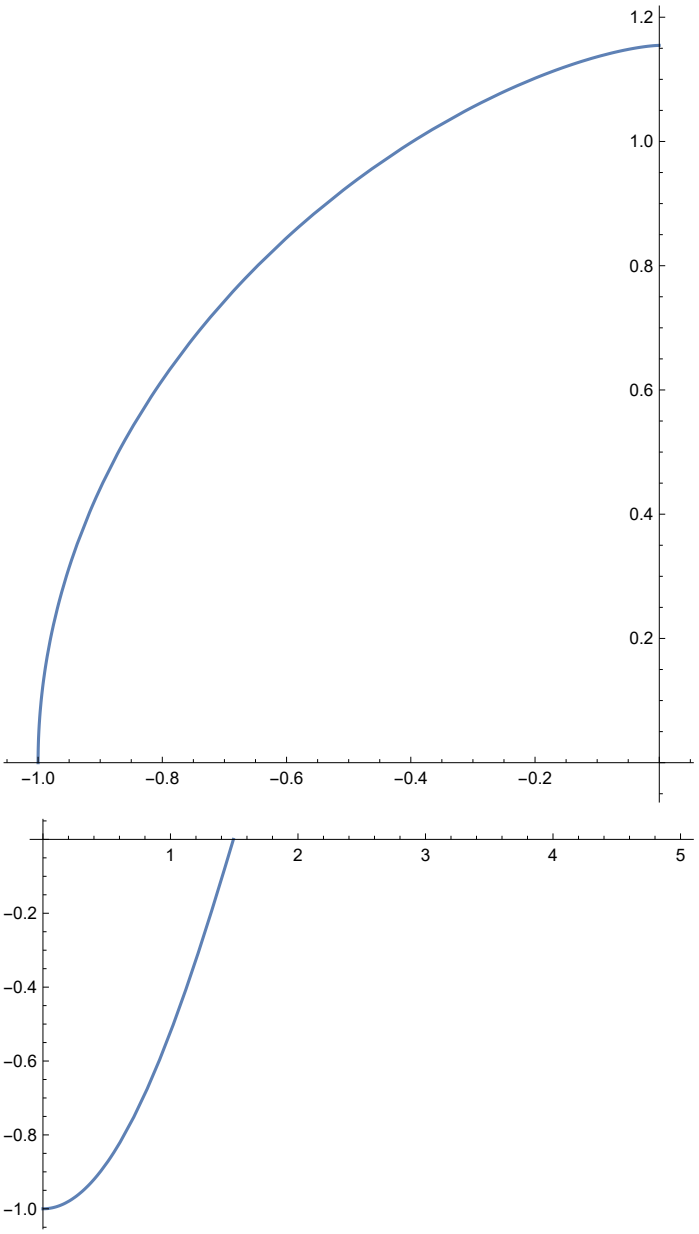
In[36]:=
```
duffEq = (x''[t])^2 + g x'[t] + x[t] (1 - a x[t]^2) == f Cos[ω t];
duff[vals_, ic_, tmax_] := Module[{nds},
  nds = NDSolve[Join[{duffEq /. vals}, ic], x, {t, 0, tmax}, MaxSteps → 10000][[1]];
  Print[duffEq /. vals];
  Print[
   ParametricPlot[Evaluate[{x[t], x'[t]} /. nds], {t, 0, 10}, AspectRatio → Automatic]];
  Print[Plot[Evaluate[x[t] /. nds], {t, 0, tmax}]];
 ]
```

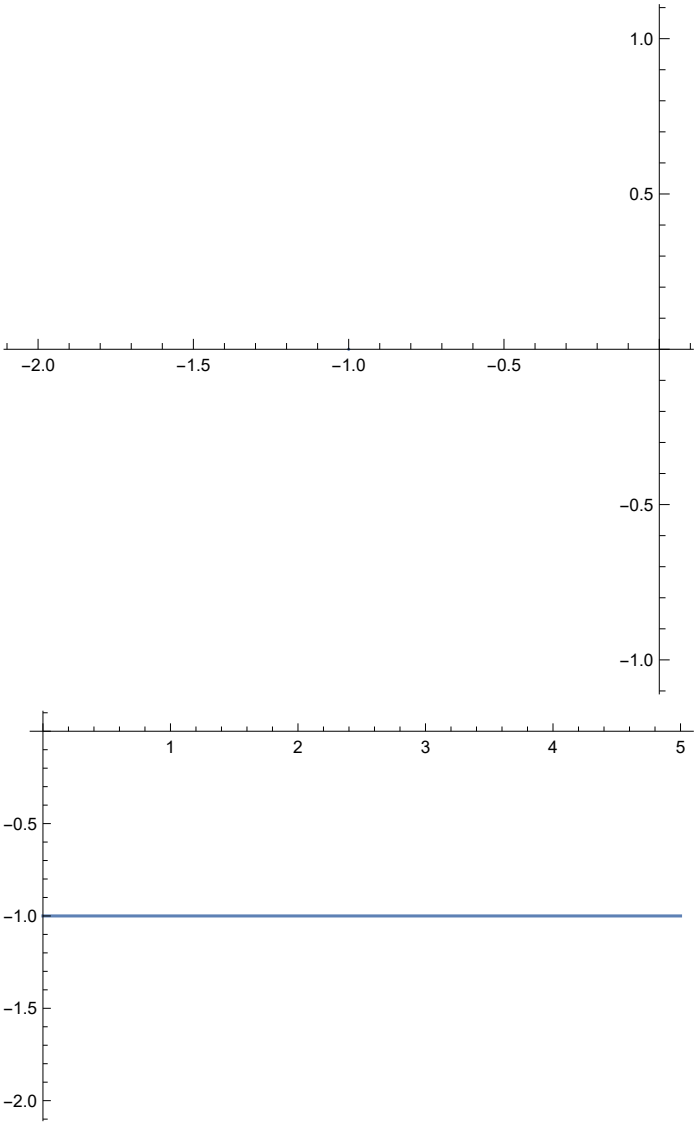
In[38]:=
```
(*a*)
duff[{g → 0, f → 0, a → 0, ω → .25}, {x[0] == -1, x'[0] == 0}, 5]
```

```
x[t] + x''[t]^2 == 0
```

