Determinant using Sarrus rule

Procedure

1. Choose an n X n matrix
   Otherwise-Pop up error – the number of rows and columns should be the same (or matrix dimension mismatched)

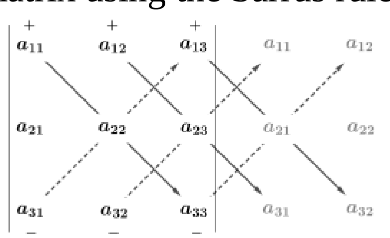
Although, Sarrus rule is not applicable for a square matrix which size is greater than 3 X 3, but also we can apply it to obtained 3 X 3 matrix after co-factor expansion of a given matrix.

2. For a given matrix,

$$\mathbf{A} = \begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix}$$

(a) If the **number of rows = number of columns = 3**, then,

The determinant of this matrix using the Sarrus rule is



Rule of Sarrus: The determinant of the three columns on the left is the sum of the products along the down-right diagonals minus the sum of the products along the up-right diagonals.

Write out the first two columns of the matrix to the right of the third column, giving five columns in a row.

Then add the products of the diagonals going from top to bottom (solid) and subtract the products of the diagonals going from bottom to top (dashed).

$\det(\mathbf{A}) = a_{11}*a_{22}*a_{33} + a_{12}*a_{23}*a_{31} + a_{13}*a_{21}*a_{32} - a_{31}*a_{22}*a_{13} - a_{32}*a_{23}*a_{11} - a_{33}*a_{21}*a_{12}$

(b) If the **number of rows = number of columns > 3**, then
Use cofactor expansion to find the determinant of given matrix
For example,

$$\mathbf{A} = \begin{bmatrix} a11 & a12 & a13 & a14 \\ a21 & a22 & a23 & a24 \\ a31 & a32 & a33 & a34 \\ a41 & a42 & a43 & a44 \end{bmatrix}$$

$$\det(\mathbf{A}) = (-1)^{i+j} * a_{11} * \begin{vmatrix} a22 & a23 & a24 \\ a32 & a33 & a34 \\ a42 & a43 & a44 \end{vmatrix} + (-1)^{i+j} * a_{12} * \begin{vmatrix} a21 & a23 & a24 \\ a31 & a33 & a34 \\ a41 & a43 & a44 \end{vmatrix} +$$

$$(-1)^{i+j} * a_{13} * \begin{vmatrix} a21 & a22 & a24 \\ a31 & a32 & a34 \\ a41 & a42 & a44 \end{vmatrix} + (-1)^{i+j} * a_{14} * \begin{vmatrix} a21 & a22 & a23 \\ a31 & a32 & a33 \\ a41 & a42 & a43 \end{vmatrix}$$

(In $(-1)^{i+j}$, 'i' denotes the row number and 'j' denotes the column number of a matrix element or entry)

Now, you can apply Sarrus rule to find determinant of a matrix which size is **3X3**. Same procedure can be applied for an **n X n** matrix.

Co-factors and minors of a matrix are already discussed. Please go thru them if you want.

Example

For a 3 X 3 matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 0 & -1 \\ 4 & -3 & 1 \end{bmatrix} = 1.0.1 + 3.(-1).4 + 5.2.(-3) - 4.0.5 - (-3).(-1).1 - 1.2.3$$

$= 0 -12 -30 - 0 - 3 - 6$

$= -51$

For a 5 X 5 matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$$= (-1)^{1+1}*1*\begin{vmatrix} 3 & 4 & 5 & 1 \\ 4 & 5 & 1 & 2 \\ 5 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{vmatrix} + (-1)^{1+2}*2*\begin{vmatrix} 2 & 4 & 5 & 1 \\ 3 & 5 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 5 & 2 & 3 & 4 \end{vmatrix} + (-1)^{1+3}*3*\begin{vmatrix} 2 & 3 & 5 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 5 & 2 & 3 \\ 5 & 1 & 3 & 4 \end{vmatrix} +$$

$$(-1)^{1+4}*4*\begin{vmatrix} 2 & 3 & 4 & 1 \\ 3 & 4 & 5 & 2 \\ 4 & 5 & 1 & 3 \\ 5 & 1 & 2 & 4 \end{vmatrix} + (-1)^{1+5}*5*\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 \\ 4 & 5 & 1 & 2 \\ 5 & 1 & 2 & 3 \end{vmatrix}$$

