Relaxation Method

Procedure

1. Choose an **n X (n+1)** matrix

Take an augmented matrix **[A: b]**; where A is an **n X n** matrix and **b** is an **n X 1** matrix.

Suppose the given equations are

$a_{11}x + a_{12}y + a_{13}z = b_1$

$a_{21}x + a_{22}y + a_{23}z = b_2$

$a_{31}x + a_{32}y + a_{33}z = b_3$

$$A = \begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix};\ b = \begin{bmatrix} b1 \\ b2 \\ b3 \end{bmatrix};$$

Otherwise- Show Pop up – please select the number of columns that is one more than the number of rows

2. Verify that the magnitude of the diagonal item in each row of the matrix A is greater than or equal to the sum of the magnitudes of all other (non-diagonal) values in that row so that

$$|a_{ii}| \geq \sum_{j \neq i} |a_{ij}| \quad \text{for all } i$$

Otherwise- Show Pop up – entered matrix is not diagonally dominant

(You can command user to rearrage the rows or do coding to rearrange the rows so that it satifies the above condition)

Ensure that all of the diagonal elements are non-zero as well.

$$a_{ii} \neq 0$$

Otherwise- Show Pop up – all of the diagonal elements must be non-zero

3. Now Define Residuals,

$R1 = b_1 - a_{11}x - a_{12}y - a_{13}z$

$R2 = b_2 - a_{21}x - a_{22}y - a_{23}z$

$R3 = b_3 - a_{31}x - a_{32}y - a_{33}z$

4. Operating table will be

	R1	R2	R3
ẟ x	$-a_{11}$	$-a_{21}$	$-a_{31}$
ẟ y	$-a_{12}$	$-a_{22}$	$-a_{32}$
ẟ z	$-a_{13}$	$-a_{23}$	$-a_{33}$

In the operating table above diagonal elements are $–a_{11}$, $-a_{22}$, and $–a_{33}$

1^{st} iteration

If x = y = z = 0 (initializing)
Then,
$R1^{(1)} = b1$
$R2^{(1)} = b2$
$R3^{(1)} = b3$

Look, which is maximum among three residuals.
Here, superscript denotes iteration number.

Let's assume, R3 is maximum among R1, R2, and R3

Then, updated value of **ẟ z** will be $R3^{(1)}/(-(-a_{33}))$ where $–a_{33}$ is the diagonal element in the 3^{rd} row

Remember that as R3 is maximum among all residuals, so we're updating the value of **ẟ z**. If R2 were maximum then we need to update the value of **ẟ y** and so on.

2^{nd} iteration

$R1^{(2)} = R1^{(1)} + (-a_{13})(R3^{(1)}/(-(-a_{33})))$
$R2^{(2)} = R2^{(1)} + (-a_{23})(R3^{(1)}/(-(-a_{33})))$
$R3^{(2)} = R3^{(1)} + (-a_{33})(R3^{(1)}/(-(-a_{33})))$

Let's assume, R2 is maximum among R1, R2, and R3 above

Then, updated value of **ẟ y** will be $R2^{(2)}/(-(-a_{22}))$ where $-a_{22}$ is the diagonal element in the 2nd row

3rd iteration

$R1^{(3)} = R1^{(2)} + (-a_{12})(R2^{(2)}/(-(-a_{22})))$
$R2^{(3)} = R2^{(2)} + (-a_{22})(R2^{(2)}/(-(-a_{22})))$
$R3^{(3)} = R3^{(2)} + (-a_{32})(R2^{(2)}/(-(-a_{22})))$

Let's assume, $R1^{(3)}$ is maximum among $R1^{(3)}$, $R2^{(3)}$, and $R3^{(3)}$

Then, updated value of **ẟ x** will be $R1^{(3)}/(-(-a_{11}))$ where $-a_{11}$ is the diagonal element in the 1st row

4th iteration

$R1^{(4)} = R1^{(3)} + (-a11)(R1^{(3)}/(-(-a_{11})))$
$R2^{(4)} = R2^{(3)} + (-a21)(R1^{(3)}/(-(-a_{11})))$
$R3^{(4)} = R3^{(3)} + (-a31)(R1^{(3)}/(-(-a_{11})))$

Let's assume, $R3^{(4)}$ is maximum among $R1^{(4)}$, $R2^{(4)}$, and $R3^{(4)}$

Then, updated value of **ẟ z** will be $R3^{(4)}/(-(-a_{33}))$ where $-a_{33}$ is the diagonal element in the 1st row

Continue the iteration process up to nth iteration until $R1^{(n)}$, $R2^{(n)}$, and $R3^{(n)}$ become very close to zeroes.

Then,
$x = \sum ẟ x$
$y = \sum ẟ y$
$z = \sum ẟ z$

$\sum$ **ꝺ x** denotes sum of all updated values of **ꝺ x** we obtain during total ‘n’ number of iteration processes

Example

10x – 2y – 2z = 6 . . . (i)
-x +10y – 2z = 7 . . . (ii)
-x – y + 10z = 8 . . . (iii)

Taking values from the aforementioned equations, we obtain,

$$A = \begin{bmatrix} 10 & -2 & -2 \\ -1 & 10 & -2 \\ -1 & -1 & 10 \end{bmatrix}$$

$$b = \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$$

To perform relaxation method diagonal element in matrix A should be greater than the sum of other elements (values without signs) in the same row.

