YSC2254: Modelling and Optimization

Midterm Project

Optimizing College Life (Arbitrarily)

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Introduction

Life... is tough. We all know it. In my case, I'm studying in Yale-NUS! Truthfully, it is an extremely intense place. While modules and work are really fun, we all know a hidden truth: we cannot do everything. Occasionally, there are things we must sacrifice. We need to **optimize** our utility here.

This serves as the inspiration for this midterm project: to examine how my utility can be optimized in Yale-NUS. To do this problem, I will attach coefficients to the number of hours amount on work, leisure, sleep, commitments and meals everyday and analyse the results and experiment with some really wacky cases of this linear program.

For this project, I will be formulating constraints based on my personal experiences and observations on my weekly lifestyle. The objective function will be based on the perceived utility of the activities that I do each day while the constraints would be based on what events are compulsory and what events can be dynamic. I will not make use of any data from the internet since it is not really needed. That being said, deciding these constraints will require a huge introspection of my living habits on my part.

Problem Setting

To begin, allow me to set this problem up. Firstly, the sub-script $i, i \in 1, 2, 3, 4, 5, 6, 7$ attached to my decision variables refers to the i-th day of the week, starting from Monday. In addition, I also define these decision variables:

- S_i refers to the number of hours spent on sleep on the i-th day of the week
- M_i refers to the number of hours spent on meals on the i-th day of the week
- L_i refers to the number of hours spent on leisure on the i-th day of the week
- C_i refers to the number of hours spent on compulsory commitments (CCAs, church, lessons, etc) on the i-th day of the week
- W_i refers to the number of hours spent on work on the i-th day of the week

With these things in mind, I wish to maximize the objective function

$$z = 1.2\left(\sum_{i=1}^{7} W_i\right) + 1.0\left(\sum_{i=1}^{7} L_i\right) + 1.1\left(\sum_{i=1}^{7} S_i\right) + 0.9\left(\sum_{i=1}^{7} C_i\right) + 0.8\left(\sum_{i=1}^{7} E_i\right)$$

where z refers to the total perceived utility obtained in a given week as a function of the amount of hours spent doing the given activity in a given day.

I define the utility to be a priority score. In other words, the higher my utility score, the more things I do that is in my priority. In this sense, I attached relative weights reflecting how much I value a certain activity. This was the justification behind my preferences, in decreasing order of coefficients:

1. Work has the highest coefficient, because it's my responsibility to finish them on time!
2. Sleeping is next on the list because who doesn't like a good rest? A good sleep is healthy and helps me prepare for the next day.
3. Leisure would be the next on the list. While I value it, I prioritize my work and sleep above it as it is a luxury.
4. Commitments would be the next because it's compulsory and binding. The constrains I will set are usually greater or equal to constraints and I wouldn't want to add more hours to my commitments if possible.
5. Eating is the lowest on the list. I consume my meals alone and i finish it at a very rapid pace. As a result, I take whatever minimum time I need and go.

Now that I am done with defining the objective function, let's move on to the constraints.

Constraints - Sanity Checks

Before moving on to the constraints, let's cover some sanity checks. The sign constraint is one of them:

$$\textbf{all } S_i, M_i, L_i, C_i, W_i \geq 0.$$

In addition, the sum of hours spent per day on the various activities (in the decision variables) should not exceed 24. Well, if someone could make a day more than 24 hours, please let me know. I'll pay the person handsomely. On a serious note, this constraint can be written as this: for $i \in 1, 2, 3, 4, 5, 6, 7$,

$$S_i + M_i + L_i + C_i + W_i = 24.$$

Constraints - Commitment

Once again, commitments refer to compulsory appointments like CCAs, peer tutoring sessions, church, lessons or sending out of e-mails. These constraints are typically $\geq$ in nature due to the binding nature of these constraints. Based on my schedule, i came up with the following constraints for each day for the events I must tend to. These gives rise to the following constraints:

1. Time spent for commitments on Monday is at least 6.5 hours
2. Time spent for commitments on Tuesday is at least 2 hours
3. Time spent for commitments on Wednesday is at least 4 hours
4. Time spent for commitments on Thursday is at least 9.0 hours
5. Time spent for commitments on Friday is at least 2 hours
6. Time spent for commitments on Saturday is at least 3.5 hours
7. Time spent for commitments on Sunday is at least 4.5 hours

Mathematically, this is:

$$
\begin{aligned}
C_1 &\geq 6.5 \\
C_2 &\geq 2 \\
C_3 &\geq 4 \\
C_4 &\geq 9.0 \\
C_5 &\geq 2 \\
C_6 &\geq 3.5 \\
C_7 &\geq 4.5
\end{aligned}
$$

Of course, these numbers vary according to day. On Tuesday, I only have proof class that takes 2 hours of my time. In contrast, on Thursday, I have data structures, calculus and modelling classes on top of contract bridge training, which total up to 9 hours.

