# Vector Field Analysis and Visualization with Robustness

## Feature Extraction, Tracking and Simplification

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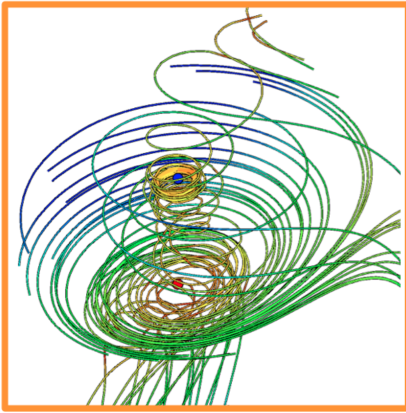
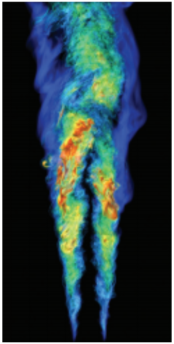
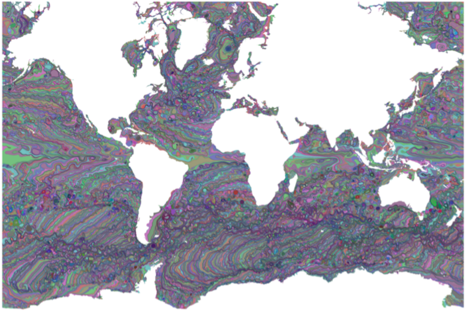
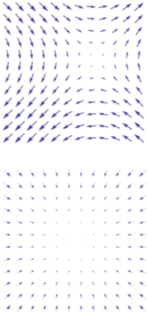
October 22, 2016



# Robust Feature Extraction and Visualization of Vector Fields

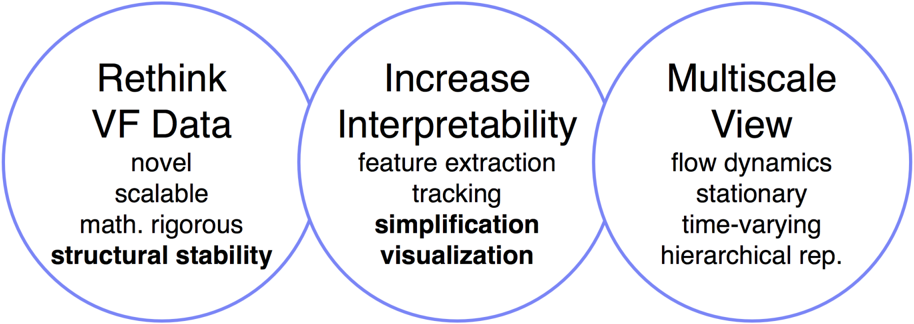
# Understanding VF is indispensable for many applications

- Turbulence combustion, global oceanic eddies simulations, etc.
- A $d$-dim VF: a function that assigns to each point a $d$-dim vector
- $f : S \subset \mathbb{R}^d \to \mathbb{R}^d$, $d = 2$ or $3$
- Critical point $x$: $f(x) = 0$

[Yu, Wang, Grout, Chen, Ma 2010] [Maltrud, Bryan, Peacock, 2010] [Levine, Jadhav, Bhatia, Pascucci, Bremer, 2012]

# Rethink VF analysis and visualization




Simplifying 2D VF: independent of topological skeleton [1]

First 3D VF simplification based on critical point cancellation [2]

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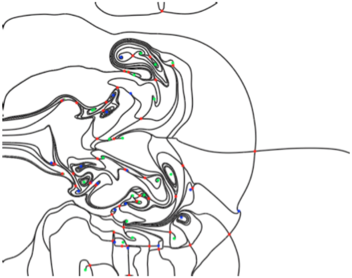
[1] Skraba, Wang, Chen and Rosen. PVis Best Paper, 2014

[2] Skraba, Rosen, Wang, Chen, Bhatia and Pascucci. PVis Best paper, 2016

# VF simplification

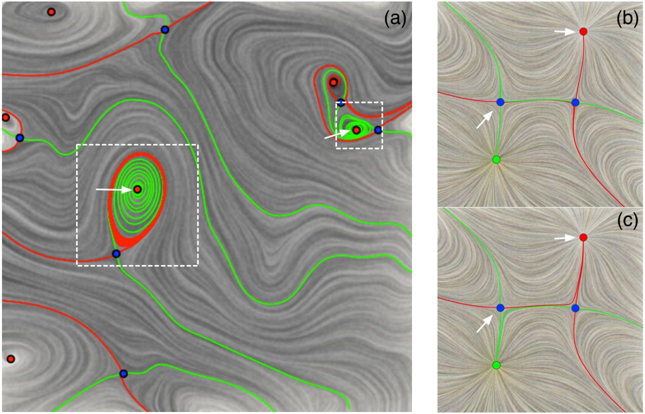
- Prior work: canceling nearby critical points based on topological skeleton: critical points connected by separatrices that divide domain into regions of uniform flow behavior
- Preserve important scientific properties of the data
- Obtain compact representation for interpretation
- Derive multi-scale view of the flow dynamics



Swirling jet simulation [Tricoche, Scheuermann, Hagen 2001]

# Challenges with prior work

Topological skeleton can be unstable due to numerical instability



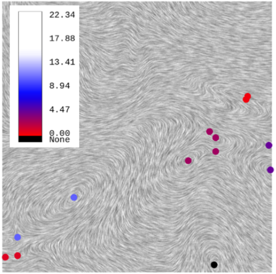

(a) Highly rotational flow, near Hopf bifurcations: diff separatrices intersect/switch.

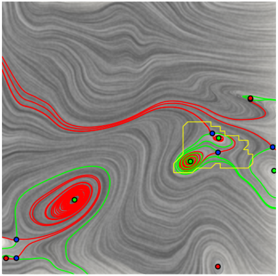
(b-c) Separatrices are unstable w.r.t perturbations.

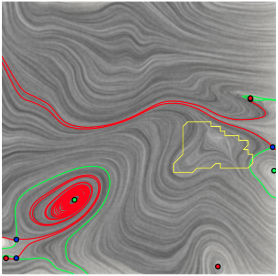
Sink, saddle-sink, saddle, source, saddle-source

# Tutorial Highlight: Robustness-based simplification

- Canceling critical points based on stability measured by robustness
- Complementary view, independent of topological skeleton
- Efficient computation for large data, avoid numerical integration
- Handle complex boundary configurations
- Analysis generalizes to higher dimensions

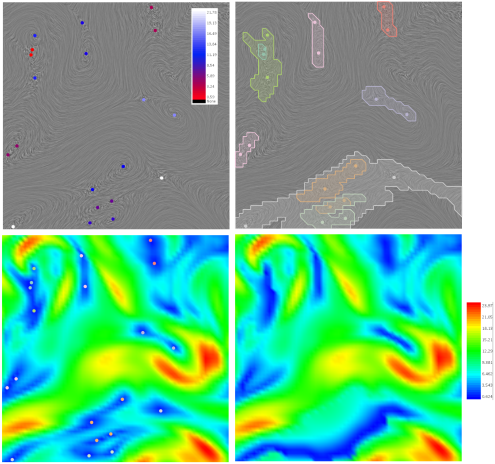


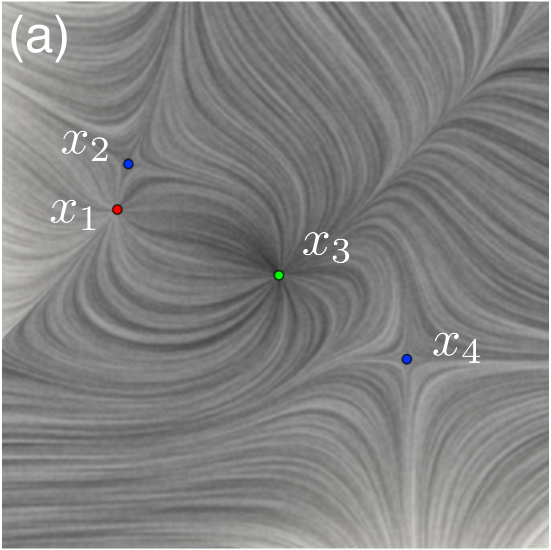


# Robustness-based simplification in a nutshell

- In the space of all VFs, find the one closest to the original VF with a particular set of critical points removed, bases on the $L_\infty$ norm
- Results are optimal: no other simplification with a smaller perturbation