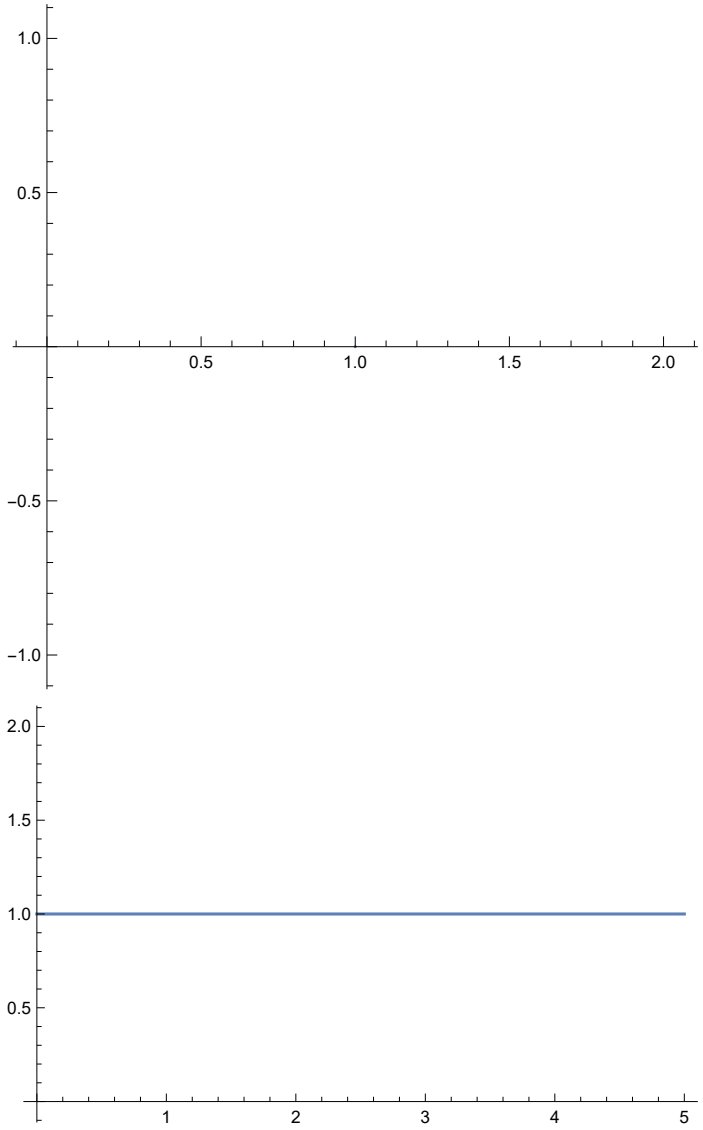


In[42]:=
```
(*b*)
duff[{g → 0, f → 0, a → 1, ω → .25}, {x[0] == -1, x'[0] == 0}, 5]
duff[{g → 0, f → 0, a → 1, ω → .25}, {x[0] == 1, x'[0] == 0}, 5]
duff[{g → 0, f → 0, a → -1, ω → .25}, {x[0] == 1, x'[0] == 0}, 5]
duff[{g → 0, f → 0, a → -1, ω → .25}, {x[0] == -1, x'[0] == 0}, 5] (*last doesn't go well*)
```

```
x[t] (1 - x[t]^2) + x''[t]^2 == 0
```

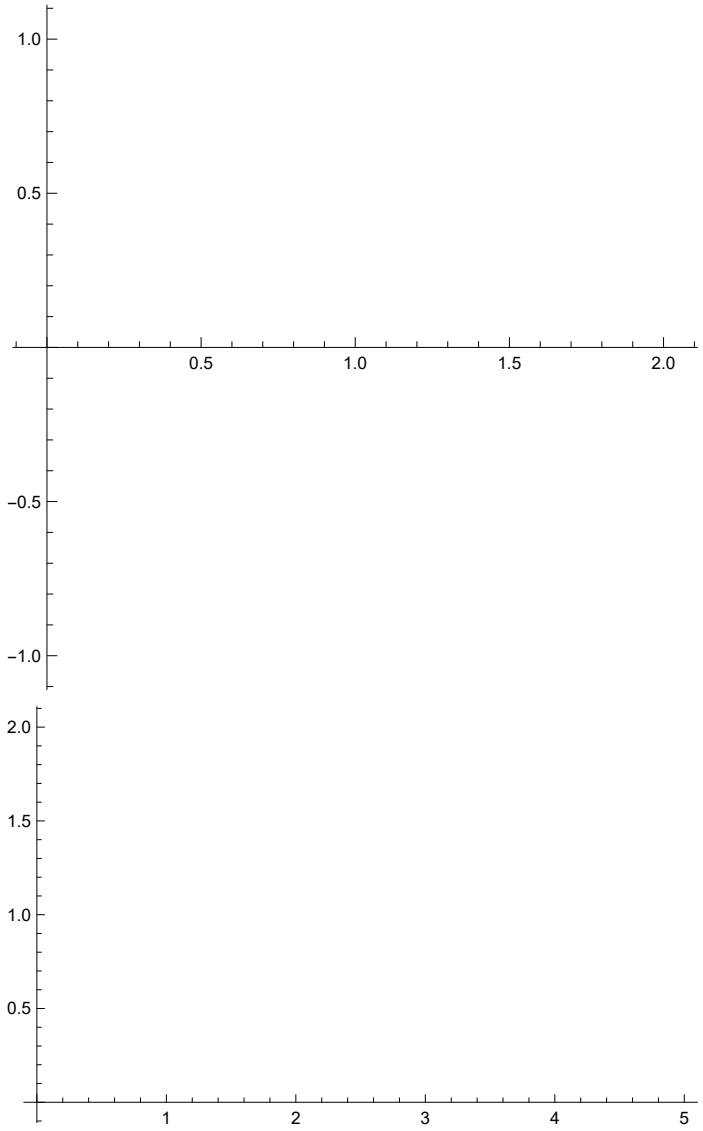


$$x[t]\left(1-x[t]^2\right)+x''[t]^2==0$$



NDSolve: Maximum number of 10000 steps reached at the point t == 0.00027356123958297647`.

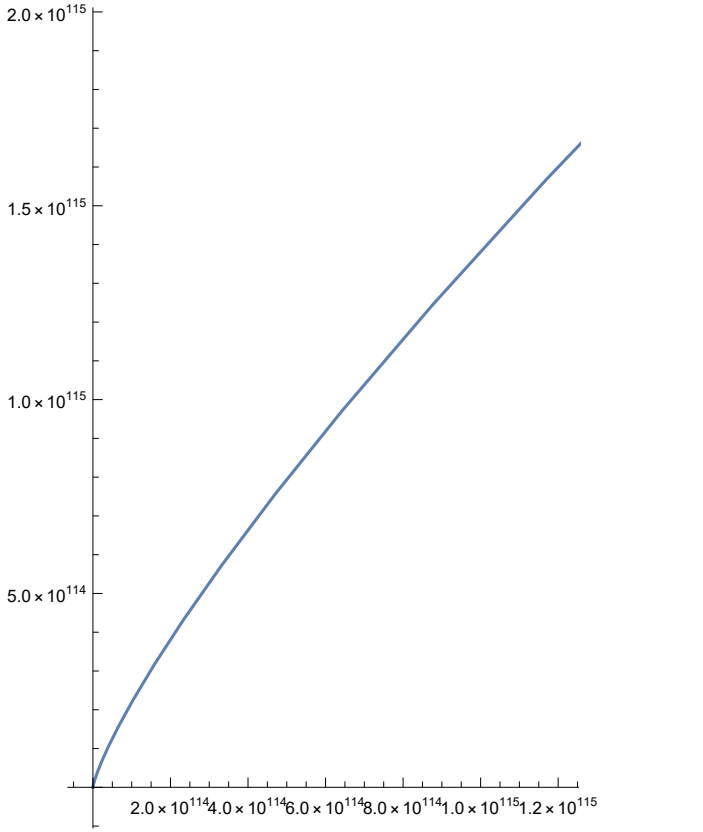
```
x[t] (1 + x[t]^2) + x''[t]^2 == 0
```

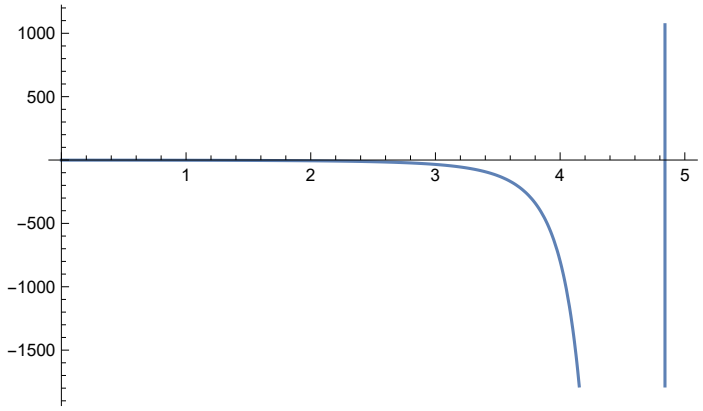


NDSolve: At t == 4.841556212546085`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 1.3087016646293348`.