Other than this, there are other variable constraints to commitment. For instance, on weekends, I could have routine family errands, but I'm not sure how long it would be, only that it would take 3 hours. This is reflected below:

$$C_6 + C_7 \geq 8 + 3 = 11$$

8 represents the sum of the minimum values C_6 and C_7 defined earlier in the constrain. 3 represents the variable commitment time I have to give to my family over the weekends.

On weekdays, I might need extra time to send e-mails to prospective employers, professors, etc. This constraint is reflected below:

$$C_1 + C_2 + C_3 + C_4 + C_5 \geq 23.5 + 2 = 25.5$$

As usual, 23.5 represents the sum of the minimum commitment time required in the weekdays while 2 represents the variable commitment time.

Constraints - Sleep

Well, this section needs no explanation! Everyone needs sleep and so do I. For me, I think that an average of 7.5hours of sleep a day is sufficient to survive the week. This totals to 52.5 hours over the week:

$$\sum_{i=1}^{7} S_i \geq 52.5.$$

Next, I'd love to sleep at least 8 hours in during the weekends where I can catch up on my sleep:

$$S_6, S_7 \geq 8.$$

The last constraint had to do with sleep stability and regularity. I realized that if I slept significantly less/ more compared to the previous day, my productivity takes a huge dip for the worse. In other words, I want to make sure that my sleep time does not deviate much; the absolute difference of my sleep-time between 2 consecutive days is less than or equal to 45 minutes, or $\frac{3}{4}$ of an hour.

In the case between Tuesday and Monday,

$$|x_2 - x_1| \leq 0.75$$

To input this as a constraint in LP form would mean simplifying the 2 modulus operator and expressing the negative and positive case:

$$x_2 - x_1 \leq 0.75$$
$$x_1 - x_2 \leq 0.75$$

We can then extend these constraints for the other days of the week:

$$
\begin{aligned}
x_3 - x_2 &\leq 0.75 \\
x_2 - x_3 &\leq 0.75 \\
x_4 - x_3 &\leq 0.75 \\
x_3 - x_4 &\leq 0.75 \\
x_5 - x_4 &\leq 0.75 \\
x_4 - x_5 &\leq 0.75 \\
x_6 - x_5 &\leq 0.75 \\
x_5 - x_6 &\leq 0.75 \\
x_7 - x_6 &\leq 0.75 \\
x_6 - x_7 &\leq 0.75 \\
x_7 - x_1 &\leq 0.75 \\
x_1 - x_7 &\leq 0.75
\end{aligned}
$$

With these constraints in mind, I can definitely sleep easier and work better.

Constraints - Work

In terms of work constraints, I have the following priorities:

- In the weekdays, a minimum amount of work should be done each day to catch up with school ($\geq$ constraints).
- In the weekends, no more than a certain amount of work should be done for rest ($\leq$ constraints).
- I spend a minimum amount of 30h per week doing work (Gross $\geq$ constraint across Mon to Sun)

Expressing the constraints in item 1 more concretely,

1. Time spent for work on Monday is at least 4.5 hours
2. Time spent for work on Tuesday is at least 7 hours
3. Time spent for work on Wednesday is at least 4 hours
4. Time spent for work on Thursday is at least 2 hours
5. Time spent for work on Friday is at least 7 hours

6. Time spent for work on Saturday is at most 2.5 hours
7. Time spent for work on Sunday is at most 3.0 hours

Expressing this mathematically gives the following:

$$\begin{aligned} W_1 &\geq 4.5 \\ W_2 &\geq 7 \\ W_3 &\geq 4 \\ W_4 &\geq 2 \\ W_5 &\geq 7 \\ W_6 &\leq 2.5 \\ W_7 &\leq 3.0 \end{aligned}$$

The constraint in item 3 can be expressed as

$$\sum_{i=1}^{7} W_i \geq 30.$$

Setting these constraints helps me to get a minimum amount of work done in one week.

