Exercise 5:

We consider a finite branch exchange that collects phone calls generated in a firm where there are 1000 phone users, each contributing Poisson traffic of 30 mErlangs. We have to design the number of the public lines from the private branch exchange to the central office of the public phone network in order to guarantee a blocking probability for new calls lower than or equal 3%. What is the increase in the number of output lines if the number of users changes to 1300, still requiring a blocking probability of 3% or lower? Also compare the percentage traffic $\Delta\ \rho\%$ increase to the percentage increase in output lines $\Delta S\%$. Assume that the duration of each is exponentially distributed with mean 20 seconds.

Solution:

Since there are 1000 independent users each generating 30 mErlangs of traffic, we consider the approximation to an infinite number of users. We know that the calls have durations exponentially distributed with mean value $\mu = 1/3$ min. The behaviour of the private branch exchange can be studied by means of an M/M/S/S system. The value of S has to be determined on the basis of the blocking probability. The input traffic density can be evaluated as:

- Each user contributes a mean arrival rate of phone calls equal to 30×10^{-3}Erlang / 3 min = 10^{-2} call/min.
- The total mean arrival rate has a rate $\lambda = 1000 \times 10^{-2} = 10$ calls/min.

Thus, the input traffic intensity is $\rho = \lambda/\mu = 30$ Erlangs. The value of S is now found out using the standard Erlang-B tables. It turns out that S = 38.

If the number of users is increased to 1300, we get the traffic input intensity as $\rho = \lambda/\mu = (1300 \times 10^{-2}$ calls/min$)^*$ 3 min = 39 Erlangs. Using the Erlang–b table again, we get a new S = 47 lines.

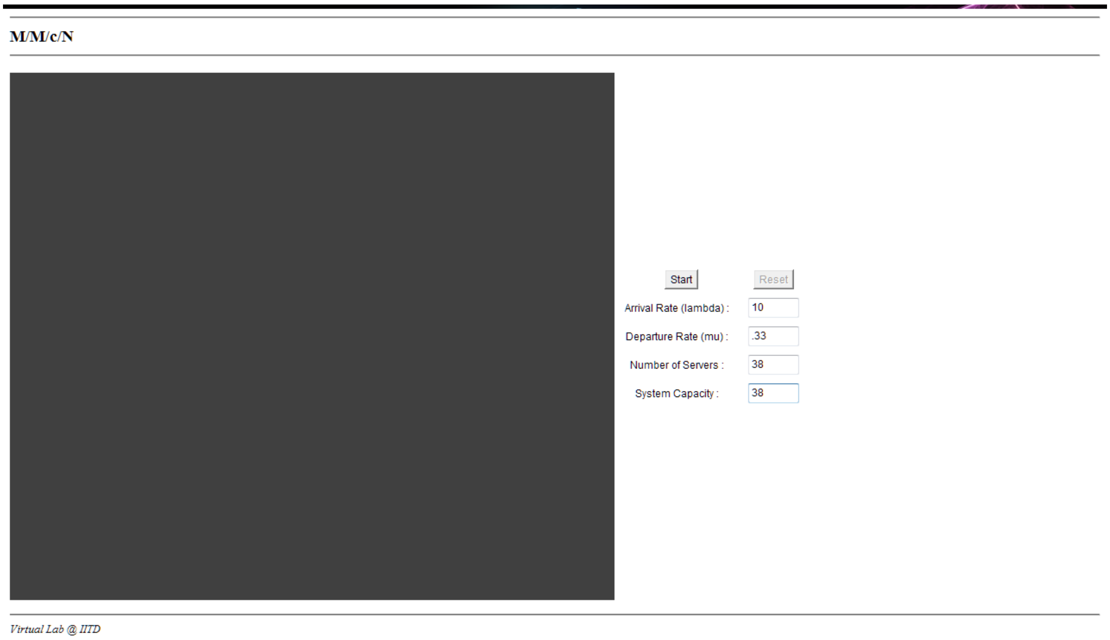
The percentage change in ρ is $\Delta\rho\% = 30\%$, while that in S is $\Delta S\% = 23.7\%$. Hence, we observe under a fixed constraint on the call blocking probability, $\Delta S\% < \Delta\rho\%$.

Simulation:

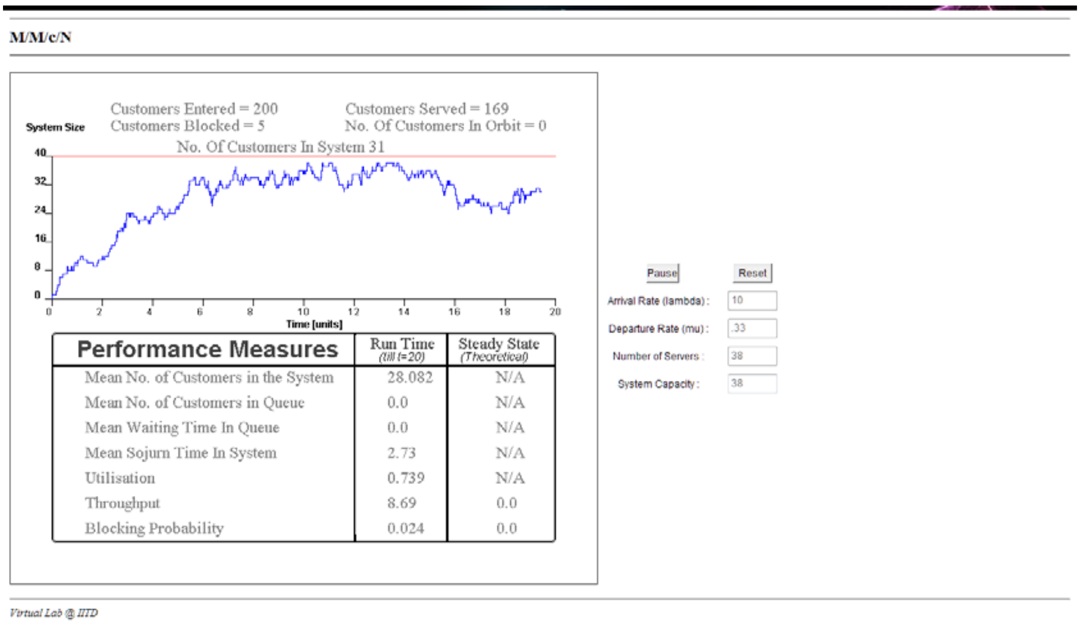
For simulation of the private branch exchange, perform the following steps:

→ Open the page where the simulation is to be performed.

→ Next feed the data as shown. Put lambda (λ) = 10, mu (μ) = .33, 38 servers and system capacity as 38.



→ Click Start. The applet will now generate a sample path for the queue.



→ We see that the runtime data obtained from the applet matches beautifully with the theoretically calculated data.