$[A]_{(i \times k)} \cdot [B]_{(k \times j)}$

$$=\begin{bmatrix} a11 & \cdots & a1n \\ a21 & \dots & a2n \\ \vdots & \ddots & \vdots \\ an1 & \cdots & ann \end{bmatrix}\begin{bmatrix} b11 & \cdots & b1n \\ b21 & \dots & b2n \\ \vdots & \ddots & \vdots \\ bn1 & \cdots & bnn \end{bmatrix}$$

$$=\begin{bmatrix} c11 & \cdots & c1n \\ c21 & \dots & c2n \\ \vdots & \ddots & \vdots \\ cn1 & \cdots & cnn \end{bmatrix} = [C]_{ij}$$

Where, $c_{ij} = \sum_1^k \quad a_{ik}b_{kj}$

$$C^T = \begin{bmatrix} c11 & \cdots & cn1 \\ c12 & \dots & cn2 \\ \vdots & \ddots & \vdots \\ c1n & \cdots & cnn \end{bmatrix} = [C]_{ji}$$

Where, $C^T$ is transpose of matrix C

**1. a.)**

$\mathbf{(A.B)^T = B^T.A^T}$

$(A.B)^T{}_{ij}$

$= (A.B)_{ji}$

$= a_{jk}b_{ki}$

$= b_{ki}a_{jk}$

$= (b^T)_{ik}\,(a^T)_{kj}$

$= (B^TA^T)_{ij}$

**(1. b.)**

$\mathbf{B^T.A^T = (A.B)^T}$

$(B^TA^T)_{ij}$

$= (b^T)_{ik} \,.\, (a^T)_{kj}$

$= (b)_{ki} \,.\, (a)_{jk}$

$= (a)_{jk} \,.\, (b)_{ki}$

$= (AB)_{ji}$

$= (AB)^T_{ij}$

Cofactor Matrix of a matrix A

$$[CO_{ij}] = (-1)^{(i+j)} * \begin{bmatrix} M11 & \cdots & M1n \\ M21 & \ldots & M2n \\ \vdots & \ddots & \vdots \\ Mn1 & \cdots & Mnn \end{bmatrix}$$

Where, $\begin{bmatrix} M11 & \cdots & M1n \\ M21 & \ldots & M2n \\ \vdots & \ddots & \vdots \\ Mn1 & \cdots & Mnn \end{bmatrix}$ *is minor matrix of A*

$M_{ij}$ is the entry in the $i^{th}$ row and $j^{th}$ column

Adjoint of a matrix, A

$Adj(A) = [CO_{ij}]^T$

Now, the inverse of the matrix A

$A^{-1} = Adj(A) / det(A)$

Where 'det' denotes the determinant

**2. $(AB)^{-1} = B^{-1}A^{-1}$**

If A and B are invertible matrices or Both A and B are **n X n** square matrices and determinants are not zeroes then

$(AB)(AB)^{-1} = I$

Where I is the identity matrix of size **n X n**

Pre-multiply by $A^{-1}$

$(A^{-1}). (AB)(AB)^{-1} = A^{-1}I$

Or, $I.(B).(AB)^{-1} = A^{-1}$

Or, $(B).(AB)^{-1} = A^{-1}$

Pre-multiply by $B^{-1}$

$(B^{-1}).(B).(AB)^{-1} = B^{-1}A^{-1}$

Or, $I(AB)^{-1} = B^{-1}A^{-1}$

Or, $(AB)^{-1} = B^{-1}A^{-1}$

## 3. Adj(A . B) = Adj(B) . Adj(A)

$(AB)^{-1}=adj(AB)/det(AB)$

Or, $adj(AB) = (AB)^{-1}\cdot det(AB)$ . . . (1)

It is also known that, $(AB)^{-1}=B^{-1}A^{-1}$

And, $det(AB)=det(A)\cdot det(B)$ . . . (2)

Also

$A^{-1}=adj(A)/det(A)$

$B^{-1}=adj(B)/det(B)$

Or, $adj(A)=A^{-1}det(A)$

Or, $adj(B)=B^{-1}det(B)$

$adj(B)\cdot adj(A)=detA\cdot detB\cdot B^{-1}\cdot A^{-1}$ . . . (3)

Putting (2) in equation (1)

$adj(AB)=det(A)\cdot det(B)\cdot B^{-1}\cdot A^{-1}$ . . . (4)

From (3) and (4)

$\mathrm{adj}(AB)=\mathrm{adj}(B)\cdot\mathrm{adj}(A)$