# Some teaser results: synthetic A



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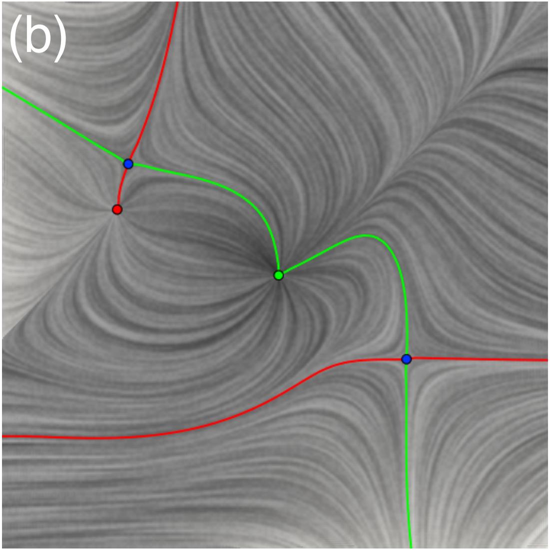

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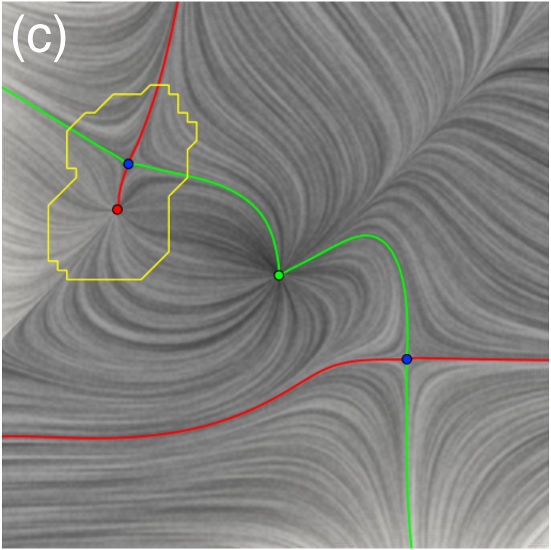

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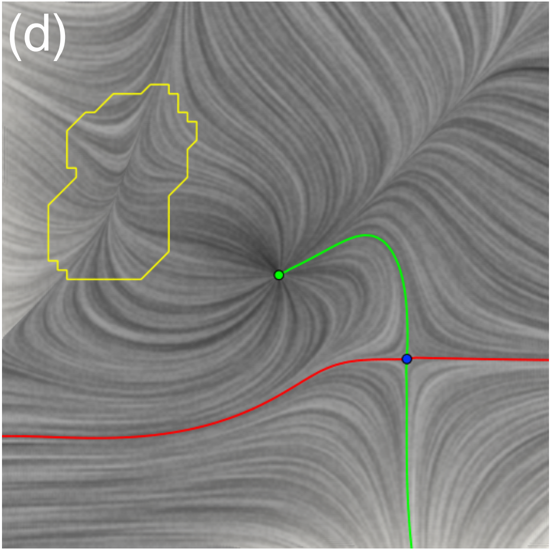

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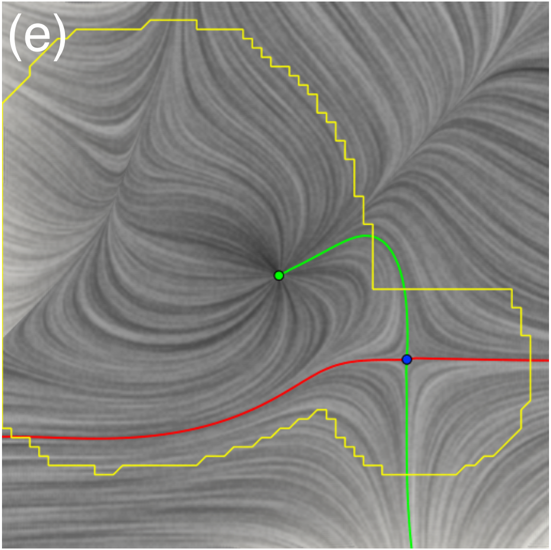

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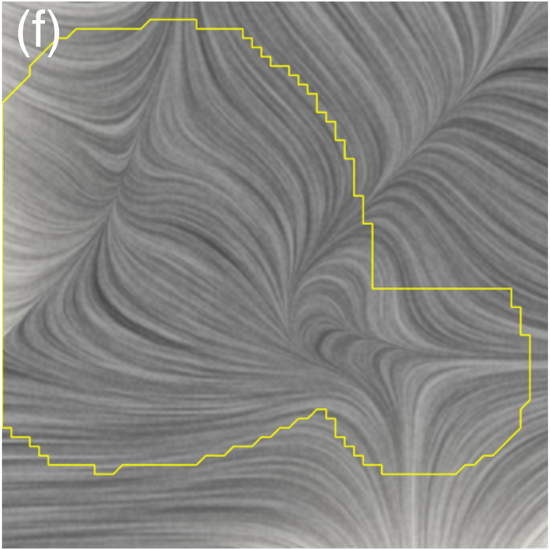

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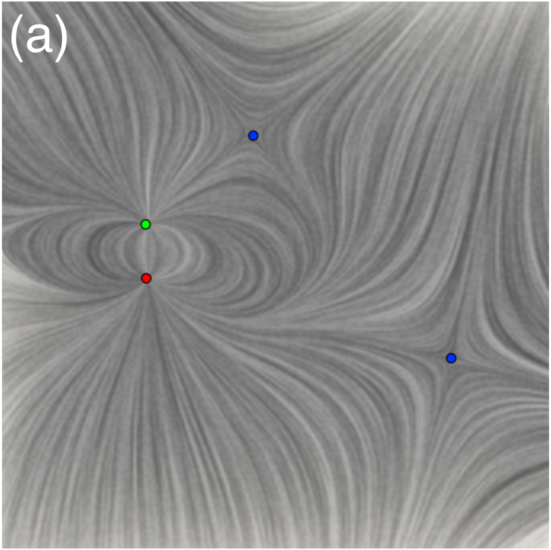

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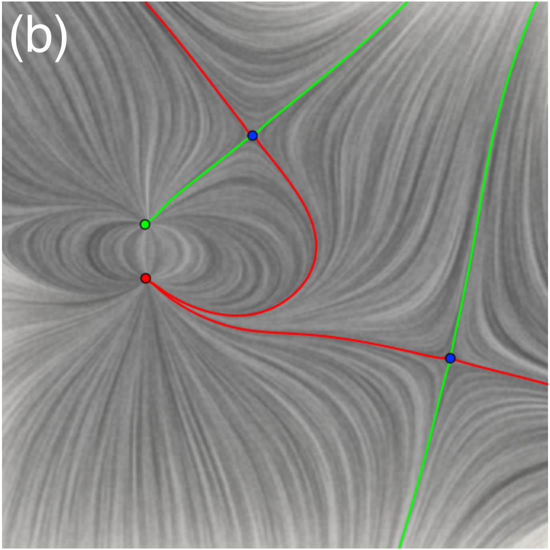

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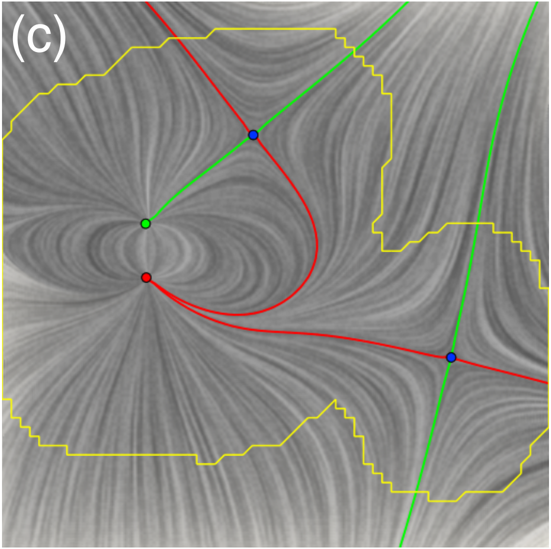

