LU Decomposition using Crout's Method

Procedure-

1. Choose a matrix (**m X n)** (e.g., 3X 3, 3 X 4, 4 X 4, etc.,)

2. ***The Crout's matrix decomposition algorithm differs slightly from the Doolittle method. Doolittle's method returns a unit lower triangular matrix and an upper triangular matrix, while the Crout's method returns a lower triangular matrix and a unit upper triangular matrix.***

3. Initialize the **L** and **U** matrices. **L** matrix size will be (**m X m**). The values of matrix elements below the main diagonal can be assigned to l_{21}, l_{31}, etc., and so on. And the matrix elements above the diagonal are zeroes.

$$\begin{bmatrix} l11 & 0 & 0 & 0 \\ l21 & l22 & 0 & 0 \\ l31 & l32 & l33 & 0 \\ l41 & l42 & l43 & l44 \end{bmatrix}$$

l_{21}, l_{31}, etc. are unknown

4. For matrix **U**, take a matrix with all diagonal elements assigned to 1, and the matrix elements below the diagonal are zeroes. Size of matrix **U** will be as same as matrix **A (or, m X n)**. Matrix **U** can be written as

$$\begin{bmatrix} 1 & u12 & u13 & u14 \\ 0 & 1 & u23 & u24 \\ 0 & 0 & 1 & u34 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

u_{11}, u_{12}, u_{13}, u_{14}, u_{22}, etc. are unknown.

5. For a given example,

$$\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{2} & \mathbf{5} & \mathbf{2} \\ \mathbf{3} & \mathbf{2} & \mathbf{5} \end{bmatrix}$$

Now, $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{2} & \mathbf{5} & \mathbf{2} \\ \mathbf{3} & \mathbf{2} & \mathbf{5} \end{bmatrix}$ = **L * U** = $\begin{bmatrix} \boldsymbol{l11} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{l21} & \boldsymbol{l22} & \mathbf{0} \\ \boldsymbol{l31} & \boldsymbol{l32} & \boldsymbol{l33} \end{bmatrix} * \begin{bmatrix} \mathbf{1} & \boldsymbol{u12} & \boldsymbol{u13} \\ \mathbf{0} & \mathbf{1} & \boldsymbol{u23} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

$$= \begin{bmatrix} \boldsymbol{l11} & (\boldsymbol{l11} * \boldsymbol{u12}) & (\boldsymbol{l11} * \boldsymbol{u13}) \\ \boldsymbol{l21} & (\boldsymbol{l21} * \boldsymbol{u12} + \boldsymbol{l22}) & (\boldsymbol{l21} * \boldsymbol{u13} + \boldsymbol{l22} * \boldsymbol{u23}) \\ \boldsymbol{l31} & (\boldsymbol{l31} * \boldsymbol{u12} + \boldsymbol{l32}) & (\boldsymbol{l31} * \boldsymbol{u13} + \boldsymbol{l32} * \boldsymbol{u23} + \boldsymbol{l33}) \end{bmatrix}$$

Next, resolve the basic matrix multiplication equations from above,

$l_{11} = 1$

$l_{11}*u_{12} = 2$

$l_{11}*u_{13} = 3$

$l_{21} = 2$

$l_{21}*u_{12}+l_{22} = 5$

$l_{21}*u_{13}+l_{22}*u_{23} = 2$

$l_{31} = 3$

$l_{31}*u_{12}+l_{32} = 2$

$l_{31}*u_{13}+l_{32}*u_{23}+l_{33}=5$

We obtain matrices **L** and **U** by solving the aforementioned equations.