EE 3305 - Signals and Systems

Lecture 7

Dr. C.K.W.Seneviratne

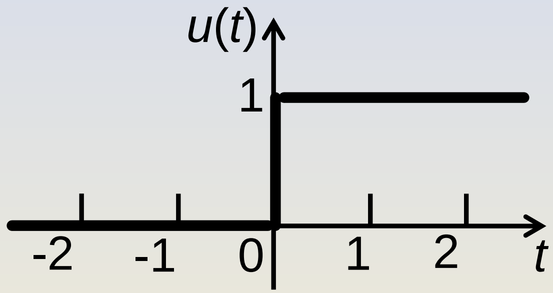
Department of Electrical and Information Engineering,

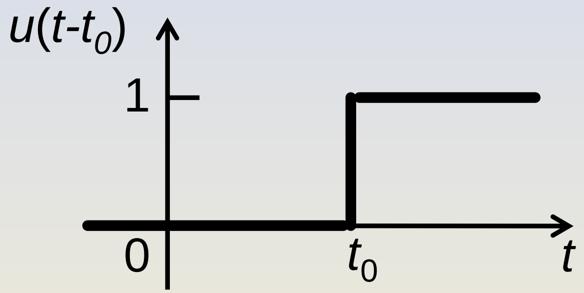
University of Ruhuna

August 07, 2020

Unit Step Function

- The unit Step function is used to keep only part of a signal (for $t > t_0$)
- Definition:





$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

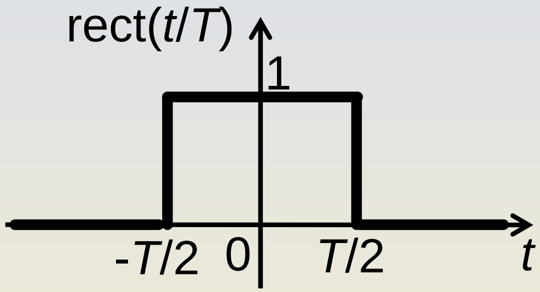
$$u(t - t_0) = \begin{cases} 1, & t > t_0 \\ 0, & t < t_0 \end{cases}$$

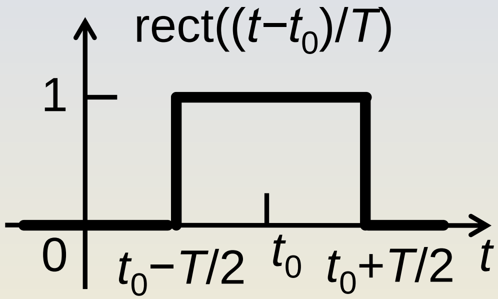
- Properties:

1. $u(t-t_0) = [u(t-t_0)]^2 = [u(t-t_0)]^k$
2. $u(at-t_0) = u(t-t_0/a) \qquad , a \neq 0$

Unit Rectangular Pulse Function

- The unit rectangular pulse function is used to keep only part of a signal (for $(t_0 - T/2) < t < (t_0 + T/2)$)
- Definition:





$$rect(t/T) = \begin{cases} 1, & -T/2 < t < T/2 \\ 0, & otherwise \end{cases}$$

$$rect(\frac{t - t_0}{T}) = \begin{cases} 1, & t_0 - \frac{T}{2} < t < t_0 + T/2 \\ 0, & otherwise \end{cases}$$

- Properties:

$$\begin{aligned} \text{rect}(t/T) &= u(t + T/2) - u(t - T/2) \\ &= u(-t + T/2) - u(-t - T/2) \\ &= u(t + T/2) \cdot u(-t + T/2) \end{aligned}$$