Constraints - Leisure

Next up is a short exploration of the leisure constraints. In college, leisure isn't given, it's a blessing given in time that cannot be confirmed. However, I'd like to spend at least 15 hours leisurely from Monday to Friday:

$$\sum_{i=1}^{5} L_i \geq 15.$$

However, the weekends is where all these changes: I **want** more leisure! With that in mind, I would like at least 5 and 6 hours of leisure on Saturday and Sunday respectively:

$$\begin{aligned} L_6 &\geq 5 \\ L_7 &\geq 6 \end{aligned}$$

Constraints - Meal

This is probably the most boring constraint of them all. Personally, I do not view meals as something that one should spend too much, unless if you are socializing. In addition, some of my days have lessons that fall on mealtimes. Therefore, my meals become very rushed. with that in mind, I came up with the following constraints that express the minimum amount of time i need to spend consuming meals:

1. Time spent for meals on Tuesday, Thursday and Friday is at least 1 hour
2. Time spent for meals on Monday and Wednesday is at least 1.5 hours
3. Time spent for meals on Saturday and Sunday is at least 2 hours

$$\begin{aligned} x_2, x_4, x_5 &\geq 1 \\ x_1, x_3 &\geq 1.5 \\ x_6, x_7 &\geq 2 \end{aligned}$$

In total, I have a rather significant number of constraints. This should be enough for me to get some useful results.

Results

In this section, I run the simplex algorithm. The following table shows the results, rounded to 2 decimal places for decimals:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Work	4.5	7.0	5.04	2	7	2.11	2.36
Sleep	9.89	10.64	9.89	9.14	8.39	8.39	9.14
Leisure	1.61	3.36	3.57	2.86	3.61	5	6
Meal	1.5	1	1.5	1	1	2	2
Commit	6.5	2.0	4.0	9.0	4.0	6.5	4.5

The maximum value of the objective function is $z = 179$. However, it does not really benefit the investigation since this objective function is arbitrary in nature. It is better to analyse the respective coefficients of sleep, leisure, meal, commitment and work.

Looking at these table of values, I feels like I can live with this. The commitment and meal constraints are very close to the minimal value that I've set earlier, which is a reflection of the low coefficients I've set for them.

Examining the leisure coefficients, I find that about 26.01 hours have been spent on them. This is pretty much very near the minimal value that I've set earlier.

However, for the sleep coefficients, I calculated based on the table of values that I'm spending an average of 9.35 hours a day sleeping. It seems rather unrealistic that a college student would be able to sleep this much.

This issue is further emphasized by the fact that based on the work constraints, I'm spending 30 hours a week working which is the minimal value I placed. This would be most definitely an ideal college lifestyle, but I think that the amount of sleep is a little too much. Perhaps I could change its priority coefficient in my objective function for a result that's more realistic than this. These things will be explored in my next function

Sensitivity to coefficient changes

In this section, I will talk about changing coefficients based on the pointers I've discussed just now and analysing the change in results. In general, I think that too much priority is placed on sleep. With regards to Tuesday, I feel that 10.64 hours is a little too unrealistic. Therefore, I'll reduce the sleep coefficient from 1.1 to 1.05. In addition, I'll bump up the work coefficient to 1.3 and see where that takes me. My objective function becomes

With these things in mind, I wish to maximize the objective function

$$z = 1.3\left(\sum_{i=1}^{7} W_i\right) + 1.0\left(\sum_{i=1}^{7} L_i\right) + 1.05\left(\sum_{i=1}^{7} S_i\right) + 0.9\left(\sum_{i=1}^{7} C_i\right) + 0.8\left(\sum_{i=1}^{7} E_i\right).$$

In addition, it seems like my sleep is a bit too high during the week. I'm supposed to sleep in for the weekends, but I'm sleeping in during the weekdays instead. I need some sort of additional constraint to reflect the fact that I should not sleep more than an average of 8.5 hours each week. In addition, I could afford to sleep 9 hours on Saturday and Sunday. Let's formulate these constraints:

$$S_6, S_7 \geq 9$$
$$\sum_{i=i}^{7} S_i \leq 59.5$$

With these 2 constraints, my sleep time should become a lot more realistic than the Utopian scenario we started with previously. Let's run the LP and see where this takes me:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Work	4.5	7.0	10.19	2	8.31	1.5	2.5
Sleep	8.31	9.06	8.31	7.56	8.25	9.0	9.0
Leisure	3.19	4.94	0	4.44	2.44	5	6
Meal	1.5	1	1.5	1	1	2	2
Commit	6.5	2.0	4.0	9.0	4.0	6.5	4.5