From the above matrix A,
a_{11} or 10 >2+2
a_{22} or 10 >1+2
a_{33} or 10 >1+1

Now, define **residuals** from the above equations
R1 = 6 – 10x + 2y + 2z
R2 = 7 + x – 10y + 2z
R3 = 8 + x + y - 10z

Then construct Operating Table from the above **residuals**

	R1	R2	R3
ꝺ x	-10	1	1
ꝺ y	2	-10	1
ꝺ z	2	2	-10

Here, the diagonal elements are -10, -10, and -10

1^{st} iteration

Let's initialize as x=0, y=0, and z=0

Then,

R1 = 6

R2 = 7

R 3 = 8

In the above R3 is maximum

Afterward, the updated value of ẟ **z** will be, ẟ **z** = 8/-(-10) = 0.8

2^{nd} iteration

R1 = 6 + (2)*(0.8) = 7.6

R2 = 7 + (2)*(0.8) = 8.6

R3 = 8 + (-10)*(0.8) = 0

R3 is maximum

So, the updated value of ẟ y will be,

ẟ y = 8.6/-(-10) = 0.86

3^{rd} iteration

R1 = 7.6 + 2*0.86 = 9.32

R2 = 8.6 + (-10)*0.86 = 0

R3 = 0 + 1*0.86 = 0.86

Max value = 9.32 (=R1)

So, updated value of **δ x** is now,

δ x = 9.32 / -(-10) = 0.932

4th iteration

R1 = 9.32 + (-10)*0.932 = 0

R2 = 0 + 1*0.932 = 0.932

R3 = 0.86 + 1*0.932 = 1.792

Max value = 1.792 (=R3)

So, updated value of δ z will be,

δ z = 1.792 / -(-10) = 0.1792

5th iteration

R1 = 0 + 2*0.1792 = 0.3584

R2 = 0.932 + 2*0.1792 = 1.2904

R3 = 1.732 + (-10)*0.1792 = 0

Max value = 1.2904 (=R2)

Now, updated δ y = 1.2904 / -(-10) = 0.12904

6th iteration

R1 = 0.3584 + 2*0.12904 = 0.6165

R2 = 1.2904 + (-10)*0.12904 = 0

R3 = 0 + 1*0.12904 = 0.129

Max value = 0.6165 (=R1)

Now, updated value of ბ x will be

ბ x = 0.6165 / -(-10) = 0.0616

7th iteration

R1 = 0.0616 + (-10)*0.0616 = 0

R2 = 0 + 1*0.0616 = 0.0616

R3 = 0.129 + 1*0.0616 = 0.1907

Max value = 0.1907

Now, updated ბ z = 0.1907 / -(-10) = 0.0191

8th iteration

R1 = 0 + 2*0.0191 = 0.0381

R2 = 0.0616 + 2*0.0191 = 0.0998

R3 = 0.1907 + (-10)*0.0191 = 0

Max value = 0.0998 (=R2)

Now, updated ბ y = 0.0998/-(-10) = 0.0098 ~ 0.01

9th iteration

R1= 0.0381 + 2*0.01 = 0.0581

R2 = 0.0998 + (-10)*0.01 = 0

R3 = 0 + 1*0.01 = 0.01

Max value is 0.0581 (=R1)

Now, updated value of **δ x** = 0.0581/-(-10) = 0.0058

10th iteration

R1 = 0.0581 + (-10)*0.0058 = 0

R2 = 0 + 1 * 0.0058 = 0.0058

R3 = 0.01 + 1*0.0058= 0.0158

Max value is 0.0158 (=R3)

Now, updated δ z = 0.0158/-(-10) = 0.0016

11th iteration

R1 = 0 + 2*0.0016 = 0.0032

R2 = 0.0058 + 2*0.0016 = 0.009

R3 = 0.0158 + (-10)*0.0016 = 0

Max value = 0.009 (=R2)

Now, updated δ z = 0.009/-(-10) = 0.0009

12th iteration

R1 = 0.0032 + 2*0.0009 = 0.005

R2 = 0.009 + (-10)*0.0009 = 0

R3 = 0 + 1*0.0009 = 0.0009

Max value = 0.005 (=R1)

Now, updated value of **δ x** = 0.005/-(-10) = 0.0005

13th iteration

R1 = 0.005 + (-10)*0.0005 = 0

R2 = 0 + 1*0.0005 = 0.0005

R3 = 0.0009 +1*0.0005 = 0.0014

Max value = 0.0014 (=R3)

Now the updated value of δ z will be

δ z = 0.0014/-(-10) = 0.0001

Here, in the 13th iteration the values of R1, R2, and R3 all are going to be very close to 0 (zero).

So, now we'll stop the iteration process.

So, solutions by Relaxation method

x = $\sum \delta$ x = 0.932 + 0.0616 + 0.0058 + 0.0005 = 0.999 ~ 1

y = $\sum \delta$ y = 0.86 + 0.129 + 0.01 + 0.0009 = 0.999 ~ 1

z = $\sum \delta$ z = 0.8 + 0.1792 + 0.0191 + 0.0016 = 0.999 ~ 1