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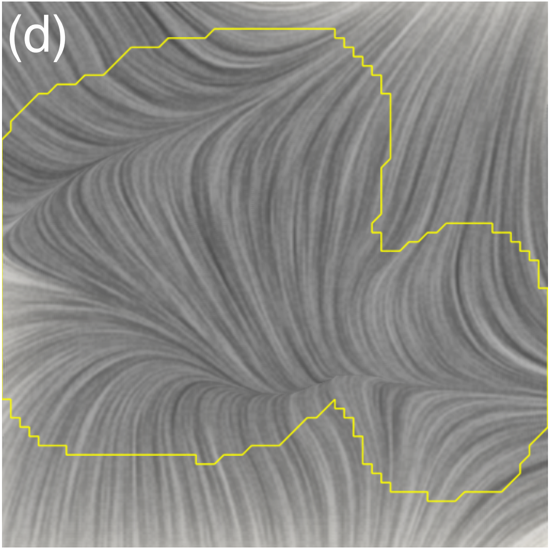
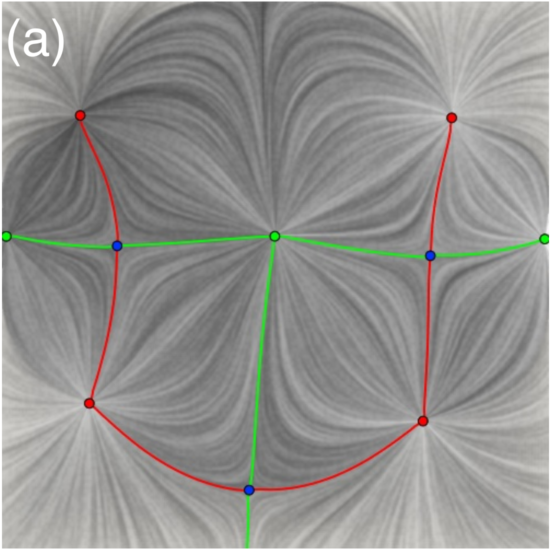


# Some teaser results: synthetic C



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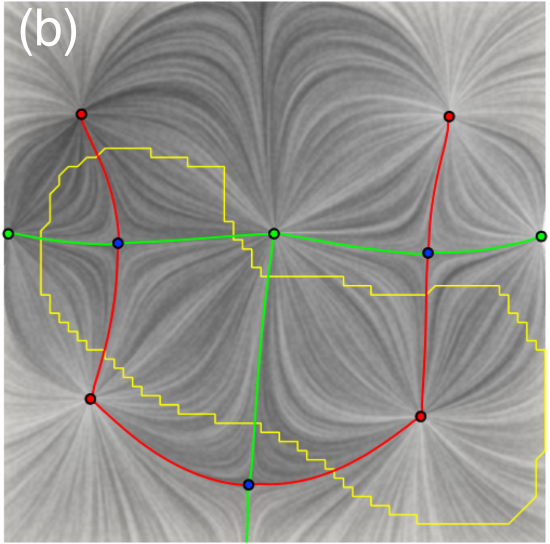

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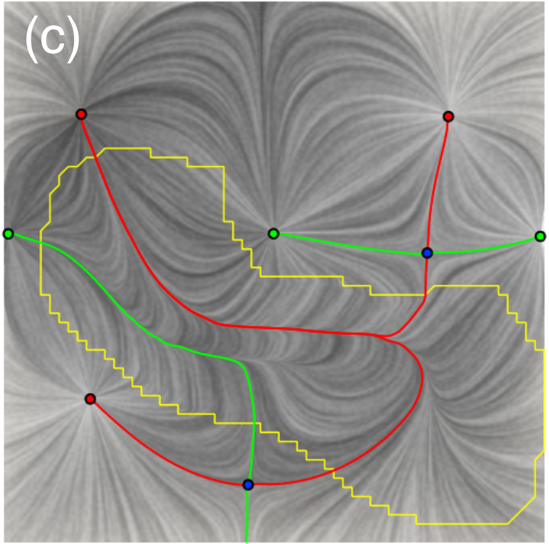
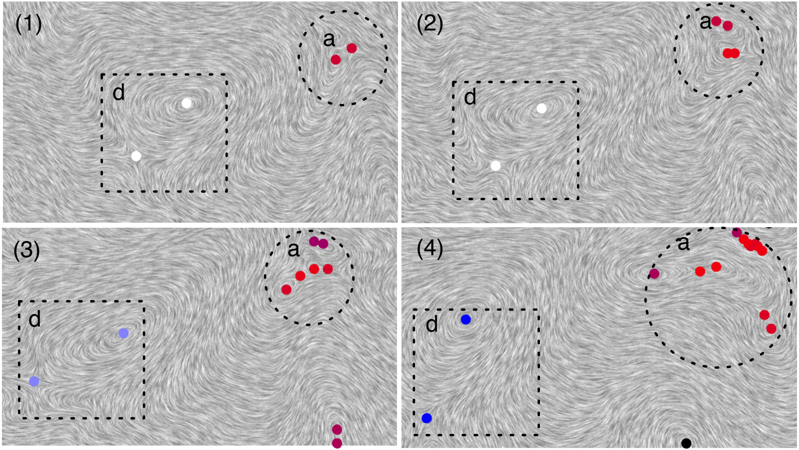


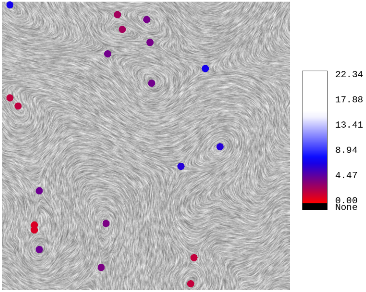
# Visualizing Robustness of Critical Points




Critical points clustered by robustness for time-varying ocean eddie simulation
[Wang, Rosen, Skraba, Bhatia and Pascucci (EuroVis) 2013]

# Robustness of critical points

- Robustness: quantify the stability of critical points
- Intuitively, the robustness of a critical point is the minimum amount of perturbation necessary to cancel it within a local neighborhood
- Well group theory [Chazal, Patel and Skraba 2011, 2012]
- Robustness computation: based on degree theory and merge tree

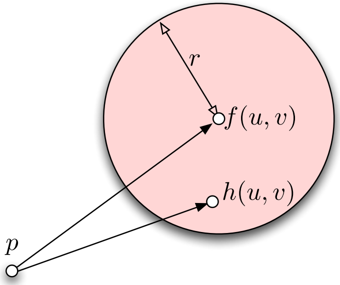


# $r$-perturbation: $L_\infty$-norm of the VF

Let $f, h : \mathbb{R}^2 \to \mathbb{R}^2$ be two continuous 2D vector fields. Define the distance between the two mapping as

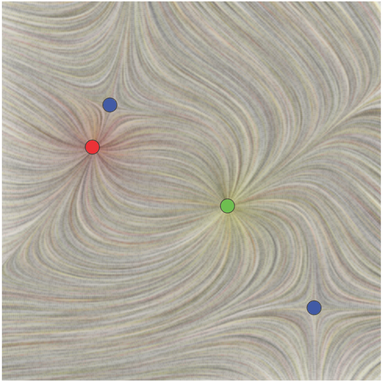
$$\mathrm{d}(f, h) = \sup_{x \in \mathbb{R}^2} ||f(x) - h(x)||_2.$$

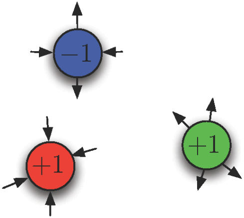
We say $h$ is an $r$-perturbation of $f$, if $\mathrm{d}(f, h) \leq r$.



