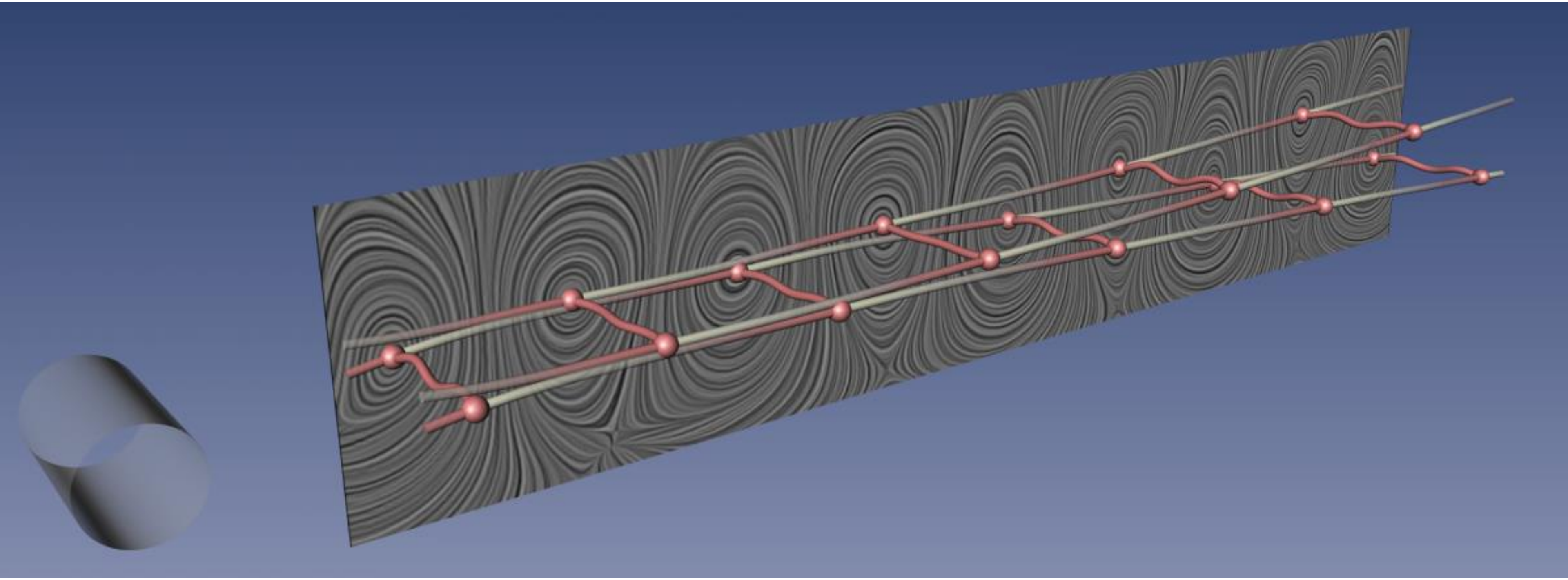
Spatio-temporal Flow Analysis



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Mario Rome

Björn Thuresson

VIC

Engineer

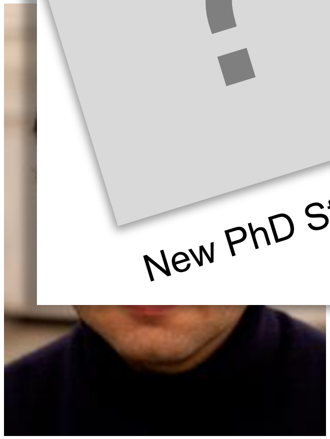
?

New PhD Student

HIRING!

?

New Post-Doc



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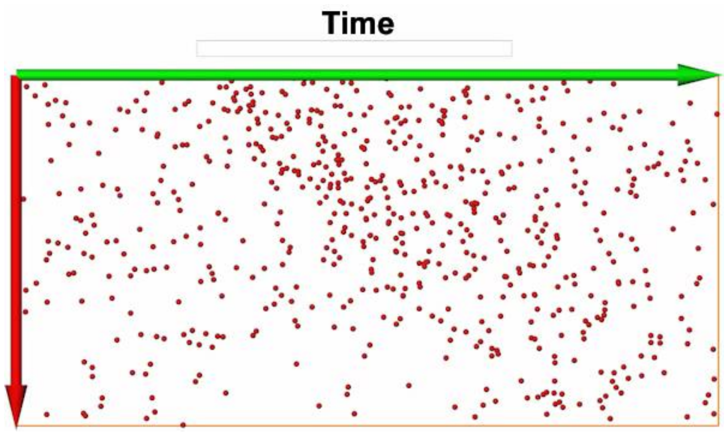
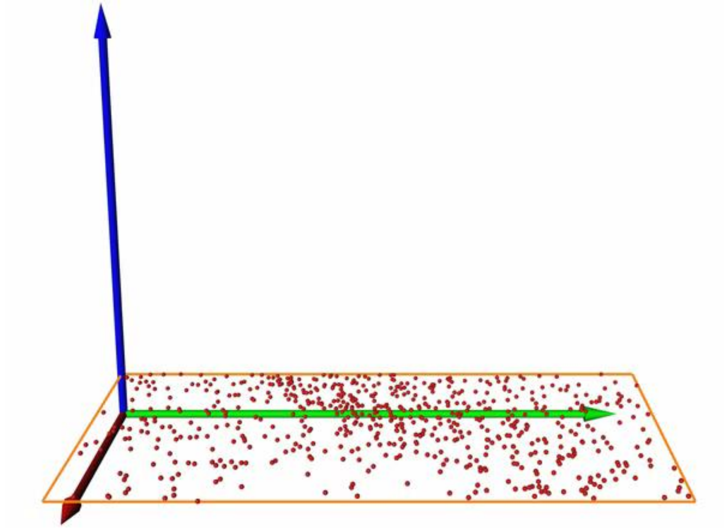
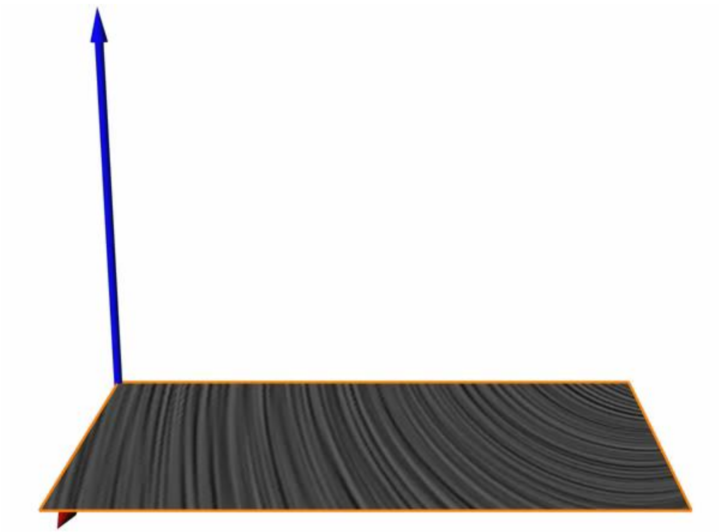
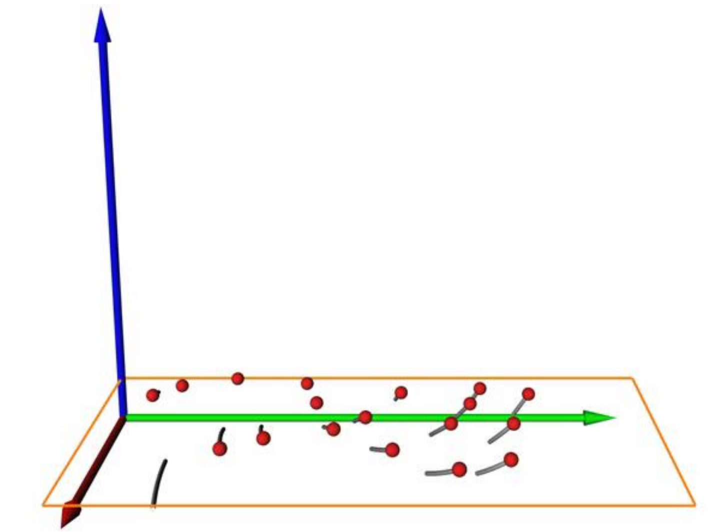




2D time-dependent vector field
particle visualization





stream lines

path lines

curve parallel to the vector field in each point for a **fixed time**

describes motion of a massless particle in an **steady** flow field

curve parallel to the vector field in each point **over time**

describes motion of a massless particle in an **unsteady** flow field

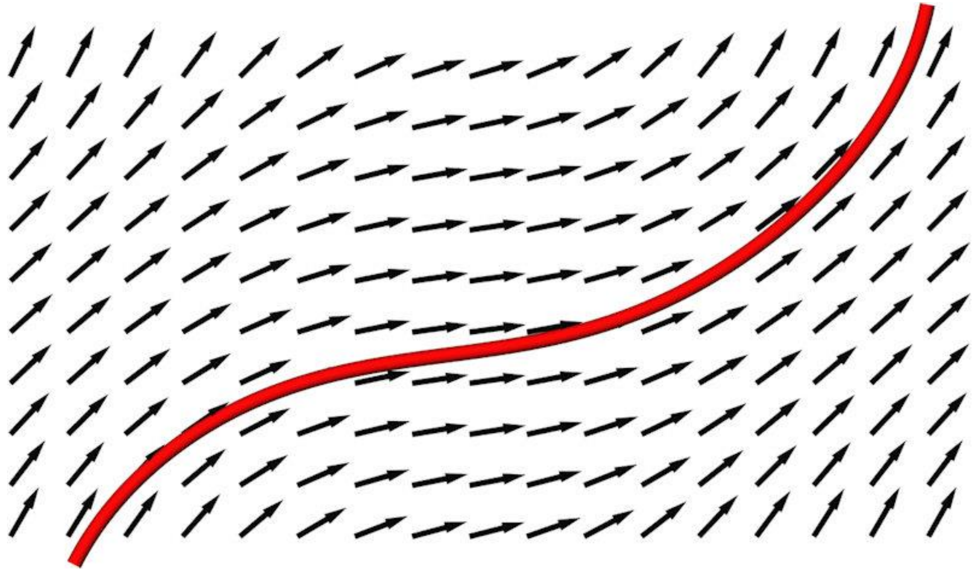
- A **tangent curve s***(t)* of the vector field **v** is a curve in $E^{2/3}$ with the property:

$$\dot{\mathbf{s}}(t) = \mathbf{v}(\mathbf{s}(t))$$

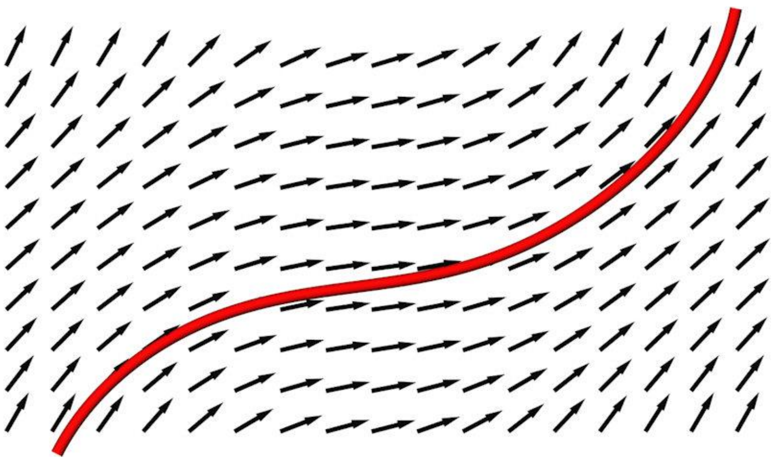
denotes the tangent vectors of **s***(t)*

for any *t* of the domain of **s**.

- Interpretation: path of a massless particle in a flow described by **v**



- **Properties of tangent curves:**
- Tangent curves do not intersect each other (except for critical points of **v**) .
- Given a point in the vector field **v**, there is one and only one tangent curve through it (except for critical points of **v**)
- A parametric description of stream lines is usually not possible ➔ numerical integration schemes (Runge Kutta)



Spatio-Temporal Flow Analysis, Tino Weinkauf, VIS 2016

- Based on tangent curves, we define 4 types of ***characteristic curves*** of a vector field

concept of tangent curves → concept of ***characteristic curves***

stream lines
path lines
streak lines
time lines

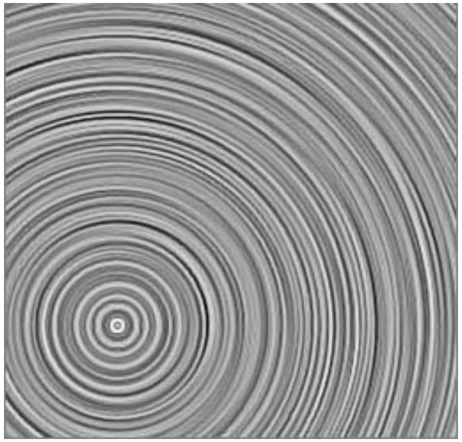
steady vector field

$$\mathbf{v}(x,y)$$

$$\frac{d}{d\tau}\mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau)) \quad \text{with } \mathbf{x}(0) = \mathbf{x}_0$$

stream lines

$$\mathbf{v}(x,y) = \begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix}$$



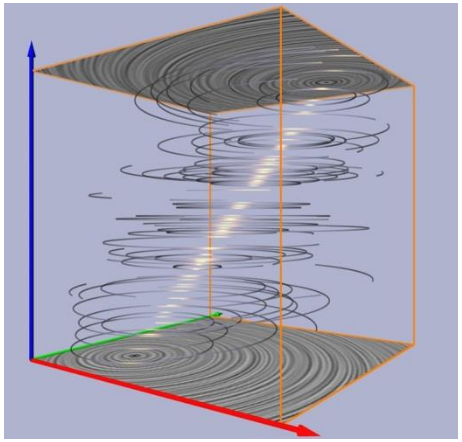
unsteady vector field

$$\mathbf{v}(x,y,t)$$

$$\frac{d}{d\tau}\mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau), t_0) \quad \text{with } \mathbf{x}(0) = \mathbf{x}_0$$

stream lines

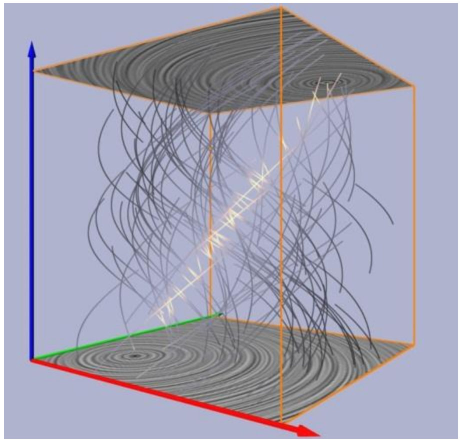
$$\mathbf{s}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \\ 0 \end{pmatrix}$$



$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{v}(\mathbf{x}(t), t) \quad \text{with } \mathbf{x}(t_0) = \mathbf{x}_0$$

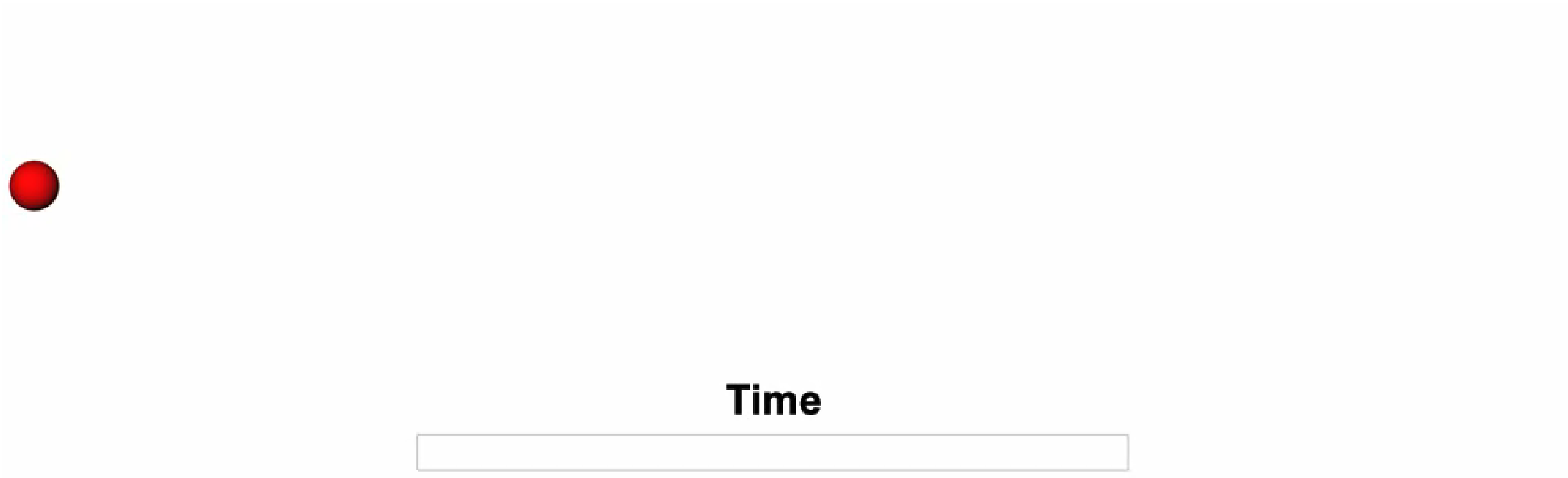
path lines

$$\mathbf{p}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \\ 1 \end{pmatrix}$$



