Yule-Walker Estimation

The **Yule-Walker Method** for spectral estimation is based on the assumption that the signal can be modeled as an *autoregressive (AR) process*. Below is the step-by-step explanation.

1. Autoregressive (AR) Model

An AR process of order p is defined as:

$x[n] = -\sum_{k=1}^{p} a_k\, x[n-k] + e[n]$

Where:

- **x[n]**: Signal at time n.
- **a_k**: AR model coefficients to be estimated.
- **p**: Order of the AR model (number of past samples used).
- **e[n]**: White noise (residual error).

2. Autocorrelation Function

The AR model is closely linked to the **autocorrelation function**, which measures how similar the signal is to itself at different time lags:

$R[k] = E[x[n]\, x[n-k]]$

For a wide-sense stationary process, R[k] depends only on the lag k, not on n.

3. Yule-Walker Equations

The Yule-Walker equations relate the AR coefficients a_k to the autocorrelation values R[k]:

$$\begin{bmatrix} R[0] & R[1] & \dots & R[p-1] \\ R[1] & R[0] & \dots & R[p-2]] \\ & \vdots & & \\ R[p-1] & R[p-2] & \dots & R[0] \end{bmatrix} \begin{bmatrix} a1 \\ a2 \\ \vdots \\ ap \end{bmatrix} = \begin{bmatrix} -[R[1] \\ -[R[2] \\ \vdots \\ -[R[p] \end{bmatrix}$$

This forms a system of linear equations:

- **Left matrix**: Toeplitz matrix of autocorrelation values.
- **Right-hand side**: Negative autocorrelation values for lags 1 to p.

4. Solving for a_k

To solve for the AR coefficients a_k, use the autocorrelation values. The residual noise variance σ_e^2 is calculated as:

$\sigma_e^2 = R[0] + \sum_{k=1}^{p} a_k R[k]$.

5. Power Spectral Density (PSD)

The power spectral density (PSD) is computed as:

$P(f) = \sigma_e^2 / |1 + \sum_{k=1}^{p} a_k e^{-j2\pi fk}|^2$

This shows how power is distributed across frequencies using the AR coefficients a_k and the noise variance σ_e^2.

Key Intuitions

- **Autoregression**: Yule-Walker assumes the signal can be predicted using its past values.
- **Autocorrelation**: Relates the past values of the signal to the current value through R[k].
- **PSD**: Transforms time-domain properties (autocorrelation) into frequency-domain characteristics (PSD).

Steps for the Calculation of PSD

1. Estimate R[k] (autocorrelation values) from the signal.
2. Solve the Yule-Walker equations to find a_k.
3. Use a_k and σ_e^2 to compute the PSD.

This is the mathematical foundation of the Yule-Walker spectral estimation method, which is efficient and works well for stationary signals.