Examples

- Plot the following signals:

$$x(t) = (t - t_0) \cdot \mathrm{u}(t - t_0)$$ → Ramp function

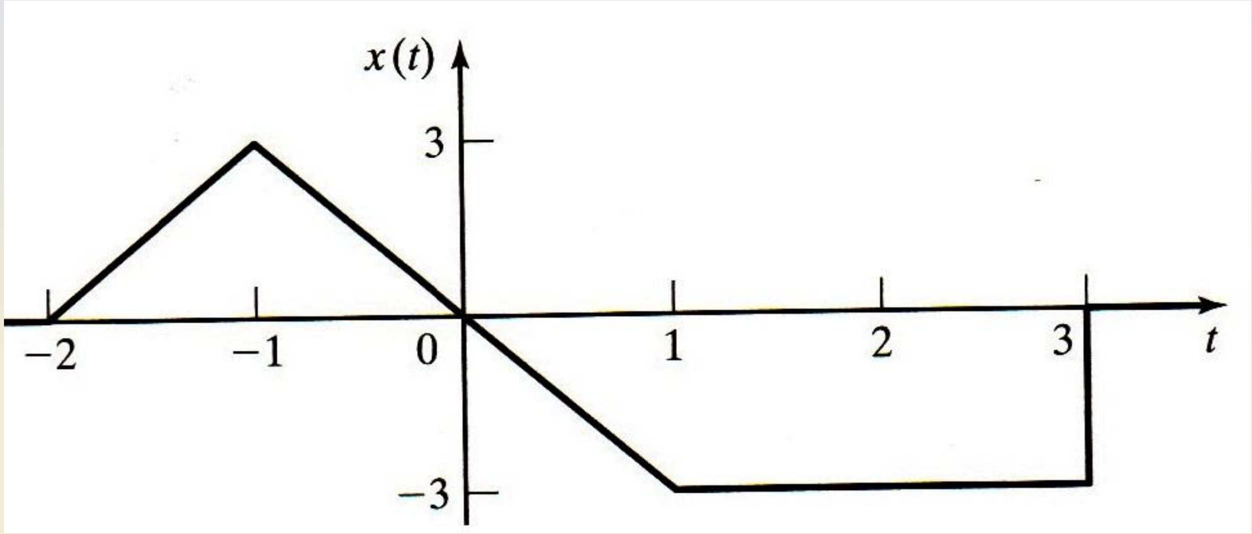
$$x(t) = e^{-t} \cdot \mathrm{u}(t+1)$$

$$x(t) = e^{-t} \cdot \mathrm{rect}(t/2)$$

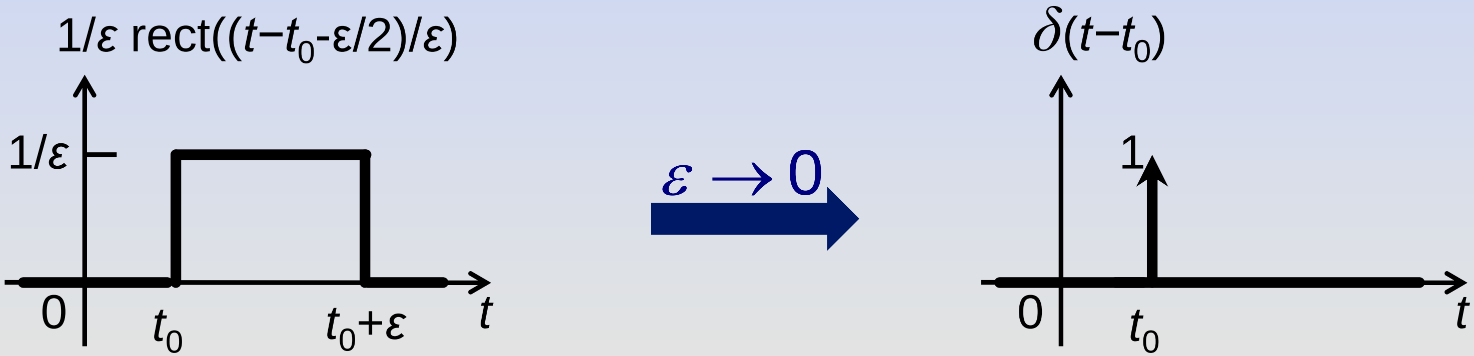
$$x(t) = \cos(t) \cdot \mathrm{rect}\left[(t-\pi)/2\pi\right]$$

Example

- Express the signal of the plot below as a summation of step functions



Unit Impulse Function (Dirac delta function)



$$\frac{1}{\varepsilon}\mathrm{rect}\left[\left(t-t_0-\frac{\varepsilon}{2}\right)\Big/\varepsilon\right]\xrightarrow[\varepsilon\to 0]{}\delta\left(t-t_0\right)$$

$$\begin{cases}\delta\left(t-t_0\right)=0, \quad t\neq t_0\\ \int_{-\infty}^{\infty}\delta\left(t-t_0\right)dt=1\end{cases}$$