# Degrees

- In 2D, $\deg(x)$ of a critical point $x$ equals its Poincaré index.
- Source $+1$, sink $+1$, saddle $-1$.
- A connected component $C$, $\deg(C) = \sum_i \deg(x_i)$.
- Corollary of Poincaré-Hopf thm: if $C$ in $\mathbb{R}^2$ has degree zero, then it is possible to replace the VF inside C with a VF free of critical points





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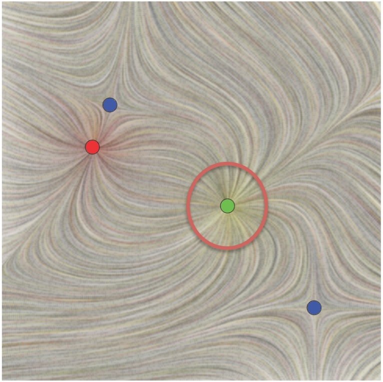

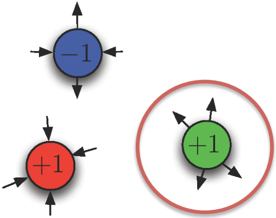

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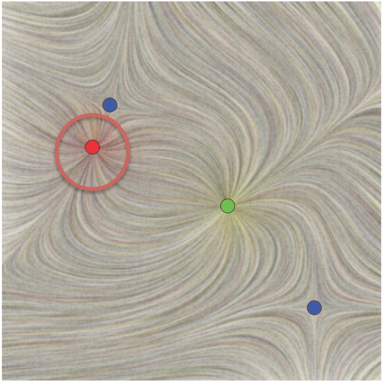

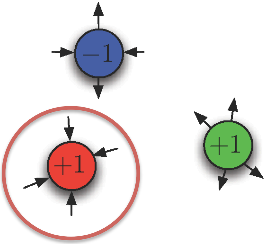

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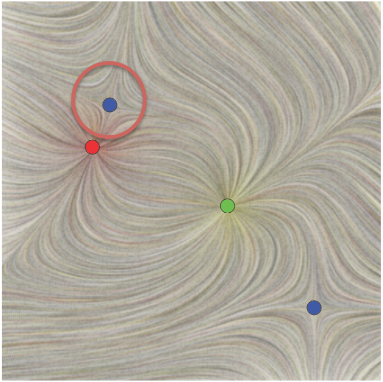

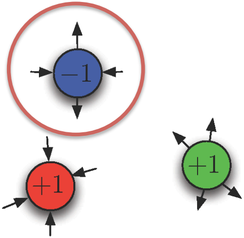

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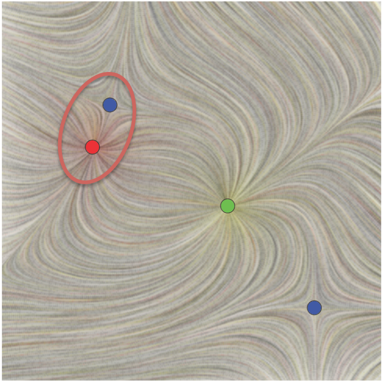
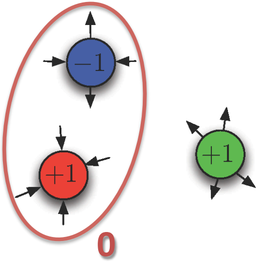




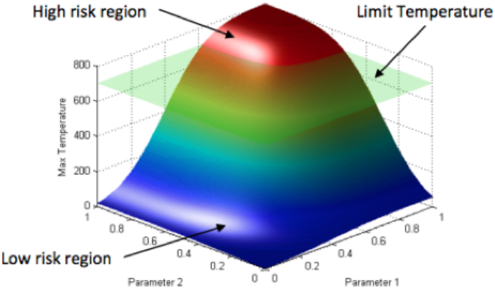
# Sublevel set

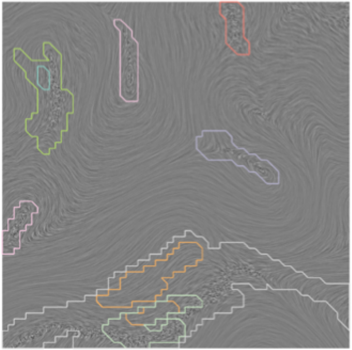
Given $f : \mathbb{R}^2 \to \mathbb{R}^2$, define its norm (speed of flow) $f_0 : \mathbb{R}^2 \to \mathbb{R}$ as

$$f_0(x) = ||f(x)||_2$$

For some $r \geq 0$, define the sublevel set of $f_0$ as

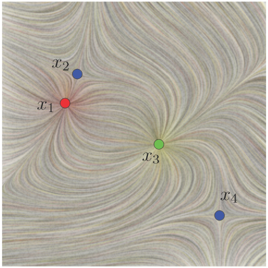
$$\mathbb{F}_r = f_0^{-1}[0, r].$$

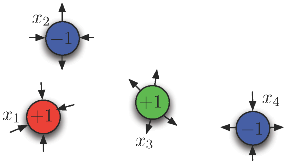


# Merge tree of $f_0$

Track components of $\mathbb{F}_r$ as they appear and merge, as $r$ increases from $0$



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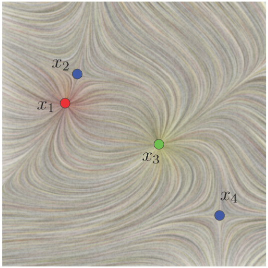


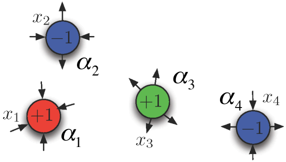


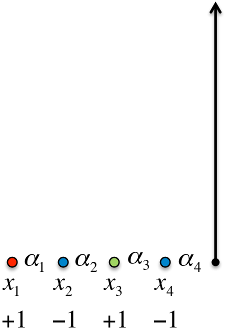

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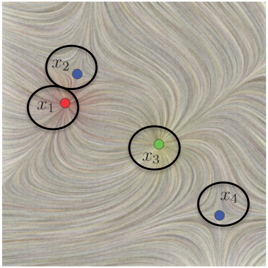


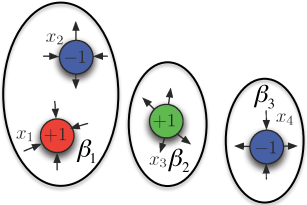


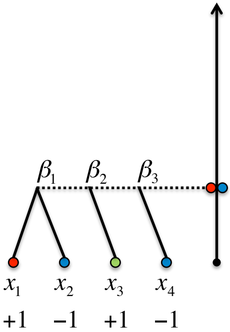

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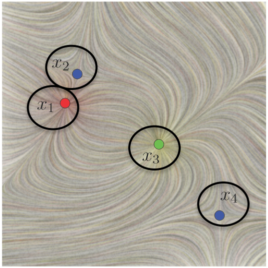


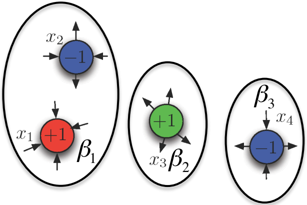


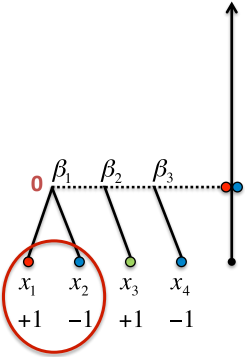

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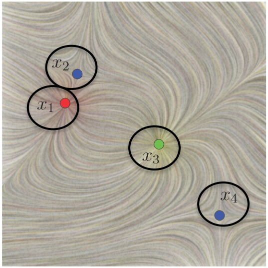


