AMME3500

Design Project 1

AMME3500
BY 470416309
Vishant Prasad

Contents

Project 1 – Cruise Control

Design Project 1

Introduction

Motivation

This report will focus on the design of a cruise control system working in conjunction with a controller that will be automatically changing lanes. This project specifically aims to design a controller allowing the vehicle to:

1. achieve any desired speed while still accounting for constant disturbances,
2. discuss the effect of a **system zero** or when the vehicle is reversing $v_0 < 0$,
3. change from one target speed to another and
4. be able to automatically change lanes.

<u>Note:</u> Even if the vehicle is influencing constant disturbances so that the true system is

$$m\dot{v} + \frac{1}{2}A\rho c_D v^2 = u + d,$$

where d is the disturbance.

Project Scope

From the objectives listed in the *motivation* section, the dynamics that will be analysed includes the change from speeds alongside force disturbances such as transitions in terrain and elevation with the examination of mass changes within the vehicle.

The first component of the control system will be the design of the cruise control which will encompass more specifically:

a. analysis of the steepness of hills and changes such as slope and
b. the analysis of the effect of additional passengers and/or cargo.

The second component of the controller design will focus on the lateral control of the system, more specifically the lane changing element.

a. Side-to-side motion of the vehicle with the wheel angle being the controller input.
b. The motion of the centre of mass and position described by the following differential equations.

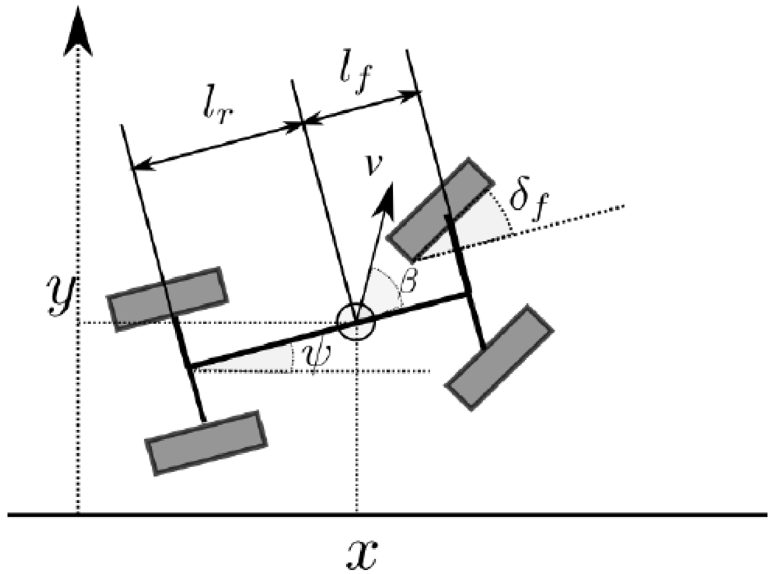


Figure 1: Lateral Control - Lane Changing

(AMME3500 Systems Dynamics and Control: Design Project 01, 2020)

Assumptions

Within this project we will assume that the vehicle (in reference to its wheelbase) is moving in the x direction.

- Use a PI controller for the cruise control design.
- Let the vehicle be moving in a straight line with its velocity described by $v(t)$ at time t.
- Assume an engine controller designed, so that the control input u is the force demanded from the engine:

$$m\dot{v} + \frac{1}{2} A\rho c_D v^2 = u$$

  - ρ is the density of air in kg/m^3 which is 1.225 constant.
  - c_D is a dimensionless drag coefficient.
  - A is the cross-sectional area of the vehicle in m^2.

Note: The values for A and c_D are specified below.

- It is the lateral position y that we want to control.
- The aim is to design a controller that will smoothly and accurately transition from lane-to-lane.
- Simulate the closed-loop system response for lane-change manoeuvres at a variety of speeds.
- The wheelbase is assumed to be $l_f + l_r$ from *figure 1* above, it is assumed that $l_f = l_r$ for simplicity.
- It is also assumed that the velocity of the vehicle is defined as: $v > 0$.

Formulations

The following is the formulas that are assumed to be a part of the control system:

1. The motion of the centre of mass (CoM) position (x, y) is described by the following differential equations.
$$\dot{y} = v\sin(\psi + \beta)$$
$$\dot{\psi} = \frac{v}{l_r}\sin\beta$$
Note: the coupling to longitudinal dynamics through $v(t)$.
2. Algebraic equation between δ_f and the centre of mas (CoM) rotation angle β:
$$\tan\beta = \frac{l_r}{l_f + l_r}\tan\delta_f$$
   a. The wheelbase is sourced and mentioned below in the parameter's sections:
$$l_f + l_r$$
3. A second-order differential equation describes $y(t)$ depends on the wheel angle function $\delta_f(t)$.
   a. The transfer function from steering-wheel angle δ_f to lateral position y that has the form:
$$G(s) = \frac{As + B}{s^2}$$

Parameters

The car that is selected as part of this project, which will be implemented with, or the cruise control system will be designed for, is the ***Ford Focus Sedan***.

Table 1: Vehicle Parameters - Ford Focus Sedan

Parameter	Measurement
Length	4647 *mm*
Width	1825 *mm*
Height	1483 *mm*
Curb Weight	1239 *kg*
c_D	0.36
v_0	$11.11ms^{-1}$
l_f & l_r	2648mm

$$\therefore A = 1.825m \times 1.483m = 2.71m^2\ (nearest\ two\ decimal\ places)(2dp)$$

Note: This is the frontal cross-sectional area of the vehicle (m^2).

System Modelling and Simulation Setup

For the Ford Focus Sedan, as mentioned above from the formularisation and assumptions, the wheelbase is $l_r + l_f$ of which $l_r = l_f$. Therefore, from the vehicle moving in the x direction, the lateral position y is what will be controlled. If the dynamics is linearized the constant speed motion with the following equations:

$v(t) \approx v_0 > 0$ with small angles; $\emptyset \approx 0, \beta \approx 0, \delta \approx 0$, we get a second-order differential equation describing how $y(t)$ depends on $\delta_t(t)$.

This is evident by the following equations:

$$\ddot{y} = v_0(\dot{\emptyset} + \dot{\beta})$$

$$\beta = \tan^{-1}\left(\frac{l_r \tan \delta_f}{(l_r + l_f)}\right) \approx \left(\frac{l_r}{l_r + l_f}\right)\delta_f \; for \; \frac{-\pi}{2} < \delta_f < \frac{\pi}{2}$$

$$\therefore \dot{\beta} = \left(\frac{l_r}{l_r + l_f}\right)\dot{\delta_f}$$

$$\dot{\emptyset} = \frac{v_0}{l_r}\sin\left(\tan^{-1}\left(\frac{l_r \tan \delta_f}{l_r + l_f}\right)\right) \approx \left(\frac{v_0}{l_r + l_f}\right)\delta_f$$

$$\therefore \ddot{y} = v_0\left[\left(\frac{v_0}{l_r + l_r}\right)\delta_f + \left(\frac{l_r}{l_r + l_f}\right)\dot{\delta_f}\right]$$

The transfer function $G(s)$ can be formulated as follows:

$$y = \Omega_1 e^{st}\delta_f = \Omega_2 e^{st}, \text{the transfer function } G(s) = \frac{\Omega_2}{\Omega_1}$$

<u>Note:</u> The steering-wheel angle δ_f to lateral position y.

$$s^2\Omega_1 e^{st} = As\Omega_2 e^{st} + B\Omega_2 e^{st}$$

$$\rightarrow s^2\Omega_1 = \Omega_2(As + B)$$

$$\therefore G(s) = \frac{As + B}{s^2}$$

<u>Note:</u>

$$A = \left(\frac{v_0}{2}\right) \;\&\; B = \left(\frac{{v_0}^2}{l_r + l_f}\right)$$

The variables of the equation that is linearized (the equation of the engine control model) using Taylor Series expansion, v and u (as mentioned $\dot{v} = f(v,u) \rightarrow f(v_c, u_c) = 0$) result as follows:

$$\mathcal{L}(f(v,u)) \approx f(v_e, u_e) + \frac{\partial f}{\partial v}(v_e, u_e)(v - v_e) + \frac{\partial f}{\partial u}(v_e, u_e)(u - u_e)$$

From the derivations above, if we let $\delta_v = v - v_e$ and $\delta_u = u - u_e$ be equations from equilibrium.

- The linear differential equation is $\therefore$ expressed as:

$$\dot{\delta_v} \approx \left(\frac{\partial f}{\partial v}\right)\delta_v + \left(\frac{\partial f}{\partial u}\right)\delta_u$$

$$\rightarrow \frac{\partial f}{\partial v}(v_e, u_e) = -\frac{A\rho c_D v}{m}\bigg|_{c = v_0}^{n/a} = \left(-\frac{A\rho c_D v_e}{m}\right)$$

$$\rightarrow \frac{\partial f}{\partial v}(v_e, u_e) = \frac{1}{m}$$

$$\therefore \dot{\delta_v} = \left(\frac{1}{m}\right)\delta_u + \left(-\frac{A\rho c_D v_e}{m}\right)\delta_v$$

Note: $\left(\frac{1}{m}\right) = \alpha \;\&\; \left(-\frac{A\rho c_D v_e}{m}\right) = \beta$ as follows above with the linearized equation.

Longitudinal Controller

Aim: To successfully design a PI controller that will adjust a vehicle's speed (with given parameters and overshoot values) to a desired input and plot the responding output.

- The vehicle speed control system must operate through disturbance forces, inclined slope disturbances and differing masses.

The model that was previously mentioned with the equation shown below, is to be considered with a constant disturbance, z, due to a gravitational force that will be acting on the vehicle. If the vehicle is modelled on an incline with the equation of gravitational force (as denoted below), the small angle approximations must be employed through the following theorems:

$$F = mg\sin\theta$$

$$\therefore \dot{\delta_v} = \left(\frac{1}{m}\right)\delta_u + \left(-\frac{A\rho c_D v_e}{m}\right)\delta_v$$

$$\sin\theta \approx \theta$$

$\therefore$ the linearized equation of the vehicle with z, can be expressed as:

$$\frac{d(v - v_e)}{dt} = \alpha(v - v_e) + \beta(u - u_e) - g\theta$$

Note: the values of α and β are given above, as determined from the original equation.

- Suitable parameters of the speed controller, as the linearized model above denotes, needs to be found.

The PI controller design that will manage the cruise control system (as described in the *assumptions* section) can be modelled with the equation:

$$u = k_p(v_c - v) + k_i\int_0^t (v_c - v(\tau))\,d\tau$$

From the *assumptions* section, it was mentioned that the velocity is constant (v_c) (vehicle sustains constant movement to the right) therefore, the error dynamics can be computed as follows:

$$velocity\ error \rightarrow E = (v_c - v)$$

Since velocity is constant:

$$\frac{dv}{dt} = -\frac{de}{dt} \;\; and \, \frac{d^2v}{dt^2} = -\frac{d^2e}{dt^2}$$

$$\frac{d(v-v_e)}{dt} = \alpha(v-v_e) + \beta(u-u_e) - g\theta \rightarrow \frac{d^2v}{dt^2} + \alpha\frac{dv}{dt} = \beta\frac{du}{dt} - g\frac{d\theta}{dt}$$

$$\therefore \frac{du}{dt} = k_p\frac{de}{dt} + k_i e$$

$$\rightarrow \frac{d^2e}{dt^2} + \left(\beta k_p - \alpha\right)\frac{de}{dt} + \beta k_i e = g\frac{d\theta}{dt}$$

$$g = 9.81ms^{-2}\ (nearest\ 2dp)$$

$$\therefore \frac{d^2e}{dt^2} + \left(\beta k_p - \alpha\right)\frac{de}{dt} + \beta k_i e = 9.81\frac{d\theta}{dt}$$

k_p and k_i can be determined for the controller parameter using the following equation:

$$\ddot{q} + 2\zeta\omega_n\dot{q} + {\omega_n}^2 q = ku$$

$$\frac{d^2e}{dt^2} + \left(\beta k_p - \alpha\right)\frac{de}{dt} + \beta k_i e = 9.81\frac{d\theta}{dt}$$

$$\therefore k_p = \frac{\alpha + 2\zeta\omega_n}{\beta}\ \&\ k_i = \frac{{\omega_n}^2}{\beta}$$

Note: The steady state error is zero if θ and e are constant.

$$rise\ time \rightarrow t_r = \frac{1.8}{\omega_n}$$

$$settling\ time \rightarrow t_s = \frac{3.93}{\sigma} = \frac{3.93}{\omega_n\zeta}$$

$$overshoot \rightarrow M = e^{-\frac{\sigma\pi}{\omega_d}} = e^{\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$peak\ time \rightarrow t_p = \frac{\pi}{\omega_d}$$

Note: The range for ζ is $0 < \zeta < 1$ for a stable system.

