Control Systems (Assignment - 1)

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The Gompertz growth model is commonly used to model tumor cell growth. The volume of tumor is given by equation -

$$\frac{dv(t)}{dt} = \lambda e^{-\alpha t} v(t)$$

where λ and α are two appropriat e constants.

Solution a)

if we integrate the given equation -

$$\int_{v_0}^{v(t)} \frac{dv}{v} dv = \int_0^t \lambda e^{-\alpha t} dt$$

Upon integrating

$$ln(\frac{v(t)}{v_0}) = \frac{\lambda}{\alpha}(e^{-\alpha t} - 1)$$

or

$$v(t) = v_0 \exp[\frac{\lambda}{\alpha}(1 - e^{-\alpha t})]$$

Solution b)

b)
in part a, we derived expression for v(t),

$$v(t) = v_0 \exp[\frac{\lambda}{\alpha}(1 - e^{-\alpha t})]$$

we can see that, as t $\rightarrow \infty$

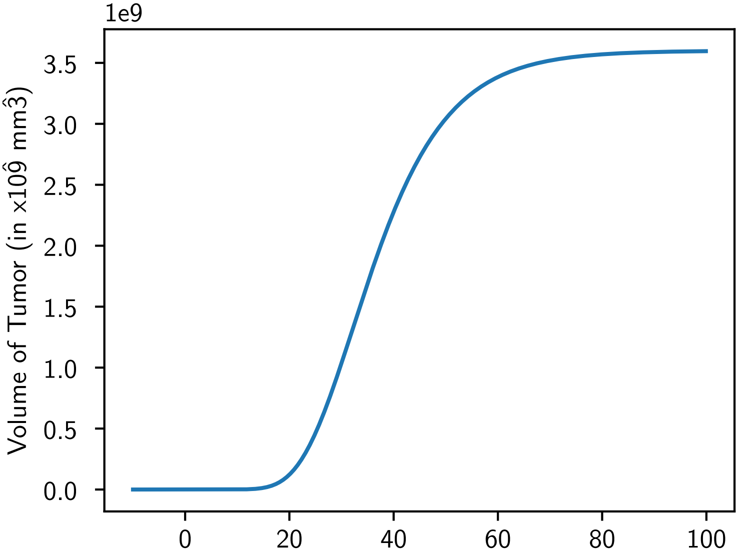
$$v = v_0 e^{\frac{\lambda}{\alpha}}$$

Solution c)

c)
For a specific mouse Tumor
$\lambda = 2.5$ days,
$\alpha = 0.1$ days
with $v_0 = 50 \times 10^{-3} mm^3$
Substituting these values into expression of v(t) from part a,

$$v(t) = 50 \times 10^{-3} \exp[25 \times (1 - e^{-0.1t})] mm^3$$

Volume of Tumor v/s Time Plot



Solution d)

d)
for λ= 2.5 days and α = 0.1 days

$$v = 50 \times 10^{-3} e^{\frac{2.5}{0.1}} \approx 3.6 \times 10^{9} mm^{3}$$

as $t \to \infty$
In the plot also, we can see that as t is sufficiently large (around 100 days), v(t) is approching the same value as we have calculated here.