First Stage DC Analysis

- Applying Thevenin representation to the following circuit:

$$R_{B1} = R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{330 \times 10^3}{2} = 165\,\mathrm{k\Omega}$$

$$V_{B1} = V_{Th} = V_{CC} \times \frac{R_2}{R_1 + R_2} - V_{EE} \times \frac{R_1}{R_1 + R_2}$$
$$= 9 \times \frac{330 \times 10^3}{2(330 \times 10^3)} - 9 \times \frac{330 \times 10^3}{2(330 \times 10^3)} = 0\,\mathrm{Volt}$$

- Apply Kirchhoff's Voltage Law (KVL) to the input loop, which includes the Thevenin resistance R_{B1}, the base-emitter junction, and the emitter resistor:

$$V_{B1} - I_{B1}R_{B1} - V_{BE} - I_{E1}(R_3 + R_4) + V_{CC} = 0$$

$$V_{B1} - I_{B1}R_{B1} - V_{BE} - I_{B1}(1 + \beta_1)(R_3 + R_4) + V_{CC} = 0$$
$$0 - I_{B1} \cdot 165k - 0.7 - I_{B1}(1 + 205)(33k + 1k) + 9 = 0$$
$$I_{B1} = \frac{9 - 0.7}{165000 + 206 \times 34000} = 1.15\,\mathrm{\mu A}$$
$$I_{C1} = \beta_1 I_{B1} = 205 \times 1.15 \times 10^{-6} = 0.235\,\mathrm{mA} \approx 0.24\,\mathrm{mA}$$
$$I_{E1} = (\beta_1 + 1)I_{B1} = 206 \times 1.15 \times 10^{-6} = 0.24\,\mathrm{mA}$$
$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.24 \times 10^{-3}}{0.026} = 9.23\,\mathrm{mA/V}$$
$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{205}{9.23 \times 10^{-3}} = 22.21\,\mathrm{k\Omega}$$

Second Stage DC Analysis

- Applying Thevenin representation:

$$R_{B2} = R_{Th2} = \frac{R_5 R_6}{R_5 + R_6} = \frac{47 \times 22}{47 + 22} = 15\,\mathrm{k\Omega}$$

$$V_{B2} = V_{Th2} = V_{CC} \times \frac{R_6}{R_5 + R_6} + (-V_{EE}) \times \frac{R_5}{R_5 + R_6}$$
$$= 9 \times \frac{22}{47 + 22} - 9 \times \frac{47}{47 + 22} = -3.26\,\mathrm{V}$$

- Apply KVL:

$$V_{B2} - I_{B2} R_{B2} - V_{BE} - I_{E2}(R_7 + R_8) + (-V_{EE}) = 0$$
$$V_{B2} - I_{B2} R_{B2} - V_{BE} - I_{B2}(1 + \beta_2)(R_3 + R_4) - V_{EE} = 0$$
$$3.26 - I_{B2} \cdot 15k - 0.7 - I_{B2}(1 + 287)(100 + 10000) - 9 = 0$$
$$I_{B2} = \frac{9 - 3.26 - 0.7}{15000 + 288 \times 10100} = 1.72\,\mathrm{\mu A}$$
$$I_{C2} = \beta_2 I_{B2} = 287 \times 1.72 \times 10^{-6} = 0.49\,\mathrm{mA} \approx 0.5\,\mathrm{mA}$$
$$I_{E2} = (\beta_2 + 1) I_{B2} = 288 \times 1.72 \times 10^{-6} = 0.5\,\mathrm{mA}$$
$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{0.5 \times 10^{-3}}{0.026} = 19.23\,\mathrm{mA/V}$$
$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = \frac{287}{19.23 \times 10^{-3}} = 14.924\,\mathrm{k\Omega}$$