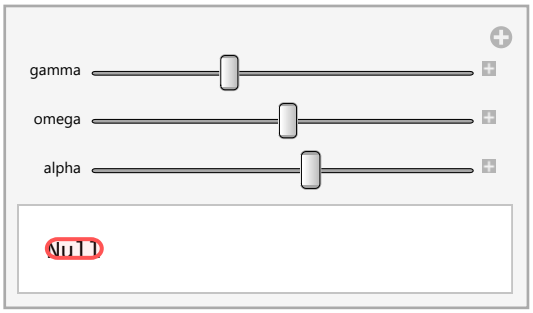
$$x[t]\left(1+x[t]^2\right)+x''[t]^2==0$$





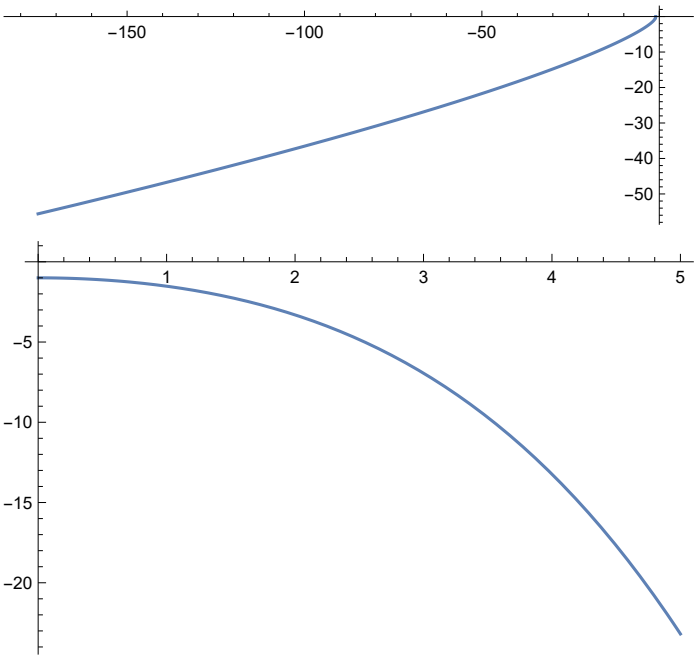
In[46]:=
```
(*C*)
Manipulate[duff[{g → gamma, f → 0, a → alpha, ω → omega}, {x[0] == -1, x'[0] == 0}, 5],
 {gamma, .01, 1}, {omega, .25, 2}, {alpha, 0, 1}]
```

Out[46]=



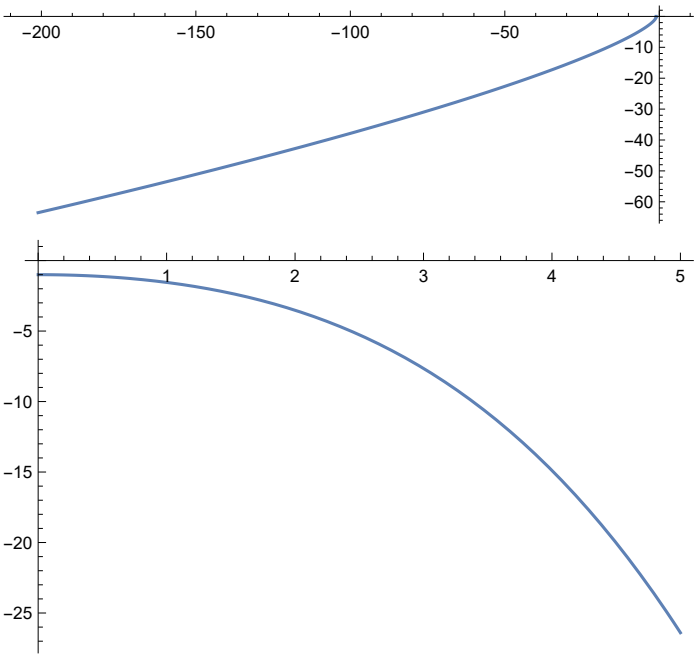
NDSolve: Maximum number of 10000 steps reached at the point t == 1.487540532783203`.

$$x[t] + 0.01\, x'[t] + x''[t]^2 == 0$$



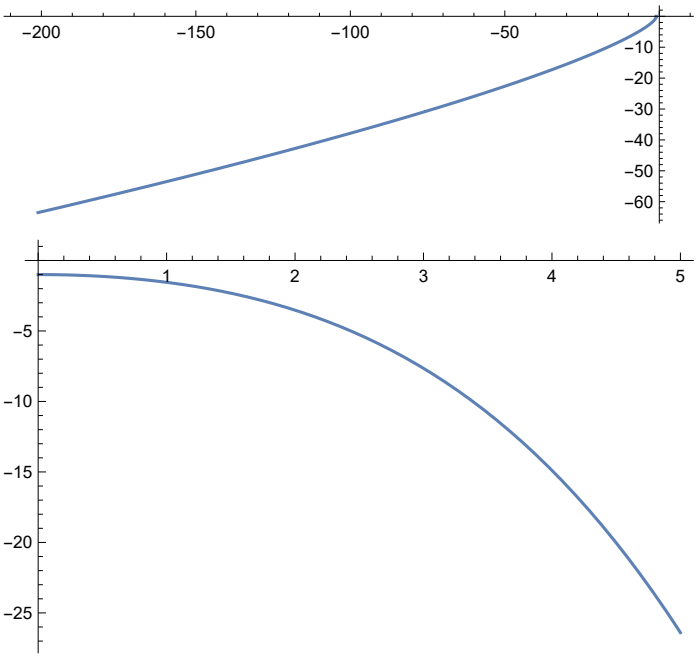
NDSolve: Maximum number of 10000 steps reached at the point t == 1.268168146162493`.

```
x[t] + 0.351 x'[t] + x''[t]^2 == 0
```



