Gaussian Elimination with Back Substitution Method

Gaussian elimination is a method in which an augmented matrix is subjected to row operations until the component corresponding to the coefficient matrix is reduced to triangular form.

Procedure

1. Choose an **n X n** matrix

   Otherwise- Show Pop up – please select number of rows equal to number of Columns

2. Here, we can perform two different types of operations to convert a given matrix into the REF (row echelon) form (a) modify a row by adding or subtracting multiples of another row. (b) multiply/divide a row by a scalar
3. Construct an upper triangular matrix from the given matrix.
4. In the next step convert the diagonal elements to 1s.

Example

Solve Equations x+2y-3z=1, 2x-y+z=1, x+4y-2z=9 using Gaussian Elimination with Back Substitution method

Solution:
Total Equations are 3

x+2y-3z=1 … (i)

2x-y+z=1 … (ii)

x+4y-2z=9 … (iii)

From the aforementioned equations,

A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 1 \\ 1 & 4 & -2 \end{bmatrix}$; **b** = $\begin{bmatrix} 1 \\ 1 \\ 9 \end{bmatrix}$;

Now, the augmented matrix,

[A:b] = $\begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 2 & -1 & 1 & : & 1 \\ 1 & 4 & -2 & : & 9 \end{bmatrix}$

Converting the given matrix into an upper triangular matrix form

R3 ← -R1 + R3

$$\begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 2 & -1 & 1 & : & 1 \\ 0 & 2 & 1 & : & 8 \end{bmatrix}$$

R2 ← -2R1 + R2

$$\begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 0 & -5 & 7 & : & -1 \\ 0 & 2 & 1 & : & 8 \end{bmatrix}$$

R3 ← 2R2 + 5R3

$$\begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 0 & -5 & 7 & : & -1 \\ 0 & 0 & 19 & : & 38 \end{bmatrix}$$

The aforementioned matrix is an upper triangular matrix.

Now, convert all diagonal elements to 1s.

R2 ← (-1/5)*R2

$$\begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 0 & 1 & -7/5 & : & 1/5 \\ 0 & 0 & 19 & : & 38 \end{bmatrix}$$

R3 ← (1/19)*R3

$$\begin{bmatrix} 1 & 2 & -3 & : & 1 \\ 0 & 1 & -7/5 & : & 1/5 \\ 0 & 0 & 1 & : & 2 \end{bmatrix}$$

The above matrix is now in the desired form.

$x + 2y – 3z = 1$ . . . (i)

$y – 7z/5 = 1/5$ . . . (ii)

$z = 2$ . . . (iii)

You have to move in a back substitution manner in order to retrieve the values of y and x.

y – 7*2/5 = 1/5 (since we already know the value of z = 2)

Or, y = 1/5 + 14/5 = 3

Then, x + 2(3) – 3(2) = 1

Or, x = 1