Reduced Row Echelon Form (RREF)

Procedure

1. Choose an m X n matrix
2. All zero rows are at the bottom.
3. Choose the leading entry in the first non-zero row and swap it with the first row if necessary.
4. Divide the first row by the leading entry so that the leading entry becomes 1.
5. Use row operations to make all entries in the first column above and below the leading entry equal to 0.
6. Repeat steps 3 through 5 for each subsequent row, working from top to bottom.
7. After all, rows have been processed, the matrix is in reduced row echelon form.

Example:

Given matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 6 & 10 \end{bmatrix}$

$$R2 \leftarrow R2 - 2*R1$$ *(R1 denotes row 1 and so on)*

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 3 & 6 & 10 \end{bmatrix}$$

$$R3 \leftarrow R3 - 3*R1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R1 \leftarrow R1 - 2*R2$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R1 \leftarrow R1 - R3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R2 \leftarrow R2 - R3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$