Target Simulation

From the above equations we can model the vehicle cruise control system as follows:

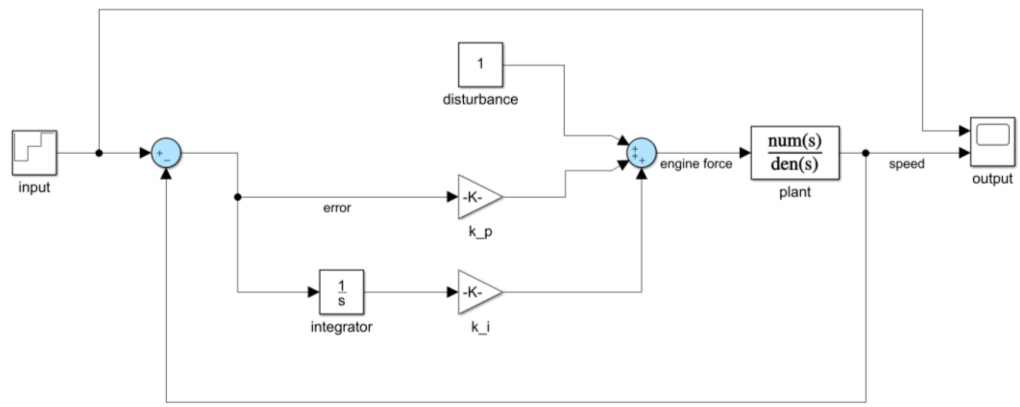


Figure 2: Longitudinal Controller Design

The above model utilizes a disturbance that is togglable with values of 1 for an existing disturbance and 0 for no disturbance. Both values result in the trace below. These traces also disregard overshoot. As the input for the cruise control is given, the vehicle accelerates steadily to the specified speed.

The following is the output of tracking the change in speeds for the PI controller when desired input speed is given:

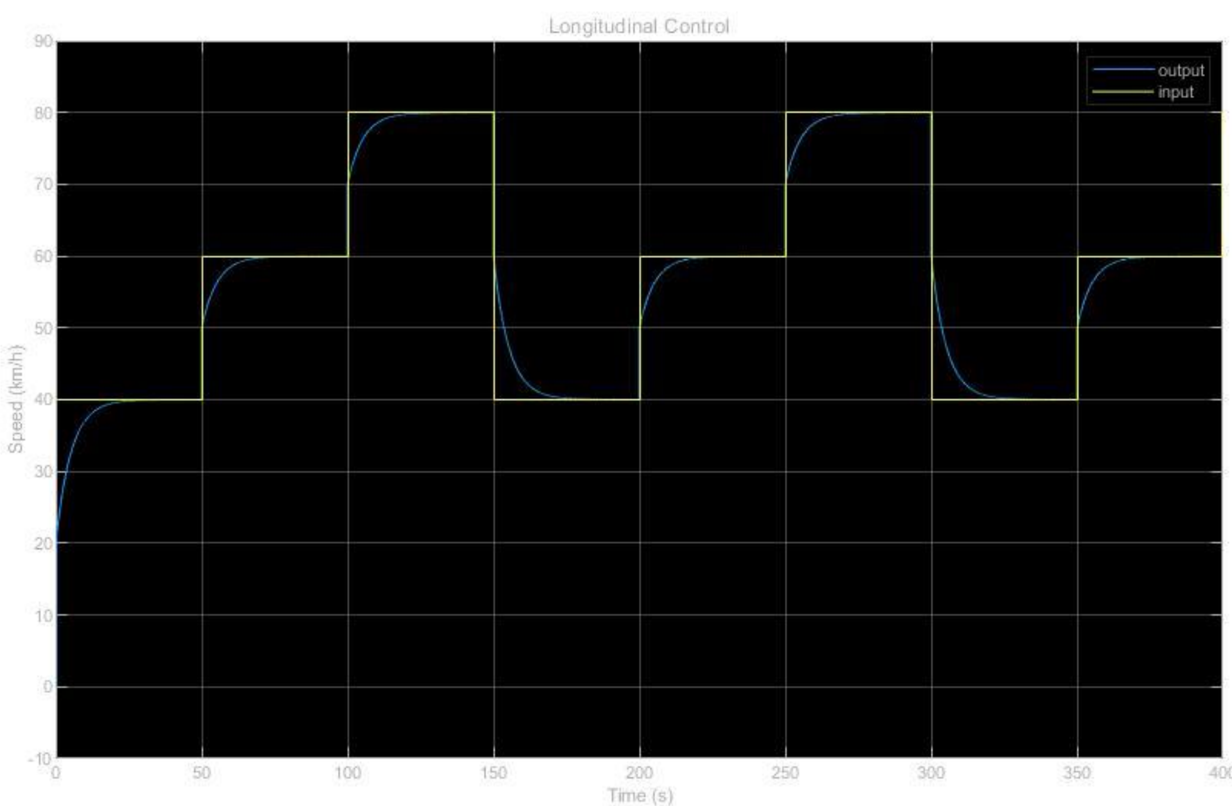


Figure 3: Longitudinal Control Output

With the addition of noise for a realistic simulation the following trace is a realistic output and input comparison.

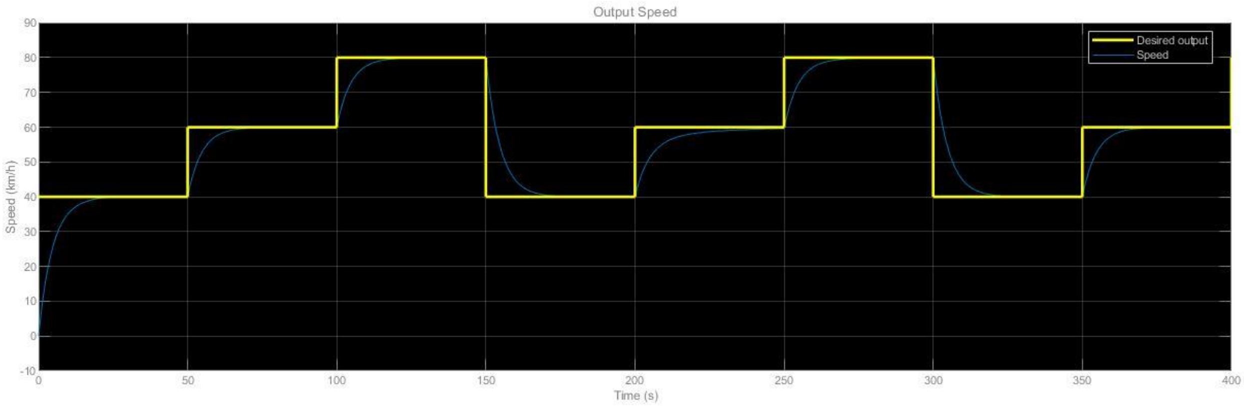


Figure 4: Noise Control Output

Solving for k_i and k_p the following calculations were made using previous equations:

Given Information:

$A = 2.71m^2$

$\rho = 1.225\ kg/m^3$

$c_D = 0.36$

$v_0 = 40\ km/h = 11.11m/s\ (2dp)$

$m = 1239kg$

Calculations:

$$k_p = \frac{\alpha + 2\zeta\omega_n}{\beta} \rightarrow \beta = \frac{(2.71)(1.225)(0.36)(11.11)}{1239} = 0.010716\ \&\ \alpha = \frac{1}{m} = \frac{1}{1239}$$

$$\zeta\omega_n = \frac{3.93}{10} \rightarrow overshoot\ of\ 5\%\ \therefore \zeta = 0.7 \rightarrow \omega_n = 0.56\ (2dp)$$

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + {\omega_n}^2 x(t) = c$$

$$Since, c = 0:$$

$$2\zeta\omega_n = Hv_0 + k_p$$

$$H = A\rho c_D$$

$$\boldsymbol{assume: overshoot = 5\% \rightarrow \therefore \zeta = 0.7}$$

$$\boldsymbol{settling\ time = 10s}$$

$${\omega_n}^2 = \frac{k_i}{m}$$

$$t_r = \frac{1.8}{\omega_n} \rightarrow t_r < 5 \rightarrow \therefore 5 > \frac{1.8}{\omega_n}$$

$$\therefore k_i > 0.1296$$

$$(2)(0.7)\left(\sqrt{k_i}\right) = Hv_0 + k_p$$

$$\therefore k_p = 0.504 - Hv_0 = 0.504 - (2.71)(1.225)(0.36)(11.11)$$

$$\therefore k_p = -12.77\ (2dp)$$

$$\therefore G(s) = \frac{k_d + k_p + k_i}{s^2(m + k_d) + s\left(Hv_0 + k_p\right) + k_i}$$

$$G(s) = \frac{-12.77 + 0.1296}{s^2 + s(0.508) + 0.1296}$$

Comparing with a Nonlinearized Response

Comparing the above results within the *simulation* section with a nonlinearized model, the output is as follows:

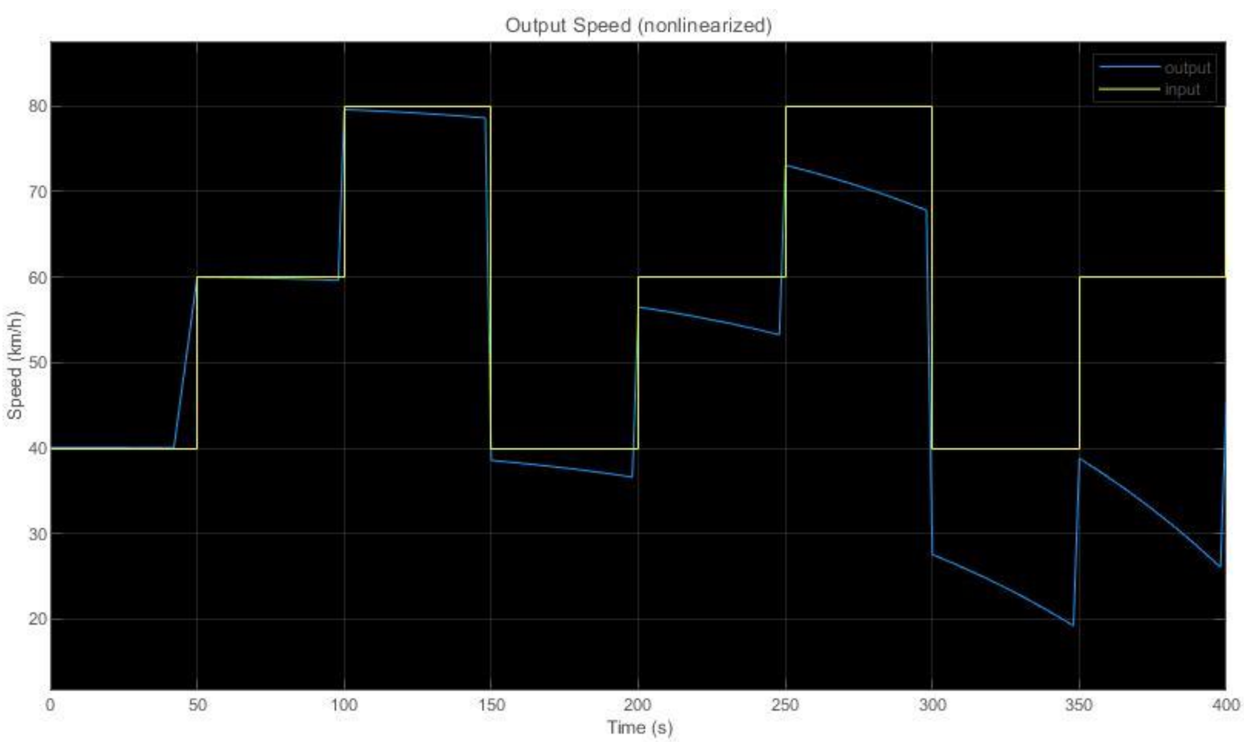


Figure 5: Output with a Nonlinearized Model

Therefore, with the nonlinearized model as mentioned partially in the *formulations* and *calculations* sections above, will as expected display an overshoot then a decline in stability due to the feedback of the control model. The disturbance was also run on the first iteration to model its affect on the speed of the vehicle. It is concluded that the nonlinearized model reaches input however, upon each sample will decrease achievable speed due to most of the parameters not being utilized in the equation for the transfer function.

$$m\dot{v} + \frac{1}{2}A\rho c_D v^2 = u$$

The above nonlinearized equation does no factor the conditions of: δ_u & δ_v. Please see above for the Taylor expansion of the equation that results in the linearized function.