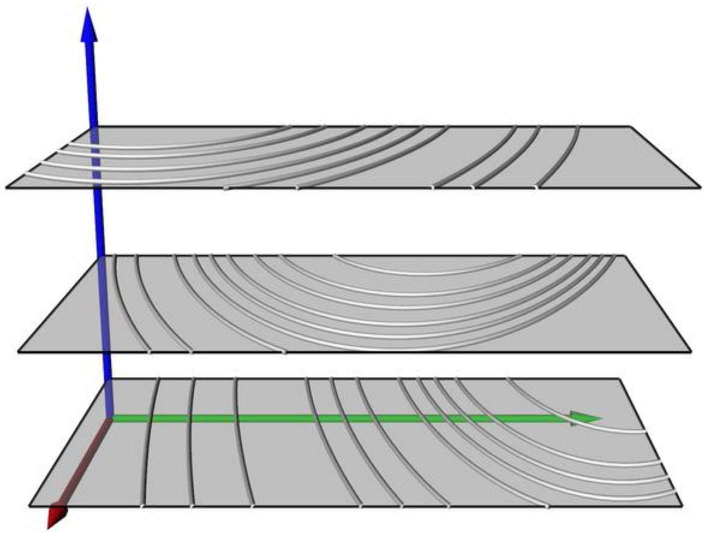
	steady vector field	unsteady vector field	
2D	$\mathbf{v}(x,y) = \begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix}$		
3D	$\mathbf{v}(x,y,z) = \begin{pmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{pmatrix}$	$\mathbf{s}(x,y,z,t) = \begin{pmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \\ 0 \end{pmatrix}$	$\mathbf{p}(x,y,z,t) = \begin{pmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \\ 1 \end{pmatrix}$

*n*D unsteady vector field
➔ *(n+1)*D steady vector field

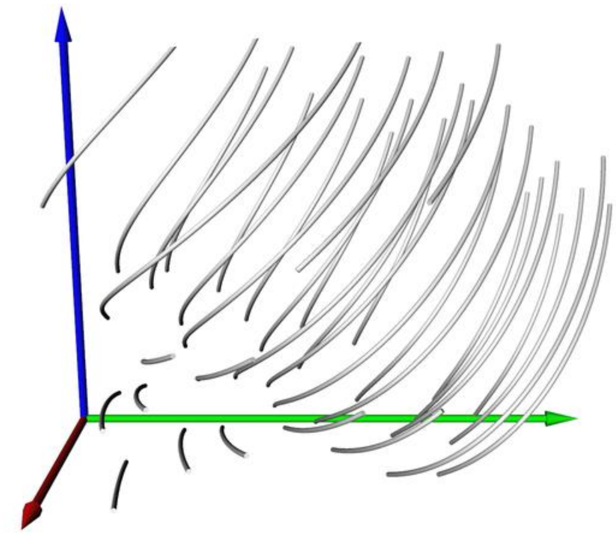


streak line

location of all particles set out at a fixed point at different times

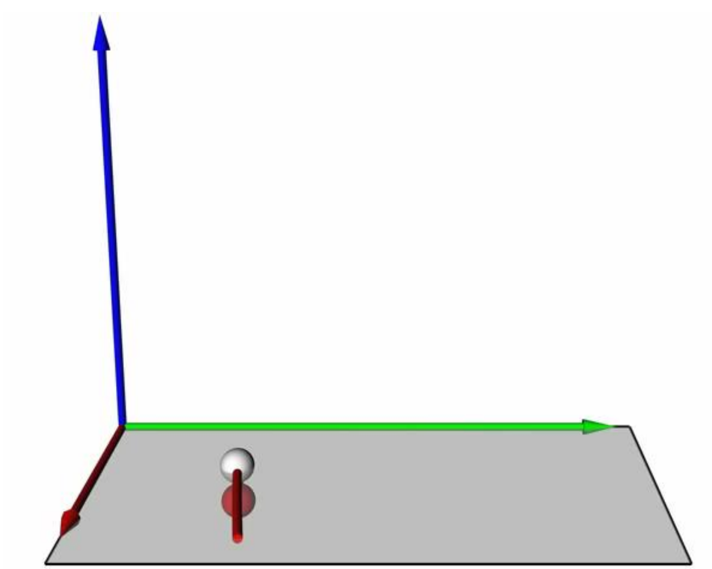


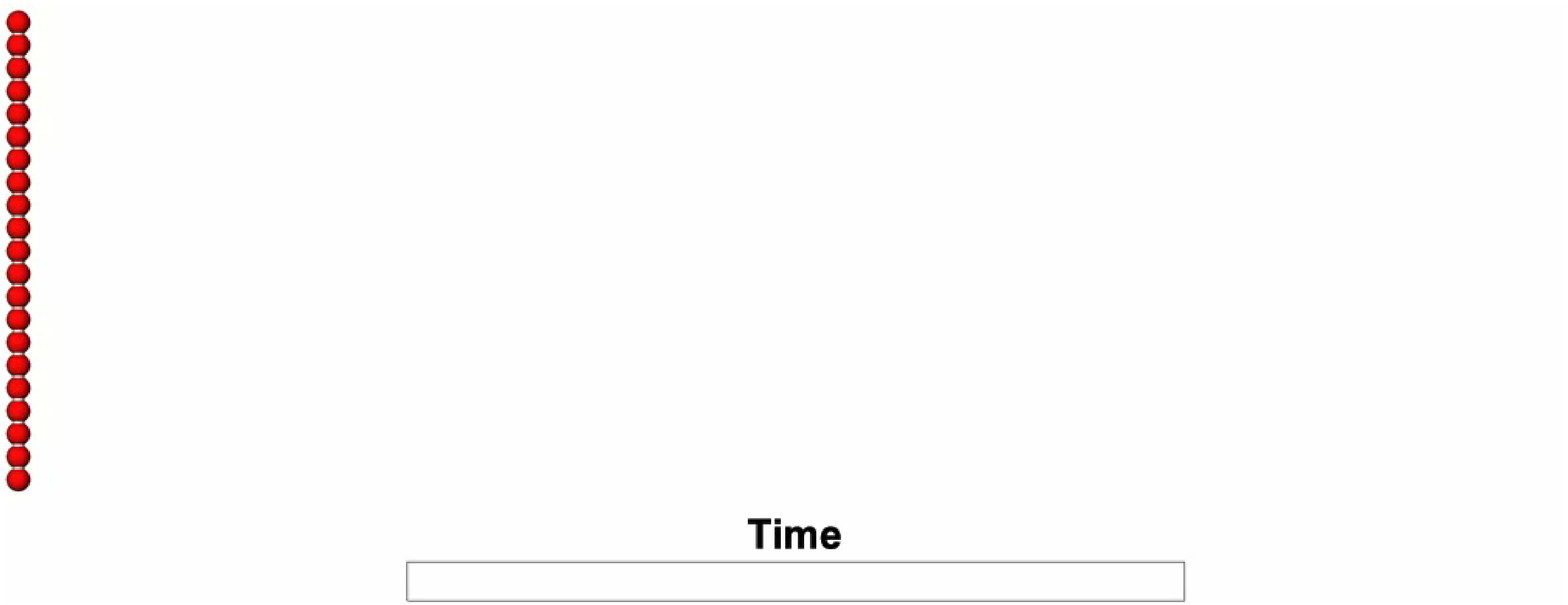
stream lines



path lines

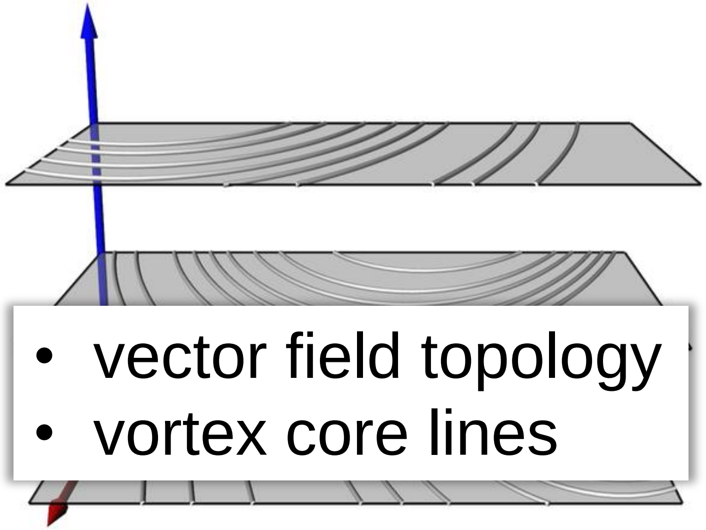
streak lines





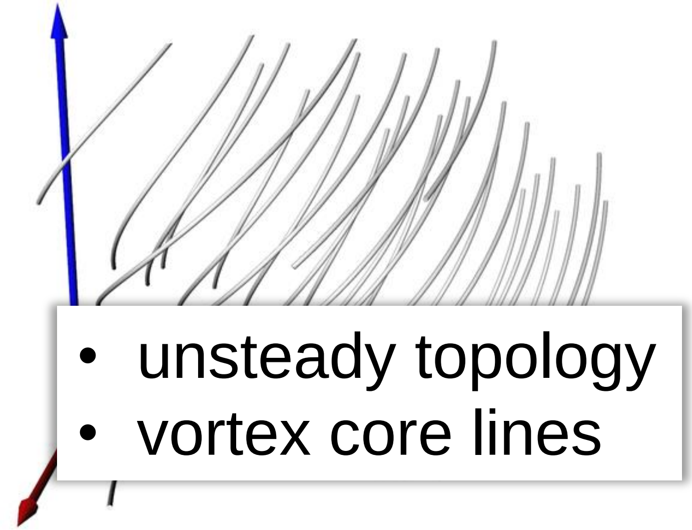
time line

location of all particles set out on a certain line at a fixed time

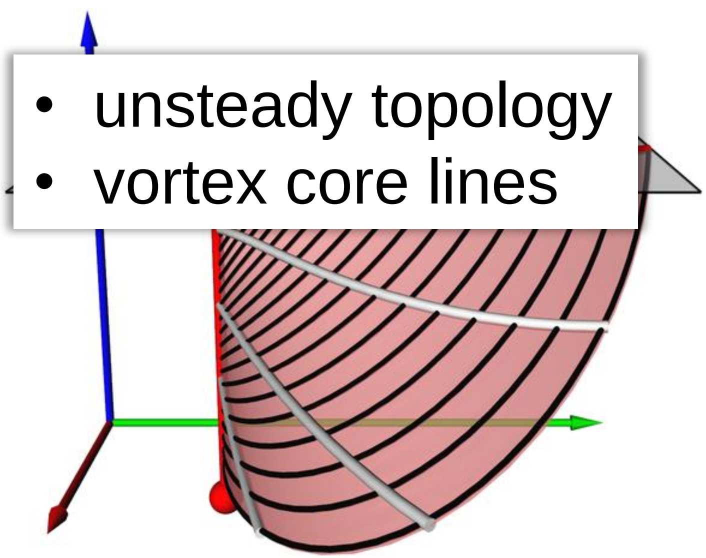


stream lines

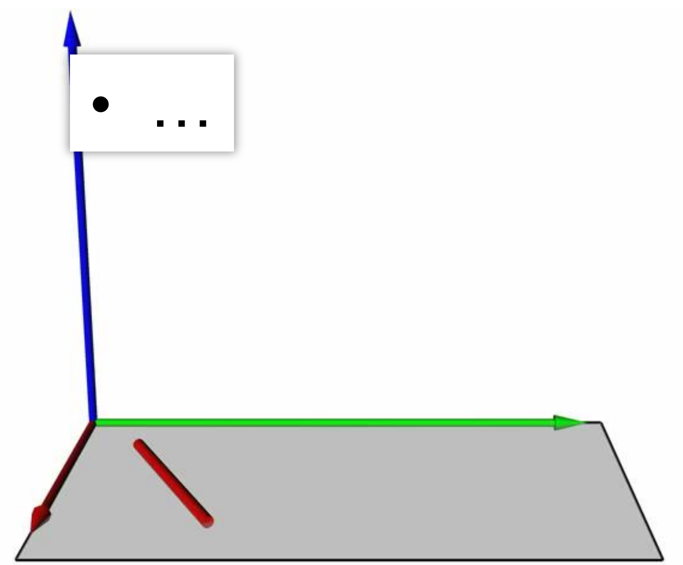
path lines



streak lines



time lines



tangent curves in a time-**independent** vector field $\mathbf{v}(\mathbf{x}):\ \frac{d}{d\tau}\mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau))$ with $\mathbf{x}(0) = \mathbf{x}_0$

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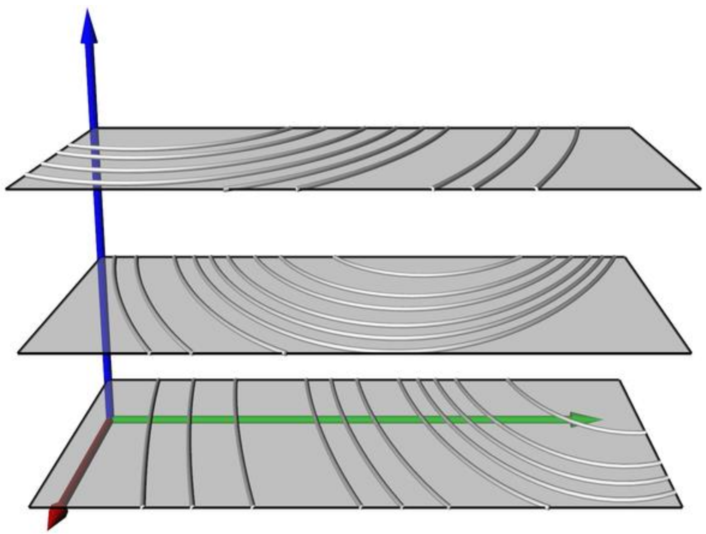
time-**dependent** vector field

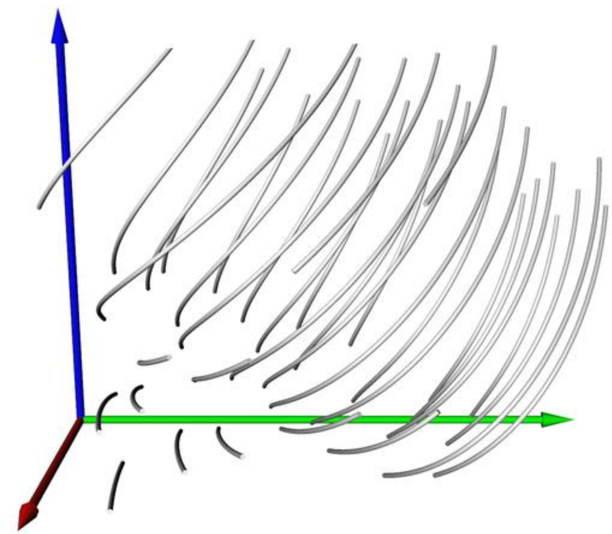
$$\mathbf{v}(\mathbf{x}, t)$$

stream lines



$$\bar{\mathbf{s}}(\mathbf{x}, t) = \begin{pmatrix} \mathbf{v}(\mathbf{x}, t) \\ 0 \end{pmatrix}$$

path lines



$$\bar{\mathbf{p}}(\mathbf{x}, t) = \begin{pmatrix} \mathbf{v}(\mathbf{x}, t) \\ 1 \end{pmatrix}$$

$$\bar{\mathbf{s}} = \begin{pmatrix} \mathbf{v} \\ 0 \end{pmatrix} \quad \longleftarrow \quad \text{tangent curves of a derived vector field} \quad \longrightarrow \quad \bar{\mathbf{p}} = \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$$

Feature extraction and analysis methods:

stream lines

path lines

$$\bar{\mathbf{s}} = \begin{pmatrix} \mathbf{v} \\ 0 \end{pmatrix}$$ stream lines $\longleftarrow$ tangent curves of a derived vector field $\longrightarrow$ path lines $$\bar{\mathbf{p}} = \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$$

Feature extraction and analysis methods:

Vector field topology
[Helman and Hesselink, IEEE Comp. 1989

Topology simplification
[Tricoche et al., Vis 2000 & 2001]

