

QATAR UNIVERSITY
Foundation Program
Math & Computer Department
Math 3, Section 1.2
Polynomials: Basic Operations
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The aim of this document is to provide a short review and self assessment of the basic operations on polynomials.

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1. Introduction

1.1. Polynomial

A **polynomial** is a term or a finite sum of terms in which all variables have whole number exponents and no variables appear in denominators.

Examples of polynomials include:

$$5x - 6, \quad 4m^3 - 5m^2p + 8, \quad \text{and} \quad -7s^2t^3$$

Some examples of expressions that are not polynomials are:

$$x^{-1} + 3x^{-2}, \quad \sqrt{9-x}, \quad \text{and} \quad \frac{1}{x}$$

1.2. Finding the degree of a Polynomial

The **degree of a term** with one variable is the exponent on the variable. The degree of a term in more than one variable is defined to be the sum of the exponents of the variables. The greatest degree of any of the terms in a polynomial is called the **degree of the polynomial**.

Example:

Polynomial	Degree
$2x^3$	3
$-x^4$	4
$15x$	1
$4x^3y^7$	$3 + 7 = 10$
$4x^3 - 2x^2 - 3x + 7$	3

2. Simplifying Polynomials

2.1. Additional Distributive property

We use the distributive property to simplify polynomials by combining terms.

1. $a(b - c) = (b - c)a = ab - ac$
2. $a(b + c + \cdots + f) = ab + ac + \cdots + af$

Example 1. Remove parentheses and combine like terms.

$$2(3x^2 - 2x + 5) + (x^2 + 3x - 7)$$

Solution:

$$\begin{aligned} 2(3x^2 - 2x + 5) + (x^2 + 3x - 7) &= 2(3x^2 - 2x + 5) + 1(x^2 + 3x - 7) \\ &= 6x^2 - 4x + 10 + x^2 + 3x - 7 \\ &= 7x^2 - x + 3 \end{aligned}$$

□

Example 2. Remove parentheses and combine like terms.

$$(x^3 - 2x - 6) - (2x^3 - x^2 + 2x - 3)$$

Solution:

$$\begin{aligned} &= 1(x^3 - 2x - 6) + (-1)(2x^3 - x^2 + 2x - 3) \\ &= x^3 - 2x - 6 - 2x^3 + x^2 - 2x + 3 \\ &= -x^3 + x^2 - 4x - 3 \end{aligned}$$

□

Example 3. Remove parentheses and combine like terms.

$$[3x^2 - (2x + 1)] - (x^2 - 1)$$

Solution:

$$\begin{aligned} [3x^2 - (2x + 1)] - (x^2 - 1) &= [3x^2 - 2x - 1] - (x^2 - 1) \\ &= 3x^2 - 2x - 1 - x^2 + 1 \\ &= 2x^2 - 2x \end{aligned}$$

□

3. Adding Polynomials

To add polynomials, combine like terms.

Example 1. Add: $(4x^2 - 5x + 2) + (-9x^2 + 3x - 7)$

Solution:

$$(4x^2 - 5x + 2) + (-9x^2 + 3x - 7) = 4x^2 - 9x^2 - 5x + 3x + 2 - 7$$

Combine like terms.

$$= -5x^2 - 2x - 5$$

□

Example 2. Add: $(3x^5 - 9x^3 + 4x^2) + (-8x^5 + 8x^3 + 2)$

Solution:

$$= 3x^5 - 8x^5 - 9x^3 + 8x^3 + 4x^2 + 2$$

Combine like terms.

$$= -5x^5 - x^3 + 4x^2 + 2$$

□

4. Subtracting Polynomials

Example 1. Subtract: $4x^2 - 3x + 5$ from $x^2 - 8$

Solution:

$$\begin{aligned}(x^2 - 8) - (4x^2 - 3x + 5) &= x^2 - 8 - 4x^2 + 3x - 5 \\ &= -3x^2 + 3x - 13\end{aligned}$$

□

Example 2. $(-6m^2 - 8m + 5) - (-5m^2 + 7m - 8)$

Solution:

$$\begin{aligned}&= -6m^2 - 8m + 5 + 5m^2 - 7m + 8 \\ &= -6m^2 + 5m^2 - 8m - 7m + 5 + 8 \\ &= -m^2 - 15m + 13\end{aligned}$$