The linearized function is as follows:

$$\therefore \dot{\delta_v} = \left(\frac{1}{m}\right)\delta_u + \left(-\frac{A\rho c_D v_e}{m}\right)\delta_v$$

<u>Note:</u> $\left(\frac{1}{m}\right) = \alpha$ & $\left(-\frac{A\rho c_D v_e}{m}\right) = \beta$

(L & A, 2007)

Changing Target Speed

<u>Aim:</u> To model the vehicle changing at different input speeds on the PI controller.

Using the specifications above onto a system as modelled by *figure 2*, the output traces are as expected, tending towards the desired speeds from the input of $40km/h, 60km/h$ & $80km/h$.

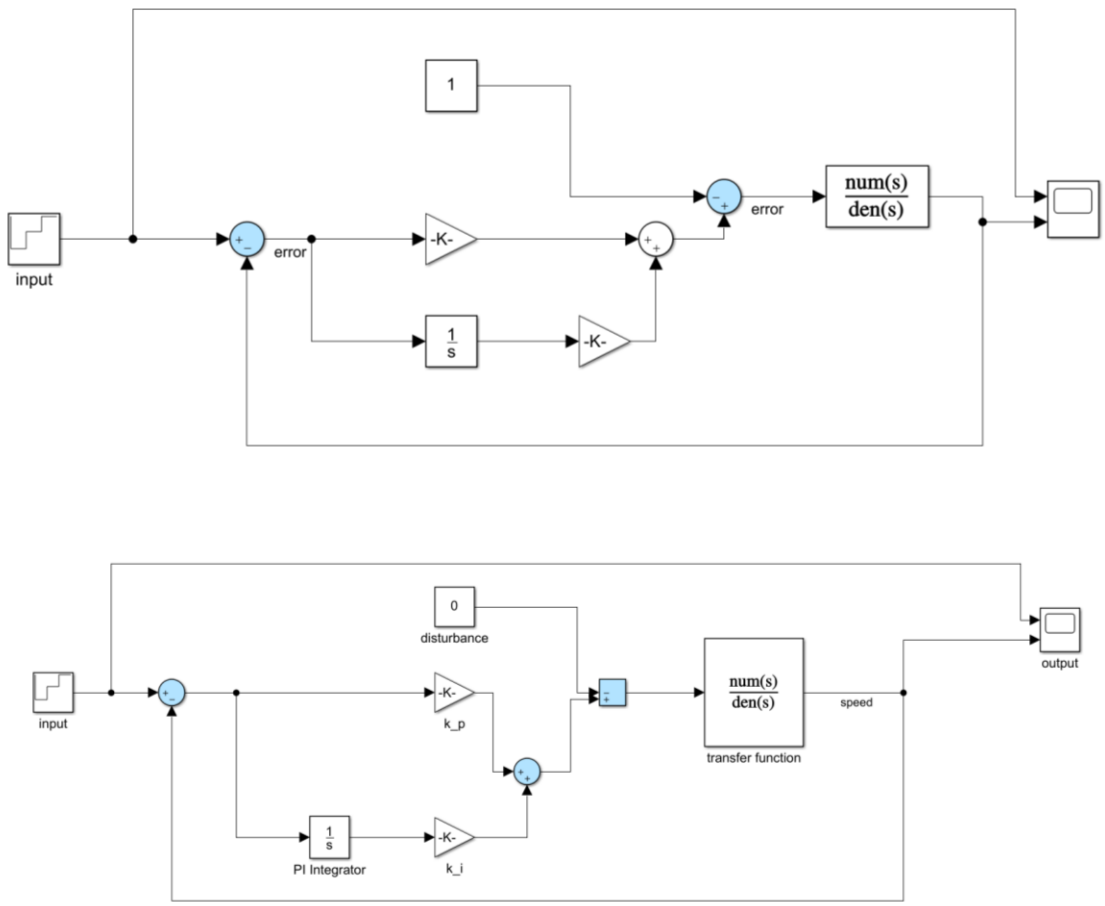


Figure 6: PI Controller Design's - Changing Speed

Using the formulation above in the linearized functions, values gained are:

$$5 = \frac{3.93}{\omega_n \zeta} \rightarrow \omega_n = 1.12\ (2dp)\ where\ \zeta = 0.7$$

$$k_i = 390.50\ (2dp)$$

$$k_p = 960.53\ (2dp)$$

Calculations:

$$G(s) = \frac{\left(\frac{k_i}{m}\right)}{s^2 + \left(\frac{k_p + A\rho c_D v_0}{m}\right)s + \left(\frac{k_i}{m}\right)}$$

Standard form characteristic polynomial:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + {\omega_n}^2 x(t) = c$$

$$\therefore 2\zeta\omega_n = \frac{k_p + A\rho c_D v_0}{m} \;\&\; {\omega_n}^2 = \frac{k_i}{m}$$

The ideal linearized output from the PI controller displays a trace as shown below:

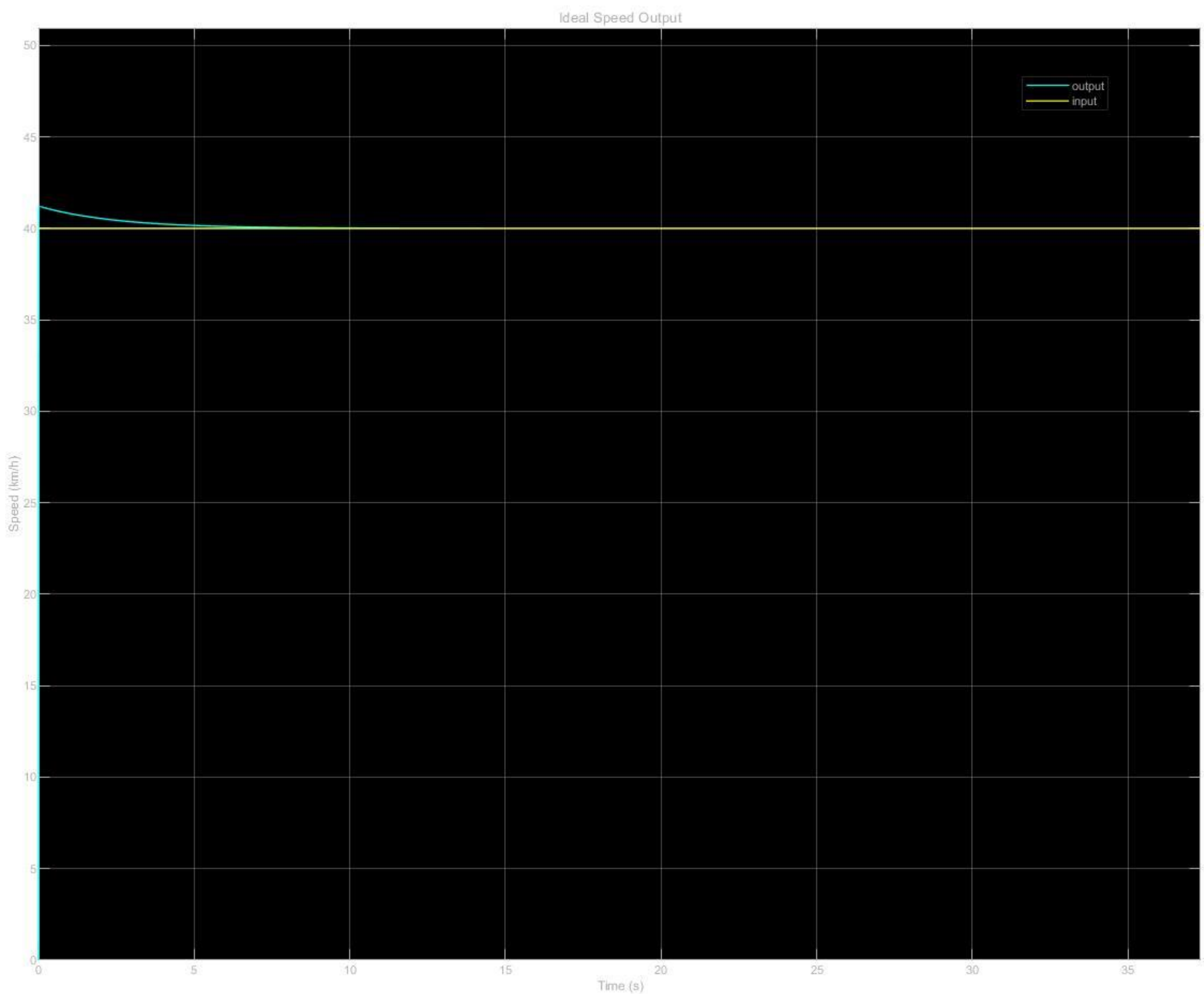
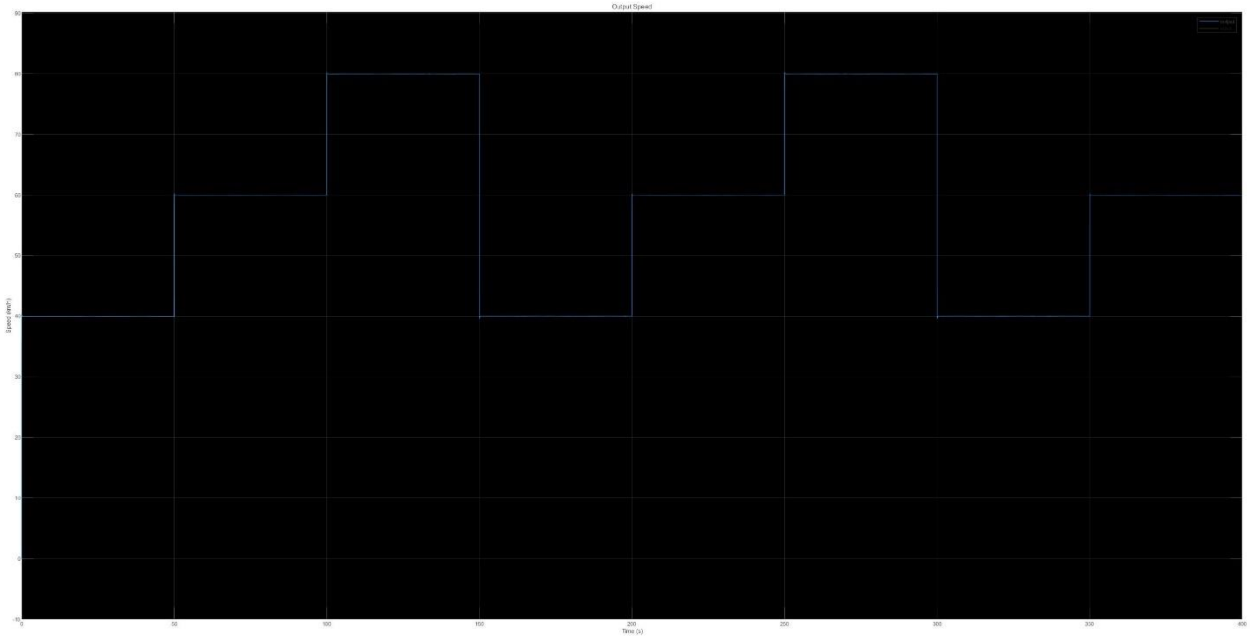


Figure 7: Full Plot PI Controller – Ideal

The full trace that was achieved from the values calculated above result in the figure below:



To more clearly illustrate the overshoot and steady state of the output speed, the figures below display the detail of the fluctuation of the speed when a $60km/h$ input is given.