Feature Flow Fields
[Theisel and Seidel, VisSym 2003]

Topological vector field construction
[Weinkauf et al., EG 2004]

Critical point tracking 3D
[Garth et al., Vis 2004]

Curvature-based seeding
[Weinkauf and Theisel, WSCG 2002]

Path line attributes
[Shi et al., TopoInVis 2007]

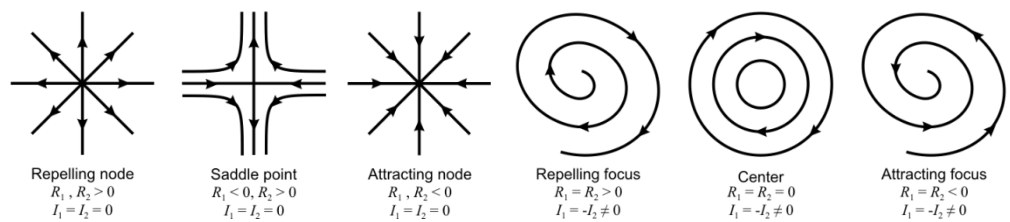
Unsteady PV criteria
[Fuchs et al., TVCG 2008]

Predictor-Corrector for SPH
[Schindler et al., Vis 2009]

Topological Methods

Critical point: $\mathbf{v} = \mathbf{0}$

Type of critical point: $\mathbf{J}(\mathbf{v})$



Helman & Hesselink, 1989

$\bar{\mathbf{s}} = \begin{pmatrix} \mathbf{v} \\ 0 \end{pmatrix}$ stream lines ⟵ tangent curves of a derived vector field ⟶ path lines $\bar{\mathbf{p}} = \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$

Feature extraction and analysis methods:

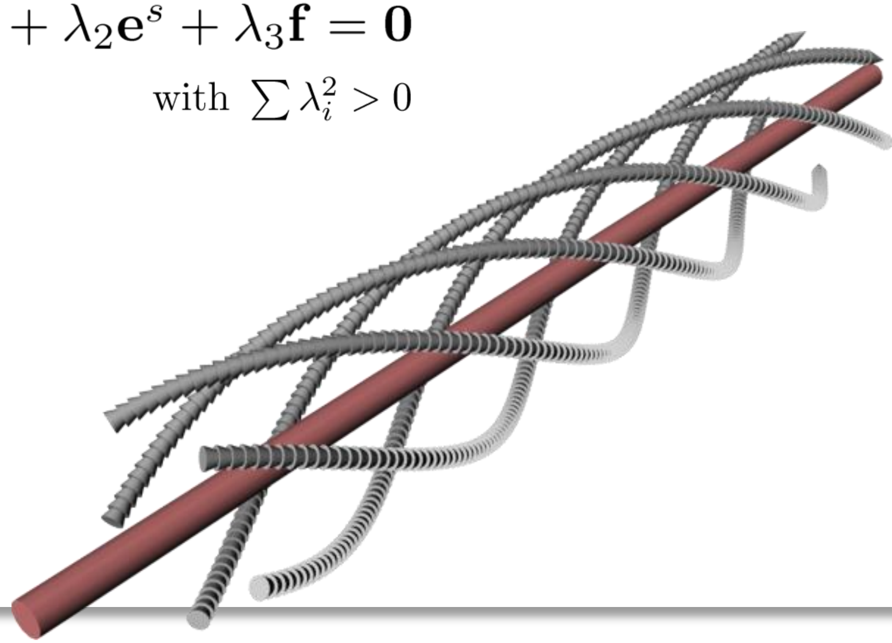
Vortex core lines

Center of swirling stream/path lines

$$\mathbf{v}(\mathbf{x}) \parallel \mathbf{e}(\mathbf{x})$$

$$\lambda_1 \bar{\mathbf{p}} + \lambda_2 \mathbf{e}^s + \lambda_3 \mathbf{f} = \mathbf{0}$$

$$\text{with } \sum \lambda_i^2 > 0$$



Eigenvector method
[Sujudi and Haimes, AIAA 1995]

Parallel Vectors operator
[Peikert and Roth, Vis 1999]

Tracking in scale space
[Bauer and Peikert, VisSym 2002]

Parallel Vectors meet FFF
[Theisel et al., Vis 2005]

Swirling Particle Cores
[Weinkauf et al., Vis 2007]

Unsteady PV criteria
[Fuchs et al., TVCG 2008]

Predictor-Corrector for SPH
[Schindler et al., Vis 2009]

Weinkauf and Theisel
Streak Lines as Tangent Curves of a Derived Vector Field
Best Paper at IEEE Visualization 2010

Weinkauf, Hege, Theisel
Advected Tangent Curves: A General Scheme for Characteristic Curves of Flow Fields
Eurographics 2012

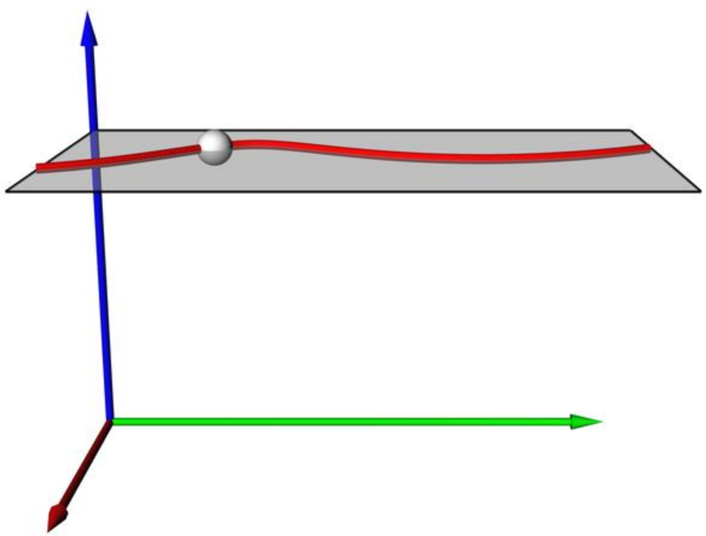
$$\bar{\bar{\mathbf{q}}}(\mathbf{x},t,\tau) = \begin{pmatrix} (\nabla \phi_t^\tau(\mathbf{x}))^{-1} \cdot \frac{\partial \phi_t^\tau(\mathbf{x})}{\partial t} + \mathbf{v}(\mathbf{x},t) \\ 0 \\ -1 \end{pmatrix}$$

$$\bar{\bar{\mathbf{q}}}(\mathbf{x},t,\tau) = \begin{pmatrix} (\nabla \phi)^{-1} \cdot \left(\mathbf{a}(\bar{\phi}) - g(\bar{\phi})\,\frac{\partial \phi}{\partial t} \right) - g(\bar{\phi}) \cdot \mathbf{w} \\ 0 \\ g(\bar{\phi}) \end{pmatrix}$$

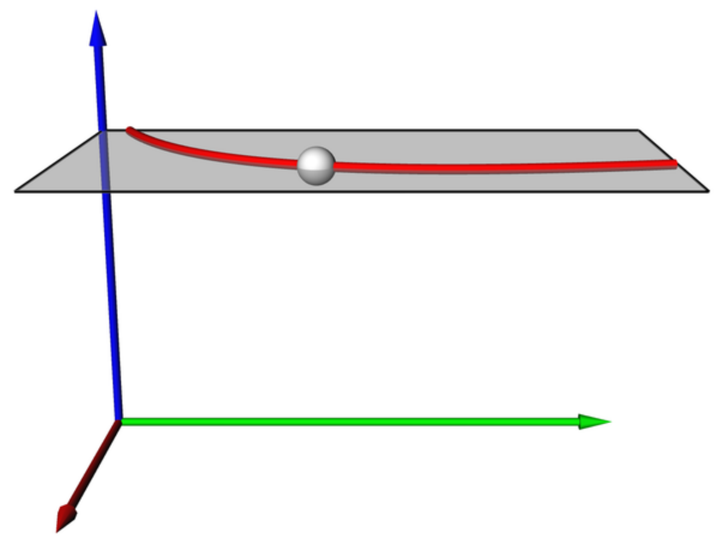
Streak Line Vector Field

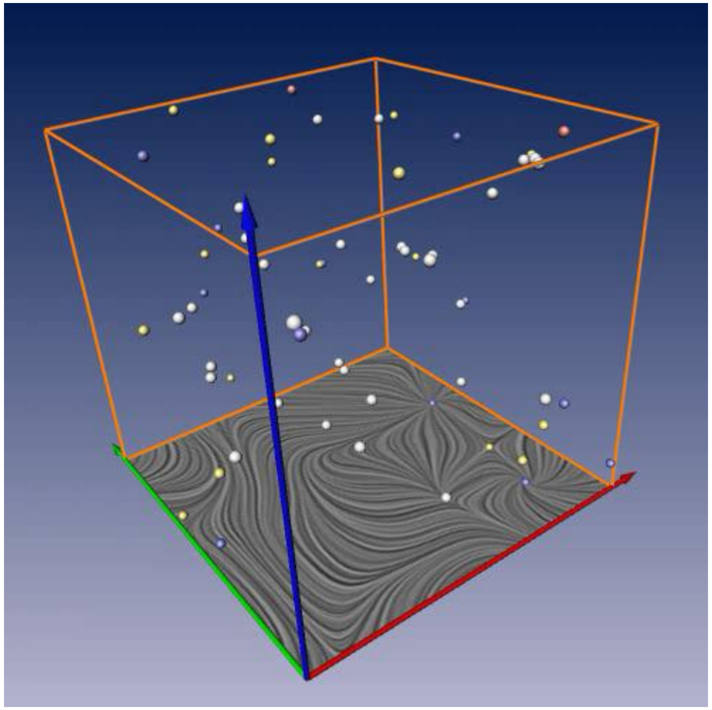
Advected Tangent Curves Vector Field

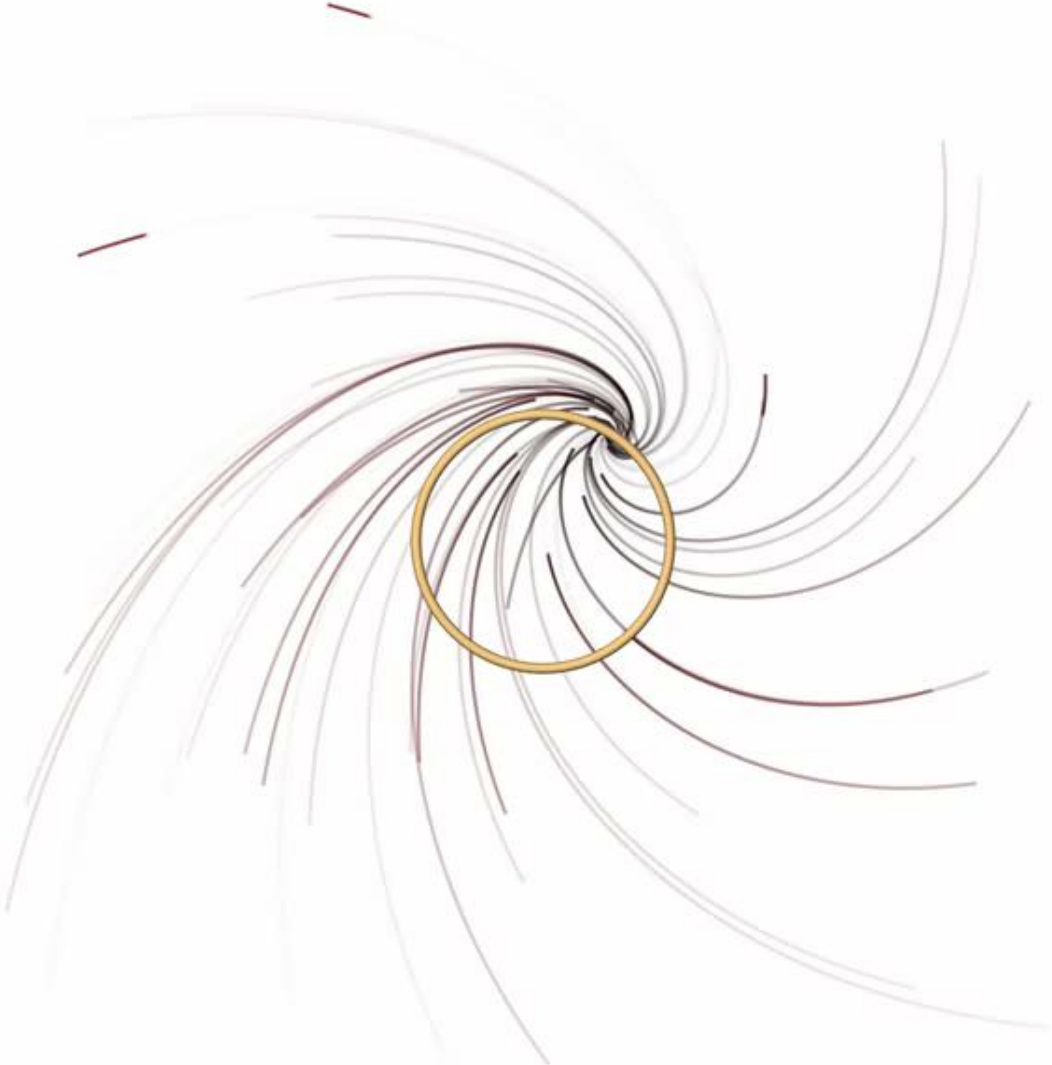
streak lines



time lines

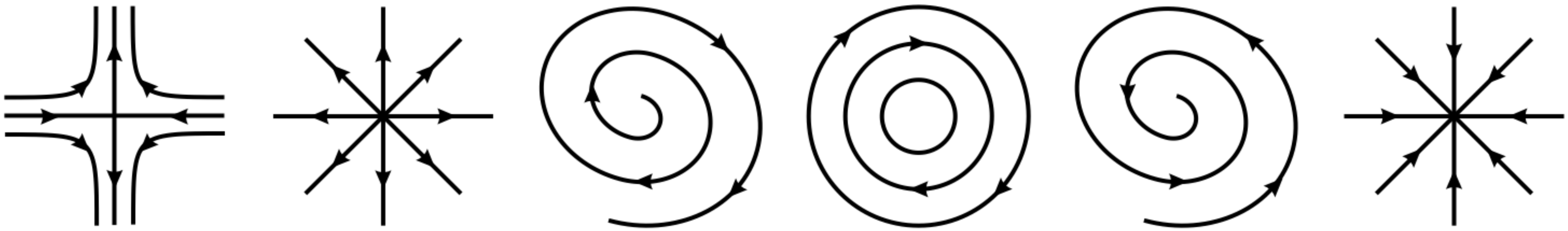




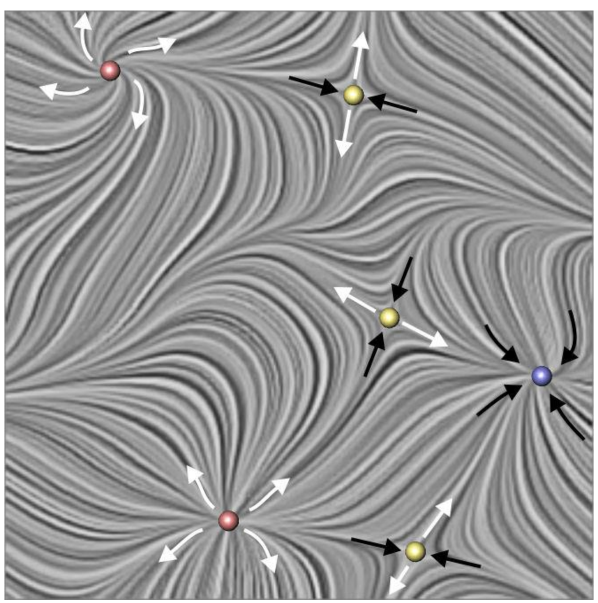


Coffee Break

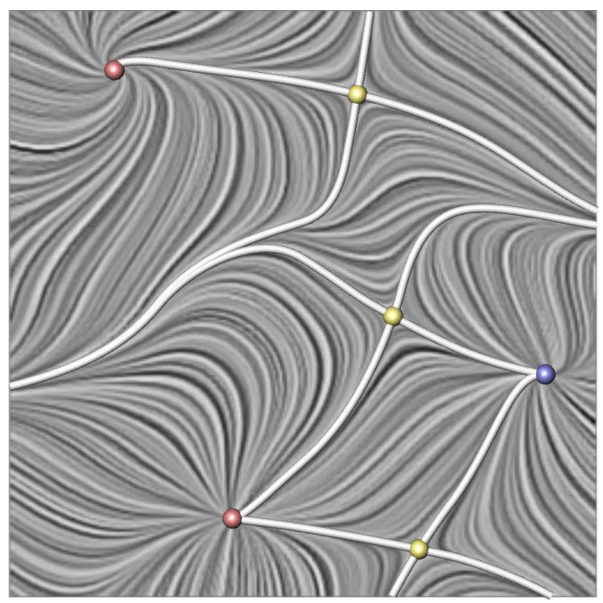
$$\mathbf{v}(\mathbf{x}_0) = \mathbf{0} \quad \text{with} \quad \mathbf{v}(\mathbf{x}_0 \pm \boldsymbol{\epsilon}) \neq \mathbf{0}$$



