

Row Echelon Form

A matrix is in row echelon form if

- All rows consisting of only zeroes are at the bottom.
- The leading entry (that is the left-most nonzero entry) of every nonzero row is to the right of the leading entry of every row above
- Some texts add the condition that the leading coefficient must be 1 while others regard this as reduced row echelon form
- These two conditions imply that all entries in a column below a leading coefficient are zeros

Procedure

1. Choose an $m \times n$ matrix
2. All zero rows are at the bottom.
3. Choose the leading entry in the first non-zero row and swap it with the first row if necessary. Or, the leading entry/element in the first row must be non-zero.
4. Divide the first row by the leading entry so that the leading entry becomes 1.
5. Use row operations to make all entries in the first column below the leading entry equal to 0.
6. Repeat steps 3 through 5 for each subsequent row, working from top to bottom.

These conditions also imply that all entries in a column below a leading coefficient are zeros

Example

Given matrix,

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -1 & 2 \\ 0 & -2 & 2 \end{bmatrix}$$

$$R2 \leftarrow R2 - R1$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

$$R3 \leftarrow R3 + 2*R2$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$R3 \leftarrow R3 / 4$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$