$E(X) = G'(1)$ $\qquad$ $Var(X) = G''(1) + G'(1) - \left(G'(1)\right)^2$

Bernoulli $\quad F(x) = p^x\,(1-p)^{1-x}$ $\qquad$ $G(x) = qs^0 + ps^1$

Binomial $\quad p(k) = \binom{n}{k} p^k (1-\mathrm{p})^{\mathrm{n-k}}$ $\qquad$ $G(x) = (q+ps)^n$

Geometric $\quad p(k) = (1-p)^{k-1}\,p$ $\qquad$ $G(x) = \frac{p}{1-qs}$

Negative Binomial $\quad p(k) = \binom{n-1}{k-1}\,p^r\,(1-p)^{k-r}$ $\qquad$ $G(x) = \left(\frac{p}{1-qs}\right)^r$

Poisson $\quad p(k) = \frac{\lambda^k\,e^{-\lambda}}{k!}$ $\qquad$ $G(x) = e^{\lambda(s-1)}$

Uniform $\quad f(x) = \frac{1}{b-a}\,, a \le x \le b$

Exponential $\quad f(x) = \lambda e^{-\lambda x}\,, x \ge 0$

Gamma $\quad g(t) = \frac{\lambda^\alpha\,t^{\alpha-1}\,e^{-\lambda x}}{\Gamma(\alpha)}$ $\qquad$ $for\ \alpha = 1$, it tends to exponential distribution

Normal $\quad \frac{1}{\sqrt{2\pi\sigma^2}}\,e^{\frac{-(x-\mu)}{2\sigma^2}}$

$Let\ us\ assume\ that\ X1, X2, \dots, Xn\ are\ independent\ random\ variables\ with\ the\ common\ cdf\ F\ and\ density\ f. Let\ U\ denote\ the\ maximum\ of\ the\ X_i\ and\ V\ the\ minimum.$

$F_U(u) = [F(u)]^n$ $\qquad$ $f_U(u) = nf(u)[F(u)]^{n-1}$

$F_V(v) = 1 - [1-F(v)]^n$ $\qquad$ $f_V(v) = nf(v)[1-F(v)]^{n-1}$

$k^{th}\ order\ stats$ $\qquad$ $f_k(x) = \frac{n!}{(k-1)!\,(n-k)!}\,f(x)F^{k-1}(x)[1-F(x)]^{n-k}$

$If\ there\ are\ p\ experiments\ and\ the\ \text{first}\ has\ n_1\ possible\ outcomes, the\ second\ n_2, \dots, and\ the\ pth\ n_p\ possible\ outcomes, then\ there\ are\ a\ total\ of\ n_1 \times n_2 \times \cdots \times n_p\ possible\ outcomes\ for\ the\ p\ experiments$

$For\ a\ set\ of\ size\ n\ and\ a\ sample\ of\ size\ r, there\ are\ n^r\ different\ ordered\ samples\ with\ replacement\ and\ n(n-1)(n-2)\cdots(n-r+1)\ different\ ordered\ samples\ without\ replacement.$

$The\ number\ of\ unordered\ samples\ of\ r\ objects\ selected\ from\ n\ objects\ without\ replacement\ is\ \binom{n}{r}.$

$The\ number\ of\ ways\ that\ n\ objects\ can\ be\ grouped\ into\ r\ classes\ with\ n_i\ in\ the\ i^{th}\ class\ is$

$$\binom{n}{n_1\,n_2 \dots n_r} = \frac{n!}{n_1!\,n_2!\dots n_r!}$$

$CDF = \int_{-\infty}^{x} PDF$ $\qquad$ $CDF = F_{\mathrm{X}}(x) = P(\mathrm{X} \le x)$

$If\ X \sim N(\mu,\sigma^2)\ and\ Y = aX+b, then\ Y \sim N(a\mu+b, a^2\sigma^2).$

$Let\ X\ have\ f(x) and\ let\ Y = g(X)$ $\qquad$ $f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\frac{d}{dy}g^{-1}(y)\right|$

$PROPOSITION\ C$ $\qquad$ $Let\ Z = F(X);\ then\ Z\ has\ a\ uniform\ distribution\ on\ [0,1].$