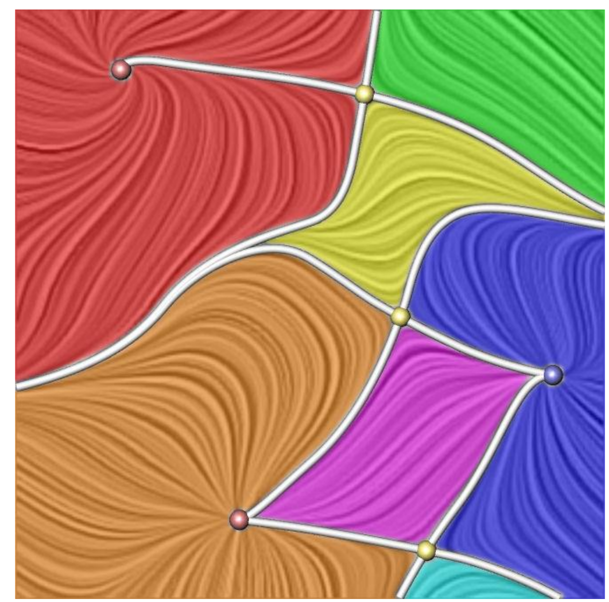
Saddle point	Repelling node	Repelling focus	Center	Attracting focus	Attracting node
$R_1 < 0, R_2 > 0$	$R_1, R_2 > 0$	$R_1 = R_2 > 0$	$R_1 = R_2 = 0$	$R_1 = R_2 < 0$	$R_1, R_2 < 0$
$I_1 = I_2 = 0$	$I_1 = I_2 = 0$	$I_1 = -I_2 \neq 0$	$I_1 = -I_2 \neq 0$	$I_1 = -I_2 \neq 0$	$I_1 = I_2 = 0$



critical points



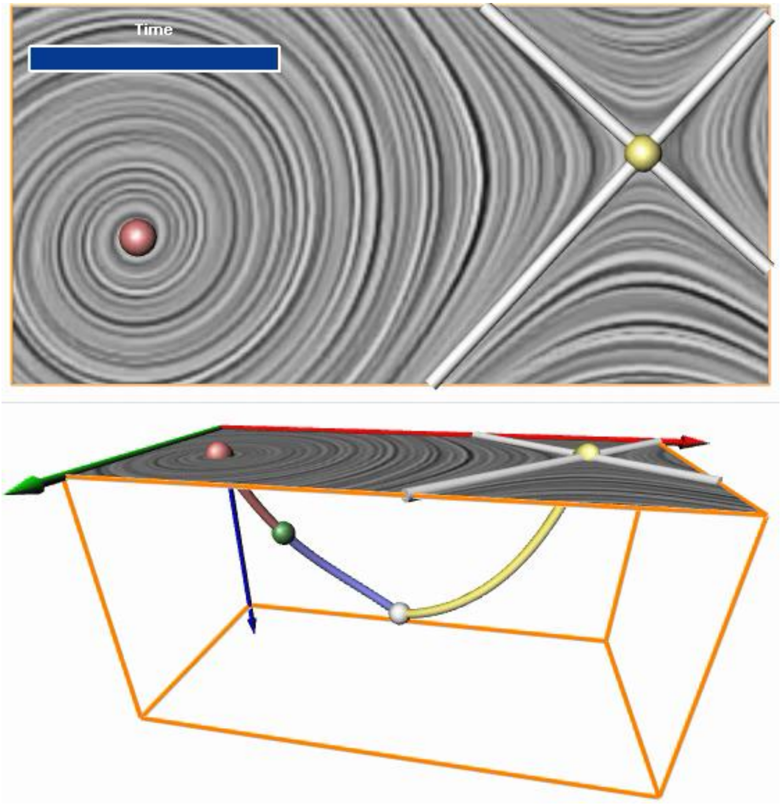
separation lines



sectors of different flow behavior

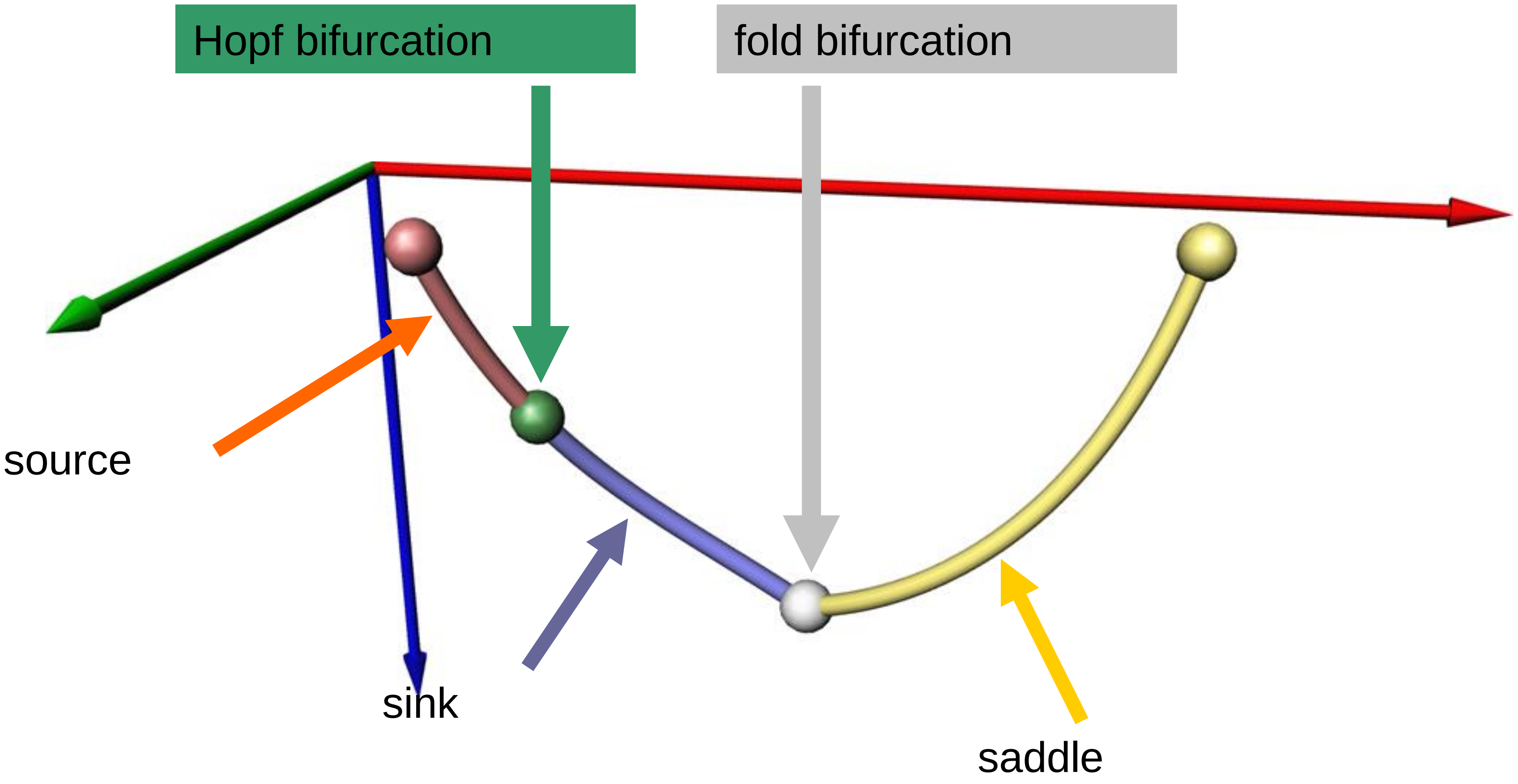
Topological Structures

unsteady vector field



Topological Structures

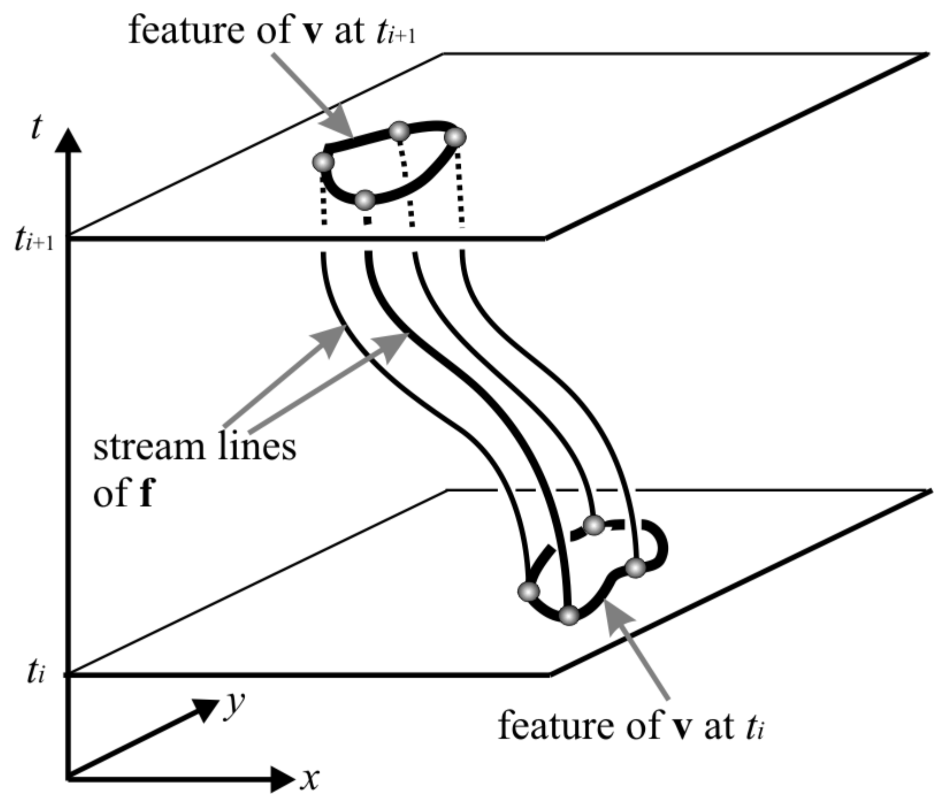
unsteady vector field



$$\mathbf{v}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \end{pmatrix}$$

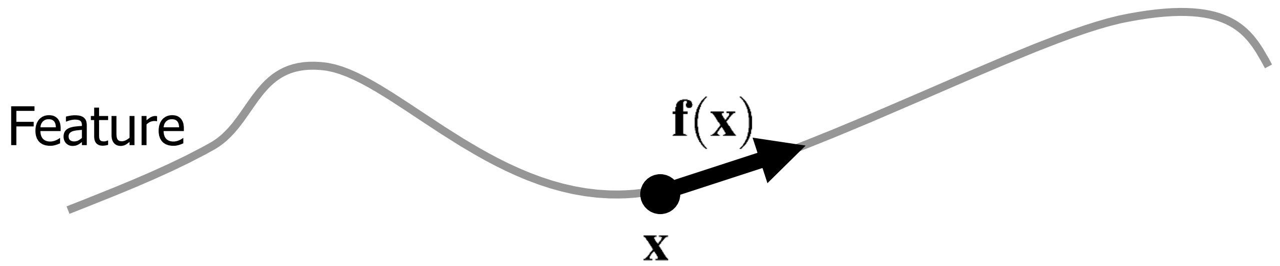
$$\mathbf{f}(x,y,t) = \begin{pmatrix} f(x,y,t) \\ g(x,y,t) \\ h(x,y,t) \end{pmatrix}$$

Feature Flow Field (FFF)



H. Theisel and H.-P. Seidel
Feature Flow Fields, VisSym 2003

- Feature Flow Field: vector field $\mathbf{f}$ at $\mathbf{x}$ pointing into direction where the feature continues



- Numerical stream line/surface integration is well-understood
- Stream object integration independent of underlying grid
- FFF gives theoretical tool for classifying local bifurcations

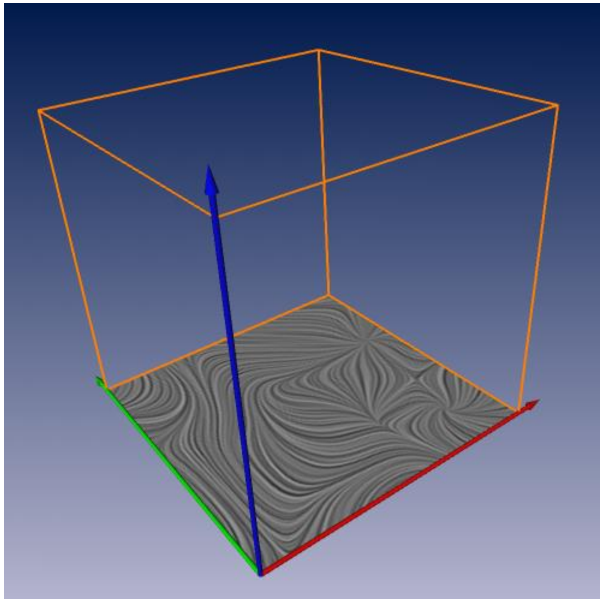
- **H. Theisel and H.-P. Seidel**
  Feature Flow Fields, VisSym 2003
- **H. Theisel, T. Weinkauf, H.-C. Hege, and H.-P. Seidel**
  Stream Line and Path Line Oriented Topology for 2D Time-Dependent Vector Fields, IEEE Vis 2004
- **T. Weinkauf, H. Theisel, H.-C. Hege, and H.-P. Seidel**
  Feature Flow Fields in Out-Of-Core Settings, TopoInVis 2005
- **X. Zheng and A. Pang**
  Topological Lines in 3D Tensor Fields and Discriminant Hessian Factorization, TVCG 2005
- **H. Theisel, J. Sahner, T. Weinkauf, H.-C. Hege, and H.-P. Seidel**
  Extraction of Parallel Vector Surfaces in 3D Time-Dependent Fields and Application to Vortex Core Line Tracking, IEEE Vis 2005
- **S. Depardon, J. J. Lasserre, L. E. Brizzi, and J. Borée**
  Automated topology classification method for instantaneous velocity fields, Experiments in Fluids 2007
- **J. Reininghaus, J. Kasten, T. Weinkauf, I. Hotz**
  Efficient Computation of Combinatorial Feature Flow Fields, TVCG 2011
- **T. Weinkauf, H. Theisel, A. Van Gelder, A. Pang**
  Stable Feature Flow Fields, TVCG 2011
- **C. Pagot, D. Osmari, F. Sadlo, D. Weiskopf, T. Ertl, J. Comba**
  Efficient Parallel Vectors Feature Extraction from Higher-Order Data, EuroVis 2011

