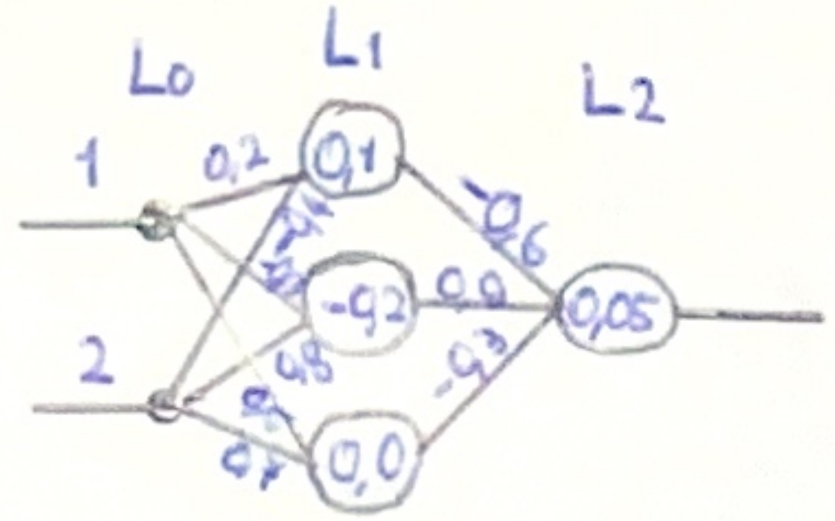
Neural Network 1: 231



$Y = 1{,}5$

$\sigma = \mathrm{ReLU}$

$$z_0^1 = w_{00}^1 \cdot a_0^0 + w_{01}^1 \cdot a_1^0 + b_0^1$$
$$= 0{,}2 \cdot 1 + (-0{,}4) \cdot 2 + 0{,}1$$
$$= -0{,}5$$
$$\sigma = 0$$

$$z_1^1 = w_{10}^1 \cdot a_0^0 + w_{11}^1 \cdot a_1^0 + b_1^1$$
$$= -0{,}3 \cdot 1 + 0{,}8 \cdot 2 - 0{,}2$$
$$= 1{,}1$$
$$\sigma = 1{,}1$$

$$z_2^1 = w_{20}^1 \cdot a_0^0 + w_{21}^1 \cdot a_1^0 + b_2^1$$
$$= 0{,}5 + 1{,}4$$
$$= 1{,}9$$
$$\sigma = 1{,}9$$

$$z_0^2 = w_{00}^2 \cdot a_0^1 + w_{01}^2 \cdot a_1^1 + w_{02}^2 \cdot a_2^1 + b_0^2$$
$$= 0{,}99 - 0{,}57 + 0{,}05$$
$$= 0{,}47$$
$$\sigma = 0{,}47$$

(1) Compute Forwards

Forward Pass

$$L = \frac{1}{2} \sum_{i=1}^{m} (y - a^L)^2$$
$$= \frac{(1{,}5 - 0{,}47)^2}{2}$$
$$= 0{,}53045$$

(2) Calculate Cost

Backpropagation

$$L = \frac{1}{2}(a^L - y)^2$$
$$a^L = \sigma(z_k^L)$$
$$z_k^L = \sum_k w_{jk}^L \cdot a^{L-1} + b_k^L$$
$$a^{L-1} = \sigma(z_k^{L-1})$$
$$z_k^{L-1} = \sum_k w_{jk}^{L-1} \cdot a^{L-2} + b_k^{L-1}$$

$$\delta^{L-1} = \delta^L \cdot \frac{\partial z_k^L}{\partial a^{L-1}} \cdot \frac{\partial a^{L-1}}{\partial z_k^{L-1}}$$
$$= \delta^L \cdot w_{jk}^L \cdot \mathrm{ReLU}'(z_k^{L-1})$$
$$= \left(\sum_k w_{jk}^L \cdot \delta^L\right) \cdot \mathrm{ReLU}'(z_k^{L-1})$$
$$= ((W^L)^T \delta^L) \odot \mathrm{ReLU}'(z_k^{L-1})$$

(3) Derive/Find Formulas

$$\delta^L = \frac{\partial L}{\partial z_k^L}$$
$$= \frac{\partial L}{\partial a^L} \cdot \frac{\partial a^L}{\partial z_k^L}$$
$$= \frac{\partial}{\partial a^L} \frac{1}{2}(a^L - y)^2 \cdot \frac{\partial}{\partial z_k^L} \sigma(z_k^L)$$
$$= (a^L - y) \cdot \mathrm{ReLU}'(z_k^L)$$

$\partial L / \partial w_{jk}^{L-1}$
$$= \delta^{L-1} \cdot \frac{\partial z_k^{L-1}}{\partial w_{jk}^{L-1}}$$
$$= \delta^{L-1} \cdot a^{L-2}$$

$$\frac{\partial L}{\partial b_k^{L-1}} = \delta^{L-1}$$

$$\frac{\partial L}{\partial w_{jk}^L} = \frac{\partial L}{\partial a^L} \cdot \frac{\partial a^L}{\partial z_k^L} \cdot \frac{\partial z_k^L}{\partial w_{jk}^L}$$
$$= \delta^L \cdot \frac{\partial z_k^L}{\partial w_{jk}^L}$$
$$= \delta^L \cdot a^{L-1}$$

$$\frac{\partial L}{\partial b_k^L} = \frac{\partial L}{\partial a^L} \cdot \frac{\partial a^L}{\partial z_k^L} \cdot \frac{\partial z_k^L}{\partial b_k^L}$$
$$= \delta^L \cdot \frac{\partial z_k^L}{\partial b_k^L}$$
$$= \delta^L$$

$$\mathrm{ReLU} = \begin{cases} 0 & \text{if } z \le 0 \\ z & \text{if } z > 0 \end{cases}$$
$$\mathrm{ReLU}' = \begin{cases} 0 & \text{if } z \le 0 \\ 1 & \text{if } z > 0 \end{cases}$$