Properties of the Unit Impulse Function

1. $\dfrac{du(t-t_0)}{dt} = \delta(t-t_0)$

2. $u(t-t_0) = \int_{-\infty}^{t} \delta(\tau - t_0)\,d\tau$

3. $f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$, f continuous at $t = t_0$

4. $\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,dt = f(t_0)$, f continuous at $t = t_0$

5. $\int_{-\infty}^{\infty} f(t-t_0)\delta(t)\,dt = f(-t_0)$, f continuous at $t = -t_0$

6. $\delta(-t) = \delta(t)$

7. $\delta(at) = \dfrac{1}{|a|}\delta(t)$, $a \neq 0$

Home Work

- Simplify the following functions:

$$\left(t^3+3\right)\delta\left(t-t_0\right)$$

$$e^{-2t}\delta\left(t-t_0\right)$$

- Calculate

$$\int_{-\infty}^{\infty}\delta\left(t-t_0\right)e^{-j\omega t}dt$$

Basic Discrete Time Signals

- Discrete time complex exponential signals

$$f[n] = A\alpha^n$$

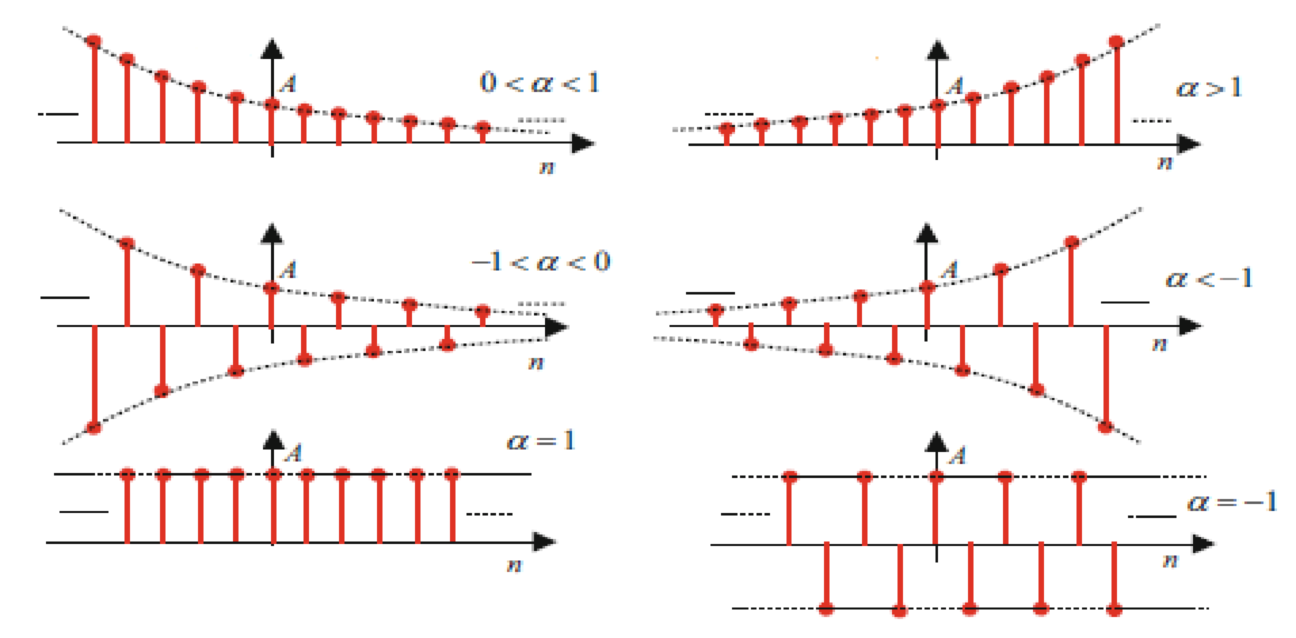
- Different types of behaviors can be seen based on the A and α
  - If A and α are real :
    - | α | > 1: f[n] grows exponentially
    - | α | < 1: f[n] decaying exponentially
    - α is positive : all values of f[n] are of the same sign
    - a is negative : sign of f[n] alternates
    - α = 1 : f[n] is constant
    - α = -1 : f[n] is alternate between +A and –A

