Theory -

The following assumptions are taken to derive the expressions for DCPF.

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \;\; \forall i \in n, \# \; (1)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \;\; \forall i \in n, \# \; (2)$$

Where, $|V_i|$ and δ_i are the voltage magnitude and angle at bus i, and $|Y_{ij}|$ and θ_{ij} are the magnitude and angle of the admittance matrix corresponding to the element at i^{th} row and j^{th} column. For a 'n' bus system, there are total '2n' load flow equations and '2n' variables.

Assumption 1: Reactive power flows and losses are ignored, $P_{ij} = -P_{ji}$ and $Q_{ij} = Q_{ji} = 0$, i.e., equation (2) is ignored.

Assumption 2: Voltage magnitudes are assumed closed to unity, i.e., $|V_i| \approx 1.0$ p.u.

With these two assumptions, the equation (1) is represented as (3), which can be further expanded to get (4).

$$P_i = \sum_{j=1}^{n} |Y_{ij}| \cos(\theta_{ij} - \delta_{ij}) \;\; \forall i \in n, \# \; (3)$$

$$\Rightarrow P_i = \sum_{j=1}^{n} |Y_{ij}|(\cos(\theta_{ij}) \cos(\delta_{ij}) + \sin(\theta_{ij}) \, sin(\delta_{ij})) \;\; \forall i \in n$$

$$\Rightarrow P_i = \sum_{j=1}^{n} G_{ij} \cos(\delta_{ij}) + B_{ij} \sin(\delta_{ij}) \;\; \forall i \in n, \# \; (4)$$

Assumption 3: Line resistances are generally much smaller than reactances due to high X/R ratio, $R_{ij} \ll X_{ij}$, i.e., $G_{ij} \ll B_{ij}$. Based on this assumption, (4) is expressed as (5).

$$P_i = \sum_{j=1}^{n} B_{ij} \sin(\delta_{ij}) \;\; \forall i \in n, \# \; (5)$$

Assumption 4: Generally, $\delta_{ij} \approx 0$, i.e., $\sin(\delta_{ij}) = \delta_{ij}$. Equation (5) can be expressed as (6).

$$P_i = \sum_{j=1}^{n} B_{ij} \delta_{ij} \;\; \forall i \in n, \# \; (6)$$

Or simply, $\mathrm{P_i} = \sum_{j=1}^{n} B_{ij}\delta_{ij} \;\; \forall i \in n$. Considering these assumptions the line power flow is simplified to $P_{ij} = \frac{\delta_i - \delta_j}{x_{ij}}$, where, P_i and δ_i are the net injected power and voltage angle at bus i, respectively. P_{ij} and x_{ij} are the power flow and reactance of the transmission line between buses i and j. B_{ij} is a constant matrix of dimension $(n-1) \times (n-1)$, removing the rows and column corresponding to slack bus. The elements of the B_{ij} matrix are negative of the imaginary part of the Y_{bus} matrix formed by ignoring the series resistances in the equivalent – π circuits of the transmission lines and setting the taps of off-nominal transformers equal to 1. Based on the power generation and demand, the power injection (excluding slack/reference bus need to be calculated), i.e., $P_i = P_i^G - P_i^D$. Assume the first bus as a slack bus, i.e., $\delta_1 = 0$, the voltage angles are calculated using $\delta_i = \left[B_{ij}\right]^{-1} P_i$. With these bus voltage angles, the power flows are calculated using $P_{ij} = \frac{\delta_i - \delta_j}{x_{ij}}$ and generation at the slack bus are calculated as in (8).

$$P_1^G = \sum_{\forall j \in 1} B_{1j}\delta_j + P_1^D, \# (7)$$

The following are the steps for performing the DCPF.

Step 1: Form the B_{ij} matrix. Form the bus injection vector (P_i).

Step 2: Solve linear equation $\delta_i = \left[B_{ij}\right]^{-1} P_i$ for all buses excluding the slack/reference bus.

Step 3: Compute the line power flows and the injection from the slack bus.