$PROPOSITION\ D$ $\qquad$ $Let\ U\ be\ uniform\ on\ [0,1], and\ let\ X = F-1(U). Then\ the\ cdf\ of\ X\ is\ F.$

$E(X) = \sum_i x_i\,p(x_i)\ \ or\ \ E(X) = \int_{-\infty}^{\infty} xf(x)dx$ $\qquad$ $Suppose\ that\ Y = g(X),\ \ replace\ x = g(x)$

$THEOREM\ A$ $\qquad$ $Markov's\ Inequality$

$If\ X\ is\ a\ random\ variable\ with\ P(X \ge 0) = 1\ and\ for\ which\ E(X) exists, then\ \ P(X \ge t) \le \frac{E(X)}{t}$

$If\ X_1, \ldots, X_n\ are\ jointly\ distributed\ random\ variables\ with\ expectations\ E(X_i) and\ Y\ is\ a\ linear\ function$

$$of\ the\ X_i, Y = a + \sum_{i=1}^{n} b_i\, E(X_i), \quad then\ E(Y) = a + \sum_{i=1}^{n} b_i\, E(X_i)$$

$$Var(X) = E(X^2) - [E(X)]^2 \quad or \quad Var(X) = E\{[X - E(X)]^2\}$$

$$If\ Var(X)\ exists\ and\ Y = a + bX, then\ Var(Y) = b^2 Var(X).$$

$THEOREM\ C \qquad Chebyshev's\ Inequality$

$Let\ X\ be\ a\ random\ variable\ with\ mean\ \mu\ and\ variance\ \sigma^2. Then, for\ any\ t > 0,$

$$P(|X - \mu| > t) \leq \frac{\sigma^2}{t^2}$$

$COROLLARY\ A \qquad If\ Var(X) = 0, then\ P(X = \mu) = 1.$

$A\ Model\ for\ Measurement\ Error$

$$X = x_0 + \beta + \varepsilon \qquad E(X) = x_0 + \beta \quad and \quad Var(X) = \sigma^2 \qquad \beta\ is\ bias$$

$$MSE = E[(X - x_0)^2] = \beta^2 + \sigma^2$$

$$Cov(X, Y) = E(XY) - E(X)E(Y)$$

$$Cov(a + X, Y) = Cov(X, Y)$$

$$Cov(aX, bY) = abCov(X, Y)$$

$$Cov(X, Y + Z) = Cov(X, Y) + Cov(X, Z)$$

$$Cov(aW + bX, cY + dZ) = acCov(W, Y) + bcCov(X, Y) + adCov(W, Z) + bd\ Cov(X, Z)$$

$$U = a + \sum_{i=1}^{n} b_i X_i \ \ and\ \ V = c + \sum_{j=1}^{n} d_j Y_j \ , then \quad Cov(U, V) = \sum_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n}} b_i d_j\ Cov(U, V)$$

$$Var\left(a + \sum_{i=1}^{n} b_i X_i\right) = \sum_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n}} b_i b_j\ Cov(X_i, X_j)$$

$$Var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} Var(X_i) \quad if\ \ X_i\ are\ independent$$

$$Correlation \quad \rho = \frac{Cov(X, Y)}{\sqrt{Var(X)Var(Y)}}$$

$$-1 \leq \rho \leq 1. Furthermore, \rho = \pm 1\ if\ and\ only\ if\ P(Y = a + bX) = 1\ for\ some\ constants\ a\ and\ b.$$

$$\binom{n}{k} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

$$|A_1 \cup A_2 \cup \ldots \cup A_p| = \sum_{1 \leq i \leq p} |A_i| - \sum_{1 \leq i_1 < i_2 \leq p} |A_{i_1} \cap A_{i_2}| + \sum_{1 \leq i_1 < i_2 < i_3 \leq p} |A_{i_1} \cap A_{i_2} \cap A_{i_3}| - \ldots + (-1)^{p-1} |A_1 \cap A_2 \cap \cdots \cap A_p|,$$

