

Assignment 2

August 31, 2020

1 Assignment 2 - EDS

1.0.1 Portfolio Optimization to maximize the idiosyncratic risk

A client that uses a factor model for their risk calculations wants us to develop a portfolio optimizer that maximizes their idiosyncratic risk. They want to specify a maximum amount of trading, a maximum overall volatility, and a maximum (idiosyncratic risk as a % of total idiosyncratic risk) for any individual ticker. Please write out the problem statement as a series of matrix inequalities.

Assumptions and Variables - Let W be a matrix with each term in W representing the amount invested in corresponding instrument. Also, let Σ represent the variance-covariance matrix between all the underlying instruments. Based on our given constraints, let the maximum amount of trading be c and the maximum overall volatility be σ_t .

Now let's define a factor model similar to CAPM (by ignoring the α), the factor model can be described as -

$$R_p - r_f = \beta' \cdot (R_F - r_f) + \epsilon$$

where R_p is the return of the portfolio, r_f is the risk free return in the market, R_F is the return for the factors and β denotes the factor loadings. All these are represented in vectors and matrices format. As an expression for the idiosyncratic risk (non systematic risk), the model above uses the standard deviation of the residuals ϵ - this is defined as σ_ϵ where -

$$\sigma_\epsilon^2 = \sigma_p^2 - \beta^2 \sigma_F^2$$

This is the total idiosyncratic risk, where -

$$\sigma_p^2 = W^T \Sigma W$$

$$\beta = W^T \beta_i$$

And, the idiosyncratic risk for any one instrument is σ_i , where

$$\sigma_{\epsilon_i}^2 = \sigma_i^2 - \beta_i^2 \sigma_F^2$$

Let the maximum idiosyncratic risk as a % of total idiosyncratic risk for any individual ticker be $r_{max} \cdot 100$ %

So our problem statement can be written as -

$$\begin{aligned}
& \max_W && W^T \Sigma W - W^T \beta_i \sigma_F^2 \\
& \text{s.t.} && W^T \Sigma W \leq \sigma_t^2 \\
& && 1^T \cdot |W| \leq c \\
& && \frac{\sigma_{\epsilon_i}}{\sigma_\epsilon} \leq r_{max} \quad \forall i
\end{aligned} \tag{1}$$