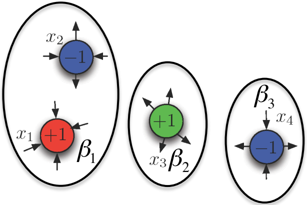


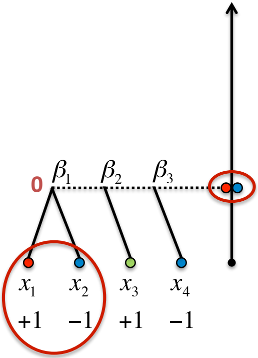

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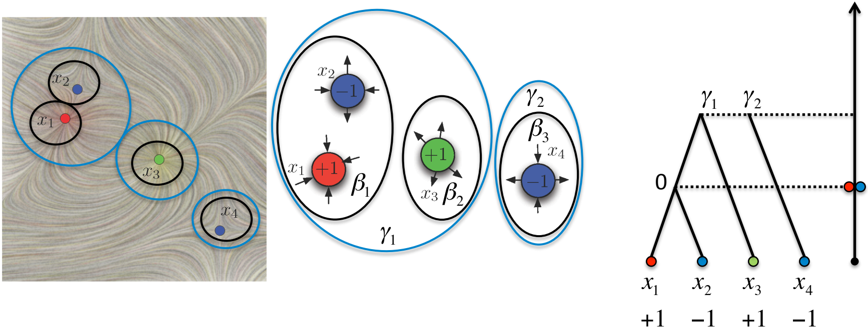

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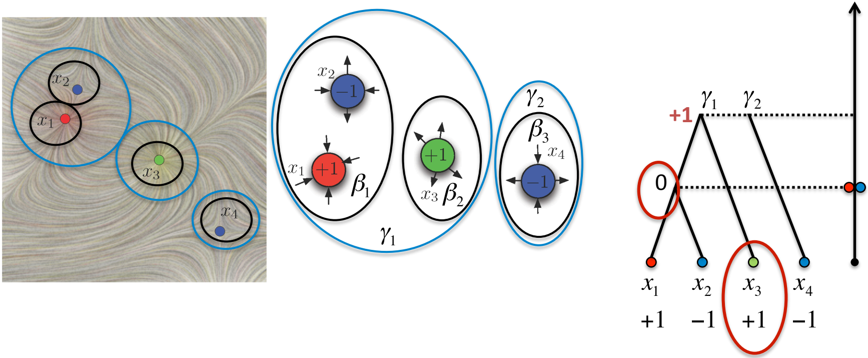

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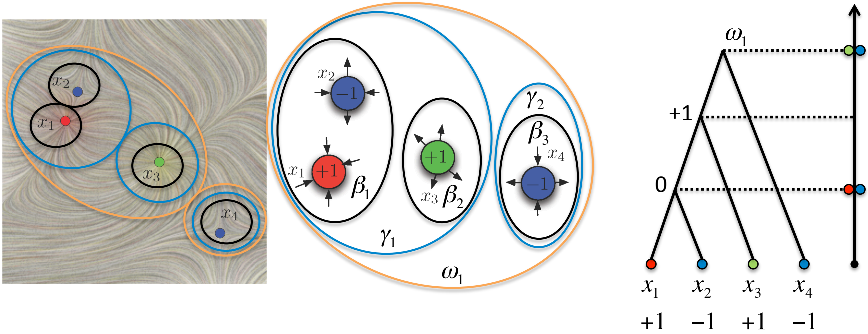

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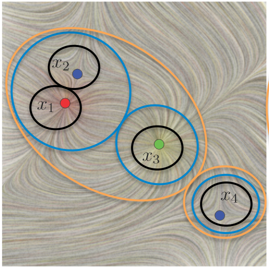
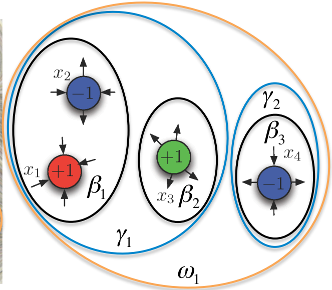
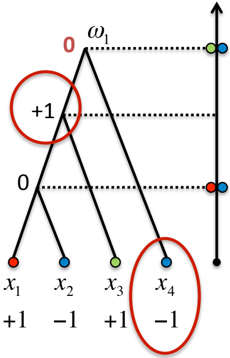
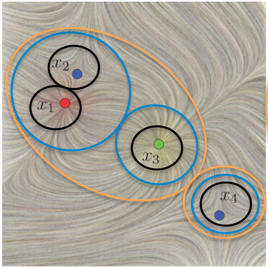


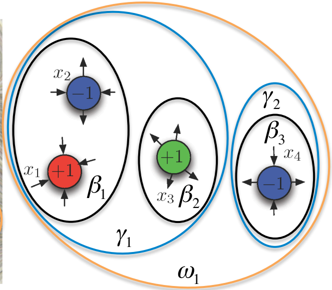



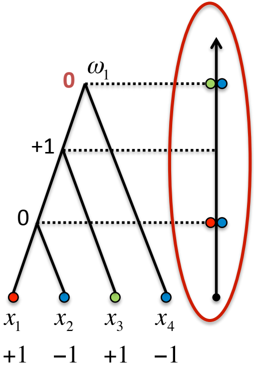



# Merge tree of $f_0$

Track components of $\mathbb{F}_r$ as they appear and merge, as $r$ increases from $0$

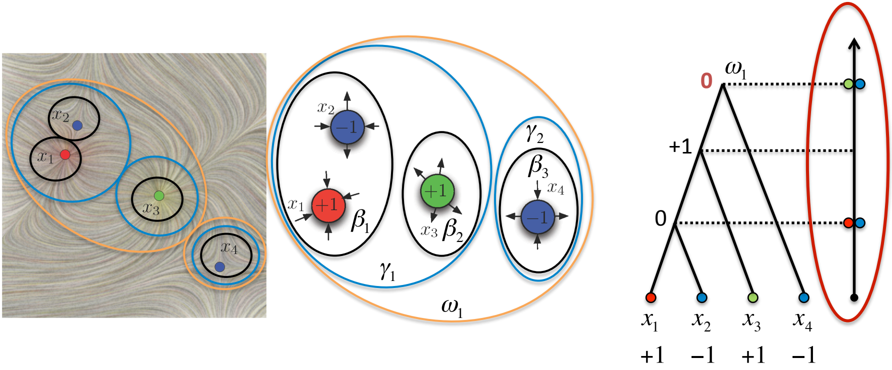


# Merge tree of $f_0$ and robustness

The robustness of a critical point is the height of its lowest degree zero ancestor in the merge tree. [Chazal, Patel, Skraba 2012]
Interpretation: robustness is the min amount of perturbation necessary to cancel a critical point.




Robustness: $\mathrm{rb}(x_1) = \mathrm{rb}(x_2)$, $\mathrm{rb}(x_3) = \mathrm{rb}(x_4)$.

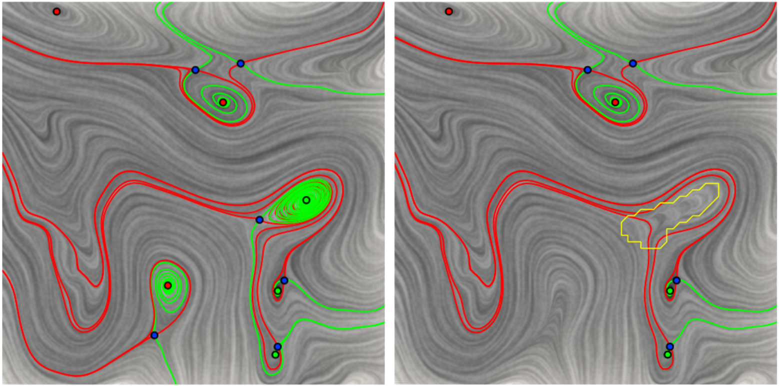
# Robustness properties

Robustness quantifies the stability of a critical point w.r.t. perturbations of the VFs.

If a critical point $x$ has a robustness $r$:

- Need $(r + \delta)$-perturbation to cancel $x$, for arbitrarily small $\delta > 0$
- Any $(r - \delta)$-perturbation is not enough to cancel $x$.