Basic Discrete Time Signals

- Discrete time complex exponential signals

$$f[n] = A\alpha^n$$

- Different types of behaviors can be seen based on the A and α
  - If A and α are real :



Basic Discrete Time Signals

- When α= $e^{\beta n}$

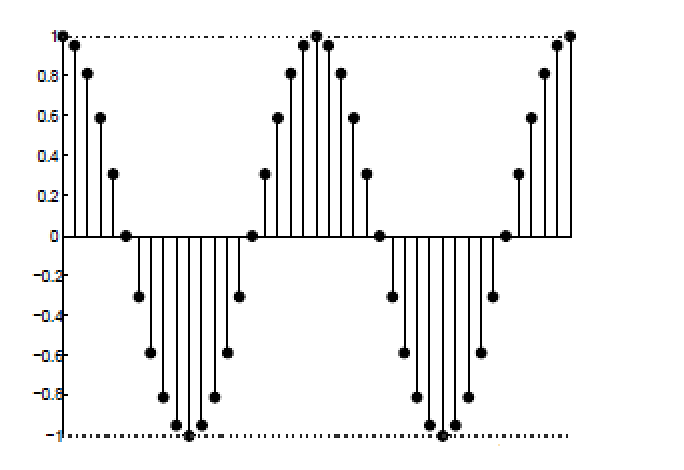
$$f[n] = Ae^{\beta n}$$

- If A is real and β is imaginary :

$$f[n] = Ae^{jn\Omega}$$

- Using Euler's expansion,

$$f[n] = Ae^{jn\Omega} = A cos(n\Omega) + \text{j } A sin(n\Omega)$$

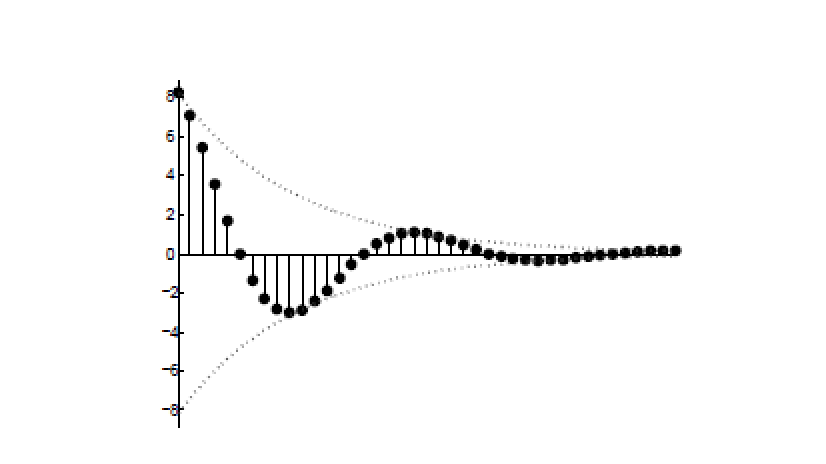


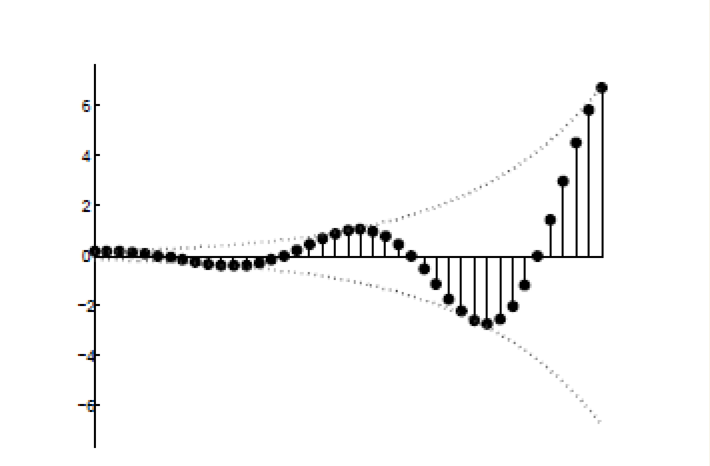
Basic Discrete Time Signals

- General complex exponential function can be written as

$$f[n] = A\alpha^n = |A||\alpha|^n cos(\Omega n + \theta) + j|A||\alpha|^n sin(\Omega n + \theta)$$