1. Find all seeds

2. Integrate

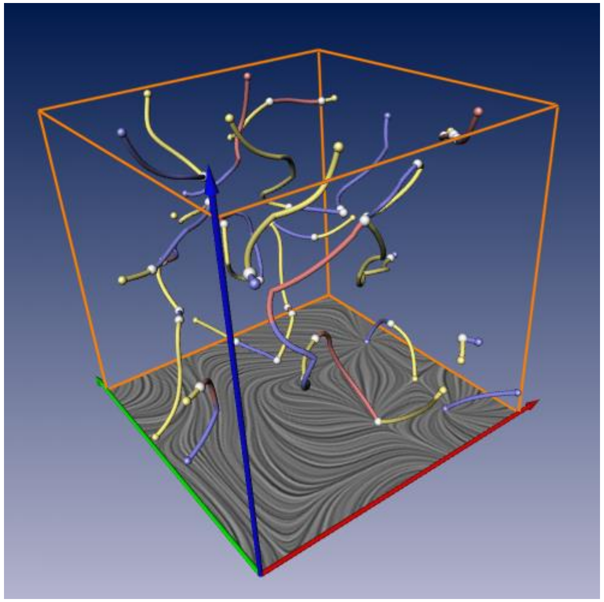
unsteady vector field

$$\mathbf{v}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \end{pmatrix}$$



FFF-based Tracking

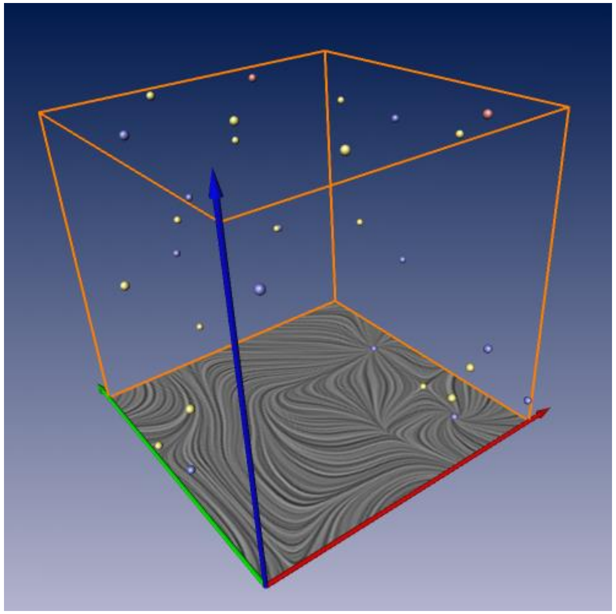
tracked critical points



1. Find all seeds

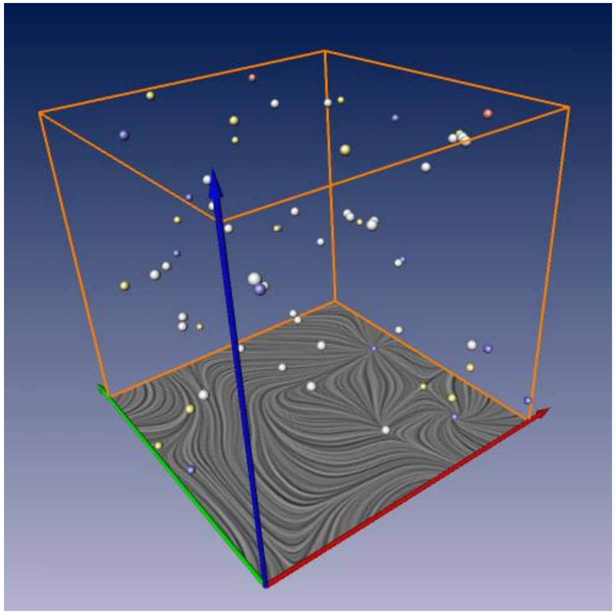
Finding Zeros

2. Integrate



Domain boundaries

$$\mathbf{v}(x, y, t_{min}) = (0,0)^T \text{ and } \mathbf{v}(x, y, t_{max}) = (0,0)^T$$
$$\mathbf{v}(x, y_{min}, t) = (0,0)^T \text{ and } \mathbf{v}(x, y_{max}, t) = (0,0)^T$$
$$\mathbf{v}(x_{min}, y, t) = (0,0)^T \text{ and } \mathbf{v}(x_{max}, y, t) = (0,0)^T$$



Fold bifurcations

$$[\ \mathbf{v}(\mathbf{x}) = (0,0)^T \quad , \quad \det(\mathbf{J_v}(\mathbf{x})) = 0\ ]$$

1. Find all seeds

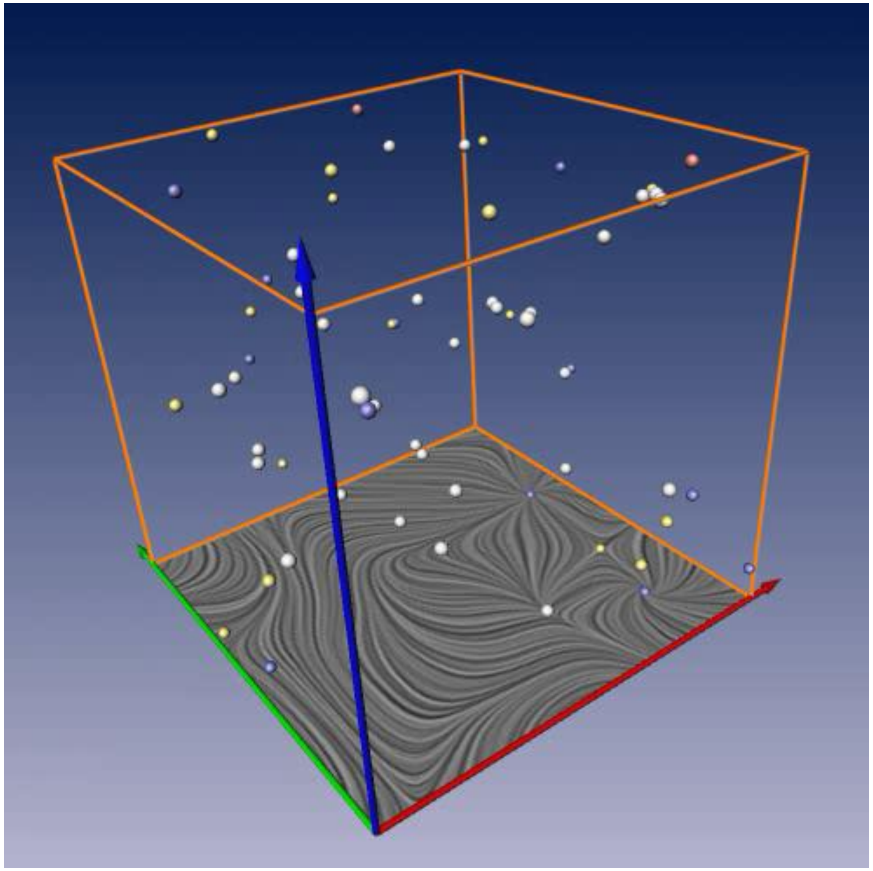
2. Integrate

Integrating Stream Objects

$$\mathbf{v}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \end{pmatrix}$$

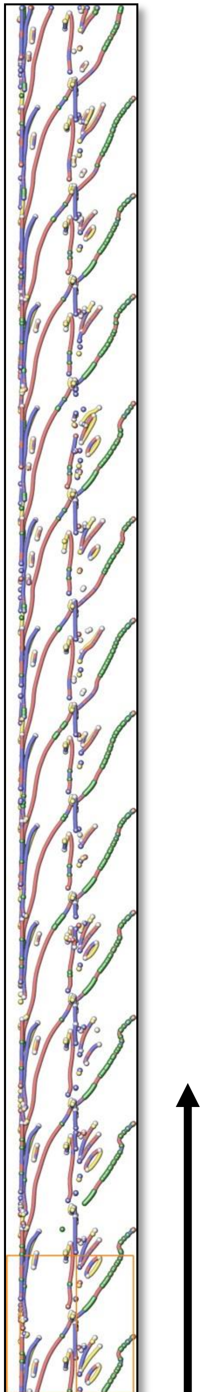
$$\mathbf{f}(x,y,t) = \begin{pmatrix} \det(\mathbf{v}_y, \mathbf{v}_t) \\ \det(\mathbf{v}_t, \mathbf{v}_x) \\ \det(\mathbf{v}_x, \mathbf{v}_y) \end{pmatrix}$$

[Theisel and Seidel; VisSym 2003]

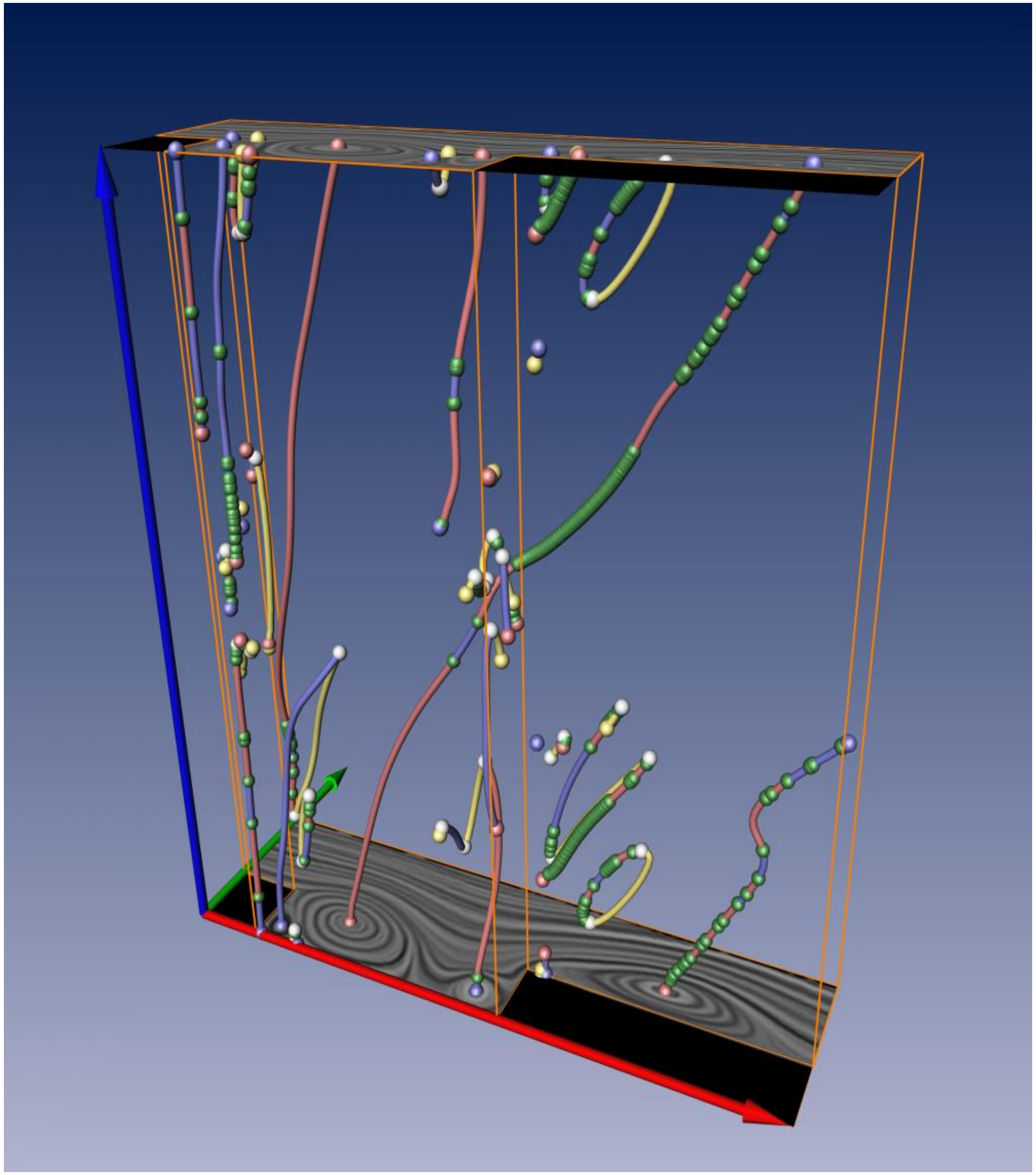


Extensions:

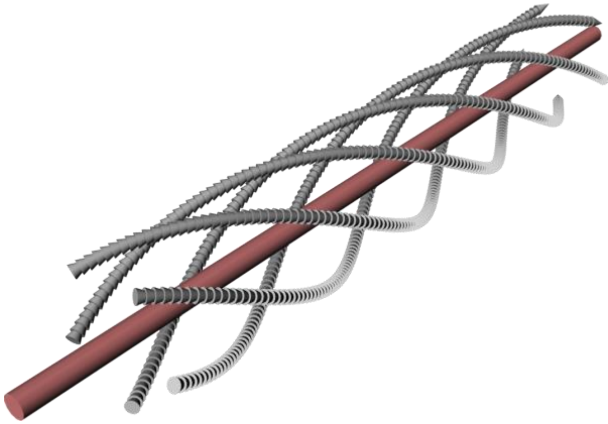
- Large data (out-of-core) TopoInVis 2007
- Stability TVCG 2011
- Combinatorial equivalent TVCG 2012



1000 time steps



Flow around a 2D Cavity. Data courtesy of Samimy, Noack.



NASA Langely Research Center

Eigenvector method
[Sujudi and Haimes, AIAA 1995]

Parallel Vectors operator
[Peikert and Roth, Vis 1999]

Tracking in scale space
[Bauer and Peikert, VisSym 2002]

Parallel Vector surfaces
[Theisel et al., Vis 2005]

stream lines

Swirling Motion Cores
[Weinkauf et al., Vis 2007]

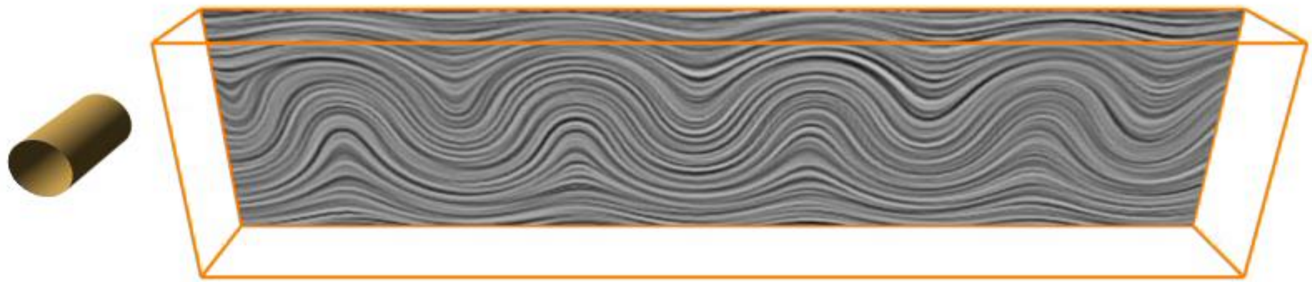
Streak Line Cores
[Weinkauf and Theisel, Vis 2010]

Advected Tangent Cores
[Weinkauf et al., EG 2012]

Vortex Cores of Inertial Particles
[Günther and Theisel, Vis 2014]

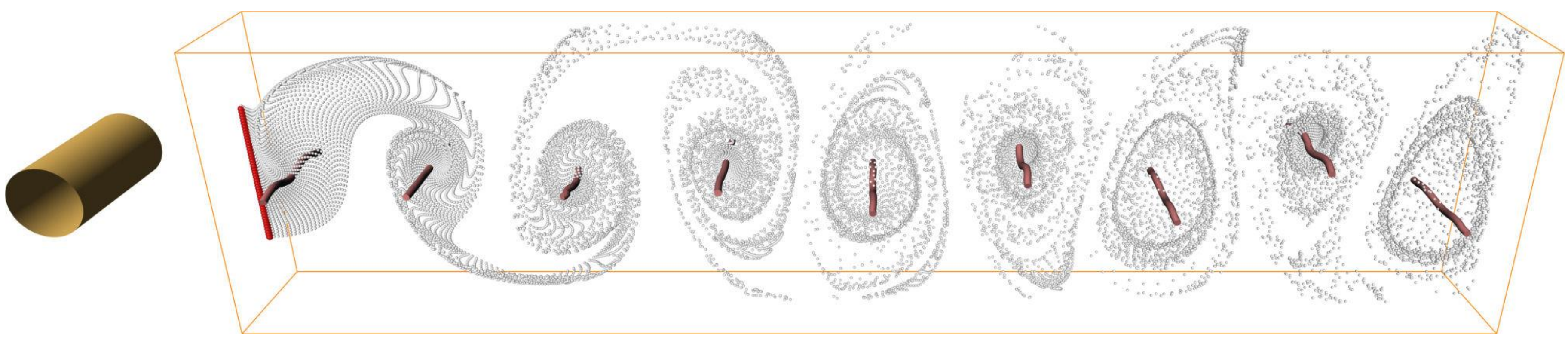
**path lines, streak lines,
other curves, inertial particles**

