Theory:

Definition: The rank of a matrix is defined as the maximum number of linearly independent rows (or columns) in the matrix. It can also be seen as the dimension of the row space or column space of the matrix.

Rank of a Matrix in Row Echelon Form (REF)

1. **Row Echelon Form (REF)**: A matrix is in row echelon form when:
   - All non-zero rows are above any rows of all zeros.
   - The leading entry (pivot) of each non-zero row is to the right of the leading entry of the row above it.
   - All entries below a pivot are zero.

2. **Finding the Rank**:
   - **Identify Non-Zero Rows**: In REF, the rank of the matrix is equal to the number of non-zero rows. This is because each non-zero row represents a linearly independent vector in the row space of the matrix.
   - **Process**: Convert the matrix to REF using row operations (row swapping, scaling rows, adding/subtracting multiples of rows) and count the number of non-zero rows to determine the rank.

Rank of a Matrix in Reduced Row Echelon Form (RREF)

Theory:

1. **Reduced Row Echelon Form (RREF)**: A matrix is in reduced row echelon form when:
   - It is in row echelon form (REF).
   - Each leading entry (pivot) is 1.
   - Each leading 1 is the only non-zero entry in its column.
   - All rows with leading 1s are above rows of all zeros.

2. **Finding the Rank**:

- **Count Leading 1s**: In RREF, the rank of the matrix is equal to the number of leading 1s. Each leading 1 represents a pivot position in a linearly independent row.
- **Process**: Convert the matrix to RREF using row operations (pivoting, scaling, and clearing entries above and below pivots) and count the number of leading 1s to determine the rank.