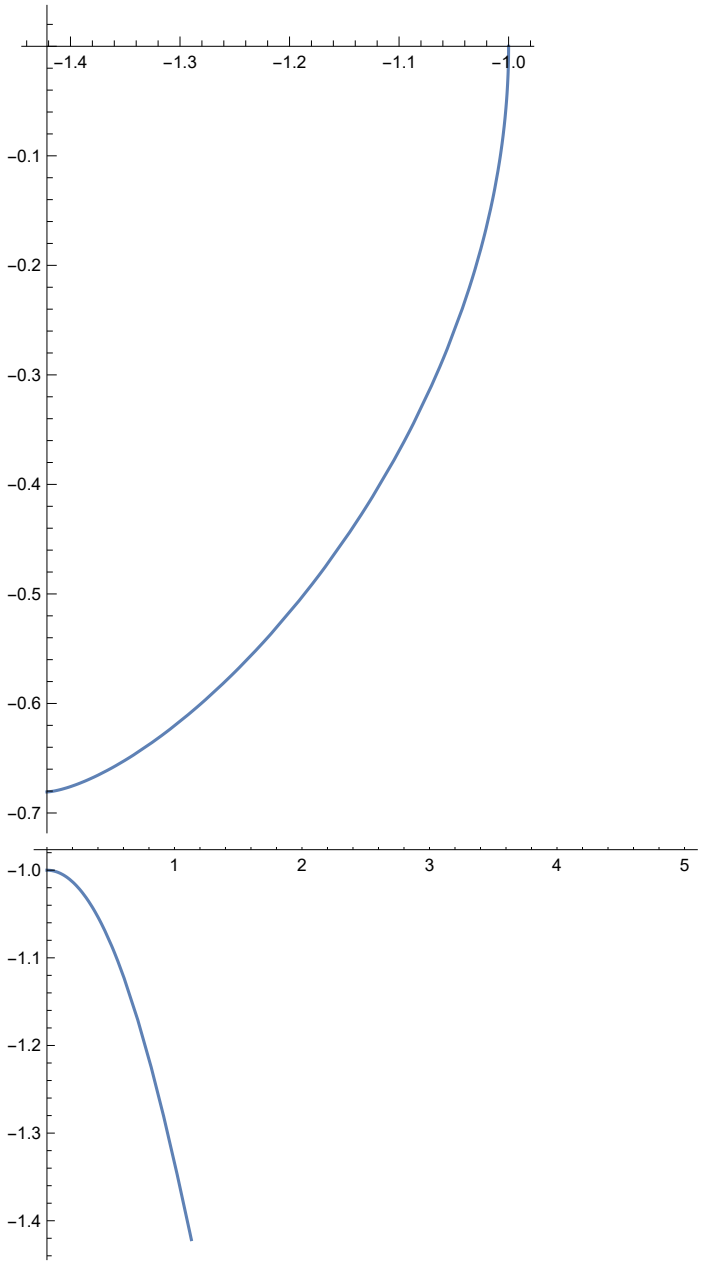
NDSolve: Maximum number of 10000 steps reached at the point t == 1.268168146162493`.

```
x[t] + 0.351 x'[t] + x''[t]^2 == 0
```



NDSolve: Maximum number of 10000 steps reached at the point t == 1.544947915520409`.

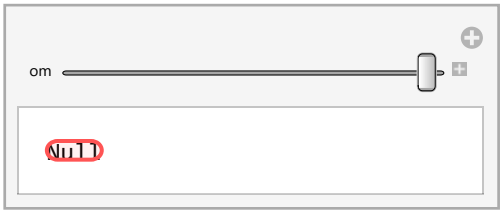
$$x[t]\left(1-0.578\,x[t]^2\right)+0.351\,x'[t]+x''[t]^2==0$$



In[47]:=

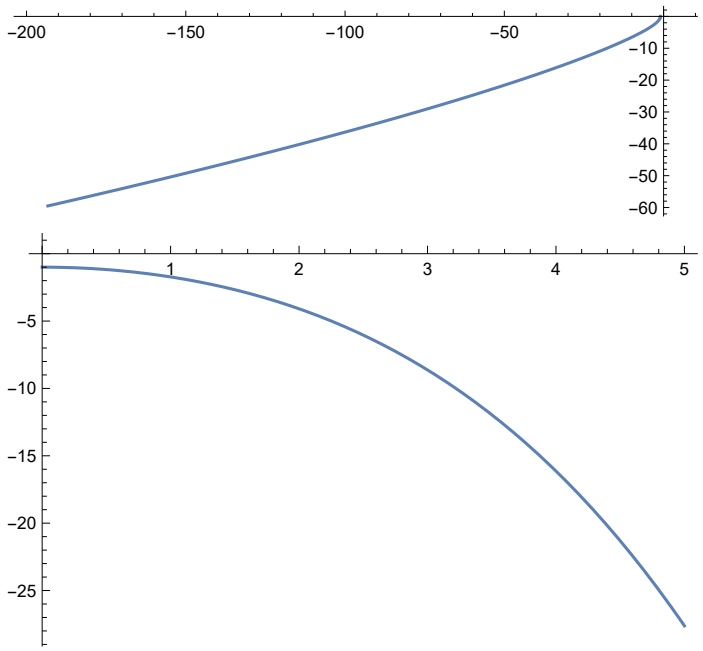
```
(*d*)
Manipulate[duff[{g → 0, f → 1, a → 0, ω → om}, {x[0] == -1, x'[0] == 0}, 5],
 {om, .5, 1.5, .5}]
```

Out[47]=



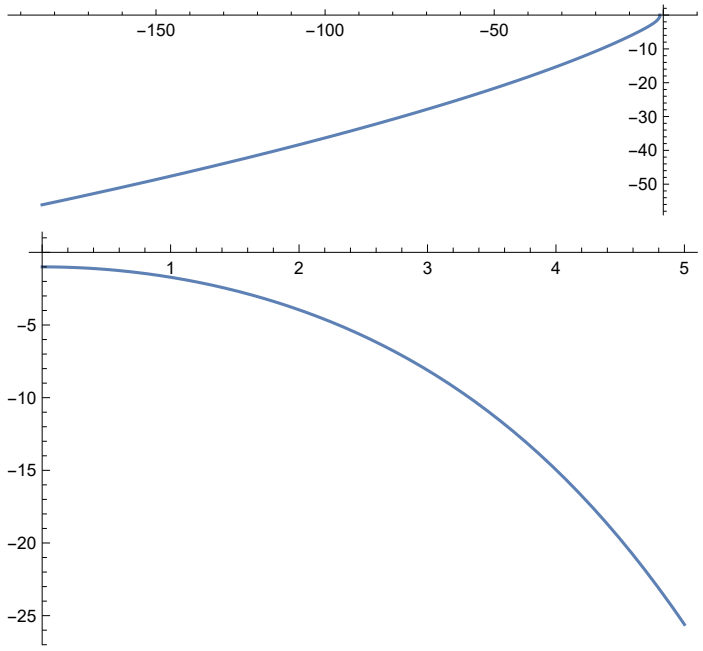
NDSolve: Maximum number of 10000 steps reached at the point t == 1.6347431137604151`.

```
x[t] + x''[t]^2 == Cos[0.5 t]
```



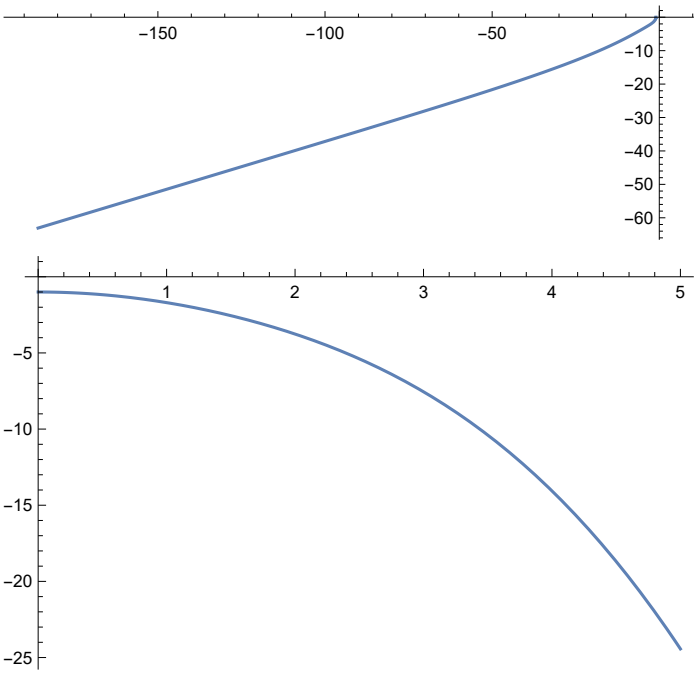
NDSolve: Maximum number of 10000 steps reached at the point t == 1.3786630237940685`.

```
x[t] + x''[t]^2 == Cos[1. t]
```



NDSolve: Maximum number of 10000 steps reached at the point t == 1.1585023409114246`.