3D unsteady flow behind a cylinder (Data courtesy of Bernd R. Noack)



stream lines

path lines

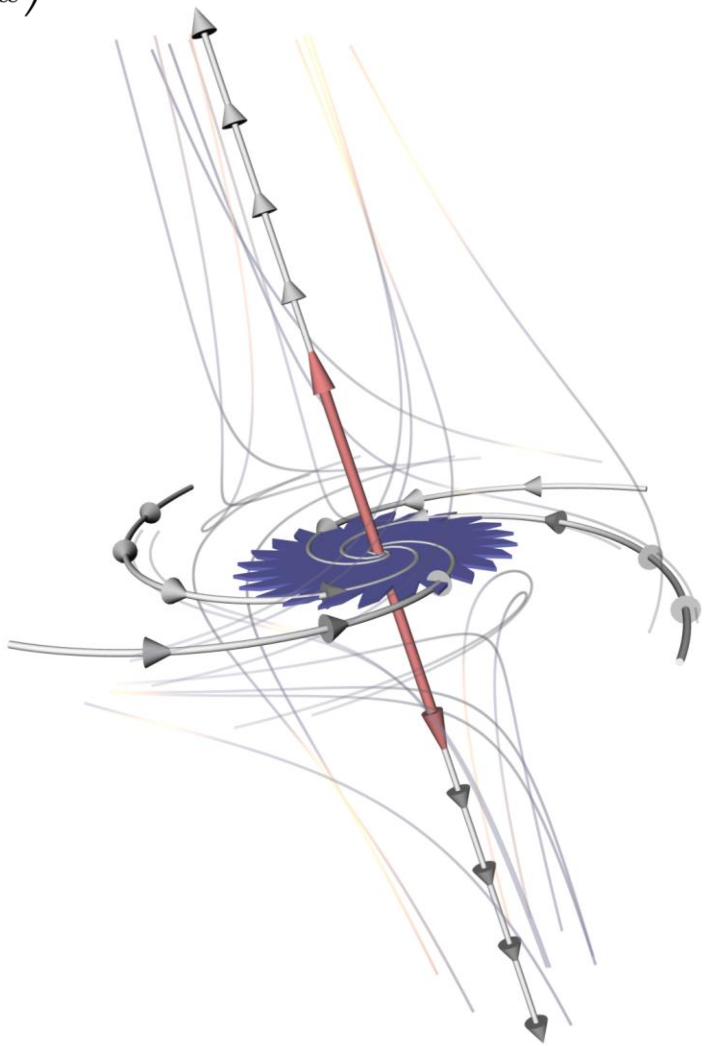


cores of swirling particle motion

Swirling motion of stream lines – 3D steady

$$\mathbf{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

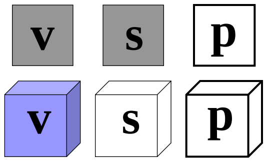
Eigenvector method
[Sujudi and Haimes, AIAA 1995]



$$\begin{aligned} \mathbf{w}(\mathbf{x}) &= \mathbf{v}(\mathbf{x}) - (\mathbf{v}(\mathbf{x}) \cdot \mathbf{e}(\mathbf{x}))\, \mathbf{e}(\mathbf{x}) \\ \mathbf{w}(\mathbf{x}) &= \mathbf{0} \end{aligned}$$

reduced velocity

Although a point **x** *on the core structure is surrounded by spiraling integral curves, the flow vector at* **x** *itself is solely governed by the non-swirling part of the flow.*



$$\mathbf{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Eigenvector method
[Sujudi and Haimes, AIAA 1995]

Parallel Vectors operator
[Peikert and Roth, Vis 1999]

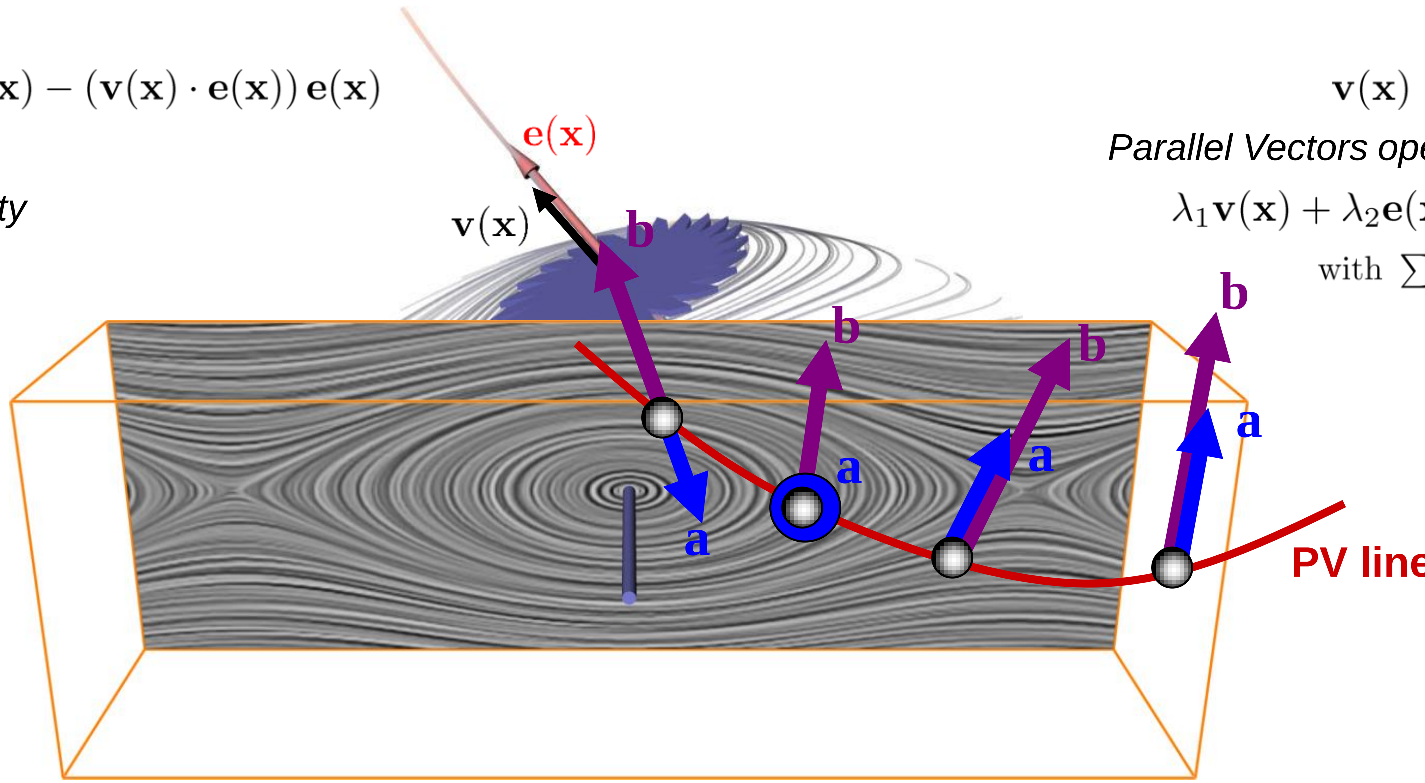
$$\mathbf{w}(\mathbf{x}) = \mathbf{v}(\mathbf{x}) - (\mathbf{v}(\mathbf{x}) \cdot \mathbf{e}(\mathbf{x}))\,\mathbf{e}(\mathbf{x})$$
$$\mathbf{w}(\mathbf{x}) = \mathbf{0}$$

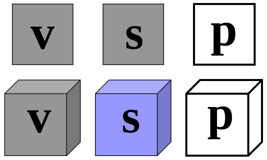
reduced velocity

$$\mathbf{v}(\mathbf{x}) \;||\; \mathbf{e}(\mathbf{x})$$

Parallel Vectors operator

$$\lambda_1 \mathbf{v}(\mathbf{x}) + \lambda_2 \mathbf{e}(\mathbf{x}) = \mathbf{0}$$
$$\text{with } \sum \lambda_i^2 > 0$$





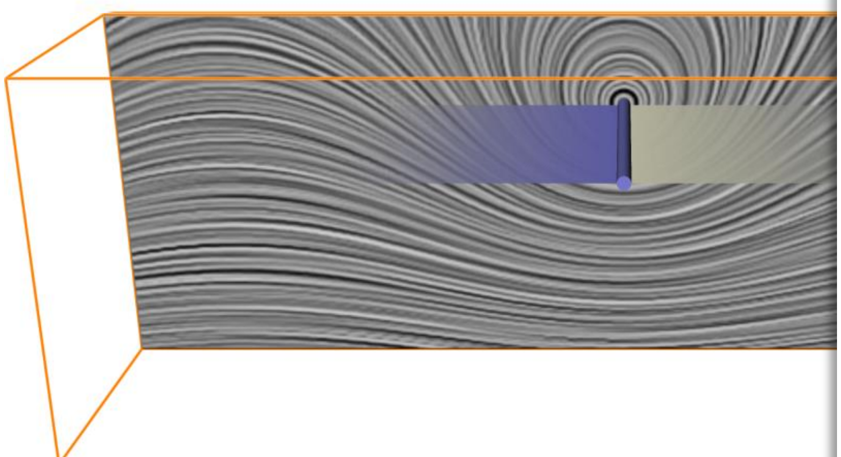
$$\mathbf{s} = \begin{pmatrix} u \\ v \\ w \\ 0 \end{pmatrix}$$

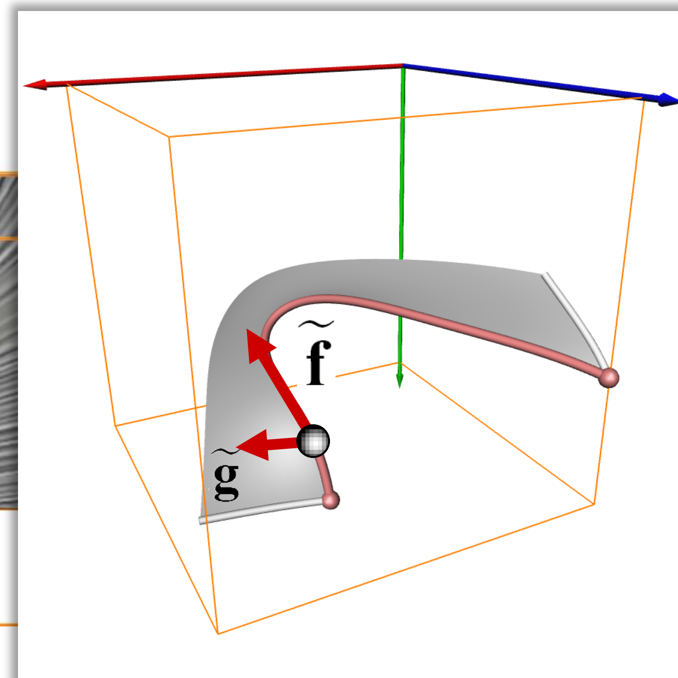
Tracking in scale space
[Bauer and Peikert, VisSym 2002]

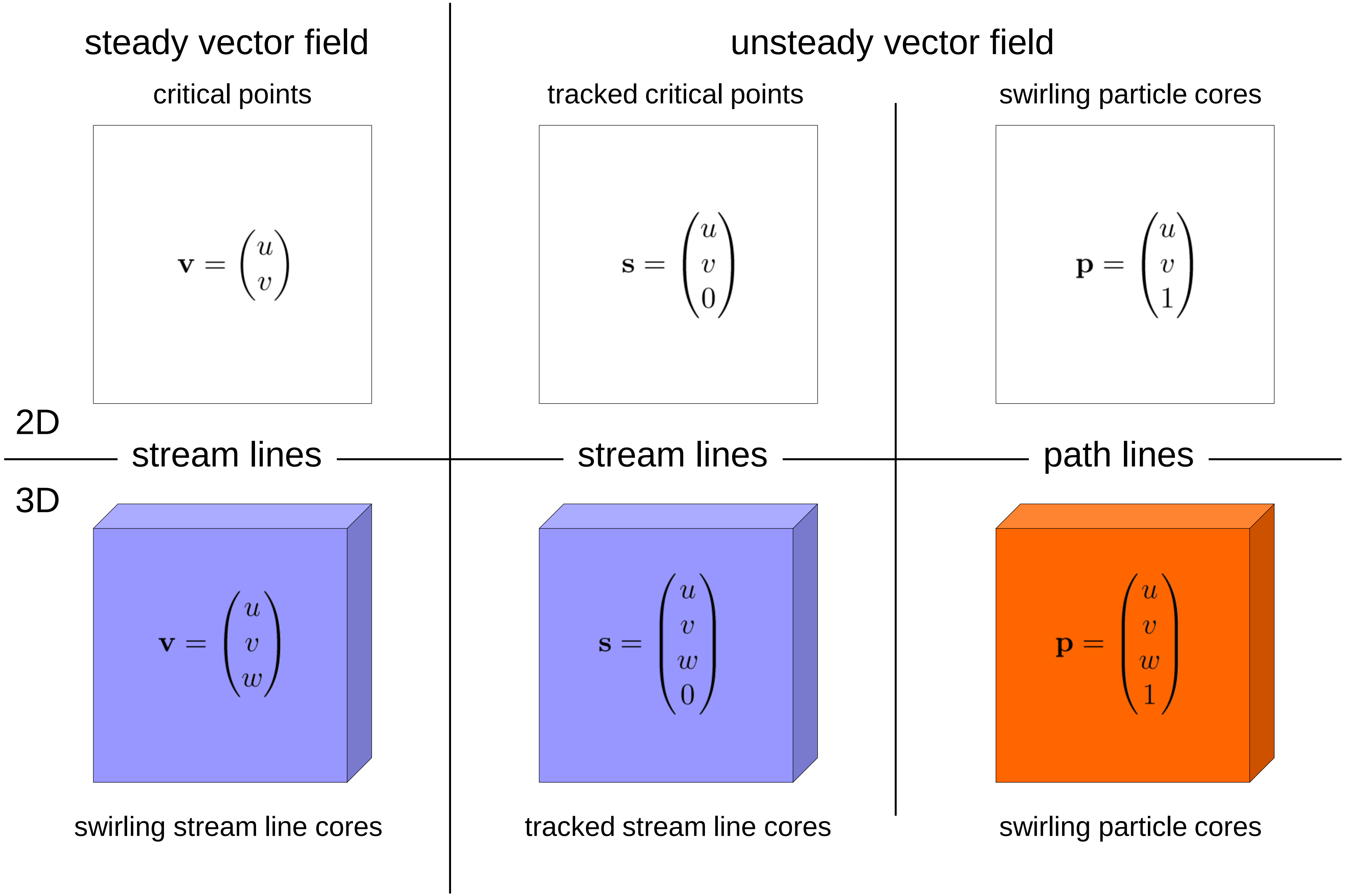
4D marching cubes like method

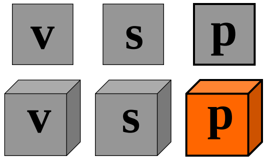
Parallel Vector surfaces
[Theisel et al., Vis 2005]

based on Feature Flow Fields



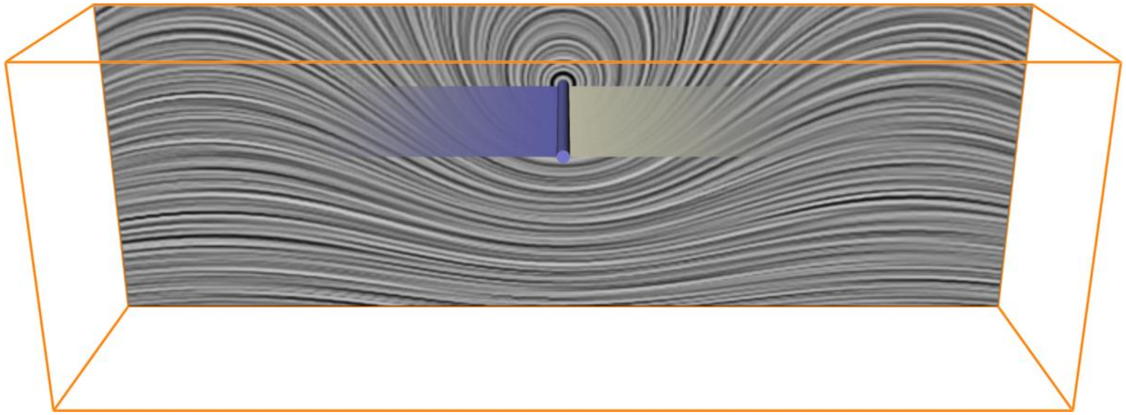



steady vector field
unsteady vector field
critical points
tracked critical points
swirling particle cores
$\mathbf{v} = \begin{pmatrix} u \\ v \end{pmatrix}$
$\mathbf{s} = \begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$
$\mathbf{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$
2D
stream lines
stream lines
path lines
3D
$\mathbf{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$
$\mathbf{s} = \begin{pmatrix} u \\ v \\ w \\ 0 \end{pmatrix}$
$\mathbf{p} = \begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix}$
swirling stream line cores
tracked stream line cores
swirling particle cores



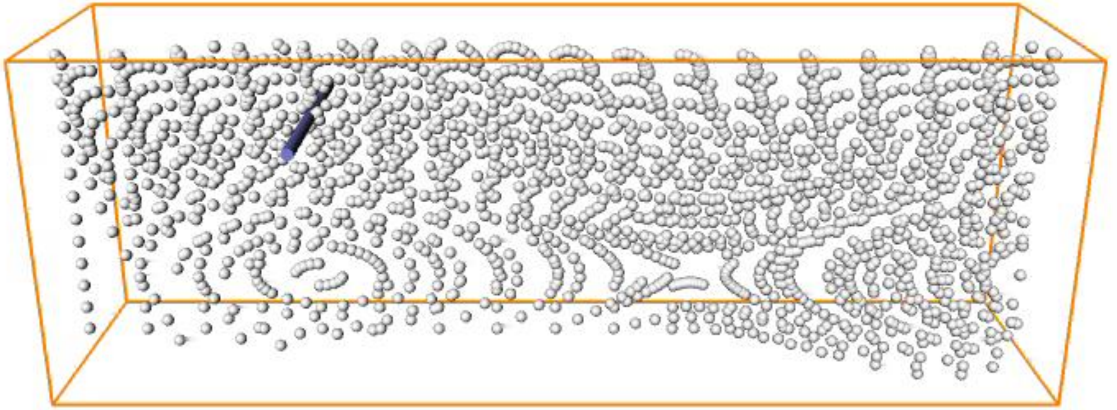
$$\mathbf{s} = \begin{pmatrix} u \\ v \\ w \\ 0 \end{pmatrix}$$

