Exercise 3:

A radio link adopts four equivalent parallel transmitters for redundancy reasons. The operational characteristics of the transmitters require that each of them be switched off (for maintenance or recovery actions) according to a Poisson process with a mean interarrival time of 1 month. The technician that performs maintenance and recovery actions requires a time exponentially distributed with mean duration of 12 hours in order to fix the problem. We consider that two such technicians are available. We are required to:

- Define a suitable model for the system.
- To determine the probability distribution of the number of down transmitters at a generic instant.
- To express the probability that no transmitter is operational on this radio link.

Solution:

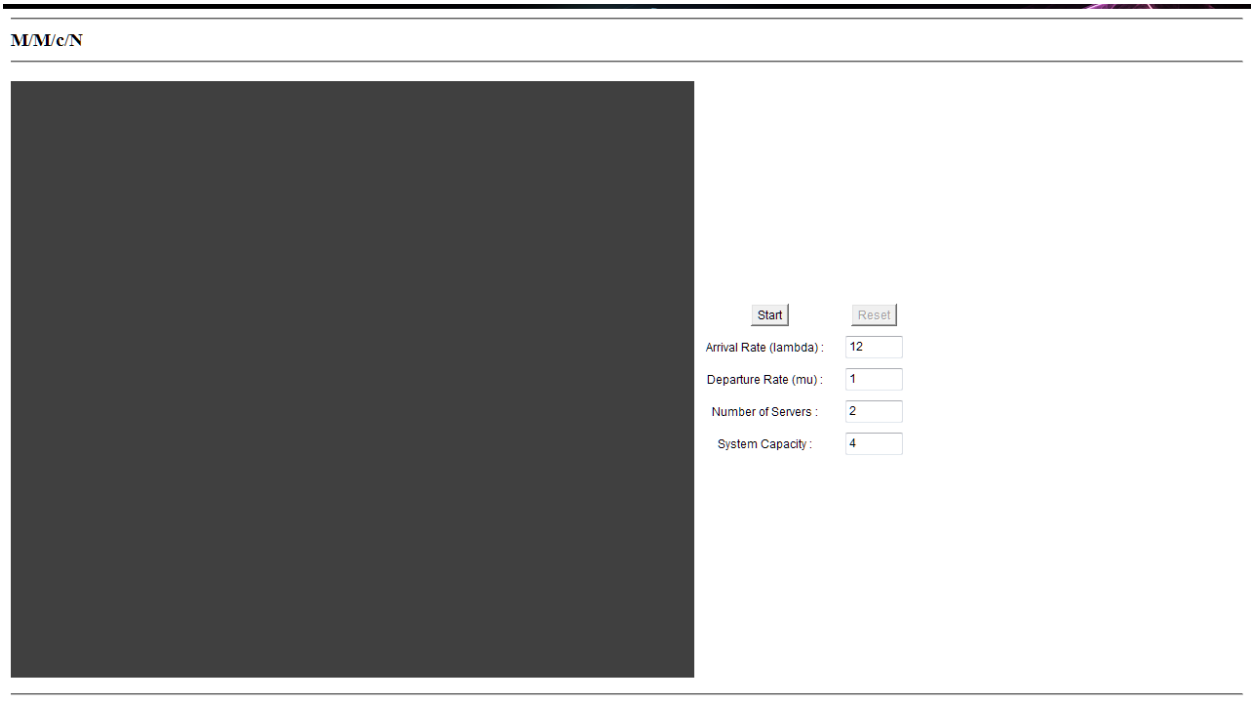
We consider the system to be composed of five states, denoting the number of non-operational transmitters. The transition from a state 'j' to a state 'j+1' is the minimum among 4-j independent times with exponential distribution and mean rate $\lambda = 1$ per month; such time is exponentially distributed with mean (4-j) λ. Transitions from state j (j = 2,3,4) to the state 'j-1' is the minimum of two exponential times with mean $\mu = 1/12$ per hour each; hence these transitions occur with a mean rate of 2μ. The transition from state 1 to state 0 is, of course, exponentially distributed with mean μ. This is thus a model of the M/M/2/4/4 type.

The cut equilibrium conditions and the normalization conditions are used to find the state probabilities. And finally, the probability that no transmitter is functional is given by the probability P_4.

Simulation:

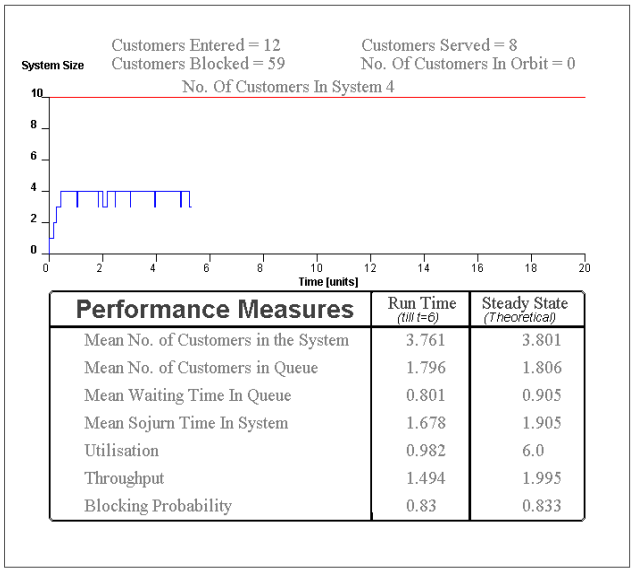
For simulation of the traffic generator, perform the following steps:

→ Open the page where the simulation is to be performed.
→ Next feed the data as shown. Put lambda (λ) = 12, mu (μ) = 1, 2 servers and system capacity = 4.



→ Click Start. The applet will now generate a sample path for the queue.

M/M/c/N



Performance Measures	Run Time (till t=6)	Steady State (Theoretical)
Mean No. of Customers in the System	3.761	3.801
Mean No. of Customers in Queue	1.796	1.806
Mean Waiting Time In Queue	0.801	0.905
Mean Sojurn Time In System	1.678	1.905
Utilisation	0.982	6.0
Throughput	1.494	1.995
Blocking Probability	0.83	0.833

Pause Reset

Arrival Rate (lambda) : 12

Departure Rate (mu) : 1

Number of Servers : 2

System Capacity : 4

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→ We see that the steady state data obtained from the applet matches beautifully with the theoretically calculated data.