Inclusion Exclusion

Derangement $n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots + (-1)^n \frac{1}{n!}\right) = n! \sum_{r=0}^{n} (-1)^r \frac{1}{r!}$

$\hat{I}(f) = \frac{1}{n}\sum_{i=1}^{n} f(X_i)$

Law of large numbers: - $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ $\quad E[\bar{X}_n] = \mu$ $\quad Var[\bar{X}_n] = \frac{\sigma^2}{n}$

Central limit theorem: - MGF = $\int_{-\infty}^{\infty} e^{tx} f(x)dx$ $\quad P(a > Y > b) = \emptyset\left(\frac{Y-b}{\sigma}\right) - \emptyset\left(\frac{Y-a}{\sigma}\right)$

$E[Y] = nE[X] \quad Var[Y] = n^2\, Var(X)$

Continuity Correction: - $P(A) = P\left(l - \frac{1}{2} \leq Y \leq u + \frac{1}{2}\right)$

Chi Square Distribution: - $\chi_n = U_1 + U_2 + \cdots + U_n \quad where\ \ U = Z^2\ \ ,\ Z\ is\ Standard\ Normal\ Distribution$

It is gamma distribution $\frac{\lambda^{\alpha}\, t^{\alpha-1}\, e^{-\lambda x}}{\Gamma(\alpha)}$ with $\alpha = \frac{n}{2}\ \ and\ \ \lambda = \frac{1}{2}\ \ for\ DOF = n$ $\qquad$ MGF = $(1-2t)^{-\frac{n}{2}}$

t Distribution: - $\frac{Z}{\sqrt{\frac{U}{n}}}$ $\qquad f(x) = \frac{\frac{\Gamma(n+1)}{2}}{\sqrt{n}\ \pi\ \Gamma(\frac{n}{2})}\left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$ $\qquad f(x) = f(-x)$

Sample Mean $\bar{X} = \sum_{i=1}^{n} X_i$ $\quad$ Sample Variance $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 \quad or \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i{}^2 - n\,\bar{X}^2)$

$\frac{(n-1)s^2}{\sigma^2} \longrightarrow chi\ square\ with\ (n-1)\ DOF$ $\qquad \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \longrightarrow t\ distribution\ with\ t_{n-1}$

For Simple Random Sampling

Population Parameter	Estimate	Variance of Estimate	Estimated Variance
μ	$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$	$\sigma^2_{\overline{X}} = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$	$s^2_{\overline{X}} = \frac{s^2}{n}\left(1 - \frac{n}{N}\right)$
p	$\hat{p} =$ sample proportion	$\sigma^2_{\hat{p}} = \frac{p(1-p)}{n}\left(\frac{N-n}{N-1}\right)$	$s^2_{\hat{p}} = \frac{\hat{p}(1-\hat{p})}{n-1}\left(1 - \frac{n}{N}\right)$
τ	$T = N\overline{X}$	$\sigma^2_T = N^2\sigma^2_{\overline{X}}$	$s^2_T = N^2 s^2_{\overline{X}}$
σ^2	$\left(1 - \frac{1}{N}\right)s^2$		

where $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$.

$\mu = \frac{1}{N}\sum_{i=1}^{n} x_i \qquad \tau = N\mu \qquad \sigma^2 = s^2 \qquad Var(\bar{X})\ with\ replacement = \frac{\sigma^2}{n}$

For Stratified Random Sampling

$L\ stratas\ have\ pop\ elements\ l = N_l\ ,\ \ N = N_1 + N_2 + \cdots + N_l\ , \qquad population\ mean\ and\ variance\ \mu_l\ and\ \sigma_l^2$

$W_l = \frac{N_l}{N} \quad , \quad x_{il} \longrightarrow i^{th}\ pop\ in\ l^{th}\ stratum \quad , \quad \mu = \sum_{l=1}^{L} W_l\, \mu_l$

$Sample\ mean\ \bar{X}_l = \frac{1}{n_l}\sum_{i=1}^{l} X_{il} \qquad Startified\ estimate\ \bar{X}_s = \sum_{l=1}^{L} W_l\,\bar{X}_l \qquad Population\ total\ T_s = N\,\bar{X}_s$