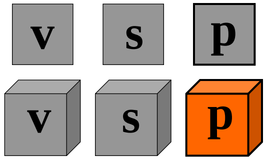
stream lines



$$\mathbf{p} = \begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix}$$

path lines
particles





$$\mathbf{p} = \begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix}$$

path lines
of a 3D unsteady flow → *no existing method*

$$\mathbf{J}(\mathbf{p}) = \begin{bmatrix} u_x & u_y & u_z & u_t \\ v_x & v_y & v_z & v_t \\ w_x & w_y & w_z & w_t \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \text{eigenvalues}$$

$$\boxed{e_1 \quad e_2} \; \boxed{e_3 \quad 0}$$

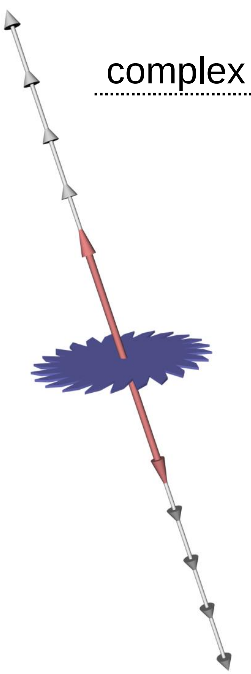
complex (necessary condition)

two real eigenvectors $\mathbf{e}^s$ $\mathbf{f}$

$$\longrightarrow \lambda_1 \mathbf{p} + \lambda_2 \mathbf{e}^s + \lambda_3 \mathbf{f} = \mathbf{0}$$

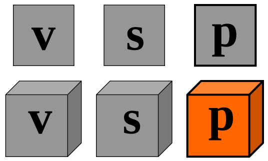
$$\text{with } \sum \lambda_i^2 > 0$$

Coplanarity of
three 4D vectors



Original idea of Sujudi/Haimes:

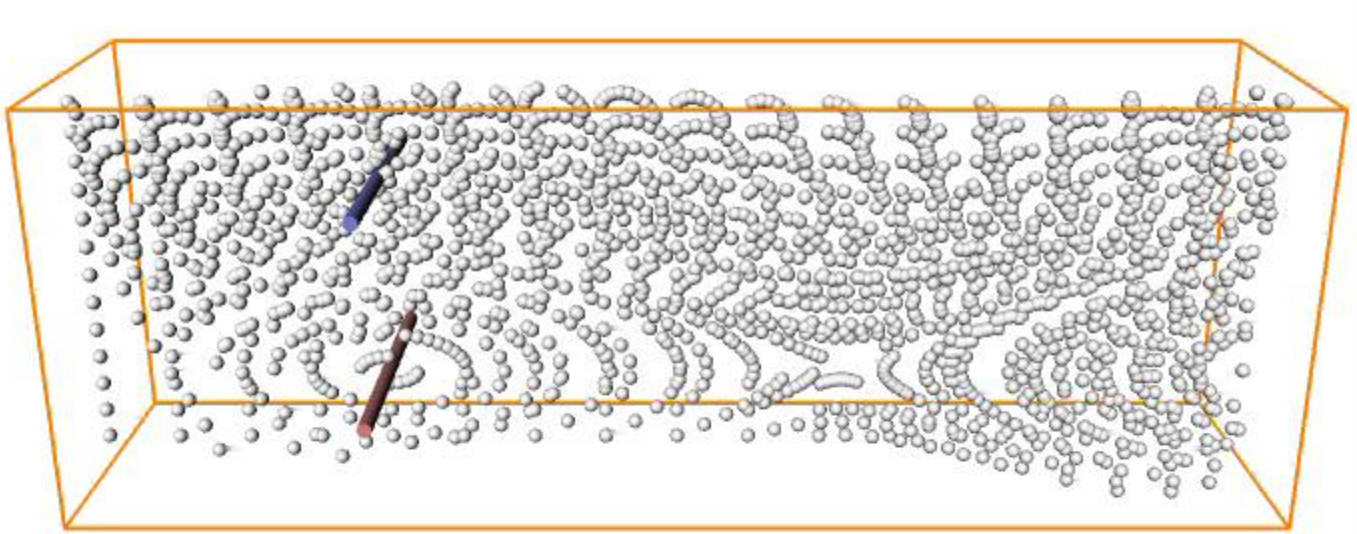
Although a point $\mathbf{x}$ *on the core structure is surrounded by spiraling integral curves, the flow vector at* $\mathbf{x}$ *itself is solely governed by the non-swirling part of the flow.*



Swirling motion of path lines – 3D unsteady

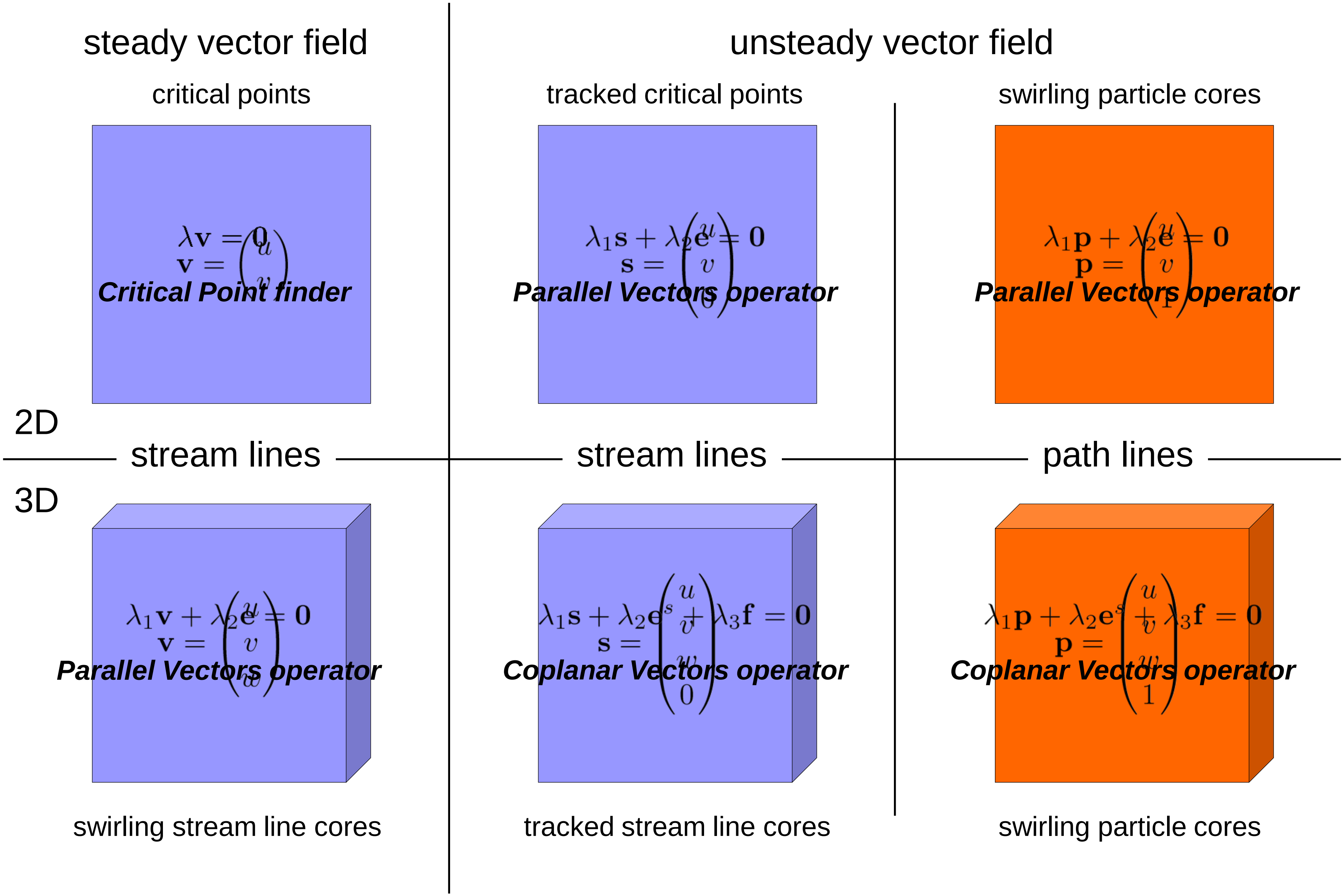
$$\mathbf{p} = \begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix}$$

path lines
of a 3D unsteady flow → *no existing method*



$$\lambda_3 \mathbf{f} = \mathbf{0}$$

with $\sum \lambda_i^2 > 0$


steady vector field
unsteady vector field
critical points
tracked critical points
swirling particle cores
$\lambda \mathbf{v} = \mathbf{0}$, $\mathbf{v} = \begin{pmatrix} u \\ v \end{pmatrix}$
Critical Point finder
$\lambda_1 \mathbf{s} + \lambda_2 \mathbf{e}^s = \mathbf{0}$, $\mathbf{s} = \begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$
Parallel Vectors operator
$\lambda_1 \mathbf{p} + \lambda_2 \mathbf{e} = \mathbf{0}$, $\mathbf{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$
Parallel Vectors operator
2D
stream lines
stream lines
path lines
3D
$\lambda_1 \mathbf{v} + \lambda_2 \mathbf{e} = \mathbf{0}$, $\mathbf{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$
Parallel Vectors operator
$\lambda_1 \mathbf{s} + \lambda_2 \mathbf{e}^s + \lambda_3 \mathbf{f} = \mathbf{0}$, $\mathbf{s} = \begin{pmatrix} u \\ v \\ w \\ 0 \end{pmatrix}$
Coplanar Vectors operator
$\lambda_1 \mathbf{p} + \lambda_2 \mathbf{e}^s + \lambda_3 \mathbf{f} = \mathbf{0}$, $\mathbf{p} = \begin{pmatrix} u \\ v \\ w \\ 1 \end{pmatrix}$
Coplanar Vectors operator
swirling stream line cores
tracked stream line cores
swirling particle cores

steady vector field

critical points

unsteady vector field

tracked critical points

swirling particle cores

unified notation of swirling motion cores

2D

3D

$$\lambda_1 \mathbf{V}(\mathbf{x}) + \sum \lambda_i \mathbf{e}_i(\mathbf{x}) = \mathbf{0}$$

$$\text{with } \sum \lambda_i^2 > 0$$

swirling stream line cores

tracked stream line cores

swirling particle cores

Streak Line Vector Field

$$\bar{\bar{\mathbf{q}}}(\mathbf{x},t,\tau) = \begin{pmatrix} (\nabla\phi_t^\tau(\mathbf{x}))^{-1}\cdot \frac{\partial \phi_t^\tau(\mathbf{x})}{\partial t} + \mathbf{v}(\mathbf{x},t) \\ 0 \\ -1 \end{pmatrix}$$

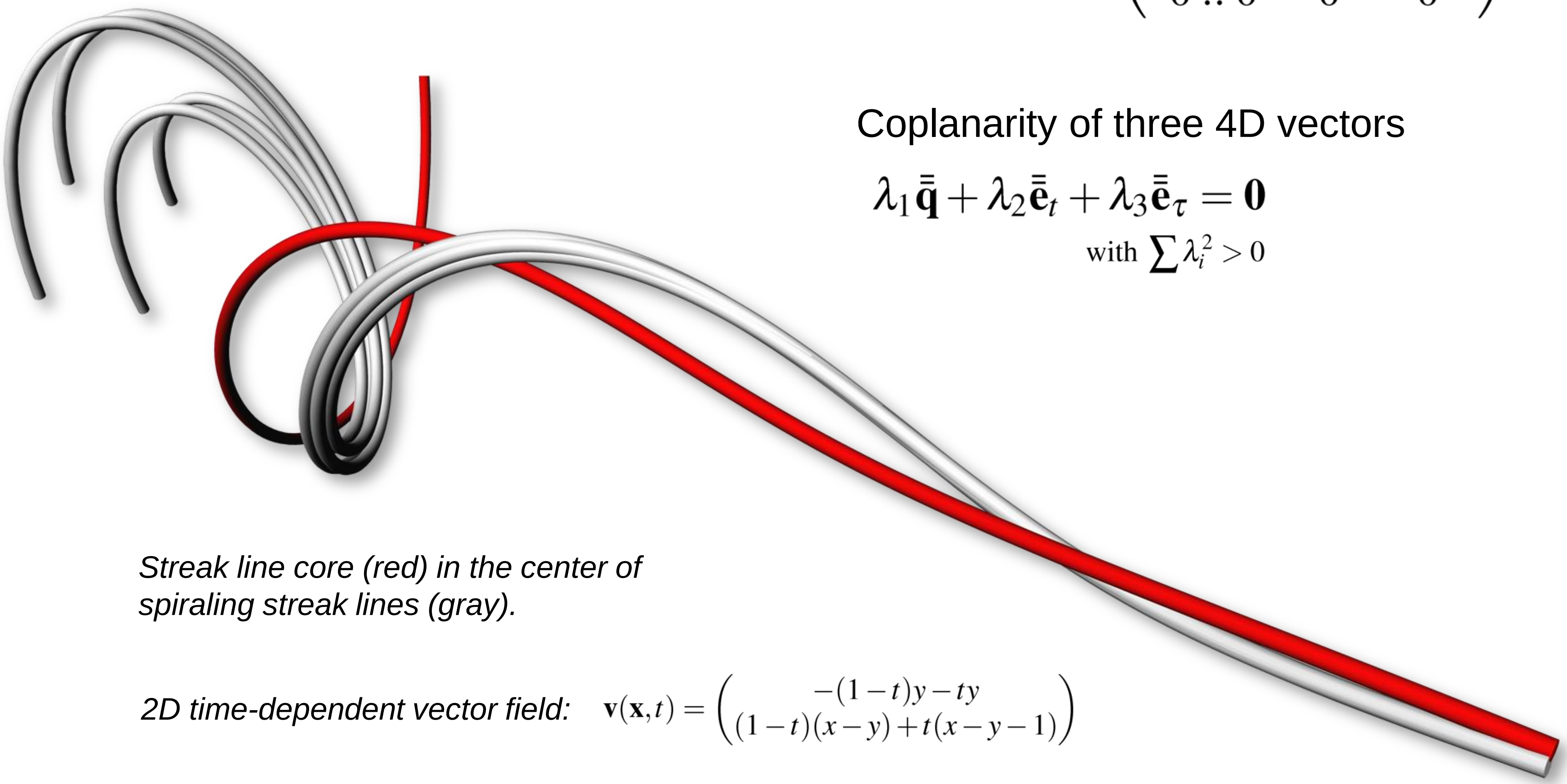
Eigenanalysis of the gradient of the streak line vector field

$$\nabla\bar{\bar{\mathbf{q}}}(\mathbf{x},t,\tau) = \begin{pmatrix} \frac{\partial \mathbf{w}}{\partial \mathbf{x}} & \frac{\partial \mathbf{w}}{\partial t} & \frac{\partial \mathbf{w}}{\partial \tau} \\ 0 \,..\, 0 & 0 & 0 \\ 0 \,..\, 0 & 0 & 0 \end{pmatrix}$$

Coplanarity of three 4D vectors

$$\lambda_1 \bar{\bar{\mathbf{q}}} + \lambda_2 \bar{\bar{\mathbf{e}}}_t + \lambda_3 \bar{\bar{\mathbf{e}}}_\tau = \mathbf{0}$$

with $\sum \lambda_i^2 > 0$



Streak line core (red) in the center of spiraling streak lines (gray).