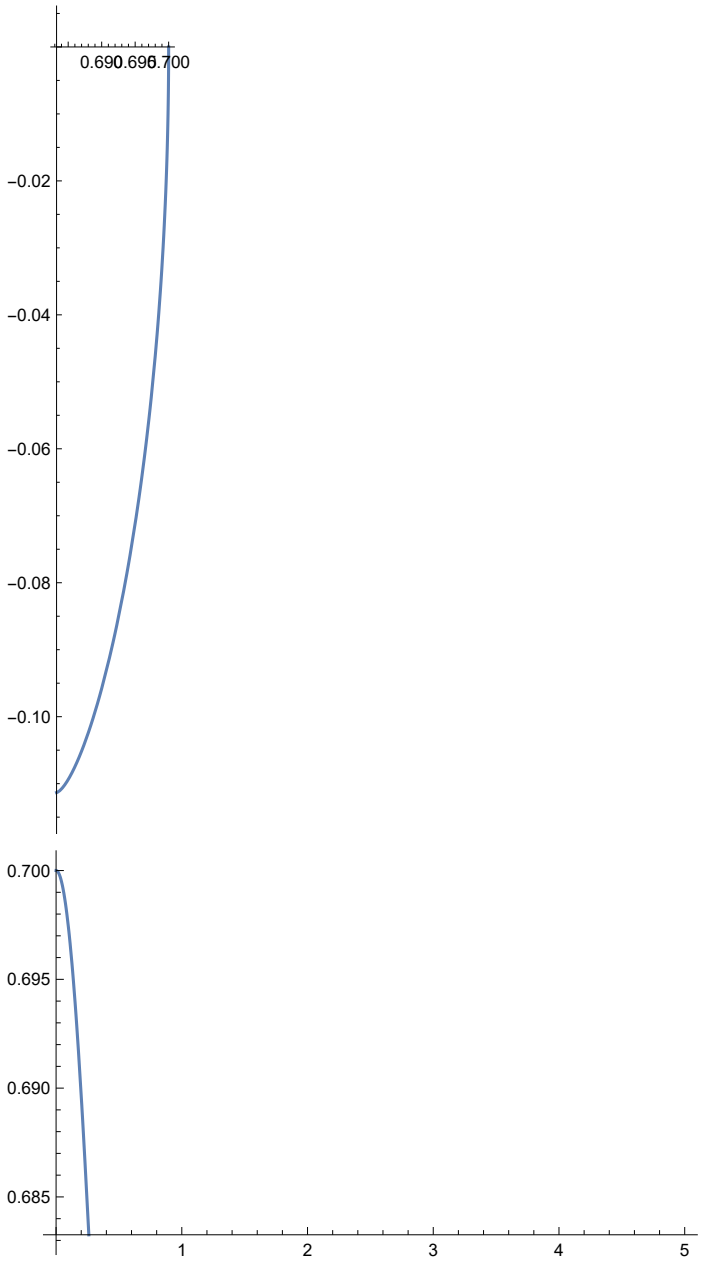
```
x[t] + x''[t]^2 == Cos[1.5 t]
```



In[49]:=
```
(*e*)
duff[{g → 0, a → 0, f → 1, ω → Pi}, {x[0] == .7, x'[0] == 0}, 5]
duff[{g → 0, a → 0, f → 1, ω → Pi}, {x[0] == -.5, x'[0] == 0}, 5]
```

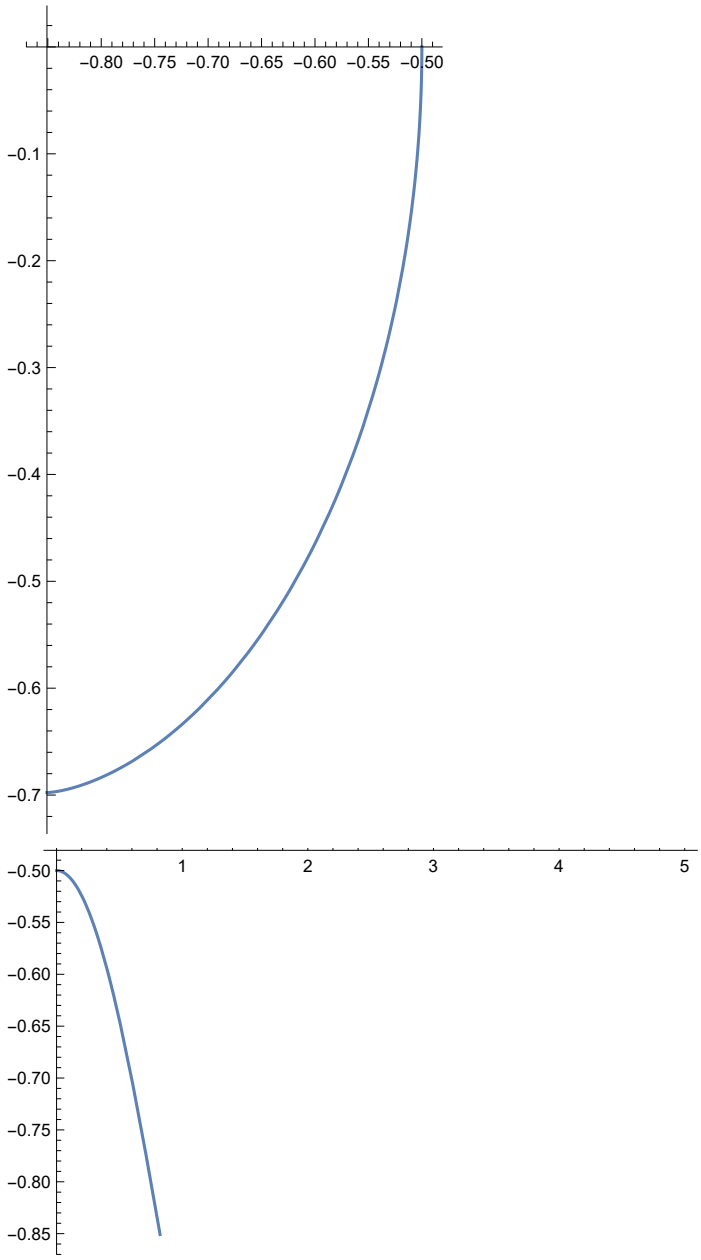
NDSolve: Maximum number of 10000 steps reached at the point t == 0.24929355937875977`.

$$x[t] + x''[t]^2 == \text{Cos}[\pi t]$$



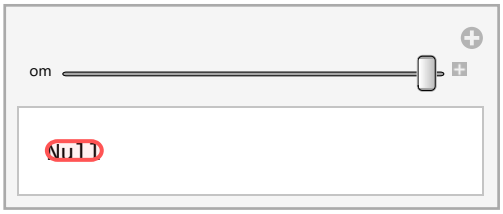
NDSolve: Maximum number of 10000 steps reached at the point t == 0.6020608239890208`.

