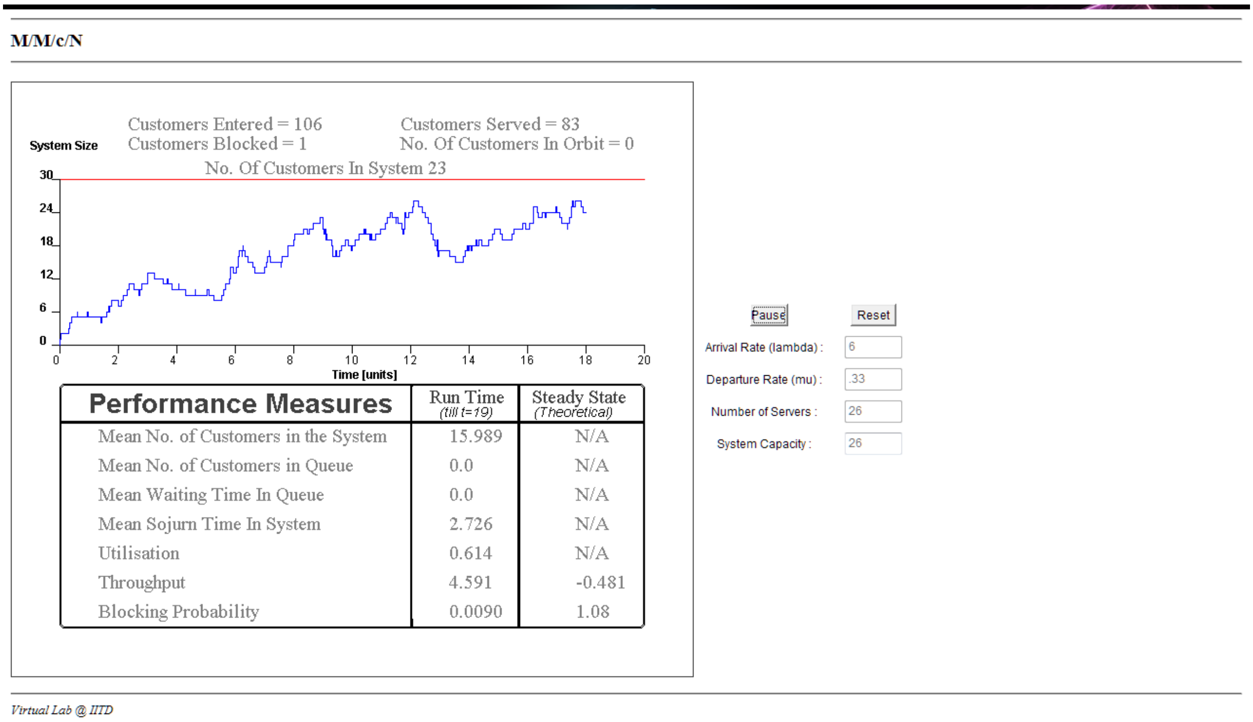
Simulation:

For simulation of the POP system, perform the following steps:

→ Open the page where the simulation is to be performed.
→ Next feed the data as shown. Put lambda (λ) = 18, mu (μ) = 1, 26 servers and system capacity as 26.



→ Click Start. The applet will now generate a sample path for the queue.



→ We see that the runtime data obtained from the applet matches beautifully with the theoretically calculated data.