2D time-dependent vector field: $\mathbf{v}(\mathbf{x},t) = \begin{pmatrix} -(1-t)y - ty \\ (1-t)(x-y) + t(x-y-1) \end{pmatrix}$

Beads Problem flow
reported by Wiebel et al., TopoInVis 2009

attractor in the flow not detectable by classic visualization methods or feature extraction methods

~~Line Integral Convolution~~

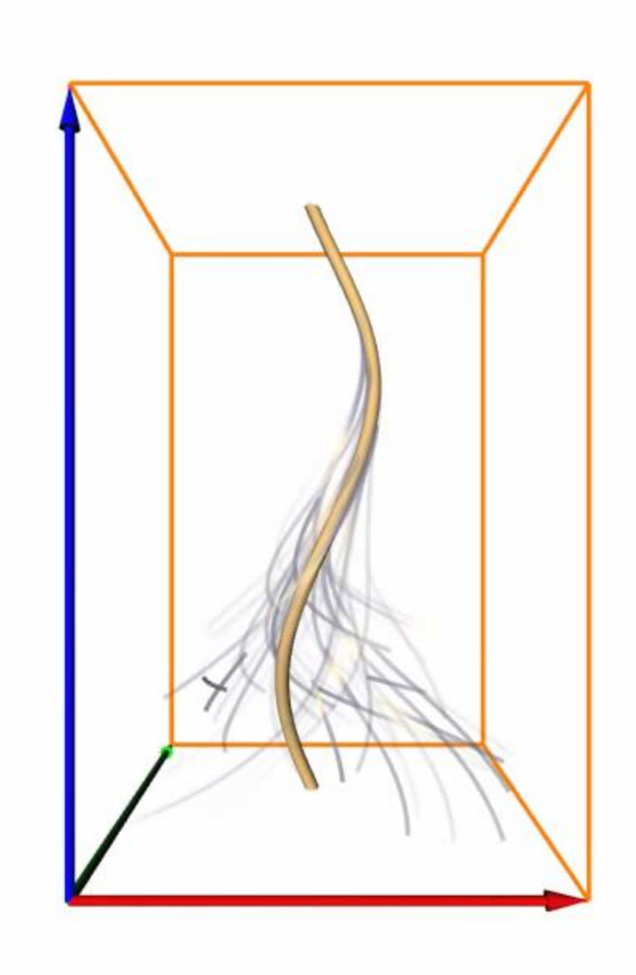
~~Vector Field Topology~~

~~Path Line Cores~~

~~Finite Time Lyapunov Exponents~~

here analytic version (Ronny Peikert) with known ground truth:

$$\mathbf{v}(x,y,t) = \begin{pmatrix} -(y-\frac{1}{3}\sin(t)) - (x-\frac{1}{3}\cos(t)) \\ (x-\frac{1}{3}\cos(t)) - (y-\frac{1}{3}\sin(t)) \end{pmatrix}$$



Path lines approaching the attractor (yellow curve).

Beads Problem flow

reported by Wiebel et al., TopoInVis 2009

attractor in the flow not detectable by classic visualization methods or feature extraction methods

~~Line Integral Convolution~~

~~Vector Field Topology~~

~~Path Line Cores~~

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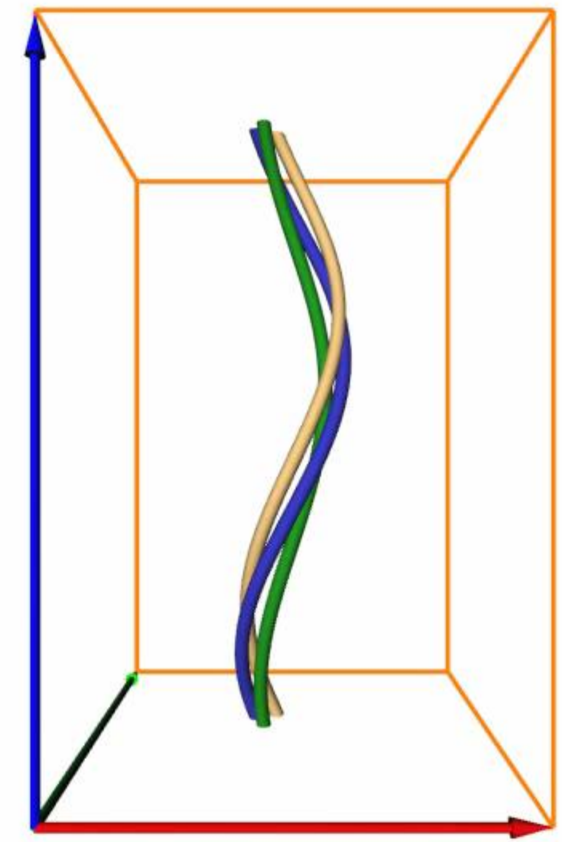

Critical points (green) and path line cores (blue) do not match the attractor.

Beads Problem flow
reported by Wiebel et al., TopoInVis 2009

attractor in the flow not detectable by classic visualization methods or feature extraction methods

~~Line Integral Convolution~~

~~Vector Field Topology~~

~~Path Line Cores~~

~~Finite Time Lyapunov Exponents~~

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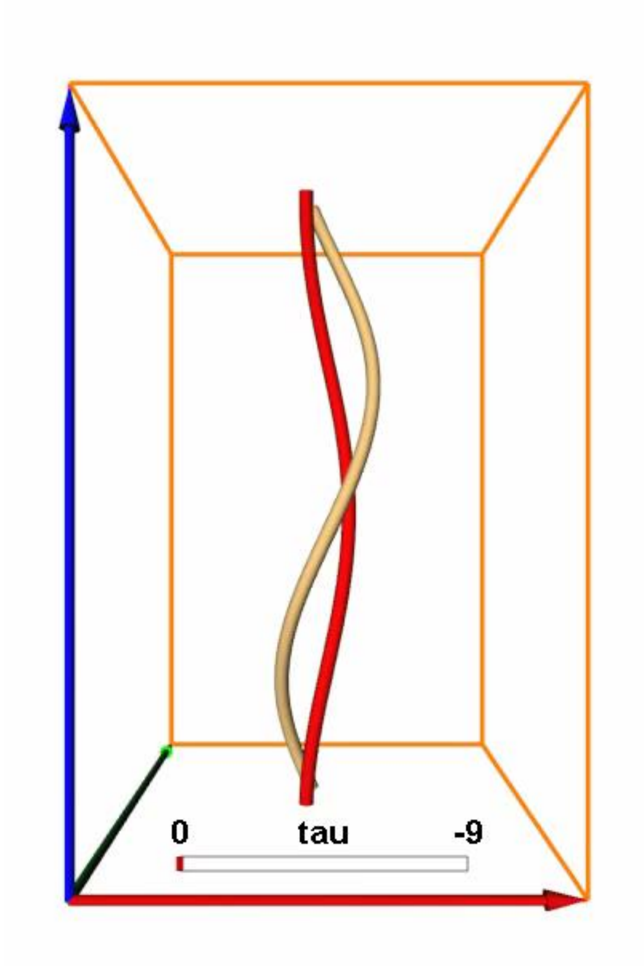
$$\mathbf{v}(x,y,t) = \begin{pmatrix} -(y-\frac{1}{3}\sin(t)) - (x-\frac{1}{3}\cos(t)) \\ (x-\frac{1}{3}\cos(t)) - (y-\frac{1}{3}\sin(t)) \end{pmatrix}$$



Streak line core (red) matches the attractor.

Beads Problem flow
reported by Wiebel et al., TopoInVis 2009

attractor in the flow not detectable
by classic visualization methods
or feature extraction methods

~~Line Integral Convolution~~

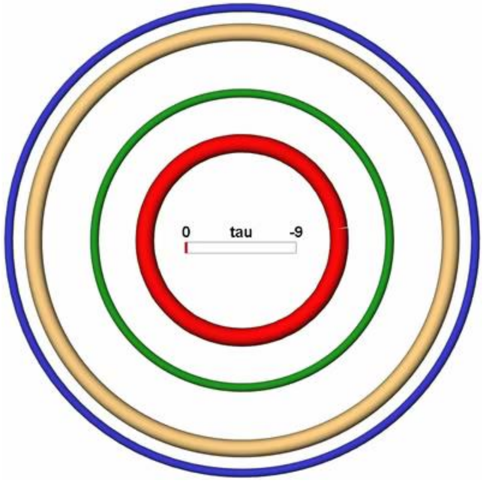
~~Vector Field Topology~~

~~Path Line Cores~~

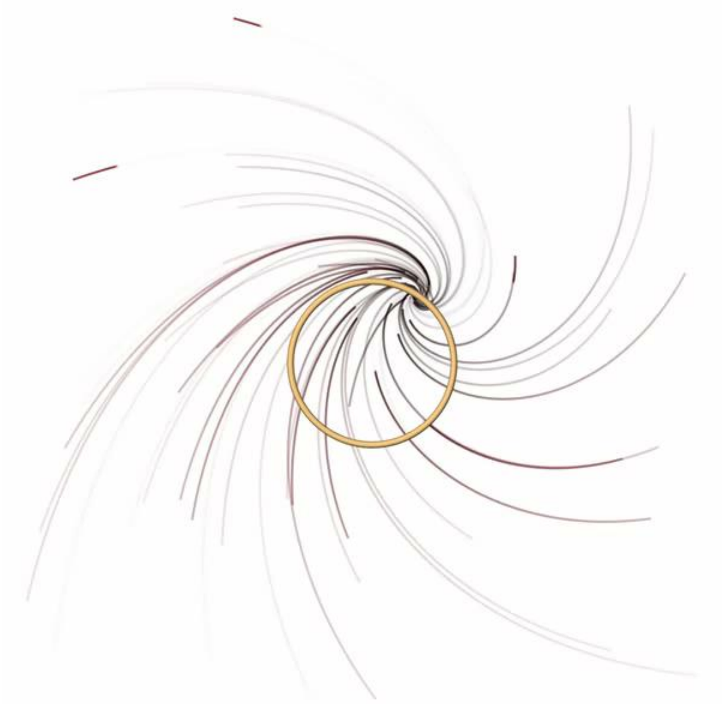
~~Finite Time Lyapunov Exponents~~

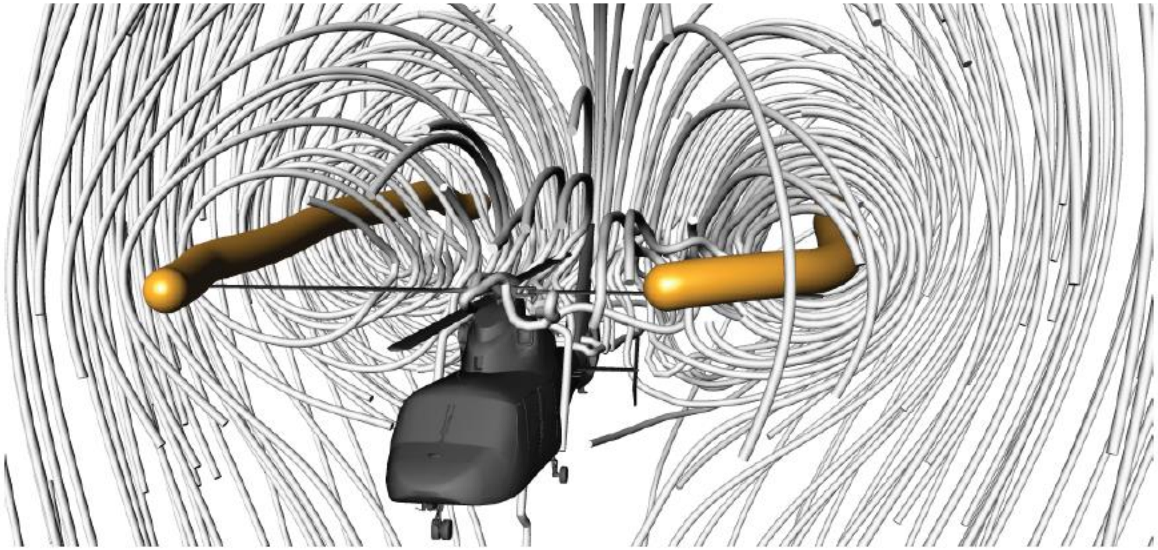
here analytic version (Ronny Peikert)
with known ground truth:

$$\mathbf{v}(x,y,t) = \begin{pmatrix} -(y - \frac{1}{3}\sin(t)) - (x - \frac{1}{3}\cos(t)) \\ (x - \frac{1}{3}\cos(t)) - (y - \frac{1}{3}\sin(t)) \end{pmatrix}$$



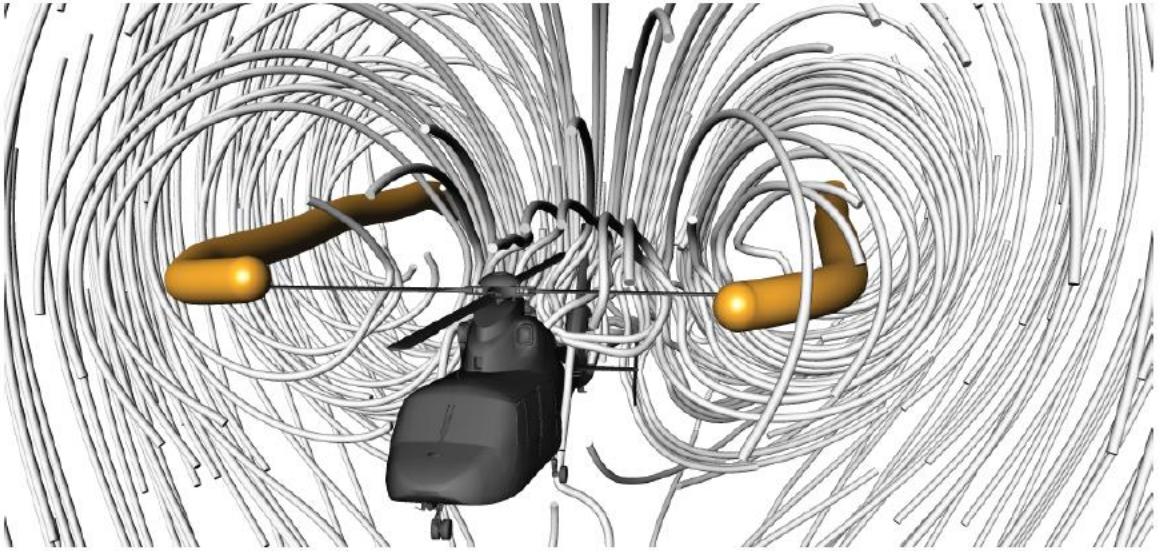
View from top.





massless

with mass

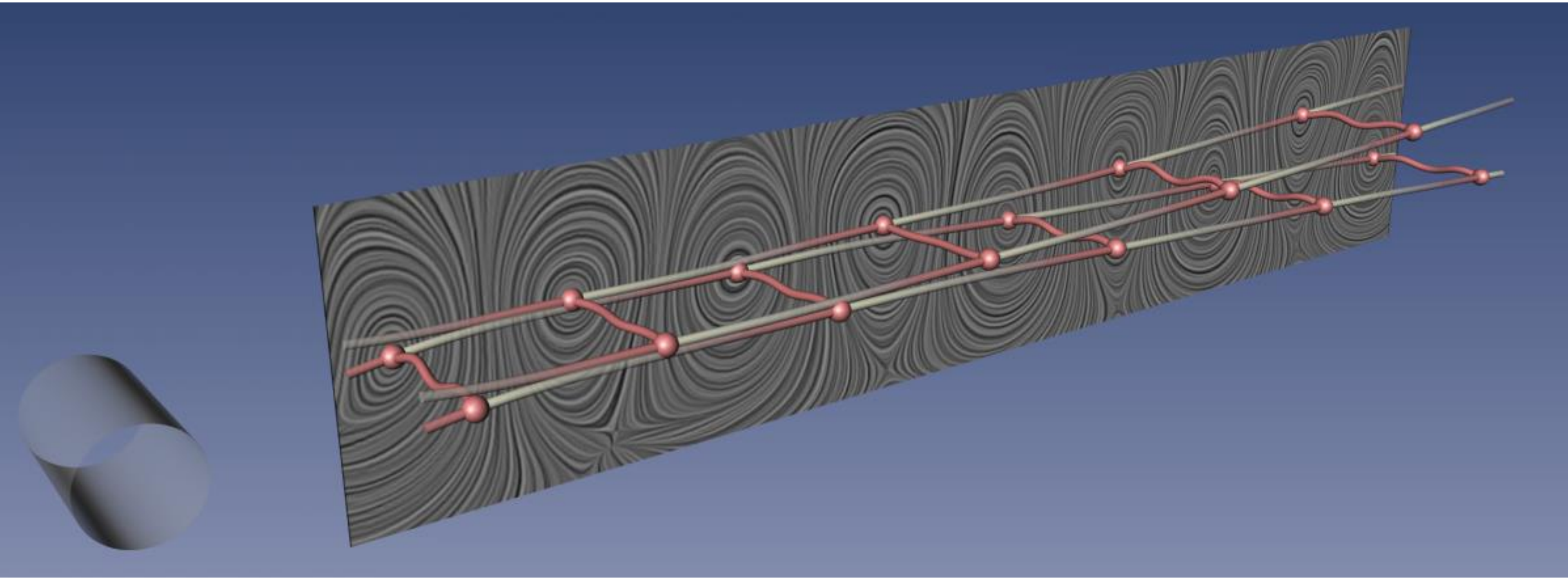


$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\mathbf{x}\\ \mathbf{v}\\ t\end{pmatrix} = \begin{pmatrix}\mathbf{v}\\ \frac{\mathbf{u}(\mathbf{x},t)-\mathbf{v}}{r}+\mathbf{g}\\ 1\end{pmatrix}$$

$$\text{with} \quad \begin{pmatrix}\mathbf{x}\\ \mathbf{v}\\ t\end{pmatrix}(0) = \begin{pmatrix}\mathbf{x}_0\\ \mathbf{v}_0\\ t_0\end{pmatrix}$$

Vortex Cores of Inertial Particles
[Günther and Theisel, Vis 2014]

Spatio-temporal Flow Analysis



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KTH Royal Institute of Technology
Stockholm, Sweden