Exercise 2:

Consider the post-office of a small town. There is only Mr. McPhee working in the post office. It has been observed that the rate at which letters comes in for postage is about 3 per hour. McPhee being highly experienced is able to work at a rate that allows him to handle postage requests at the rate of 5 per hour. The interarrival time for letters is approximately exponential, and so the time McPhee takes to handle a postage request. Also, note that since the post office is small, lack of space does not allow McPhee to put more than 4 requests on hold. What is the probability that a person coming to send a letter is unable to do so?

Solution:

The postage system can be modelled as a M/M/1/5 FIFO queue (5 because one for the request being serviced and 4 on hold). The arrival is a Poisson process with arrival rate $\lambda = 3$ per hour, the service is exponentially distributed with mean rate $\mu = 5$ per hour. The net input traffic is therefore $\rho = \lambda/\mu = 3/5 = 0.6$.

The cut equations and the normalization constant allow us the find the state probabilities:

$P_0 = .42$
$P_1 = .25$
$P_2 = .15$
$P_3 = .09$
$P_4 = .054$
$P_5 = .033$

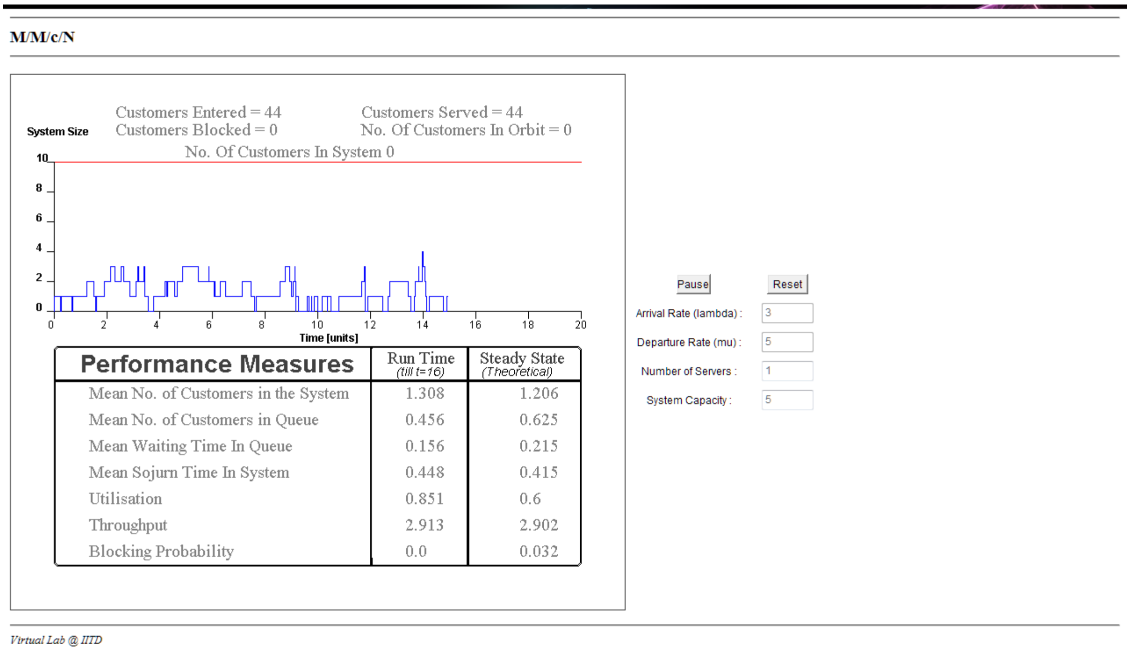
The probability that a person will be unable to use the post office is same as the probability that the queue has 4 people (i.e. the system has 5 people, P_5).

Simulation:

For a simulation of the post office, perform the following steps:

→ Open the page where the simulation is to be performed.
→ Next feed the data as shown. Put lambda (λ) = 3, mu (μ) = 5, 1 server and system capacity 5.

→ Click Start. The applet will now generate a sample path for the queue.



Performance Measures	Run Time (till t=16)	Steady State (Theoretical)
Mean No. of Customers in the System	1.308	1.206
Mean No. of Customers in Queue	0.456	0.625
Mean Waiting Time In Queue	0.156	0.215
Mean Sojurn Time In System	0.448	0.415
Utilisation	0.851	0.6
Throughput	2.913	2.902
Blocking Probability	0.0	0.032

We see that the steady state data obtained from the applet matches beautifully with the theoretically calculated data.