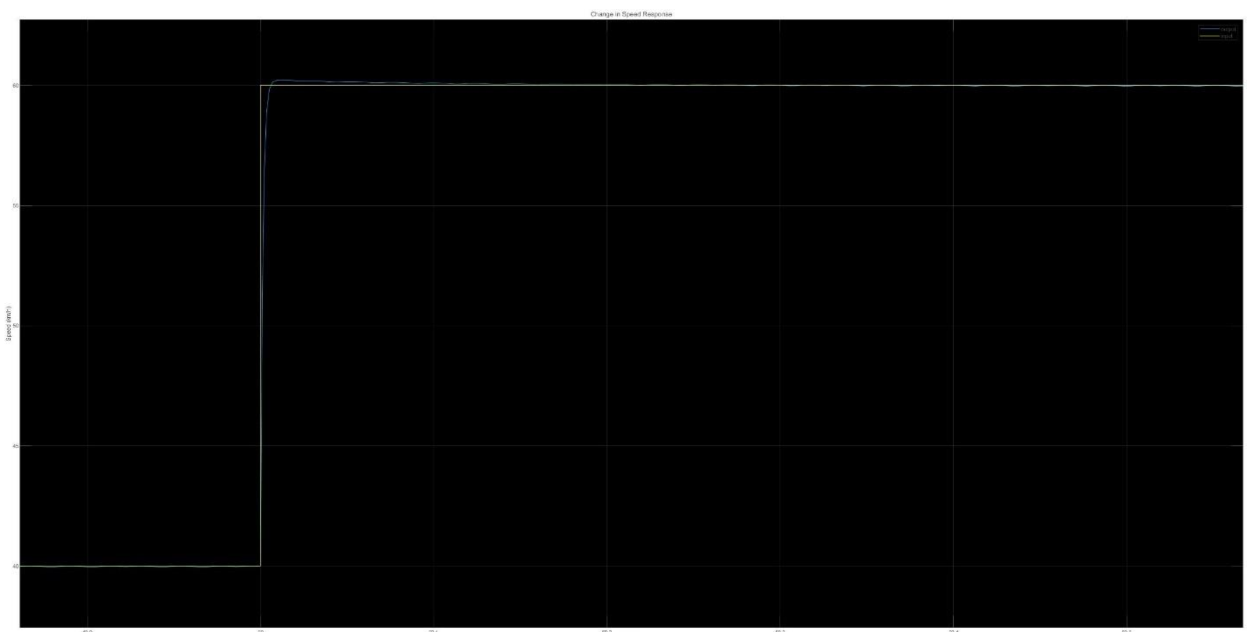


Figure 8: Detail of Overshoot - 60km/h

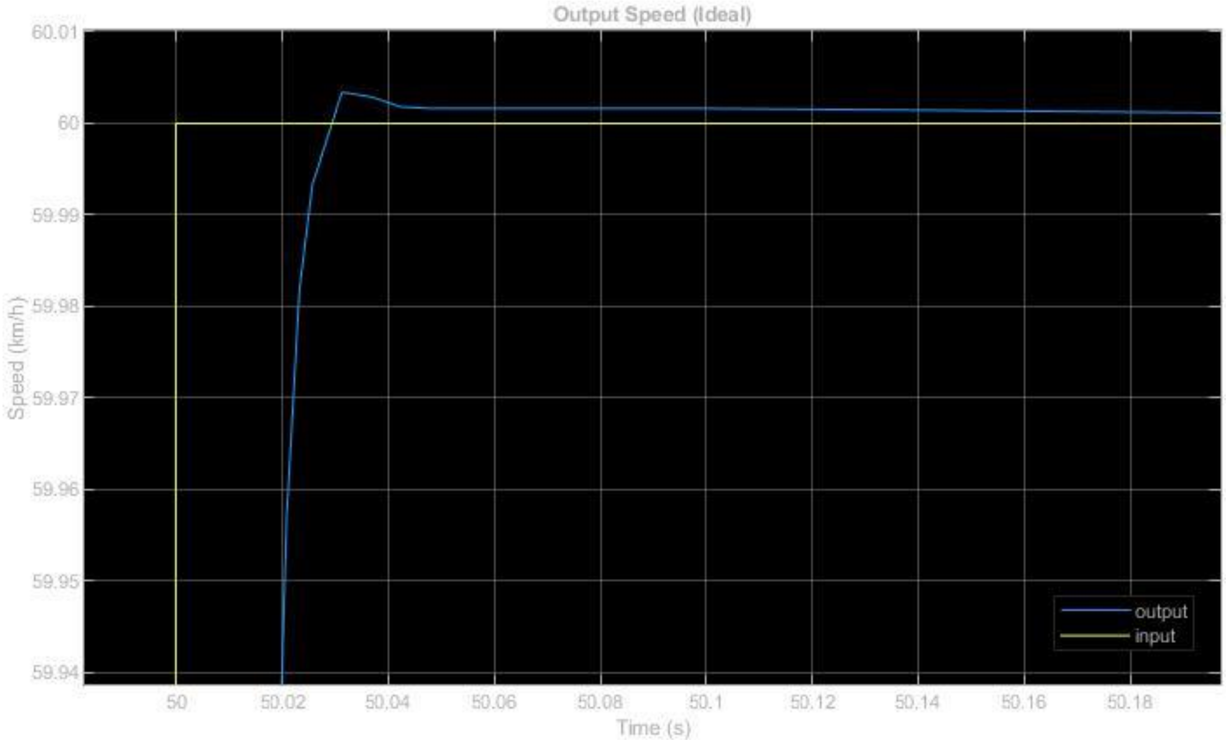


Figure 9: Overshoot PI Controller – Ideal

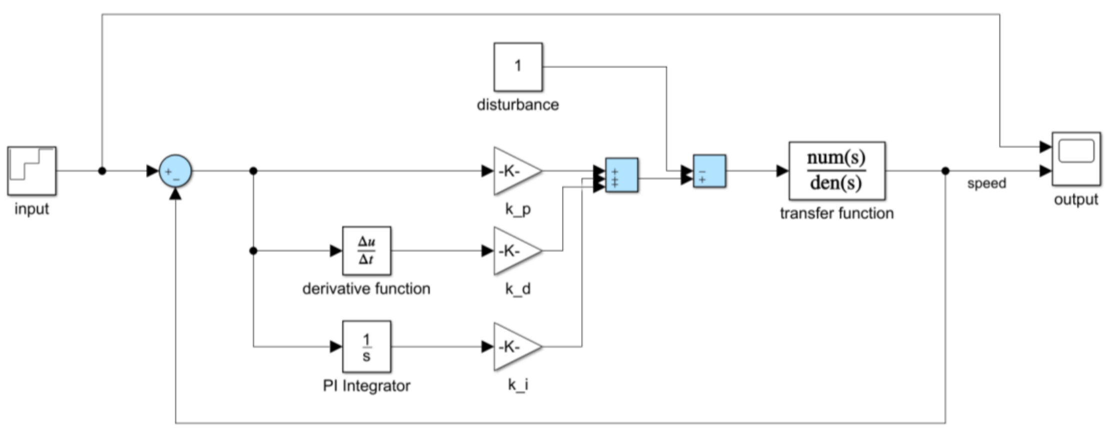


Figure 10: PID Controller Design

Steep inclination Slop Analysis

<u>Aim:</u> To simulate a 20% grad slope that the vehicle inclines on at $0 - 40km/h$ to investigate the force of gravity as a disturbance.

Using a controller with the disturbance re-designed as a simulated force of gravity acts on the vehicle. The following section highlights the equation utilized to simulate this force:

$$F_g = mg\sin\theta$$

$$Inclination\ Angle, \theta = \tan^{-1}\frac{slope\ gradient}{100}$$

$$\tan^{-1} 0.2 = 11.31$$

$$F_g = 1239 \times 9.81 \times \sin 11.31 = 2383.73N$$

(L & A, 2007)