Third Stage DC Analysis

- Apply KVL:

$$V_{CC} - I_{B3}R_{B3} - V_{BE} - I_{E3}R_{E3} = -V_{EE}$$

$$V_{CC} - I_{B3}R_{B3} - V_{BE} = I_{B3}(1+\beta_3)R_{E3}$$

$$9 - I_{B3} \cdot 100k - 0.7 = I_{B3}(1+214) \cdot 390$$

$$I_{B3} = \frac{9+9-0.7}{100000 + 215 \times 390} = 94.1\,\mu\text{A}$$

$$I_{C3} = \beta_3 I_{B3} = 214 \times 94.1 \times 10^{-6} = 20.1\,\text{mA}$$

$$I_{E3} = (\beta_3 + 1)I_{B3} = 215 \times 94.1 \times 10^{-6} = 20.2\,\text{mA}$$

$$g_{m3} = \frac{I_{C3}}{V_T} = \frac{20.1 \times 10^{-3}}{0.026} = 773\,\text{mA/V}$$

$$r_{\pi 3} = \frac{\beta_3}{g_{m3}} = \frac{214}{773 \times 10^{-3}} = 276.84\,\Omega$$

AC Analysis

$R_{in} = R_{B1} \parallel (r_{\pi 1} + (1+\beta_1)R_{E1}) = 165 \parallel (22.21 + (1+205)\times 1) = 165 \parallel 228.21 = 95.76\,\mathrm{k\Omega}$

$$R_{out} = R_{E3} \parallel R_L \parallel \left(\frac{R_{C1} \parallel R_{B2} + r_{\pi 3}}{\beta_3 + 1}\right) = 390 \parallel 10000 \parallel \left(\frac{100000 \parallel 6800 + 276.84}{215}\right) = 1.312\,594\,\Omega$$

$$v_{\pi 1} = v_{in} \times \frac{r_{\pi 1}}{r_{\pi 1} + (1+\beta_1)R_{E1}}$$

$$v_{\pi 2} = -g_{m1} v_{\pi 1} \times \left(\frac{R_{C1} \parallel R_{B2}}{(R_{C1} \parallel R_{B2}) + r_{\pi 2} + (1+\beta_2)R_{E2}}\right) \times r_{\pi 2}$$

$$v_{\pi 3} = -g_{m2} v_{\pi 2} \times \left(\frac{R_{C2} \parallel R_{B3}}{(R_{C2} \parallel R_{B3}) + r_{\pi 3} + (1+\beta_3)(R_{E3} \parallel R_L)}\right) \times r_{\pi 3}$$

$$v_o = (R_{E3} \parallel R_L) \times \left(\frac{v_{\pi 3}}{r_{\pi 3}} + g_{m3} v_{\pi 3}\right) = (R_{E3} \parallel R_L) \times v_{\pi 3} \left(\frac{1}{r_{\pi 3}} + g_{m3}\right)$$

$$A_v = \frac{v_o}{v_{in}} = -g_{m3} g_{m2} v_{in} \times \frac{r_{\pi 1}}{r_{\pi 1} + (1+\beta_1)R_{E1}} \times \frac{R_{C1} \parallel R_{B2}}{(R_{C1} \parallel R_{B2}) + r_{\pi 2} + (1+\beta_2)R_{E2}} \times r_{\pi 2}$$

$$\times \frac{R_{C2} \parallel R_{B3}}{(R_{C2} \parallel R_{B3}) + r_{\pi 3} + (1+\beta_3)(R_{E3} \parallel R_L)} \times r_{\pi 3} \times (R_{E3} \parallel R_L) \times \left(\frac{1+\beta_3}{r_{\pi 3}}\right)$$

$$= \beta_1 \beta_2 (1+\beta_3) \times \left(\frac{(R_{C1} \parallel R_{B2})(R_{E3} \parallel R_L)(R_{C2} \parallel R_{B3})}{(r_{\pi 1} + (1+\beta_1)R_{E1})((R_{C1} \parallel R_{B2}) + r_{\pi 2} + (1+\beta_2)R_{E2})(R_{C2} \parallel R_{B3}) + r_{\pi 3} + (1+\beta_3)(R_{E3}}\right.$$

$$= 371.98\,\mathrm{V/V} = 51.41\,\mathrm{dB}$$

$$\frac{A_v}{\sqrt{2}} = 263.02\,\mathrm{V/V} = 48.4\,\mathrm{dB}$$