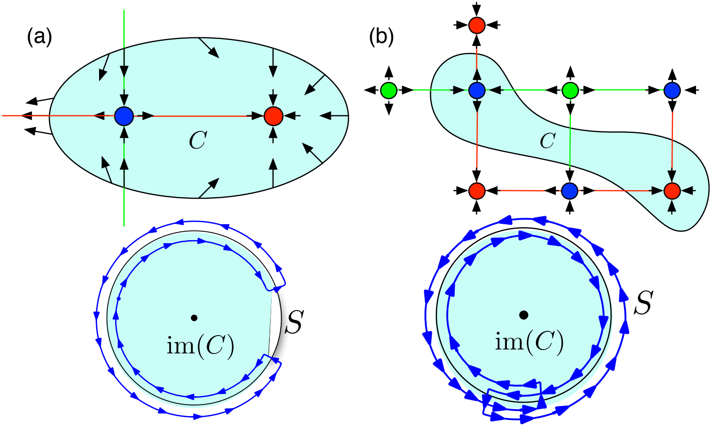
# 2D VF Simplification Based on Robustness



[Skraba, Wang, Chen and Rosen (PVis Best Paper) 2014] [Skraba, Wang, Chen and Rosen (TVCG) 2015]

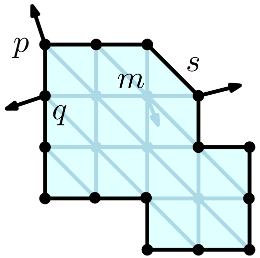
# Image space $\mathrm{im}\,(C)$ of a zero-degree component $C \subseteq \mathbb{F}_r$

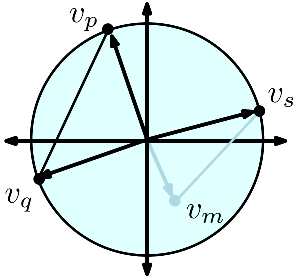
- Map each vector in $C$ to its vector coordinates
- Critical points map to the origin of $\mathrm{im}\,(C)$
- $\mathrm{im}\,(C)$ is part of a disk of radius r, whose boundary $S$ could be uncovered/covered.



# PL Image space

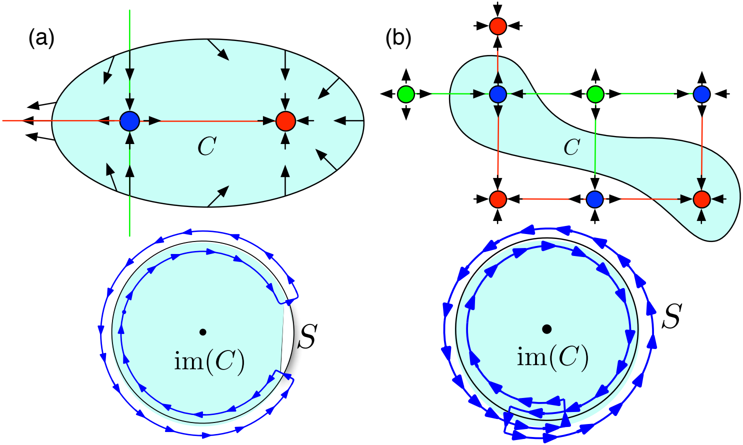
$f : K \to \mathbb{R}^2$, $K$ is a triangulation of $C$
Linear interpolation: edges and triangles in $K$ map to thoese in $\mathrm{im}\,(C)$.



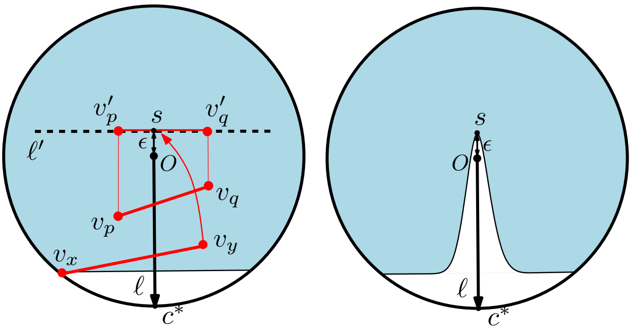



# Simplification: Key ideas

- A region contains critical points if its image space contains the origin
- Simplification: deform the VF to create a void surrounding the origin
- Simple boundary: boundary of $\mathrm{im}\,(C)$ is uncovered
- Complex boundary: boundary of $\mathrm{im}\,(C)$ is covered

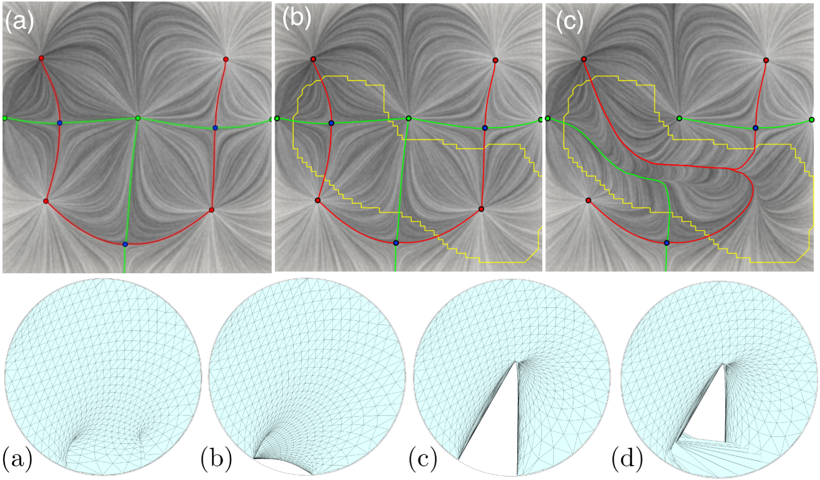


Deform $\mathrm{im}\,(C)$ to create a void surrounding the origin.
$c^*$: cut point



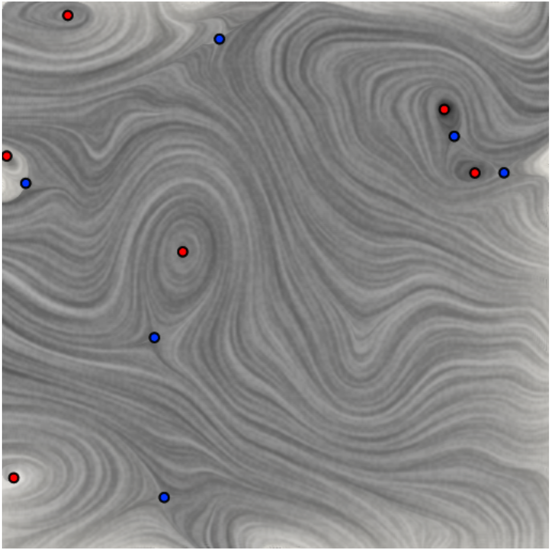

By construction: amount of perturbation $< r + \epsilon$

# Example revisited: Synthetic C complex boundary

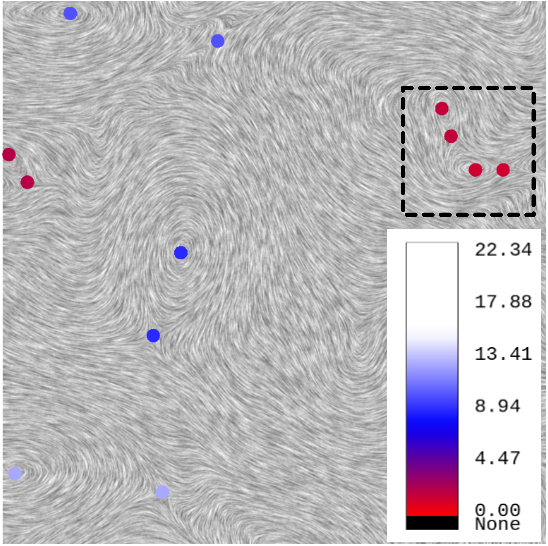



(a) original, (b) after **Unwrap**, (c) after **Cut** and (d) final output after **Restore**

