Properties:

1. The identity element is unique.

Proof: Let e and e' be 2 ~~identities~~ identity elements of a group G.

Since e is an identity element,

$$\therefore \quad ae = ea = a \quad \forall a \in G.$$

In particular for $e' \in G$

$$e'e = ee' = e' \quad \text{——— } (1)$$

Since e' is an identity element,

$$\therefore \quad ae' = e'a = a \quad \forall a \in G$$

In particular for $e \in G$

$$ee' = e'e = e \quad \text{——— } (2)$$

From (1) + (2),

$$e = e'.$$

$\therefore$ The identity element is unique.

2. Each element in a group has a unique inverse.

Proof:

Let $a \in G$. Let a' and a'' be two inverses of a.

As a' is an inverse of a

$$\therefore \; a a' = a' a = e \quad \text{—— (1)}$$

As a'' is an inverse of a

$$\therefore \; a a'' = a'' a = e \quad \text{—— (2)}$$

$$\begin{aligned} a'' &= a'' e \\ &= a''(a a') \quad , \text{from (1) } a = a a' \\ &= (a'' a) a' \quad , \text{by associative low} \\ &= e a' \quad , \text{by (2)} \\ &= a' \end{aligned}$$

$\therefore$ The inverse is unique.