$$x[t] + x''[t]^2 == \text{Cos}[\pi t]$$



In[51]:=
```
(*f*)
Manipulate[duff[{g → 0, f → 1, a → -1, ω → om}, {x[0] == -1, x'[0] == 0}, 5],
 {om, .5, 1.5, .5}]
```

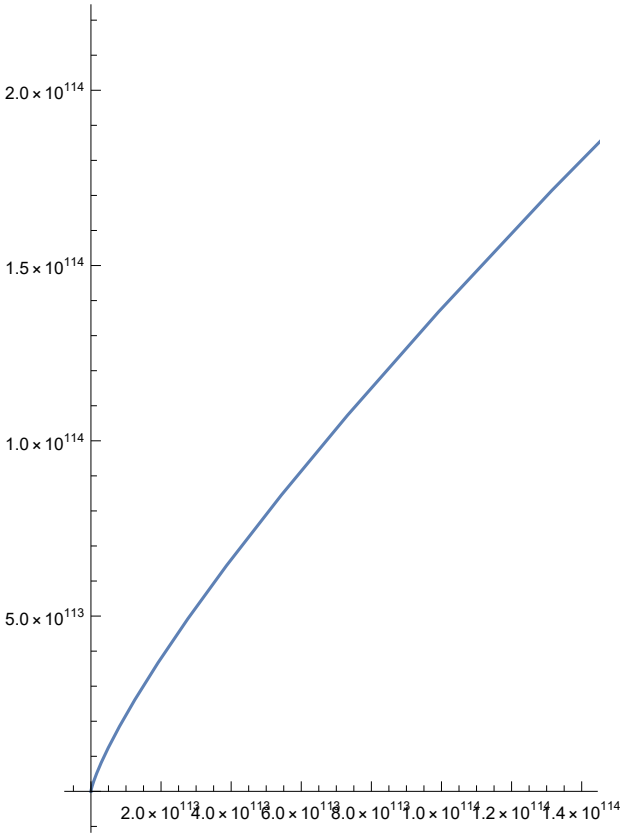
Out[51]=

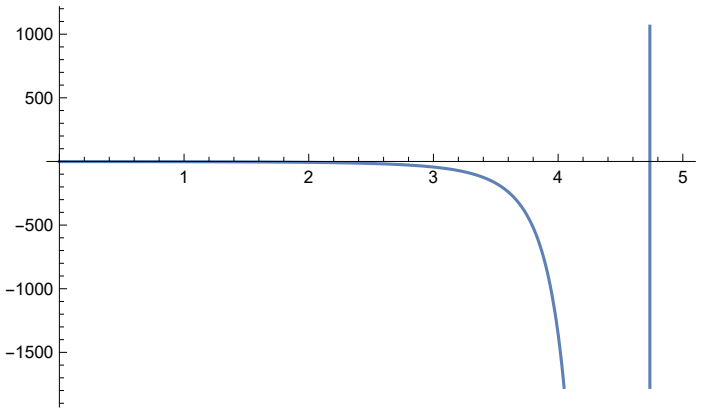


NDSolve: At t == 4.736858100770539`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 1.4670471075590004`.

$$x[t]\left(1+x[t]^2\right)+x''[t]^2==\text{Cos}[0.5\,t]$$

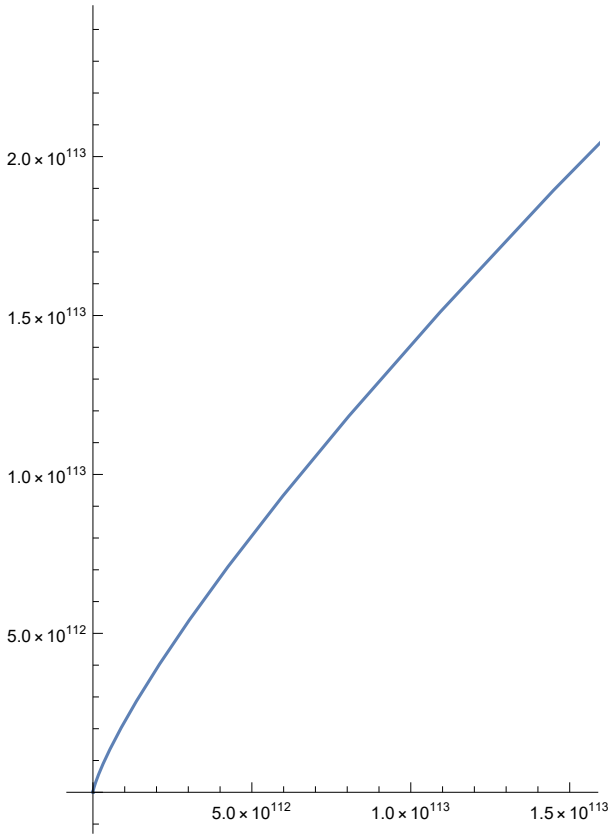


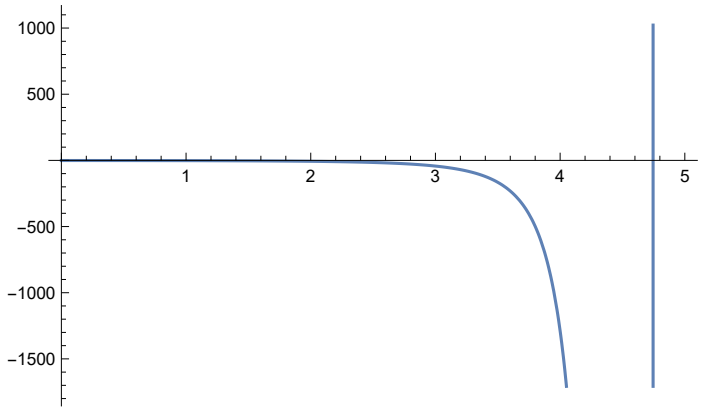


NDSolve: At t == 4.746471668340311`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 1.3094467976187019`.

```
x[t] (1 + x[t]^2) + x''[t]^2 == Cos[1. t]
```

