Model

$$
\begin{aligned}
P(\theta_j, \sigma, \mu_0, \varphi, \tau, K_j | z_{\log(y_j)}) &\propto P(z_{\log(y_j)} | \theta_j, \sigma, \mu, \varphi, \tau, K_j) \cdot P(\theta_j, \sigma, \mu_0, \varphi, \tau, K_j) \\
&\propto P(z_{\log(y_j)} | \theta_j, \sigma)\ P(\theta_j | \mu_0, \varphi, \tau, K_j)\ P(\sigma)\ P(\mu_0)\ P(\varphi)\ P(\tau)
\end{aligned}
$$

K_j has no prior since it is the predictor, i.e. it is not a parameter.

The assumed shapes of the priors and likelihoods are as follows

$$
\begin{aligned}
P(\sigma), P(\tau) &\sim \mathcal{U}\ ]0, +\infty) \\
P(\mu), P(\varphi) &\sim \mathcal{U}(-\infty, +\infty) \\
P(z_{\log(y_j)} | \theta_j, \sigma) &\sim \mathcal{N}(\theta_j, \sigma^2) \\
P(\theta_j | \mu_0, \varphi, \tau, K_j) &\sim \mathcal{N}(\mu_0 + \varphi \cdot K_j, \tau^2)
\end{aligned}
$$

Note, that the priors for $P(\sigma)$ and $P(\tau)$ exclude 0.

For slice sampling, the joint posterior over all individuals was used:

$$
\prod_j^J P(\theta_j, \sigma, \mu_0, \varphi, \tau, K_j | z_{\log(y_j)}) \propto \prod_j^J P(z_{\log(y_j)} | \theta_j, \sigma)\ P(\theta_j | \mu_0, \varphi, \tau, K_j)\ P(\sigma)\ P(\mu_0)\ P(\varphi)\ P(\tau)
$$

For the slice sampler, the log-pdfs of the assumed distributions were used.

Transformation

For each individual, j, the logarithmised observations, $\log(y_j)$, which are also mean-centred and standardised, $z_{\log(y_j)}$, are distributed as follows:

$$\begin{aligned} z_{\log(y_j)} &\sim \mathcal{N}(\theta_j, \sigma^2) \\ z_{\log(y_j)} &= \theta_j + \sigma \cdot X && \text{where } X \sim \mathcal{N}(0,1) \end{aligned}$$

$$\begin{aligned} \theta_j &\sim \mathcal{N}(\mu_0 + \varphi \cdot K_j, \tau^2) && \text{where } K_j = \begin{cases} 1 & \text{if } j \text{ is a kid} \\ 0 & \text{else} \end{cases} \\ \theta_j &= \mu_0 + \varphi \cdot K_j + \tau \cdot \Xi && \text{where } \Xi \sim \mathcal{N}(0,1) \\ z_{\log(y_j)} &= \mu_0 + \varphi \cdot K_j + \tau \cdot \Xi + \sigma \cdot X \end{aligned}$$

assuming independence, we can convolute the addition of the two normally distributed random variables, Ξ and X, and merge them into depending only on one of the, say X:

$$z_{\log(y_j)} = \mu_0 + \varphi \cdot K_j + \sqrt{\sigma^2 + \tau^2} \cdot X$$

De-standardising from the sample variance, $\bar{\sigma}_y$ and un-centering from the sample mean, $\bar{y}$, yields:

$$\begin{aligned} z_{\log(y_j)} = \frac{\log(y_j) - \bar{y}}{\bar{\sigma}_y} &= \mu_0 + \varphi \cdot K_j + \sqrt{\sigma^2 + \tau^2} \cdot X \\ \log(y_j) &= \underbrace{\bar{\sigma}_y \mu_0 + \bar{y}}_{\mu_{0,\text{unscaled}}} + \underbrace{\bar{\sigma}_y \varphi}_{\varphi_{\text{unscaled}}} \cdot K_j + \sqrt{\underbrace{\bar{\sigma}_y^2 \sigma^2}_{\sigma^2_{\text{unscaled}}} + \underbrace{\bar{\sigma}_y^2 \tau^2}_{\tau^2_{\text{unscaled}}}} \cdot X \end{aligned}$$

and therefore,

$$\begin{aligned} \log(y_j) &\sim \mathcal{N}(\mu_{0,\text{unscaled}} + \varphi_{\text{unscaled}} \cdot K_j, \quad \sigma^2_{\text{unscaled}} + \tau^2_{\text{unscaled}}) \\ y_j &\sim logNormal(\mu_{0,\text{unscaled}} + \varphi_{\text{unscaled}} \cdot K_j, \quad \sigma^2_{\text{unscaled}} + \tau^2_{\text{unscaled}}) \end{aligned}$$

The expected value at the **group** level is thus (from Wiki page):

$$\begin{aligned} \mu_{\text{unscaled,nonlog}} = E[y_j] &= e^{\mu_{0,\text{unscaled}} + \varphi_{\text{unscaled}} \cdot K_j + \frac{\sigma^2_{\text{unscaled}}}{2} + \frac{\tau^2_{\text{unscaled}}}{2}} \\ &= \underbrace{e^{\varphi_{\text{unscaled}} \cdot K_j}}_{\text{effect}} \quad e^{\mu_{0,\text{unscaled}} + \frac{\sigma^2_{\text{unscaled}}}{2} + \frac{\tau^2_{\text{unscaled}}}{2}} \end{aligned}$$

And at the **individual** level:

$$\theta_{\text{unscaled,nonlog}} = E[y_j] = e^{\theta_{\text{unscaled}} + \frac{\sigma^2_{\text{unscaled}}}{2}}$$

The multiplicative effect of being a kid acts only on the group level.