For each individual, j, the logarithmised observations, $\log(y_j)$, which are also mean-centred and standardised, $z_{\log(y_j)}$, are distributed as follows:

$$\begin{aligned}
z_{\log(y_j)} &\sim \mathcal{N}(\theta_j, \sigma^2) \\
z_{\log(y_j)} &= \theta_j + \sigma \cdot X \qquad\qquad \text{where } X \sim \mathcal{N}(0,1)
\end{aligned}$$

$$\begin{aligned}
\theta_j &\sim \mathcal{N}(\mu_0 + \varphi \cdot K_j, \tau^2) \qquad\qquad \text{where } K_j = \begin{cases} 1 & \text{if } j \text{ is a kid} \\ 0 & \text{else} \end{cases} \\
\theta_j &= \mu_0 + \varphi \cdot K_j + \tau \cdot \Xi \qquad\qquad \text{where } \Xi \sim \mathcal{N}(0,1) \\
z_{\log(y_j)} &= \mu_0 + \varphi \cdot K_j + \tau \cdot \Xi + \sigma \cdot X
\end{aligned}$$

assuming independence, we can convolute the addition of the two normally distributed random variables, Ξ and X, and merge them into depending only on one of the, say X:

$$z_{\log(y_j)} = \mu_0 + \varphi \cdot K_j + \sqrt{\sigma^2 + \tau^2} \cdot X$$

De-standardising from the sample variance, $\bar{\sigma}_y$ and un-centering from the sample mean, $\bar{y}$, yields:

$$\begin{aligned}
z_{\log(y_j)} = \frac{\log(y_j) - \bar{y}}{\bar{\sigma}_y} &= \mu_0 + \varphi \cdot K_j + \sqrt{\sigma^2 + \tau^2} \cdot X \\
\log(y_j) &= \underbrace{\bar{\sigma}_y \mu_0 + \bar{y}}_{\mu_{0,\text{unscaled}}} + \underbrace{\bar{\sigma}_y \varphi}_{\varphi_{\text{unscaled}}} \cdot K_j + \sqrt{\underbrace{\bar{\sigma}_y^2 \sigma^2}_{\sigma^2_{\text{unscaled}}} + \underbrace{\bar{\sigma}_y^2 \tau^2}_{\tau^2_{\text{unscaled}}}} \cdot X
\end{aligned}$$

and therefore,

$$\begin{aligned}
\log(y_j) &\sim \mathcal{N}(\mu_{0,\text{unscaled}} + \varphi_{\text{unscaled}} \cdot K_j, \quad \sigma^2_{\text{unscaled}} + \tau^2_{\text{unscaled}}) \\
y_j &\sim logNormal(\mu_{0,\text{unscaled}} + \varphi_{\text{unscaled}} \cdot K_j, \quad \sigma^2_{\text{unscaled}} + \tau^2_{\text{unscaled}})
\end{aligned}$$

The expected value at the **group** level is thus (from Wiki page):

$$\begin{aligned}
\mu_{\text{unscaled,nonlog}} = E[y_j] &= e^{\mu_{0,\text{unscaled}} + \varphi_{\text{unscaled}} \cdot K_j + \frac{\sigma^2_{\text{unscaled}}}{2} + \frac{\tau^2_{\text{unscaled}}}{2}} \\
&= \underbrace{e^{\varphi_{\text{unscaled}} \cdot K_j}}_{\text{effect}} \; e^{\mu_{0,\text{unscaled}} + \frac{\sigma^2_{\text{unscaled}}}{2} + \frac{\tau^2_{\text{unscaled}}}{2}}
\end{aligned}$$

And at the **individual** level:

$$\theta_{\text{unscaled,nonlog}} = E[y_j] = e^{\theta_{\text{unscaled}} + \frac{\sigma^2_{\text{unscaled}}}{2}}$$

The multiplicative effect of being a kid acts only on the group level.