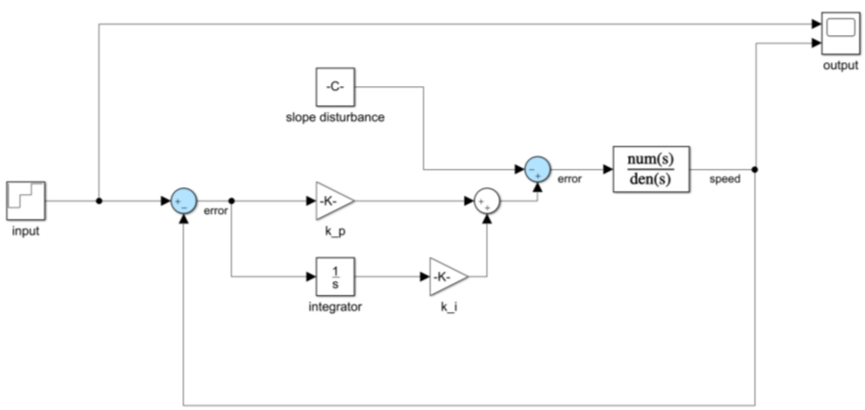


Figure 11: Slope Disturbance Control Model

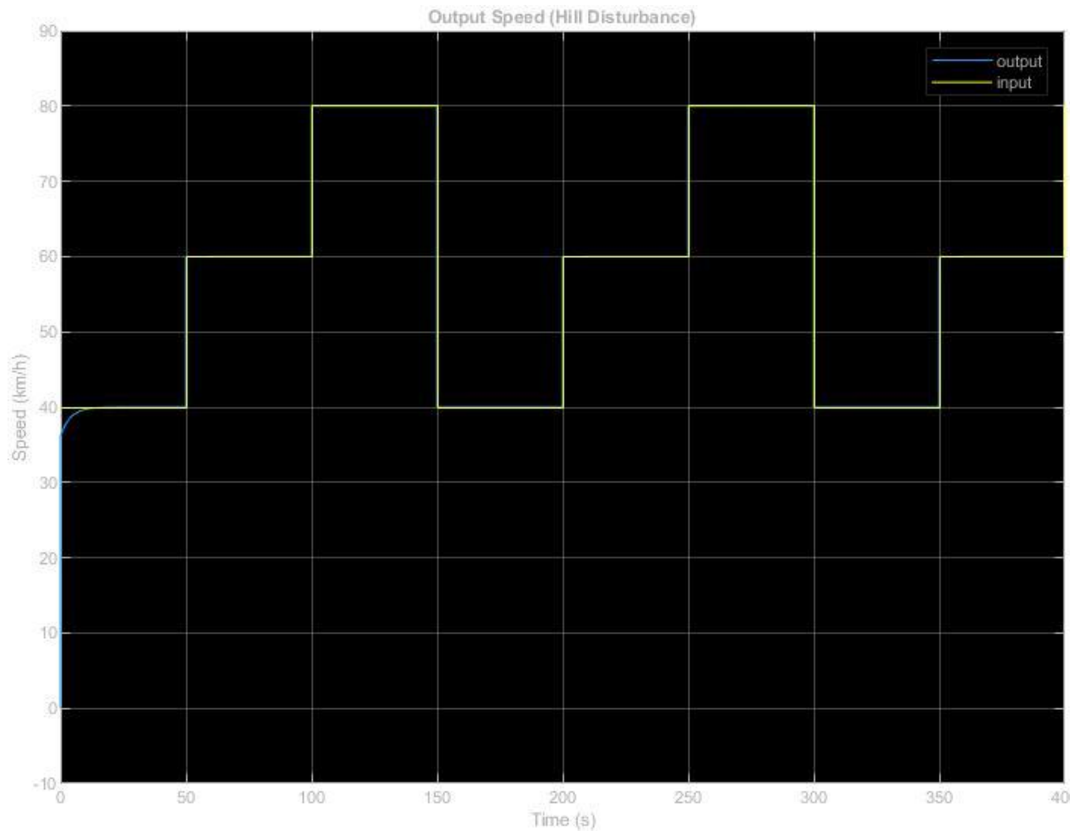


Figure 12: Initial Slope Disturbance

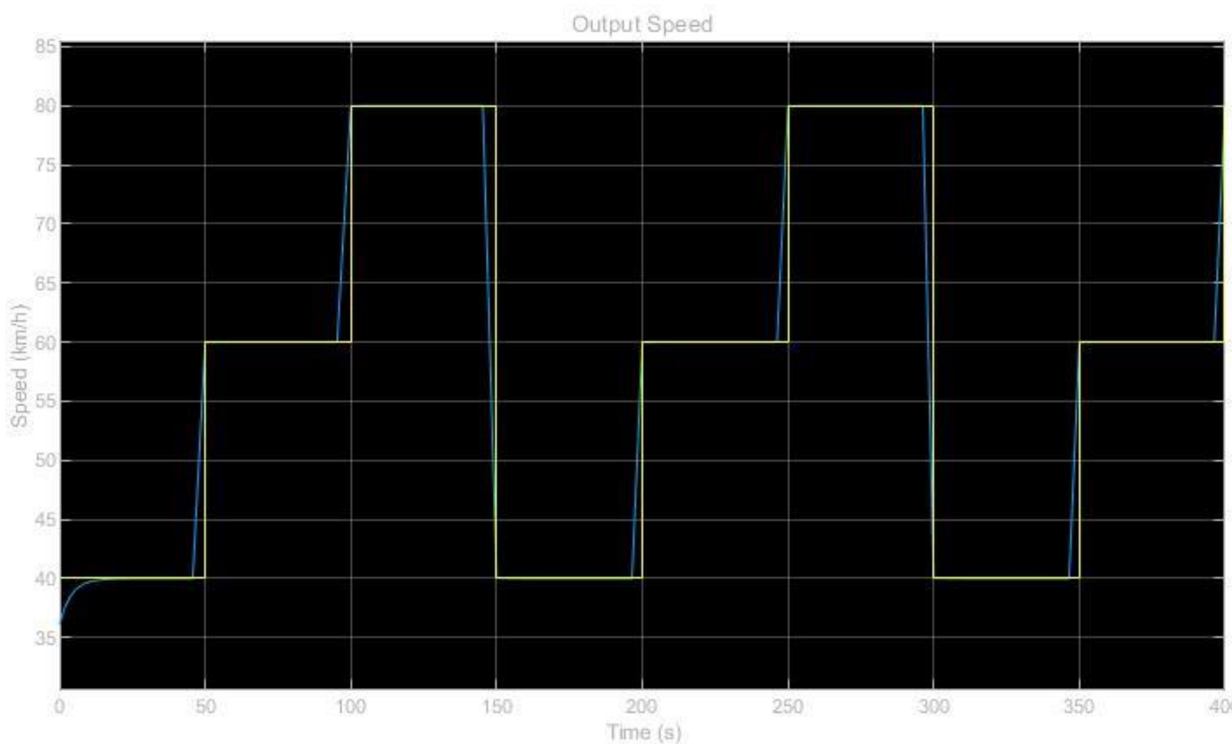


Figure 13: Output Slope Disturbance Speed

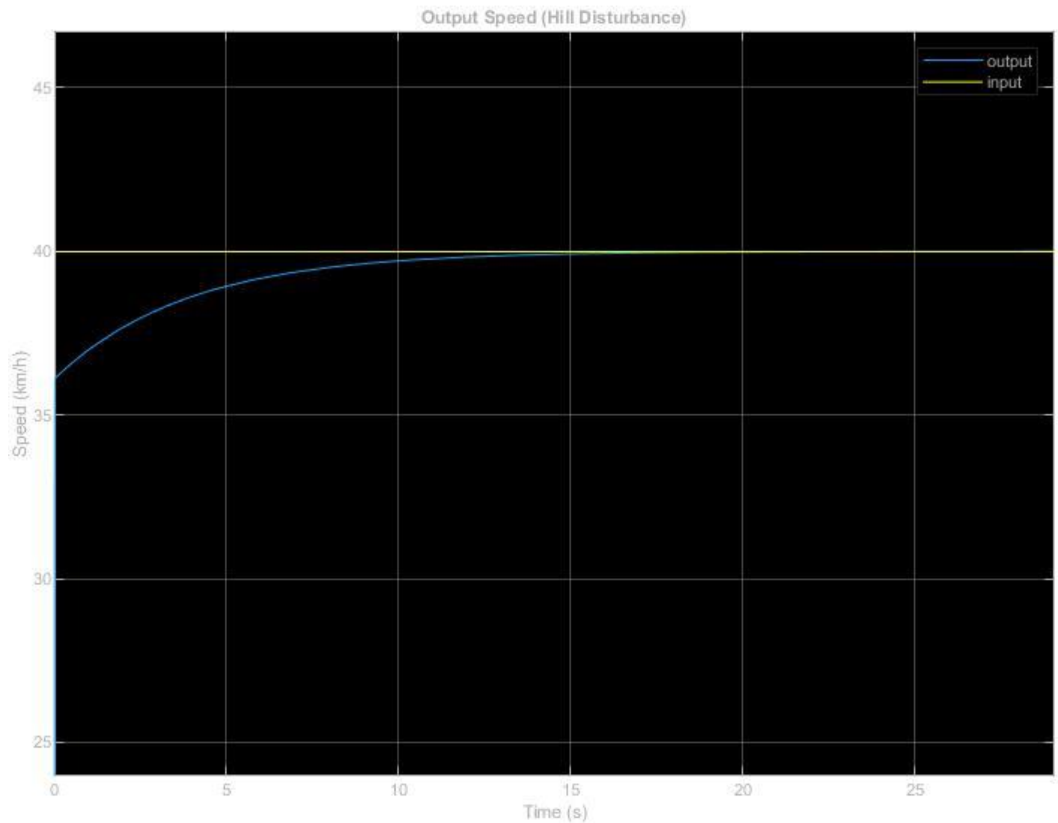


Figure 14: Zoomed in Slope Disturbance

As seen from the response of the slope introduced that the vehicle speed control is disturbed by, the response is gradual due to the increase in force also highlighted within the formulations which are employed to model through the subtraction at a specific time. To more clearly see the time that the disturbance has occurred, see the figure below which highlights an input speed of $40km/h$ on a 20% grade (uphill slope):

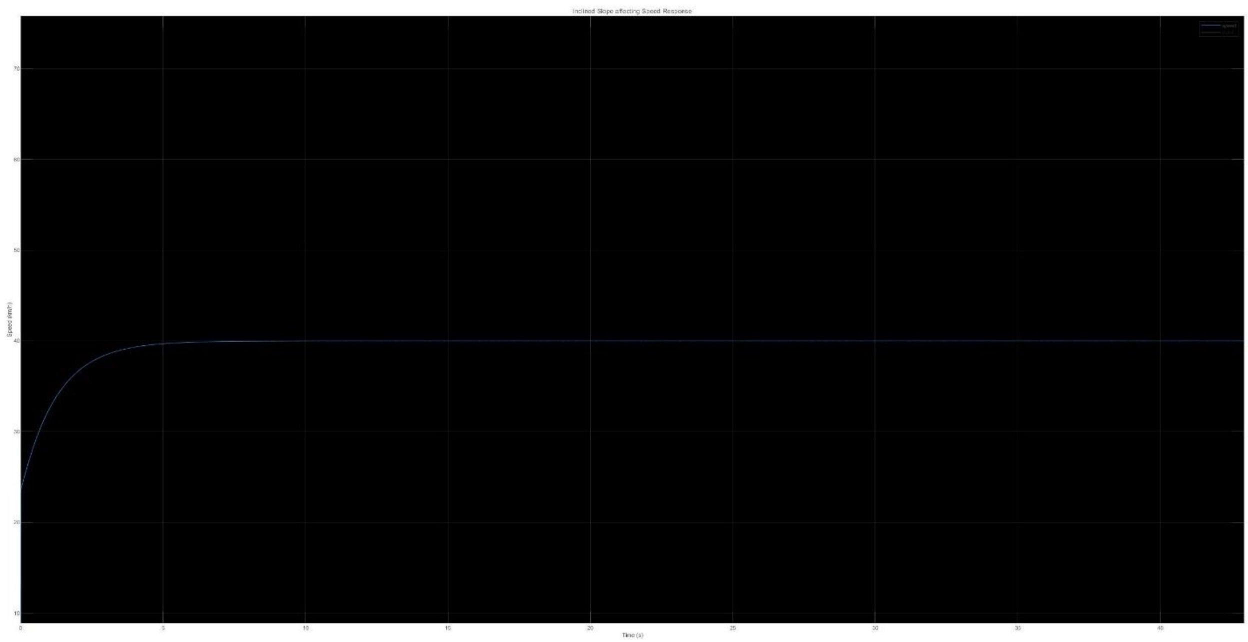


Figure 15: Slope Speed Response - 40km/h

To more clearly identify the affect that is taking place please refer to the forces on the body diagram *(FBD)* below denoting the change in acceleration and ultimately response time of the system.

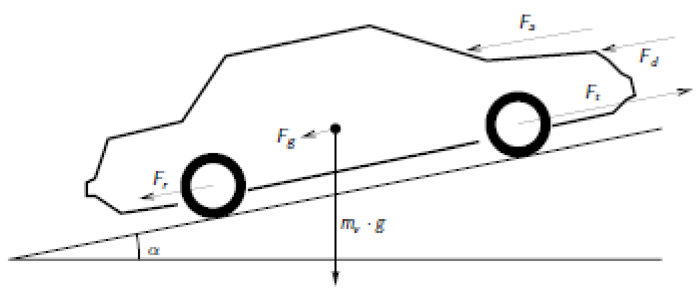


Figure 16: Vehicle Propulsion

(L & A, 2007)

<u>Note:</u>

$$m_v \frac{d}{dt} v(t) = F_t(t) - \left(F_a(t) + F_r(t) + F_g(t) + F_d(t)\right)$$

However, for this project only F_a and F_g is considered, respectively referring to the drag and gravitational forces.

Uncertainty in Mass Analysis

<u>Aim:</u> To investigate the effects of an increasing mass as modelled with one to three passengers.

If the similar equations are altered with the weight of an estimate $1-3$ passengers (gradually increasing), the following section displays the affects of the increase in mass on the output and speed response of the control system.

$$G(s) = \frac{\left(\frac{k_i}{m}\right)}{s^2 + \left(\frac{k_p + A\rho c_D v_0}{m}\right)s + \left(\frac{k_i}{m}\right)}$$

$$\therefore 2\zeta\omega_n = \frac{k_p + A\rho c_D v_0}{m} \;\&\; {\omega_n}^2 = \frac{k_i}{m}$$

It can be noticed from the above linearized equations that the mass is proportional to k_i therefore, the k_i value increases with the mass increase within the vehicle. Using average weights of three adults, the model was tested with a mass increase of three stages:

Note: The mass increase was taken from an average weight across a census of individuals who were 25-year-old males. The following masses are the total mass with an individual(s) and the vehicle.

$$m_{average} = 71.8kg$$

$$m_1 = 1310.8kg, m_2 = 1382.6kg, m_3 = 1454.4kg$$

From the linearized equation:

$$\therefore \dot{\delta_v} = \left(\frac{1}{m}\right)\delta_u + \left(-\frac{A\rho c_D v_e}{m}\right)\delta_v$$

Note: $\left(\frac{1}{m}\right) = \alpha \;\&\; \left(-\frac{A\rho c_D v_e}{m}\right) = \beta$

It is evident that the mass will cause a decrease in acceleration and hence, slow the speed response, while also causing fluctuations and instability, increasing the steady state response time.