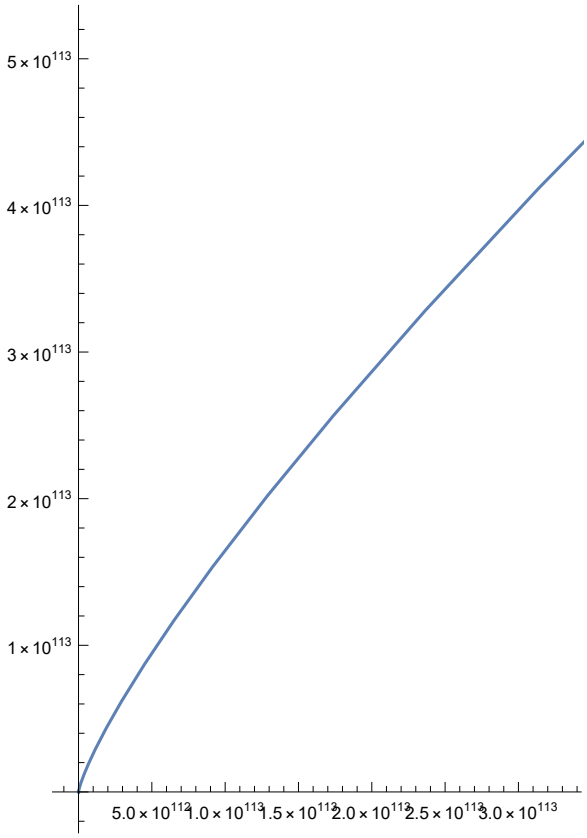


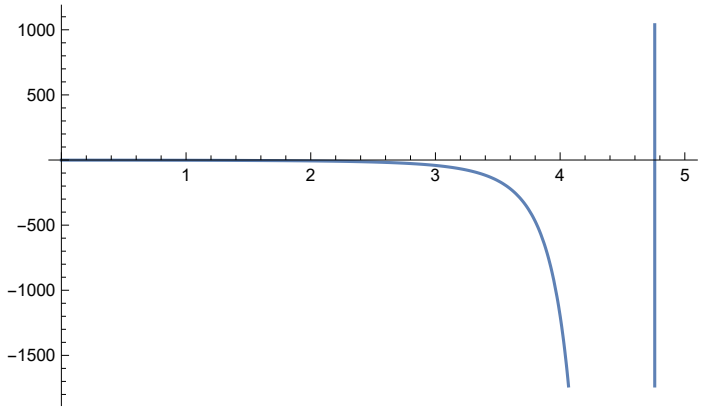


NDSolve: At t == 4.760436376266248`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 1.1119280693440132`.

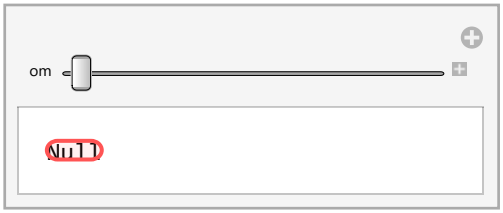
$$x[t]\left(1+x[t]^2\right)+x''[t]^2==\text{Cos}[1.5\,t]$$





In[52]:=
```
Manipulate[duff[{g → 0.1, f → 10, a → -1, ω → om}, {x[0] == -1, x'[0] == 0}, 5],
 {om, .5, 1.5, .5}]
```

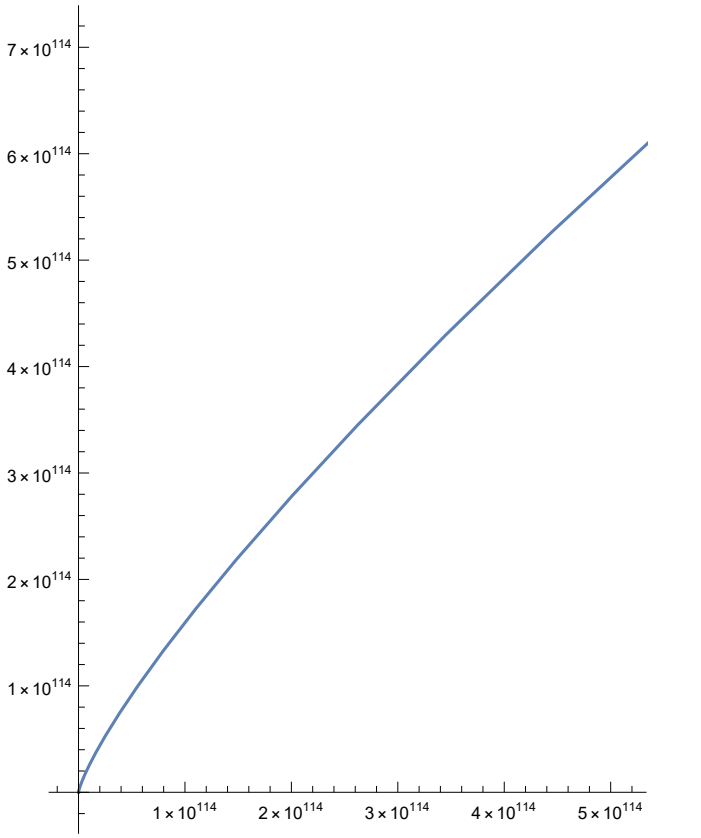
Out[52]=

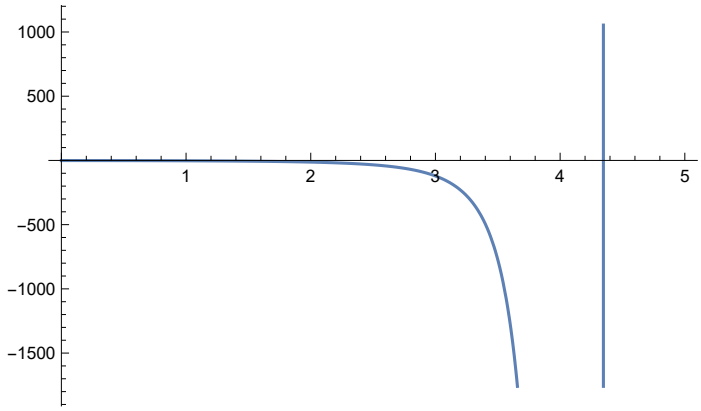


NDSolve: At t == 4.3482199086456506`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 1.3190843287491432`.

$$x[t]\left(1+x[t]^2\right)+0.1\,x'[t]+x''[t]^2==10\,\text{Cos}[0.5\,t]$$

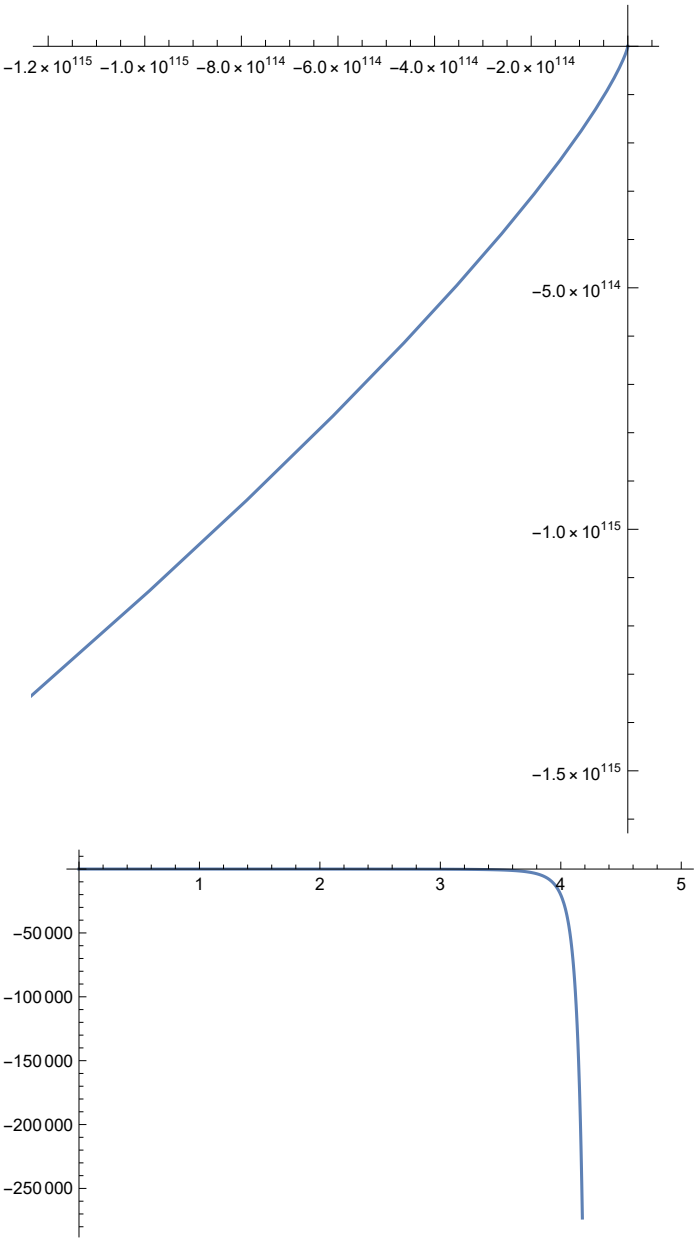




NDSolve: At t == 4.374799533740119`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 1.2069744467899333`.