(4) Calculate values of L2

$$\delta^L = 0{,}47 - 1{,}5 \cdot 1$$
$$= -1{,}03$$

$$a^{L-1} = [0 \quad 1{,}1 \quad 1{,}9]$$

$$\frac{\partial L}{\partial w_{00}^2} = 0 \qquad \frac{\partial L}{\partial w_{01}^2} = -1{,}133 \qquad \frac{\partial L}{\partial w_{02}^2} = -1{,}957$$

$$\frac{\partial L}{\partial b_0^2} = -1{,}03$$

⑤ Calculate Values of L1

$$\delta^{L-1} = ((W^L)^T \delta^L) \odot ReLU'(z_k^{l-1})$$
$$= [-0{,}6 \quad 0{,}9 \quad -0{,}3] \, [-1{,}03]$$
$$= \begin{bmatrix} 0{,}618 \\ -0{,}927 \\ 0{,}309 \end{bmatrix} \cdot (0 \ 1 \ 1) \implies \begin{matrix} z_0^1 = 0 \Rightarrow 0 \\ z_1^1 = 1{,}1 \Rightarrow 1 \\ z_2^1 = 1{,}9 \Rightarrow 1 \end{matrix}$$
$$= \begin{bmatrix} 0 \\ -0{,}927 \\ 0{,}309 \end{bmatrix} //$$

$$\frac{\partial L}{\partial w_{00}^1} = 0 \quad \frac{\partial L}{\partial w_{01}^1} = 0 \quad \frac{\partial L}{\partial b_0^1} = 0 //$$

$$\frac{\partial L}{\partial w_{10}^1} = -0{,}927 \quad \frac{\partial L}{\partial w_{11}^1} = -1{,}854 \quad \frac{\partial L}{\partial b_1^1} = -0{,}927 //$$

$$\frac{\partial L}{\partial w_{20}^1} = 0{,}309 \quad \frac{\partial L}{\partial w_{21}^1} = 0{,}618 \quad \frac{\partial L}{\partial b_2^1} = 0{,}309 //$$

⑥ Optimizing ($\alpha = 0{,}1$)

$$w_{00}^1 = w_{00}^1 - \alpha \cdot \frac{\partial L}{\partial w_{00}^1} = 0{,}2 //$$

$$w_{01}^1 = -0{,}4 - \alpha \frac{\partial L}{\partial w_{01}^1} = -0{,}4 //$$

$$b_0^1 = b_0^1 - \alpha \frac{\partial L}{\partial b_0^1} = 0{,}1 //$$

$$w_{10}^1 = -0{,}3 - 0{,}1 \cdot (-0{,}927) = 0{,}2073 //$$

$$w_{11}^1 = 0{,}8 - 0{,}1 \cdot (-1{,}854) = 0{,}9857 //$$

$$b_1^1 = -0{,}2 - 0{,}1 \cdot (-0{,}927) = -0{,}1073 //$$

$$w_{20}^1 = 0{,}5 - 0{,}1 \cdot 0{,}309 = 0{,}4691 //$$

$$w_{21}^1 = 0{,}7 - 0{,}1 \cdot 0{,}618 = 0{,}6382 //$$

$$b_2^1 = -0{,}1 \cdot 0{,}309 = -0{,}0309 //$$

$$w_{00}^2 = -0{,}6 - 0{,}1 \cdot 0 = -0{,}6 //$$

$$w_{01}^2 = 0{,}9 - 0{,}1 \cdot 1{,}133 = 0{,}7867 //$$

$$w_{02}^2 = -0{,}3 - 0{,}1 \cdot (-1{,}957) = -0{,}1043 //$$

$$b_0^2 = 0{,}05 - 0{,}1 \cdot (-1{,}03) = 0{,}153 //$$

⑦ Forward Pass

$$z_0^1 = 0{,}2 \cdot 1 + (-0{,}4) \cdot 2 + 0{,}1 = -0{,}5 \qquad \sigma = 0 /$$

$$z_1^1 = 0{,}2073 + 0{,}9857 \cdot 2 - 0{,}1073 = 2{,}0714 //$$

$$z_2^1 = 0{,}4691 + 0{,}6382 \cdot 2 - 0{,}0309 = 1{,}7146 //$$

$$z_0^2 = -0{,}6 \cdot 0 + 0{,}7867 \cdot 2{,}0714 - 0{,}1043 \cdot 1{,}7146 + 0{,}153 = 1{,}6037376 //$$

⑧ Calculate Cost

$$L = \frac{1}{2}(y - \hat{y})$$
$$= \frac{(1{,}5 - 1{,}6037376)^2}{2}$$
$$= 0{,}00538074482 7 //$$