As expected and as can be seen in the figures below, the change of the mass results in the response to be slower (takes more time to reach the desired inputted speed) and the stability or time to steady state is increased).

The response for the masses are evident below:

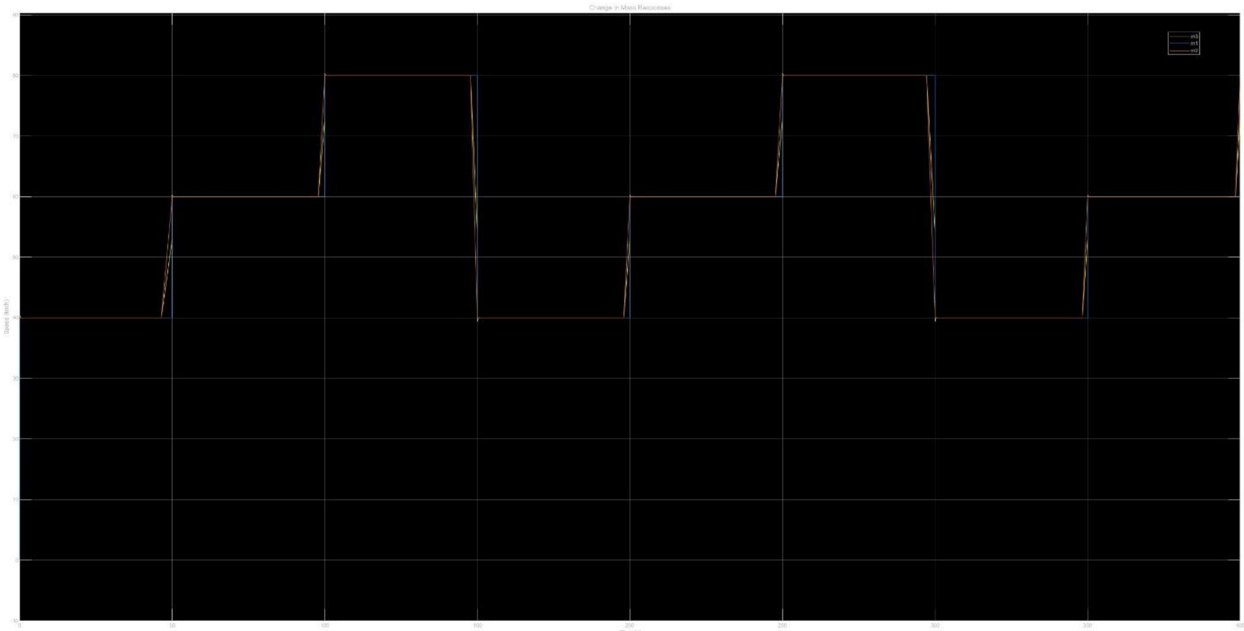


Figure 17: Change in Mass Response

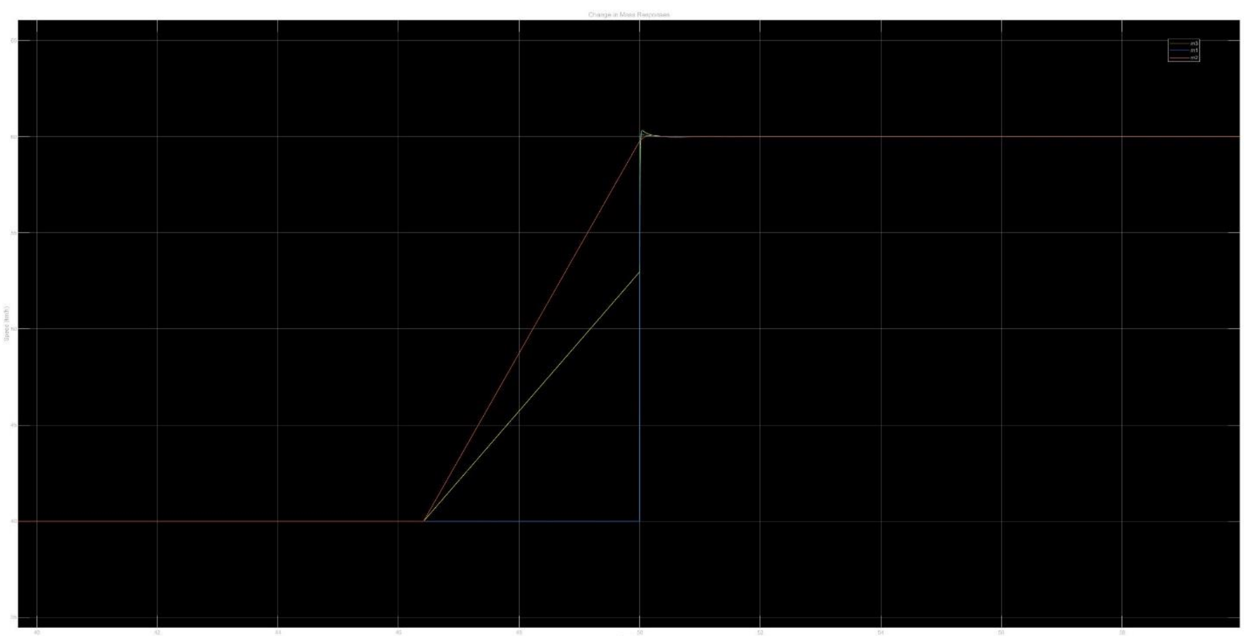


Figure 18: Stability from 40 to 60km/h

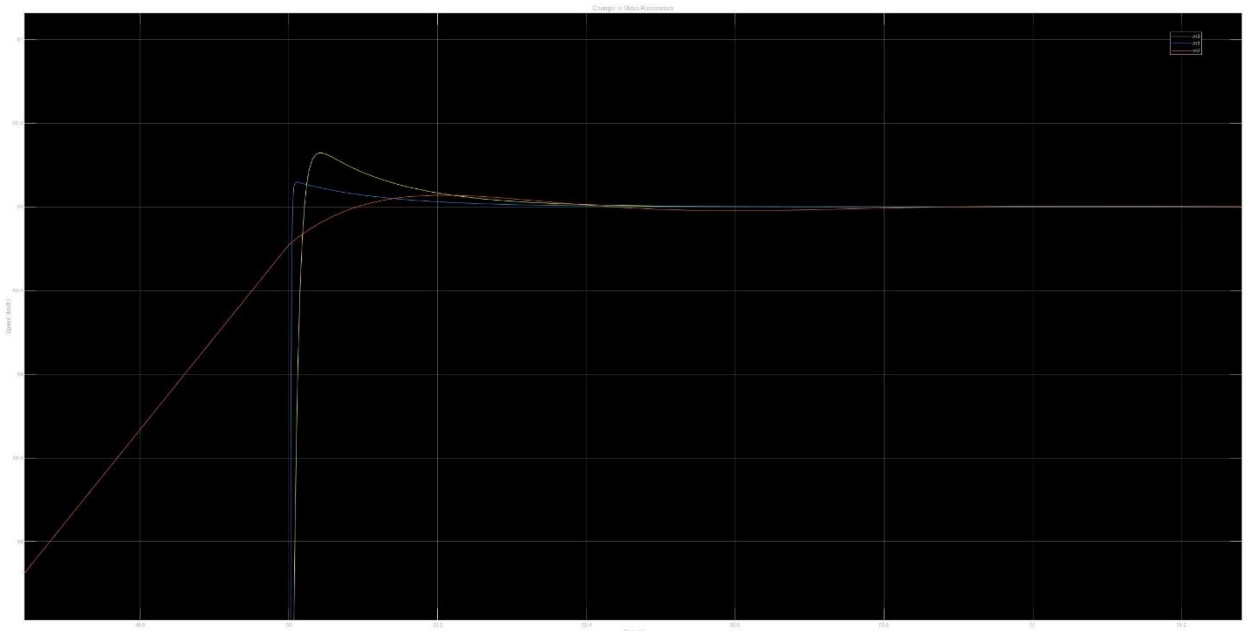


Figure 19: Steady State - Instability

For more tracers and outputs with a modified control using the same calculations, see Appendix for a clear indication of the change of target speed trace, slope trace and differing masses.

Lateral Controller

Aim: To create a PID controller to simulate the smooth and accurate transition from lane-to-lane lateral position changes of the vehicle.

- Simulate the closed-loop system response at varying speeds of the cruise control system.

Note: The equations and derivations have been partially stated in the *simulation and model setup* section of this report.

The design of the PID controller was formulated as follows:

$$m\ddot{v} = \frac{H}{2}v^2 = u + d$$

$$Let\ f(v) = v^2$$

$$m\ddot{v}(t) = \frac{H}{2}f(v) = u + d$$

$$m\ddot{v}(t) = \frac{H}{2}(\delta_v(t) + v_0) = \delta_u + u + d_0 + \delta_v$$

$$at\ steady\ state, let\ \delta = 0:$$

$$\frac{H}{2}f(v_0) = u_0 + d_0$$

$$Let\ f(v) - f(v_0) \approx \dot{\delta}_v \approx \frac{\partial f}{\partial v}\bigg| v = v_0\ (v - v_0)$$

$$\frac{\partial f}{\partial v}\big(f(v)\big) = 2v \rightarrow 2v_0\delta_v$$

$$m\ddot{v}(t) + \frac{H}{2}(2v_0\delta_v + v_0) = \delta_u + \delta_d + \frac{H}{2}(v_0)$$

$\boldsymbol{v}$	$\boldsymbol{r}$
$v = Ae^{st}$	$r = Be^{st}$
$\dot{v} = sAe^{st}$	$\dot{r} = sBe^{st}$
$\ddot{v} = s^2Ae^{st}$	$\ddot{r} = s^2Be^{st}$

Figure 20: Derivatives

<u>Note:</u>

$\delta_v(t) = v(t) - v_0 \rightarrow v(t) = \delta_v(t) + v_0$

$\delta_d = d(t) - d_0$

$d(t) = \delta_d + d_0$

$\delta_u = u(t) - u_0 \rightarrow u(t) = \delta_u + u_0$

$$msAe^{st} + \frac{H}{2}(2v_0sAe^{st} + v_0) = \dot{u} + \dot{d} + \frac{H}{2}(v_0)$$

$$ms^2Ae^{st} + \frac{H}{2}(2v_0sAe^{st}) = k_p(\dot{r} - \dot{v}) + k_d(\ddot{r} - \ddot{v}) + k_i(r - v)$$

$$m\ddot{v} + \frac{H}{2}(2v_0\dot{v}) + k_p\dot{v} + k_iv + k_d\ddot{v} = k_p\dot{r} + k_d\ddot{r} + k_ir$$

$$\therefore G(s) = \frac{k_d + k_p + k_i}{s^2(m + k_d) + s\big(Hv_0 + k_p\big) + k_i}$$

Calculations

The following section is the equation processing for the lateral position control of the vehicle:

$$\ddot{y} = \dot{v_0}\sin\alpha + v_0\dot{\alpha}\cos\alpha$$

$$\sin\dot{v_0} = 0$$

$$\ddot{y} = v_0\dot{\alpha}\cos\alpha$$

$$\ddot{y} = v_0\left(\frac{v_0}{l_r}\beta + \dot{\beta}\right)$$

$$\ddot{y} = v_0\left(\frac{v_0}{l_r}\left(\frac{\delta f}{2}\right) + \frac{\dot{\delta} f}{2}\right)$$

$$\ddot{y} = v_0\frac{\dot{\delta} f}{2} + \frac{{v_0}^2}{l_r}\left(\frac{\delta f}{2}\right)$$