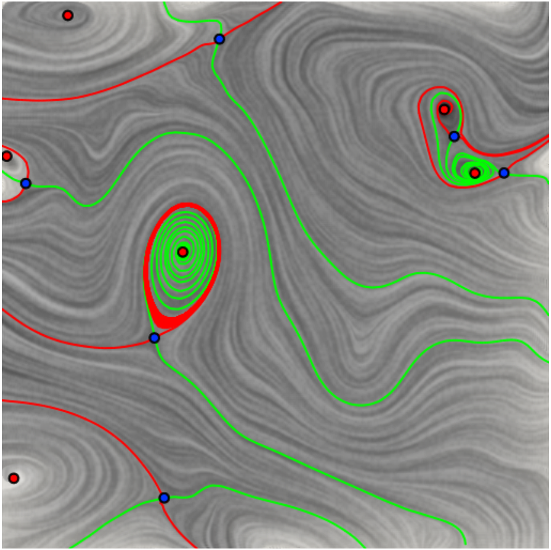
# Ocean eddie simulation



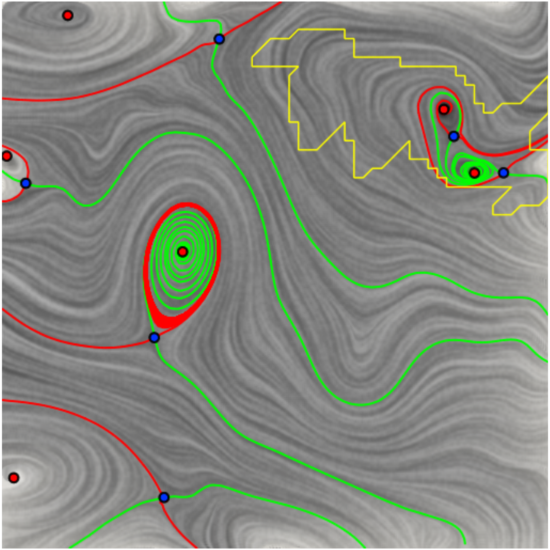
# Ocean eddie simulation



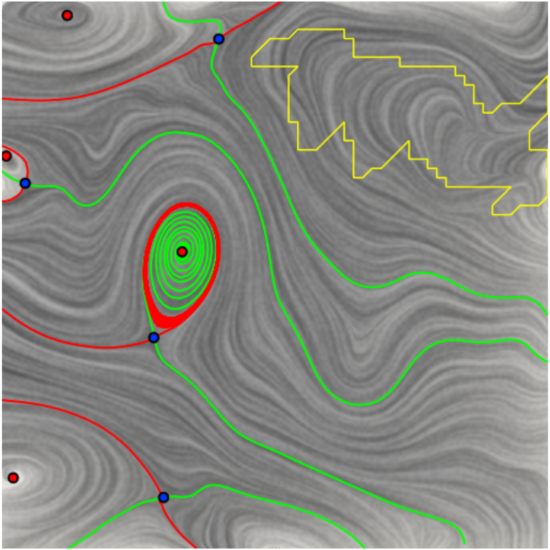
# Ocean eddie simulation



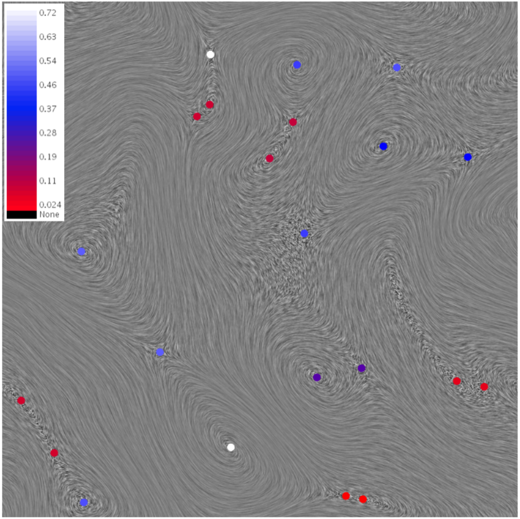
# Ocean eddie simulation



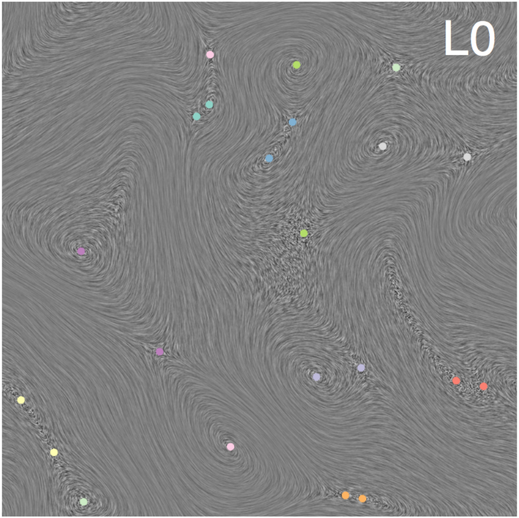
# Ocean eddie simulation



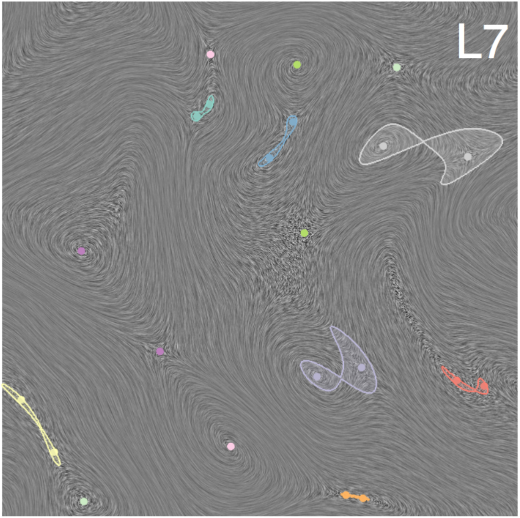
# Combustion simulation: Hierarchical simplification



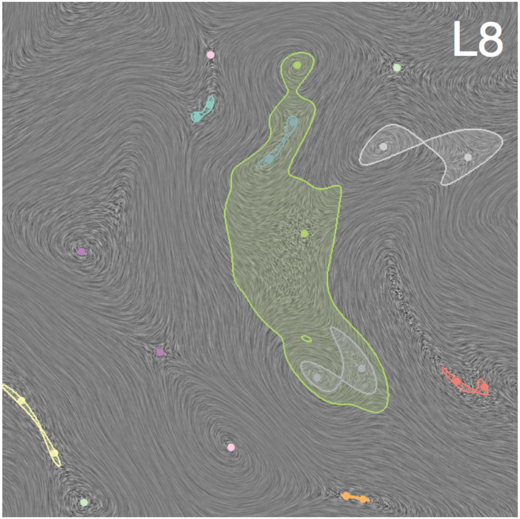
# Combustion simulation: Hierarchical simplification



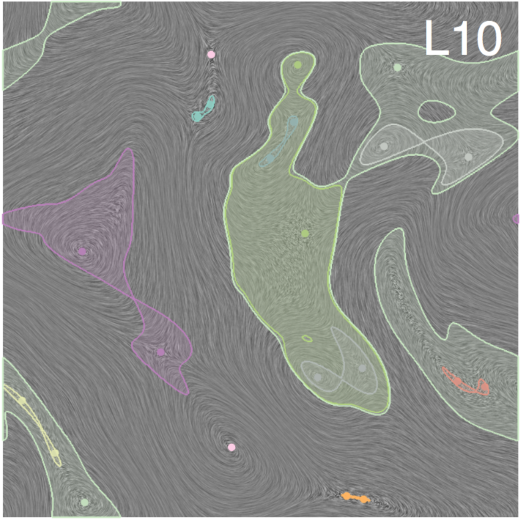
# Combustion simulation: Hierarchical simplification



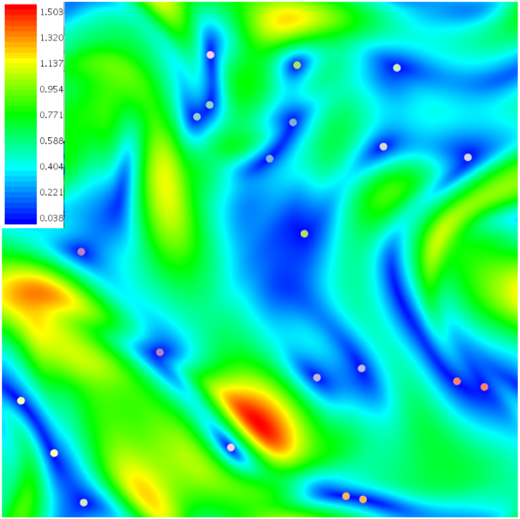
# Combustion simulation: Hierarchical simplification



