

Gauss Jordan Elimination Method

Often Gauss Jordan Elimination (GJE) is used to get a matrix to reduced echelon form so it is easy to solve a linear equation. Linear systems can have many variables. These systems can be solved as long as we have one unique equation/variable.

For example- two variables need two equations

Three variables need three equation to find a unique solution.

And ten variables need ten equation and so on.

In the same way 4 variables need 4 equation to find a unique solution.

Although, Gauss-Jordan elimination works on matrices of any size, they don't have to be square. But the number of independent linear equations must not be less than number of unknown variables. However, we actually don't need. For solving 'n' number of unknown variables 'n' number of independent linear equations are enough. On the other hand, the given matrix needs to be square if you are using it to calculate the inverse of the matrix.

Procedure

1. Choose an $n \times n$ matrix

Otherwise- Show Pop up – please select number of rows equal to number of Columns

2. Swap the rows so that all rows with all zero entries are on the bottom.
3. Swap the rows so that the row with the largest, leftmost nonzero entry is on top.
4. Multiply / Divide the top row by a scalar so that top row's leading entry becomes 1.
5. Add/subtract multiples of the top row to the other rows so that all other entries in the column containing the top row's leading entry are all zero.
6. Repeat steps 3-5 for the next leftmost nonzero entry until all the leading entries are 1.

7. Swap the rows so that the leading entry of each nonzero row is to the right of the leading entry of the row above it.

Example

Solve Equations $2x+5y+z=17$, $3x+y+z=12$, $x+y+z=6$ using Gauss-Jordan Elimination method

Solution:

Total Equations are 3

$$2x+5y+z=17 \dots(i)$$

$$3x+y+z=12 \dots(ii)$$

$$x+y+z=6 \dots(iii)$$

Converting given equations into matrix form

$$\mathbf{AX} = \mathbf{b}$$

Where,

$$\mathbf{A} = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 17 \\ 12 \\ 6 \end{bmatrix}$$

Now, we can generate the augmented matrix like that

$$[\mathbf{A} : \mathbf{b}] = \begin{bmatrix} 2 & 5 & 1 & : & 17 \\ 3 & 1 & 1 & : & 12 \\ 1 & 1 & 1 & : & 6 \end{bmatrix}$$

Now, we're swapping the rows R1 & R2 as leftmost element / entry of R2 is the largest.

$$R1 \leftrightarrow R2$$

$$\begin{bmatrix} 3 & 1 & 1 & : & 12 \\ 2 & 5 & 1 & : & 17 \\ 1 & 1 & 1 & : & 6 \end{bmatrix}$$

Now, divide the top row 1 by a scalar so that top row's leading entry becomes 1.

$$R1 \leftarrow (R1/3)$$

$$\begin{bmatrix} \textcolor{red}{1} & 0.3333 & 0.3333 & : & 4 \\ 2 & 5 & 1 & : & 17 \\ 1 & 1 & 1 & : & 6 \end{bmatrix}$$

Now, Add/subtract multiples of the top row to the other rows so that all other entries in the column 1 containing the top row's leading entry are all zeroes.

$$R2 \leftarrow R2 - (2 \times R1)$$

$$\begin{bmatrix} 1 & 0.3333 & 0.3333 & : & 4 \\ 0 & 4.3333 & 0.3333 & : & 9 \\ 1 & 1 & 1 & : & 6 \end{bmatrix}$$

$$R3 \leftarrow R3 - R1$$

$$\begin{bmatrix} \textcolor{red}{1} & 0.3333 & 0.3333 & : & 4 \\ \textcolor{red}{0} & 4.3333 & 0.3333 & : & 9 \\ \textcolor{red}{0} & 0.6667 & 0.6667 & : & 2 \end{bmatrix}$$

Now you can see all other entries in the column 1 containing the top row's leading entry are all zero. Apply the same process to convert the leading non zero element in row 2 to 1. Then attempt

$$R2 \leftarrow (0.2308 \times R2)$$

$$\begin{bmatrix} 1 & 0.3333 & 0.3333 & : & 4 \\ 0 & \textcolor{red}{1} & 0.0769 & : & 2.0769 \\ 0 & 0.6667 & 0.6667 & : & 2 \end{bmatrix}$$

$$R1 \leftarrow (R1 - 0.3333 \times R2)$$

$$\begin{bmatrix} 1 & 0 & 0.3077 & : & 3.3077 \\ 0 & 1 & 0.0769 & : & 2.0769 \\ 0 & 0.6667 & 0.6667 & : & 2 \end{bmatrix}$$

$$R3 \leftarrow (R3 - 0.6667 \times R2)$$

$$\begin{bmatrix} 1 & 0 & 0.3077 & : & 3.3077 \\ 0 & 1 & 0.0769 & : & 2.0769 \\ 0 & 0 & 0.6154 & : & 0.6154 \end{bmatrix}$$

$$R3 \leftarrow R3 \times 1.625$$

$$\begin{bmatrix} 1 & 0 & 0.3077 & : & 3.3077 \\ 0 & 1 & 0.0769 & : & 2.0769 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

$$R1 \leftarrow R1 - (0.3077 \times R3)$$

$$\begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0.0769 & : & 2.0769 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

$$R2 \leftarrow R2 - (0.0769 \times R3)$$

$$\begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

Now, we get, $x=3$, $y=2$, $z=1$

(Solution by Gauss Jordan Elimination Method)