□

5. Multiplying Polynomials

Example 1. Multiply: $(2x - 3)(2x^2 + 3x - 2)$

Solution:

$$\begin{aligned}(2x - 3)(2x^2 + 3x - 2) &= 2x(2x^2 + 3x - 2) - 3(2x^2 + 3x - 2) \\ &= 4x^3 + 6x^2 - 4x - 6x^2 - 9x + 6 \\ &= 4x^3 - 13x + 6\end{aligned}$$

□

Example 2. $(3x^2 - 2x + 1)(x + 4)$

Solution:

$$\begin{aligned}(3x^2 - 2x + 1)(x + 4) &= 3x^3 + 12x^2 - 2x^2 - 8x + x + 4 \\ &= 3x^3 + 10x^2 - 7x + 4\end{aligned}$$

□

5.1. Special Products

Products of certain binomial factors occur so frequently that it is useful to remember formulas for their products.

1. $(x - y)(x + y) = x^2 - y^2$
2. $(x + y)^2 = x^2 + 2xy + y^2$
3. $(x - y)^2 = x^2 - 2xy + y^2$

Example 3. $(2xy + 3)(2xy - 3)$

Solution:

$$\begin{aligned}(2xy + 3)(2xy - 3) &= 4x^2y^2 - 6xy + 6xy - 9 \\ &= 4x^2y^2 - 9\end{aligned}$$

□

Example 4. $(6x + y)^2$ *Solution:*

$$\begin{aligned}(6x + y)^2 &= (6x)^2 + 2(6x)(y) + y^2 \\ &= 36x^2 + 12xy + y^2\end{aligned}$$

□

Example 5. $(5x - 3)^2$ *Solution:*

$$\begin{aligned}(5x - 3)^2 &= (5x)^2 - 2(5x)(3) + 3^2 \\ &= 25x^2 - 30x + 9\end{aligned}$$

□

6. Combined Operations

We now consider examples that use all the operations just discussed.

Example 1. Perform the indicated operations and simplify:

$$2x - \{7 - 2[x - x(4 + x)]\}$$

Solution:

$$\begin{aligned} 2x - \{7 - 2[x - x(4 + x)]\} &= 2x - \{7 - 2[x - 4x - x^2]\} \\ &= 2x - \{7 - 2[-3x - x^2]\} \\ &= 2x - \{7 + 6x + 2x^2\} \\ &= 2x - 7 - 6x - 2x^2 \\ &= -2x^2 - 4x - 7 \end{aligned}$$

□

7. Quiz

Test your understanding of the lesson by trying the quizzes that follow.

You must initialize the Exam by clicking on **Start**

When you finish the exam, click on **End** to view your scores and then click on **Correct** to get the correct answers.

1. The **degree** of the polynomial $4x^3 - 2x^2 + 7x - 1$ is:

6

7

3

4

2. The **degree** of the polynomial $3x^2y^5 - 2xy^2 + 4xy$ is:

12

7

3

4

3. Subtract $-5x^2 + 7x + 10$ from $-3x^2 - 8x + 6$

$$2x^2 - 15x - 4$$

$$2x^2 - x + 16$$

$$-2x^2 + 15x + 4$$

$$-2x^2 - x + 16$$

4. $(x^3 + x + 4) + (-2x^2 + 5x - 6) + (-x - 2) =$

$$-2x^3 + 5x - 8$$

$$x^3 + 2x^2 + 7x - 4$$

$$x^3 - 2x^2 - 7x - 4$$

$$x^3 - 2x^2 + 5x - 4$$

5. $(3x^2 - 2x + 1)(x + 4) =$

$$3x^3 - 8x + 4$$

$$3x^3 - 7x + 4$$

$$3x^3 + 10x^2 - 7x + 4$$

$$x^2 + 4$$

6. $(3x - 7y)(3x + 7y) =$

$$6x^2 - 14y^2$$

$$9x^2 - 42xy + 49y^2$$

$$9x^2 - 49y^2$$

$$9x^2 - 21xy + 49y^2$$

7. $(x + 3y)(3x - 5y) =$

$$3x^2 + 14xy - 15y^2$$

$$3x^2 + 4xy - 15y^2$$

$$3x^2 + 14xy - 15y$$

$$3x^2 - 14xy - 15y^2$$

8. $10 - 3[1 + 2(x - 5)] =$

$$6x + 37$$

$$-6x + 37$$

$$6x - 37$$

$$-6x - 37$$