$Variance\ of\ startified\ mean\ Var(\bar{X}_s) = \sum_{l=1}^{L} W_l^2\ \left(\frac{1}{n_l}\right)\left(1 - \frac{n_l - 1}{N_l - 1}\right)\sigma_l^2$

$s_l^2 = \frac{1}{n_l - 1}\sum_{i=1}^{n_l}(X_{il} - \bar{X}_l)^2 \qquad s^2_{\bar{X}_s} = \sum_{l=1}^{L} W_l^2\ \left(\frac{1}{n_l}\right)\left(1 - \frac{n_l}{N_l}\right) s_l^2$

Neyman Allocation $n_l = n\,\frac{W_l\,\sigma_l}{\sum_{k=1}^{L} W_k\,\sigma_k}$ $\qquad proportional\ allocation\ n_l = n\frac{N_l}{N}$

$Var(\bar{X}_{sp}) = \frac{1}{n} \ (\sum_{l=1}^{L} W_l\, \sigma_l^2)^2 \qquad Var(\bar{X}_{so}) = \frac{1}{n} \sum_{l=1}^{L} W_l\, \sigma_l^2 \qquad Var(\bar{X}_{sp}) - Var(\bar{X}_{so}) = \sum_{l=1}^{L} W_l\, (\sigma_l - \bar{\sigma})^2$

$MLE\ l(\theta) = \sum_{i=1}^{n} \log(f(X_i|\theta)) \qquad MLE\ multinomial\ Cell\ Prbabilities\ \ f(x_i \mid p_m) = \frac{n!}{\sum x_m!} \sum (p_m)^{x_m}$

$Confidence\ intervals\ for\ MLE$

$$P\left(\bar{X} - \frac{S}{\sqrt{n}}\, t_{n-1}\left(\frac{\alpha}{2}\right) \leq \ \mu \leq \ \bar{X} + \frac{S}{\sqrt{n}}\, t_{n-1}\left(\frac{\alpha}{2}\right)\right) = 1 - \alpha$$

$$P\left(\frac{n\,\hat{\sigma}^2}{\chi^2_{n-1\left(\frac{\alpha}{2}\right)}} \leq \ \sigma^2 \leq \ \frac{n\,\hat{\sigma}^2}{\chi^2_{n-1\left(\frac{1-\alpha}{2}\right)}}\right) = \ 1 - \alpha$$

$Hypothesis\ Testing$

$Null\ Hypothesis\ (H_0) \qquad Alternate\ Hypothesis\ (H_A)$

$P(reject\ H_0\ |H_0) = P(X > c \mid H_0) \longrightarrow Type\ I\ error\ (\alpha) \quad Significance\ Level = \ \alpha$

$P(accept\ H_0\ |H_1) = p(X \leq c\ |H_1) \longrightarrow \ Type\ II\ error\ (\beta)\ Power = 1 - \ \beta$

$$Z(\alpha) = \frac{x_0 - \ \mu_0}{\frac{\sigma}{\sqrt{n}}} \qquad\qquad Z(\alpha) = \frac{\bar{X} - \ \mu_0}{\frac{S}{\sqrt{n}}}$$

$Likelihood\ Ratio\ Tests$

$$\hat{p_i} = \frac{x_i}{n} \qquad \Lambda = \ \prod_{i=1}^{m}\left(\frac{P_i(\hat{\theta})}{\hat{P_i}}\right)^{x_i} \qquad x_i = observed\ count\ in\ m\ cells,\ \ \hat{\theta} = MLE\ of\ \theta$$

$p_i\ (\hat{\theta}) = corresponding\ probabilities$

$$-2\log\Lambda = 2\sum_{i=1}^{m} O_i \log\left(\frac{O_i}{E_i}\right) \qquad O_i = n\,\hat{p}_i \quad and \quad E_i = \ n\,p_i\,(\hat{\theta})$$

$$Pearson's\ chi-square\ statistic \quad \chi^2 = \ \sum \frac{\left[x_i - \ n\,p_i\,(\hat{\theta})\right]^2}{n\,p_i\,(\hat{\theta})}$$