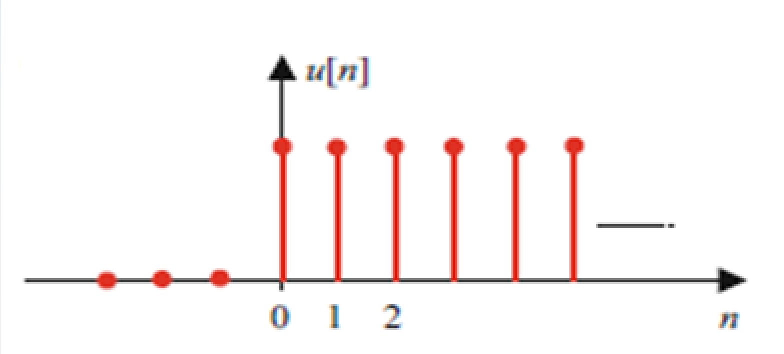
- For | α | = 1 : real and imaginary parts of a complex exponential are sinusoidal
- For | α | < 1 : sinusoidal sequences are multiplied by a decaying exponential
- For | α | > 1 : sinusoidal sequences are multiplied by a growing exponential





Basic Discrete Time Signals

- Discrete time unit step function : the signal has a value of unity only where the argument zero.

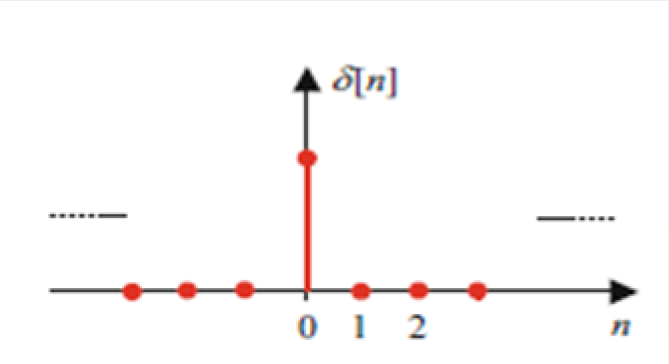


- Mathematically defined as

$$\mathrm{u}[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases} \qquad u[n-K] = \begin{cases} 0, & n < K \\ 1, & n \geq K \end{cases}$$

Basic Discrete Time Signals

- Discrete time unit impulse function : the signal has a value of unity only where the argument zero.



- Mathematically defined as

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases} \qquad \delta[n-K] = \begin{cases} 0, & n \neq K \\ 1, & n = K \end{cases}$$

Basic Discrete Time Signals

- Properties of the DT impulse function

	Summation/Product	Result
1	$\delta[n]$	$u[n] - u[n-1]$
2	$\sum_{k=-\infty}^{\infty} \delta[k]$	1
3	$\sum_{k=-\infty}^{\infty} \delta[-k]$	1
4	$\sum_{k=-\infty}^{n} \delta[k]$	$u[n]$
5	$\sum_{k=-\infty}^{n} \delta[k-M]$	$u[n-M]$
6	$\sum_{k=-\infty}^{\infty} x[k]\delta[k-M]$	$x[M]$
7	$x[M]\sum_{k=-\infty}^{\infty} \delta[k-M]$	$x[M]$
8	$\sum_{k=-\infty}^{\infty} x[k-M]\delta[k]$	$x[-M]$
9	$\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$	$x[n]$
10	$x[n]\delta[n]$	$x[0]\delta[n]$

Examples

- Evaluate the following summations

$$\sum_{k=-\infty}^{\infty} \delta[k-4]u[k]$$

$\sum_{k=-\infty}^{4} r[k]\delta[k+6]$ where r[k] is the DT unit ramp function

$\sum_{k=-\infty}^{4} r[k]\delta[k-6]$ where r[k] is the DT unit ramp function

$$\sum_{k=-\infty}^{\infty} \cos(k\frac{\pi}{6})\delta[k-4]$$

$$\sum_{k=-\infty}^{\infty} \mathrm{x[k]}\delta[k-4]$$