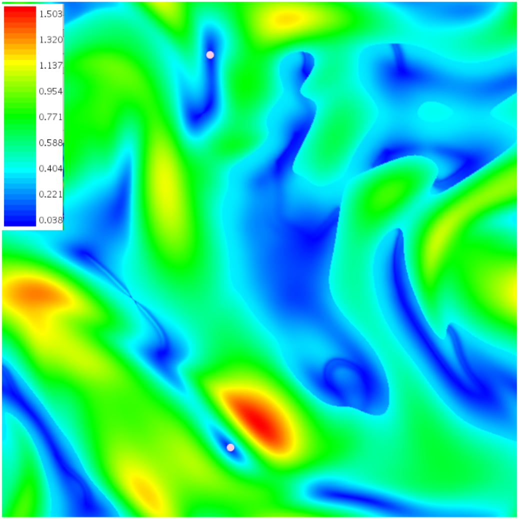
# Combustion simulation: Hierarchical simplification



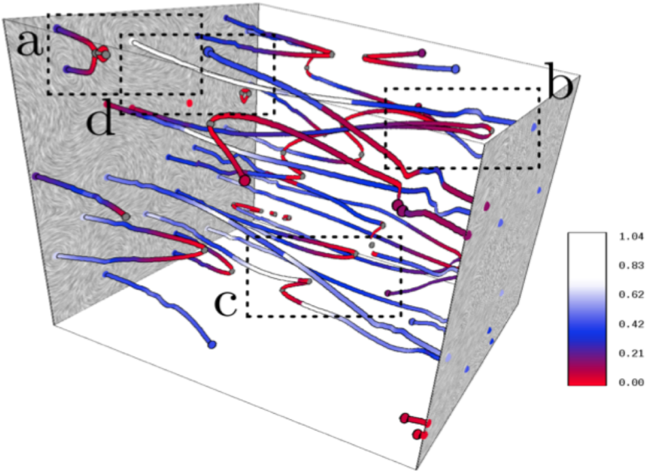
# Combustion simulation: Hierarchical simplification



# Combustion simulation: Hierarchical simplification

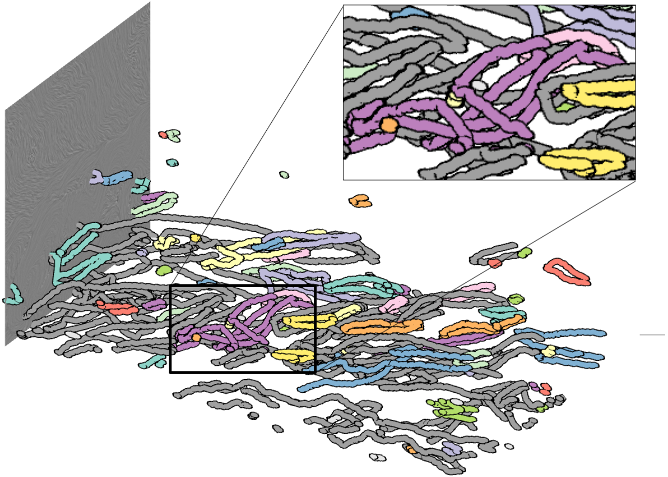


# Feature Tracking for 2D Time-Varying VF



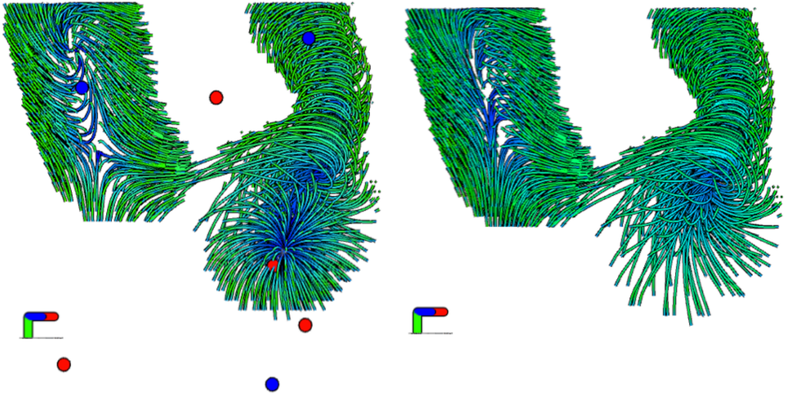

Stable critical points could provably be tracked more easily and more accurately in the time-varying setting.
[Skraba, Wang (TopoInVis/Book Chapter), 2014]

# Simplifying 2D Time-Varying VF



[Skraba, Wang, Chen, Rosen (TVCG), 2015]

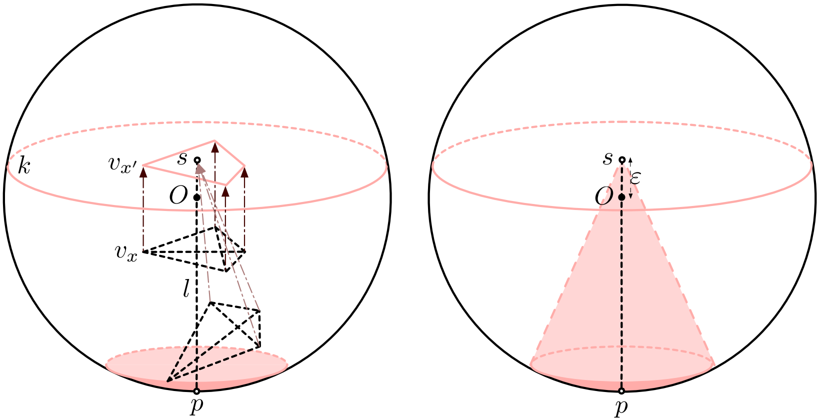
# Simplifying 3D VF



[Skraba, Rosen, Wang, Chen, Bhatia, Pascucci (PacificVis/TVCG), 2016]

# Cut: Finding the cut point

Randomly "stabbing" to find the uncovered area



# Unwrap: Challenging

- Iterative smoothing procedure along a sphere
- Peeling back ("un-wrinkle") the boundary until a point is uncovered
- Intuition (Hopf degree theorem): any map $f : \mathbb{S}^n \to \mathbb{S}^n$ is homotopic to a constant map if and only if it has degree 0
- Guarantee to converge

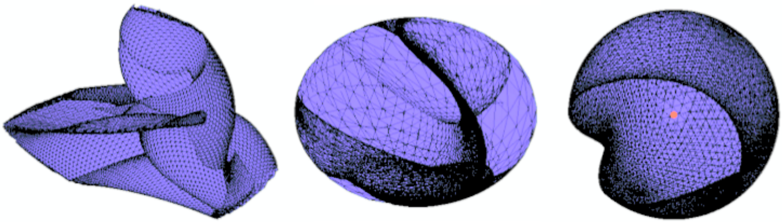


Image space: original, normalized and after unwrapping

# Future directions: VF analysis and visualization

- Define and quantify stability of vortices and vortex cores
- Robustness in fluid dynamics
- Uncertainty across VF ensembles
- Robustness for tensor fields!!! (With Ingrid Hotz)

# References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- Interpreting Feature Tracking Through the Lens of Robustness. Primoz Skraba and Bei Wang. Topological Methods in Data Analysis and Visualization III: Theory, Algorithms, and Applications, pages 19-38, 2014.
- Visualizing Robustness of Critical Points for 2D Time-Varying Vector Fields. Bei Wang, Paul Rosen, Primoz Skraba, Harsh Bhatia and Valerio Pascucci. Eurographics Conference on Visualization (EuroVis) 2013. Computer Graphics Forum (CGF), 32(2), pages 221-230, 2013.

# Acknowledgement


- NSF-IIS-1513616, INL
- Primoz Skraba (Jozef Stefan Institute, Slovenia), Guoning Chen (U Houston), Paul Rosen (U South Florida), Roxana Bujack (LANL), Harsh Bhatia (LLNL), Valerio Pascucci (U of U)

# Thank you!

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I am looking for PhD students in Visualization!