Above is the nonlinearized equation, if we linearize this equation the following occurs:

$$\ddot{y} = \frac{v_0}{2}\dot{\delta f} + \frac{{v_0}^2}{2l_r}\delta f$$

$$Let\ f = Ae^{st}$$

$$\delta f = sAe^{st} \rightarrow s$$

$$\delta f^2 = s^2Ae^{st} \rightarrow s^2$$

$$Let\ u = k_p$$

$$\ddot{y} = \frac{v_0}{2}(s^2Ae^{st}) + \frac{{v_0}^2}{2l_r}\ Ae^{st}$$

$$\ddot{y} = \frac{v_0}{2}\dot{u} + \frac{{v_0}^2}{2l_r}$$

$$G(s) = \frac{\frac{v_0}{2}s + \frac{{v_0}^2}{2l_r}}{s^2}$$

Note: $H = A\rho c_D$

Comparing with the denominator:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + {\omega_n}^2x(t) = c$$

$$2\zeta\omega_n = Hv_0 + k_p\ \&\ {\omega_n}^2 = k_i$$

$$k_d = 1 - m$$

$$l_f\ \&\ l_r = 2.648m$$

Transition from Lane-to-Lane

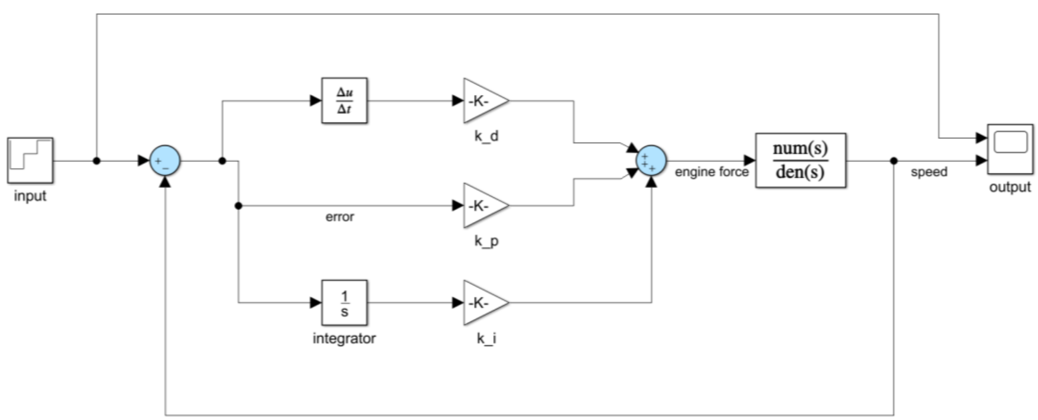


Figure 21: Lateral Controller Design – PID Controller

The following plot displays the controllers speed that must now be account for:

- Since the controller is adjusting the speed of the vehicle during the lane change, the force causes the reduction in the cruise control.
  - Therefore, this reduction in speed must be accounted for.

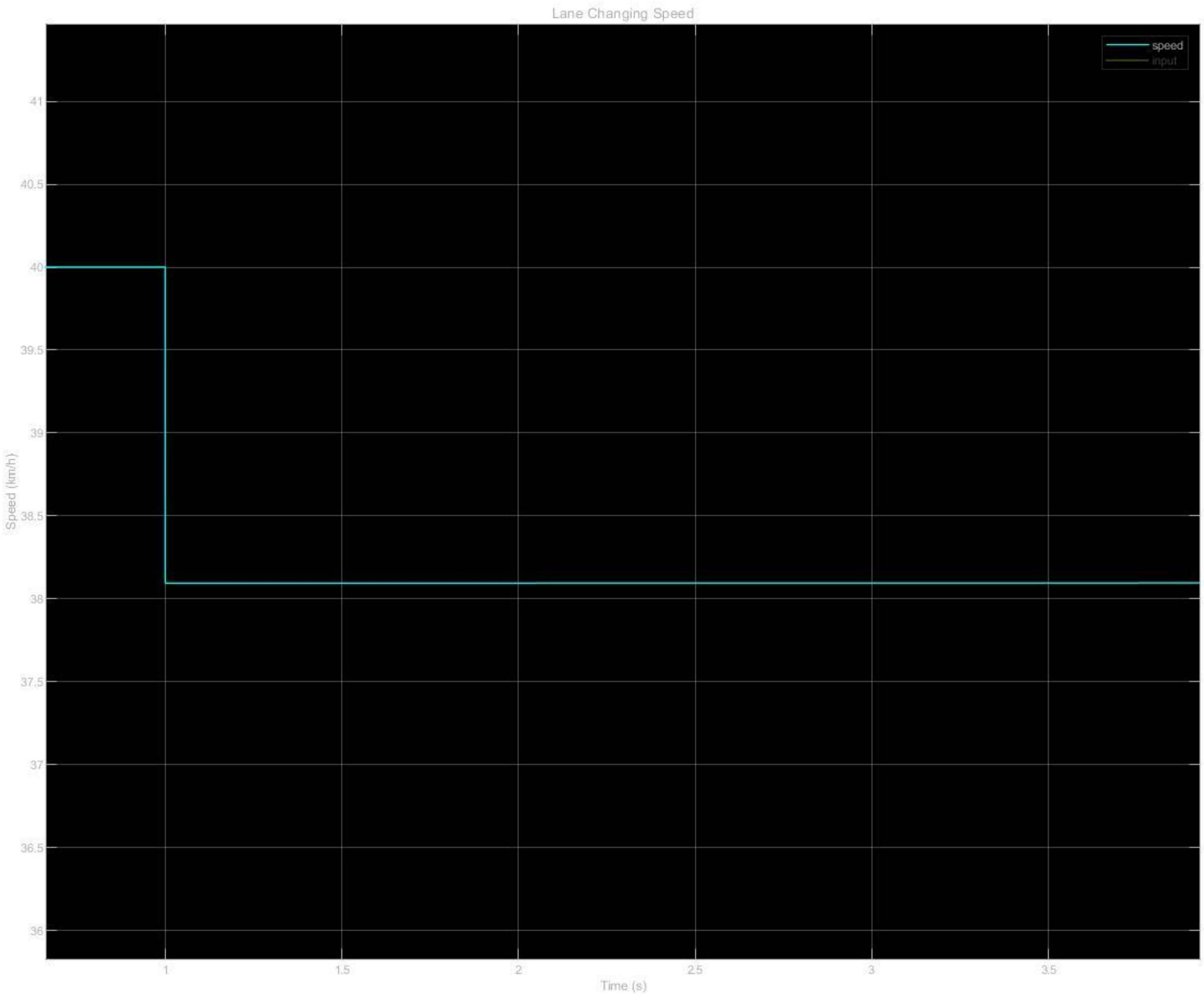


Figure 22: Lane Changing - Speed Decrease

For the changing of lanes, the speed reduction can be accounted for and the transfer function modified which will result in the vehicle overshooting the target inputted speed of $40km/h$ but then stabilising after a short fluctuation to steady state (input).

Closed Loop System Response

The following section below displays the speed response of the lane-change manoeuvres at variety of speeds including $40km/h$, $60km/h$ and $80km/h$.