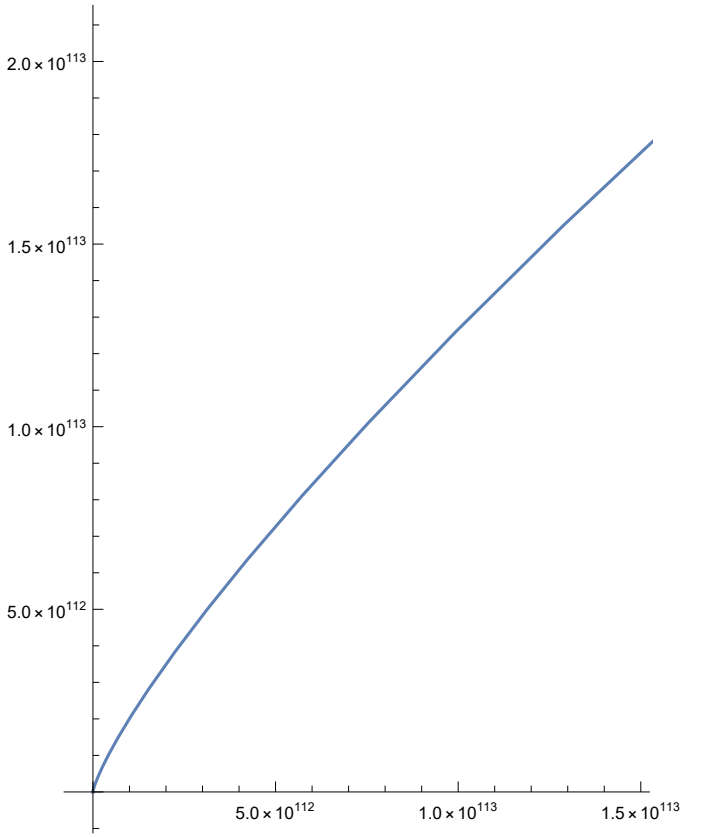
```
x[t] (1 + x[t]^2) + 0.1 x'[t] + x''[t]^2 == 10 Cos[1. t]
```

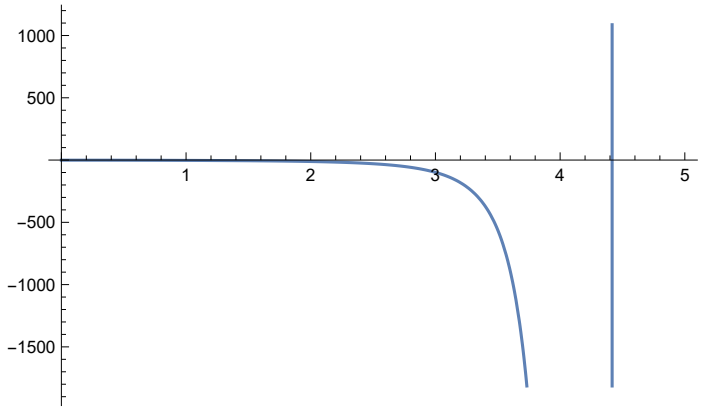


NDSolve: At t == 4.418479884833727`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 0.9903297724597246`.

```
x[t] (1 + x[t]^2) + 0.1 x'[t] + x''[t]^2 == 10 Cos[1.5 t]
```

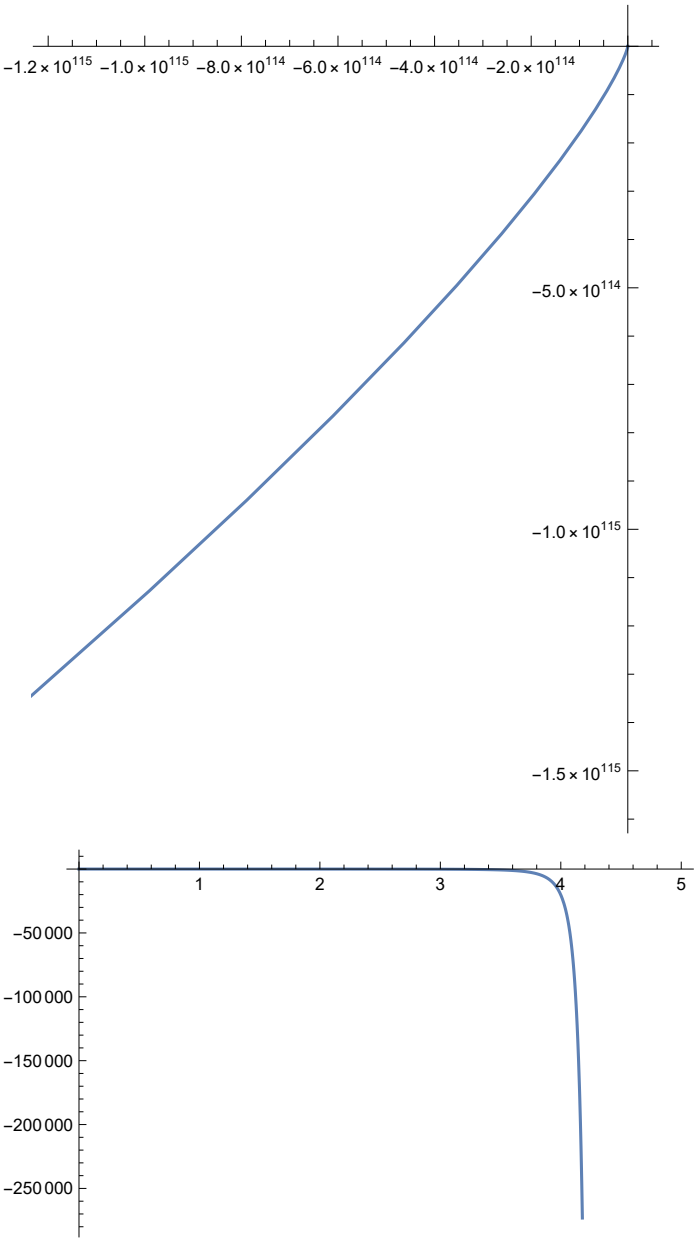




NDSolve: At t == 4.374799533740119`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 1.2069744467899333`.

```
x[t] (1 + x[t]^2) + 0.1 x'[t] + x''[t]^2 == 10 Cos[1. t]
```



NDSolve: At t == 4.3482199086456506`, step size is effectively zero; singularity or stiff system suspected.

NDSolve: Maximum number of 10000 steps reached at the point t == 1.3190843287491432`.

```
x[t] (1 + x[t]^2) + 0.1 x'[t] + x''[t]^2 == 10 Cos[0.5 t]
```