Now, this produces an optimal value of $z = 179$ again. Looking at the respective values, the sleep-time looks a lot more like what I'd be living with. I have no other comments about mealtimes and commitment times, but the work time also seems a lot more realistic than before. While the algorithm tells me I have to pull off long work days, these are also the days where I'm relatively free (Tue, Fri).

A rather significant time is spent on leisure, but I guess that's a good thing. Since I could use leisure time to do anything, these large numbers tell me that I have some leeway with the coefficients that I'm giving to my algorithm.

Above all, I think that this shows the importance of assigning the appropriate coefficients to my objective function. As you can see, a small change in coefficients leads to a relatively large shift in time allocated from sleep to leisure and work. Since I am not doing goal-oriented LP which balances a few goals, this form of LP is highly sensitive to small changes in coefficients. Fine-tuning these coefficients becomes something important, yet non-trivial.

Overall, I think I will stick with this objective function with the new constraints. It is a lot more sustainable and balances my mental health with work. I'll share more reflections in the conclusions section.

Easter eggs - Sleep is just a number

Formulating constraints is a very important part in Modelling and Optimizing real-world problems. In this section, I will talk about a very funny situation when certain constraints are removed which gives unrealistic, but hilarious outputs.

Say that in the sleep constraint, I am careless and removed some constraints. In particular, I remove the sleep stability constraint and the maximum sleep constraint. Then, my sleep constraints would look like this:

$$\sum_{i=1}^{7} S_i \geq 52$$
$$S_6 \geq 9$$
$$S_7 \geq 9$$

Running the LP produces a greater objective value of $z = 187$. It seems that my priority score is higher, but the table of values tell a very disastrous story:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Work	8.5	7	18.5	2	7	0	0
Sleep	4.5	14.0	0	12.0	0	10.5	11.5
Leisure	3	0	0	0	12	5	6
Meal	1.5	1	1.5	1	1	2	2
Commit	6.5	2.0	4.0	9.0	4.0	6.5	4.5

As you can expect, this algorithm was a total disaster. This is because with the minimum bounding constraint for sleep removed, the objective function will prioritize work whenever possible. This is why its value increased so rapidly. However, everything was balanced around work. This was why sleep hours were so irregular (Especially Wednesday, where 18.5 hours of work is allocated).

This Easter Egg taught me a lesson: to never be convinced that the machine can do all the heavy lifting. When doing these optimization problems, it is important as humans to think about the implications of assigning coefficients to something. If not done properly, disaster could strike like this.

Assumption & Further Exploration

These areas of exploration were things that were too complicated to explore in this midterm project, but here are some ideas that extends this project:

- Randomize some constraints, with regards to commitments and meals, since we don't know exactly how long they'll take.
- Assign individual coefficients to W_i and not a blanket coefficient for the entire week. Since for instance, I get more utility from work on Wednesday than Saturday simply because I'm more productive.
- Treat this problem as a 0-1 knapsack problem down to the hour.

In addition, I made many assumptions about the perceived coefficients of utility. By assigning these values, I'm actually putting a constant rank on the perceived utility of these activities. However, in real life, these "priorities" are constantly in flux and we cannot pin a coefficient to them.

I've also assumed that within a day, I can allocate these respective activities at any time of the day. For instance, when the algorithm tells me to sleep 5h on a Monday, I could do it in the afternoon or night and they still meet the sleep requirements. It does not account for when I should do a certain activity. This could be circumvented using a 0-1 knapsack format which helps segregate each hour into blocks.

In general, these are the overarching assumptions that I've made in my model. Tightening these assumptions would have certainly made for a better output, but with what I have right now, it is definitely usable.

Conclusion

This project helped me to understand some of the complexities of modelling and optimizing real-life problems. It was a pretty enjoyable process as I explored how my life could be made mathematical via optimization. At the same time, It also illuminated some of the difficulties faced in modelling, particularly productive and practical formulation of constraints.

These are things that I will definitely take with me into the future.