$$= 1*\begin{vmatrix} 3 & 4 & 5 & 1 \\ 4 & 5 & 1 & 2 \\ 5 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{vmatrix} -2*\begin{vmatrix} 2 & 4 & 5 & 1 \\ 3 & 5 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 5 & 2 & 3 & 4 \end{vmatrix} +3*\begin{vmatrix} 2 & 3 & 5 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 5 & 2 & 3 \\ 5 & 1 & 3 & 4 \end{vmatrix} -4*\begin{vmatrix} 2 & 3 & 4 & 1 \\ 3 & 4 & 5 & 2 \\ 4 & 5 & 1 & 3 \\ 5 & 1 & 2 & 4 \end{vmatrix} +$$

$$5*\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 \\ 4 & 5 & 1 & 2 \\ 5 & 1 & 2 & 3 \end{vmatrix}$$

Now, $\begin{vmatrix} 3 & 4 & 5 & 1 \\ 4 & 5 & 1 & 2 \\ 5 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{vmatrix} = 3*\begin{vmatrix} 5 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} -4*\begin{vmatrix} 4 & 1 & 2 \\ 5 & 2 & 3 \\ 1 & 3 & 4 \end{vmatrix} +5*\begin{vmatrix} 4 & 5 & 2 \\ 5 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} -1*\begin{vmatrix} 4 & 5 & 1 \\ 5 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

= 3 * **{(5.2.4 + 1.3.2 + 2.1.3) – (2.2.2 + 3.3.5 + 4.1.1)}** -4* **{(4.2.4+1.3.1+2.5.3) - (1.2.2 + 3.3.4 + 4.5.1)}** + 5* **{(4.1.4 + 5.3.1 + 2.5.2) – (1.1.2 + 2.3.4 + 4.5.5)}** – 1* **{(4.1.3 + 5.2.1 + 1.5.2) – (1.1.1 + 2.2.4 + 3.5.5)}**
= -350

(Here in the above we've used Sarrus rule to find determinants of 3 X 3 matrices)

Similarly, $-2*\begin{vmatrix} 2 & 4 & 5 & 1 \\ 3 & 5 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 5 & 2 & 3 & 4 \end{vmatrix} = -2*[2*\begin{vmatrix} 5 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} -4*\begin{vmatrix} 3 & 1 & 2 \\ 4 & 2 & 3 \\ 5 & 3 & 4 \end{vmatrix} + 5*\begin{vmatrix} 3 & 5 & 2 \\ 4 & 1 & 3 \\ 5 & 2 & 4 \end{vmatrix}$

$-1*\begin{vmatrix} 3 & 5 & 1 \\ 4 & 1 & 2 \\ 5 & 2 & 3 \end{vmatrix}]$

= -2* [2***{(5.3.4 + 1.3.2 + 2.1.3) – (2.2.2 + 3.3.5 + 4.1.1)}** -4* **{(3.2.4 + 1.3.5 + 2.4.3) – (5.2.2 + 3.3.3 + 4.4.1)}** +5* **{(3.1.4 + 5.3.5 + 2.4.2) – (5.1.2 + 2.3.3 + 4.4.5)}** -1* **{(3.1.3 + 5.2.5 + 1.4.2) – (5.1.1+ 2.2.3 + 3.4.5)}**]

(Here also we've used Sarrus rule to find determinants of 3 X 3 matrices)

= -2 *[-25] = 50

Similarly, 3 * $\begin{vmatrix} 2 & 3 & 5 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 5 & 2 & 3 \\ 5 & 1 & 3 & 4 \end{vmatrix}$ = 3 * [25] = 75

-4 * $\begin{vmatrix} 2 & 3 & 4 & 1 \\ 3 & 4 & 5 & 2 \\ 4 & 5 & 1 & 3 \\ 5 & 1 & 2 & 4 \end{vmatrix}$ = -4 * [-25] = 100

And,

5 * $\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 \\ 4 & 5 & 1 & 2 \\ 5 & 1 & 2 & 3 \end{vmatrix}$ = 5*[400] =2000

So, **A** = $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{bmatrix}$ = -350 + 50 + 75 +100 + 2000 = 1875