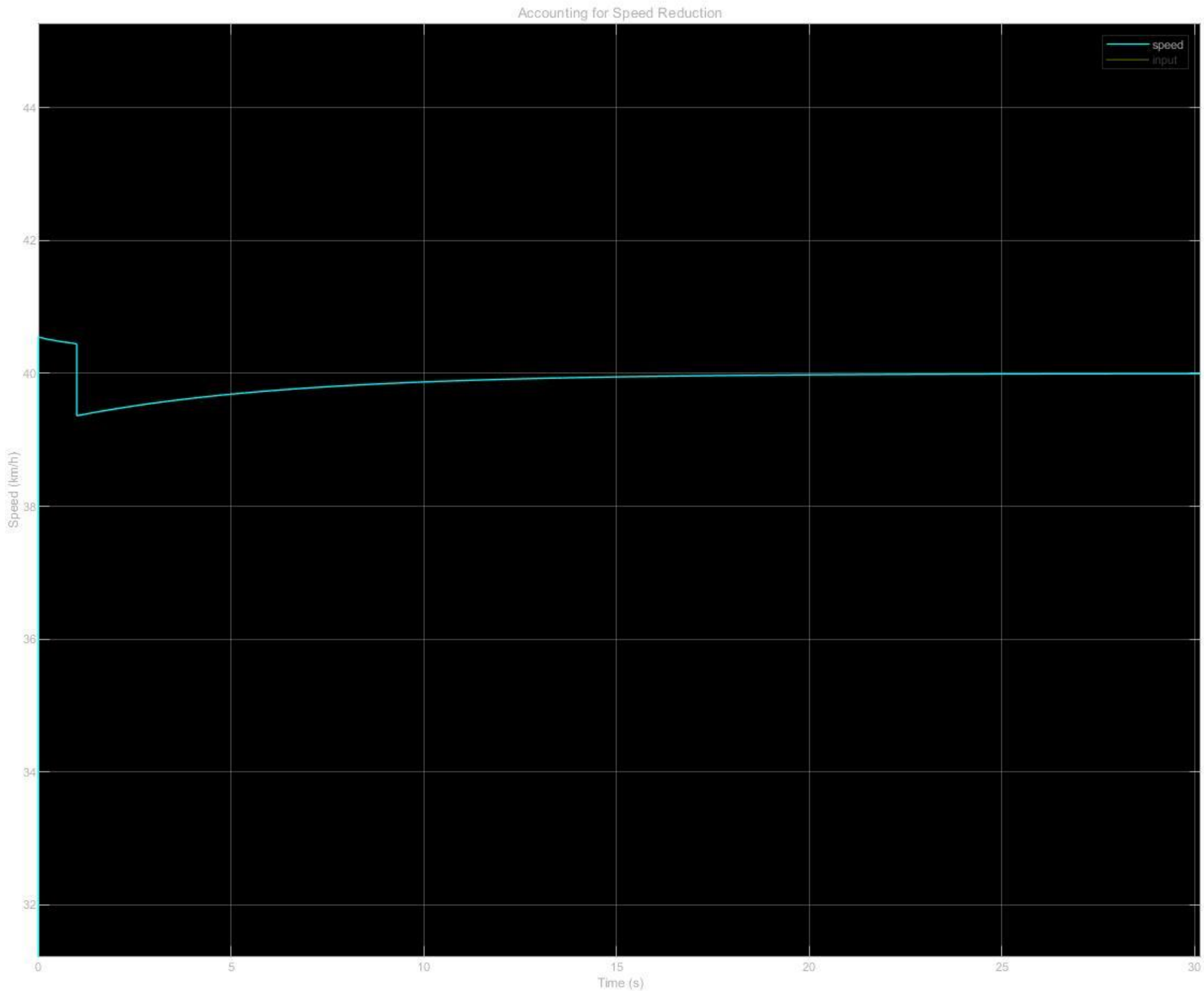


Figure 23: Lane Changing Speed Response - 40km/h

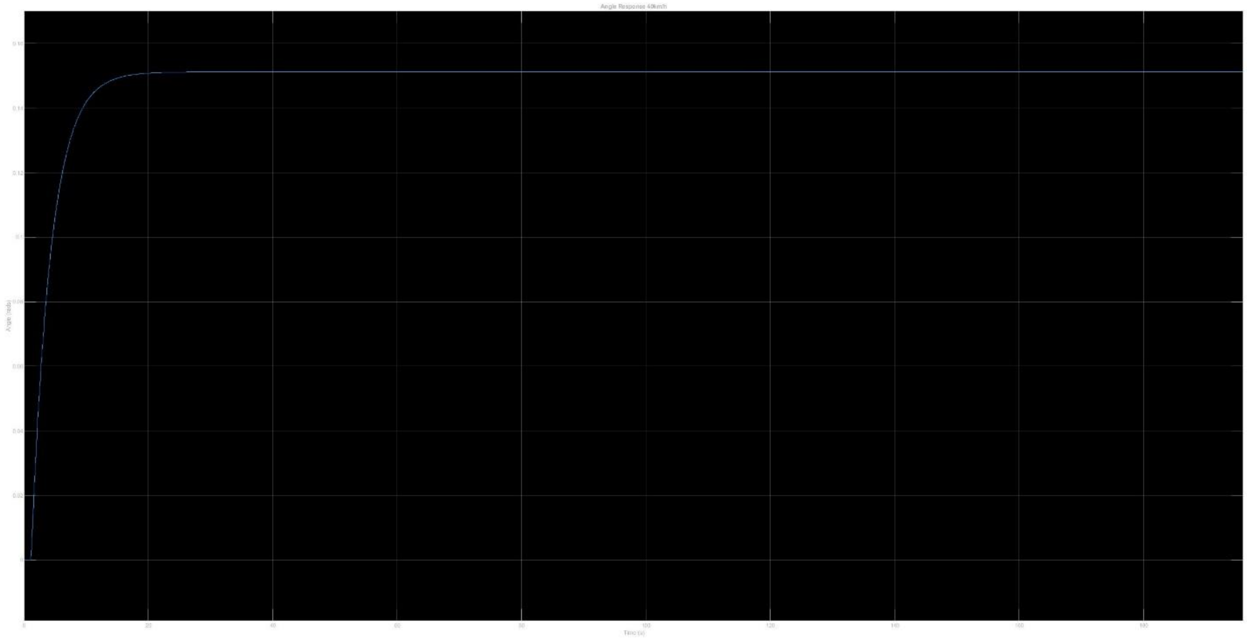


Figure 24: Lane Changing Angular Response - 40km/h

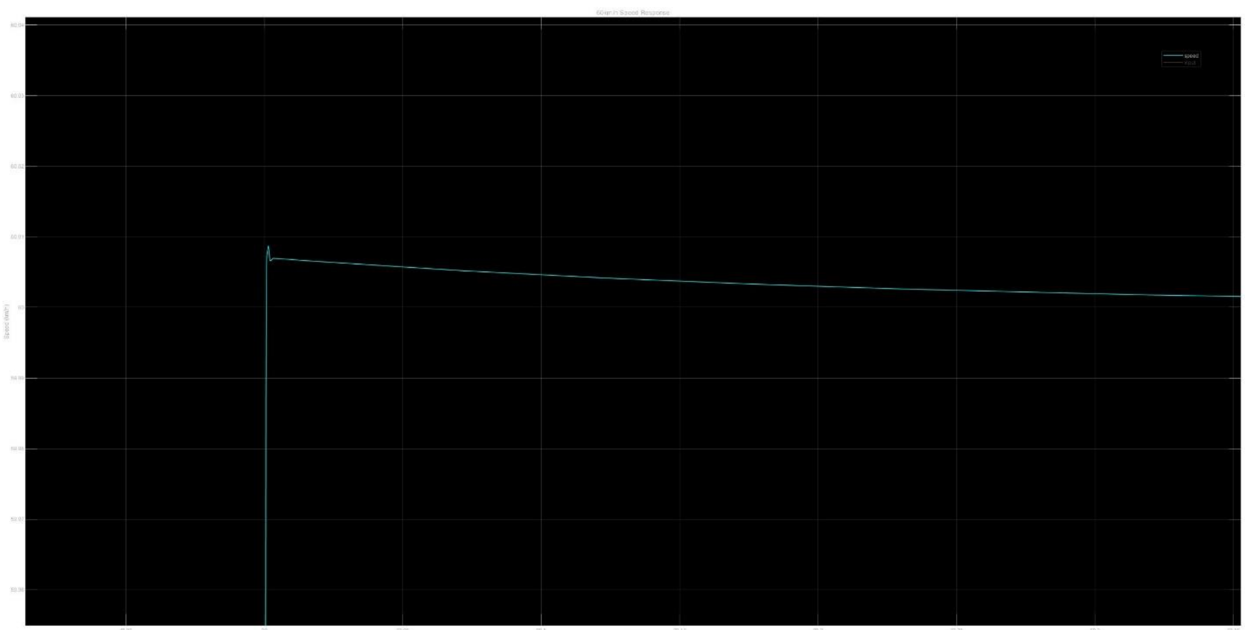


Figure 25: Lane Changing Speed Response - 60km/h

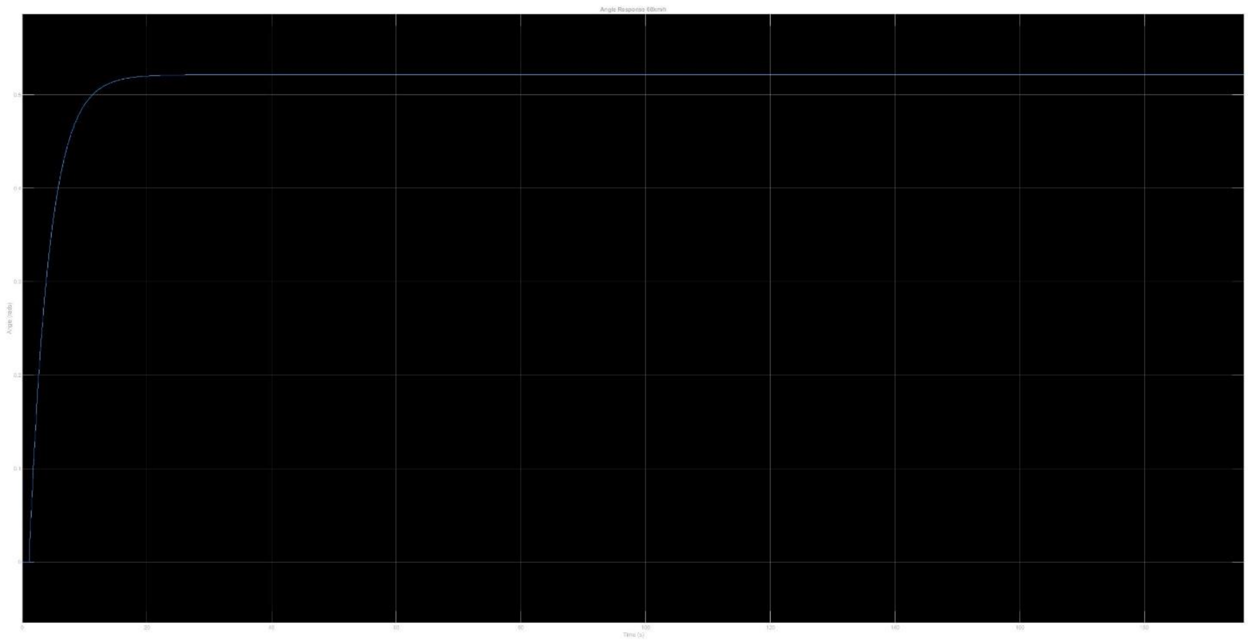


Figure 26: Lane Changing Angular Response - 60km/h

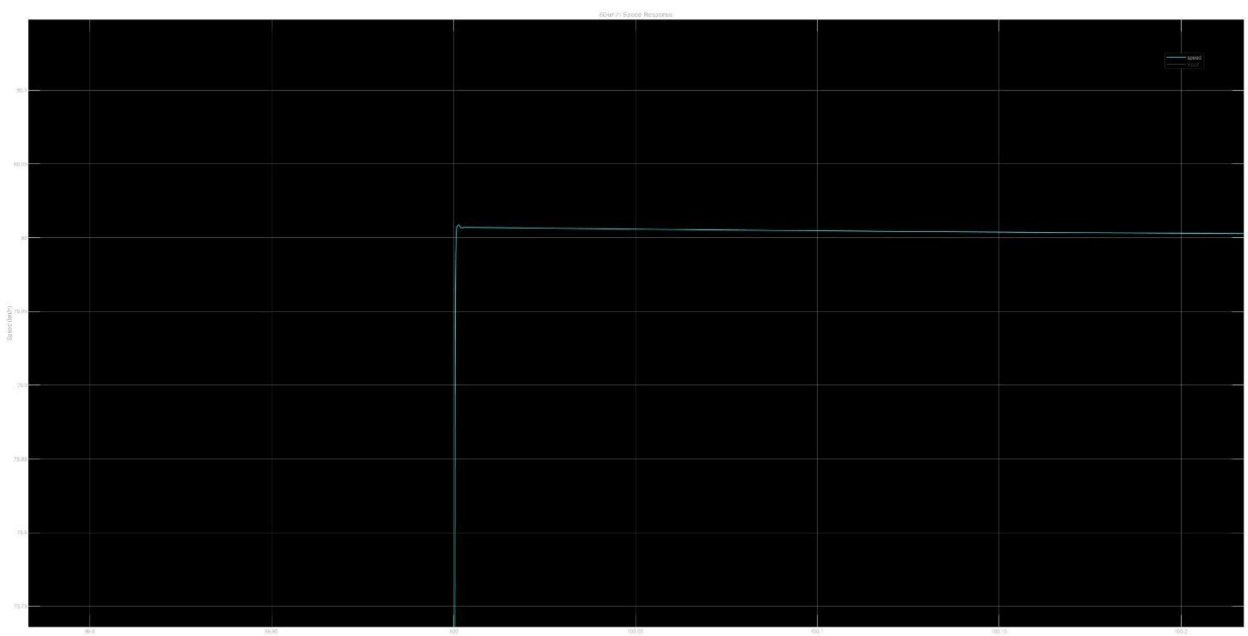


Figure 27: Lane Changing Speed Response - 80km/h

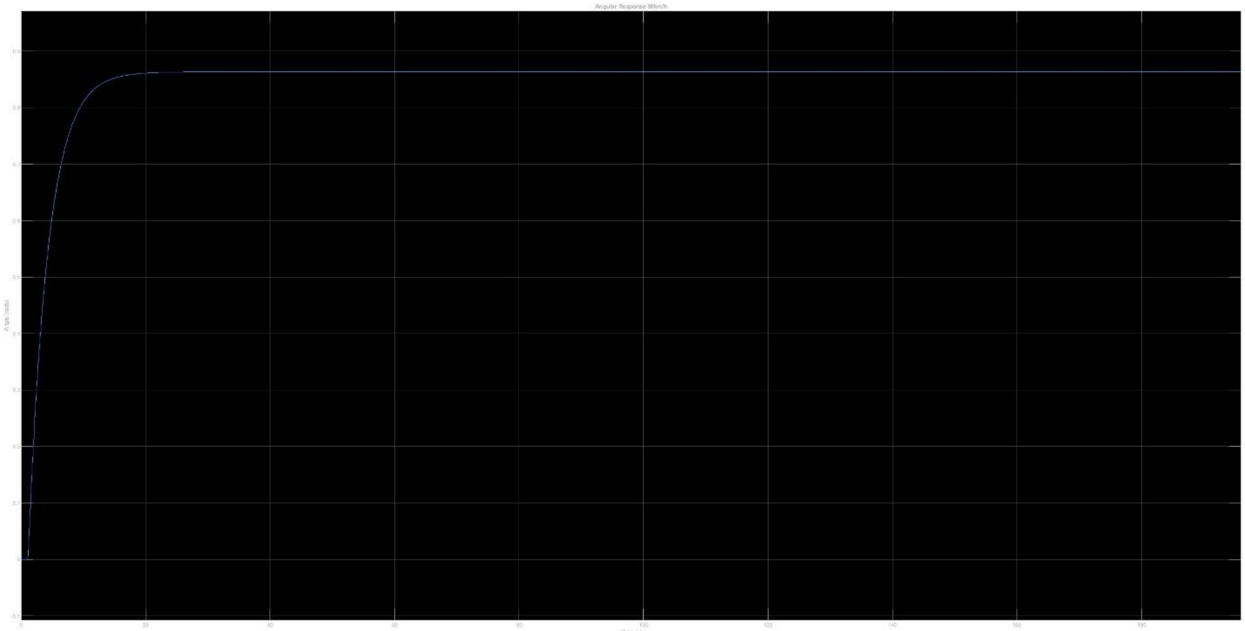


Figure 28: Angular Response - 80km/h

From the collected data, it can be concluded that with an increase in speed inputted, the angular response of the lateral position of the vehicle is increased (steeper angle) in contrast to a lower speed lane change which is modelled to occur at a reduced angular steepness.

Angular Response

The angular response of an 80 degree turn in the control system in the linearized dynamics displays an expected time scale of the manoeuvre, hence, limits of this applicability in the real-world would be the above speed response would need to be accounted for as the speed will be affected through traction forces acting on the plant in the control system. Therefore, realistically, this high angle turn would cause instability in speed.

- Ultimately the control system will attempt to maintain the angle with the speed resulting in instability in the speed response as denoted above in *figure 23*.

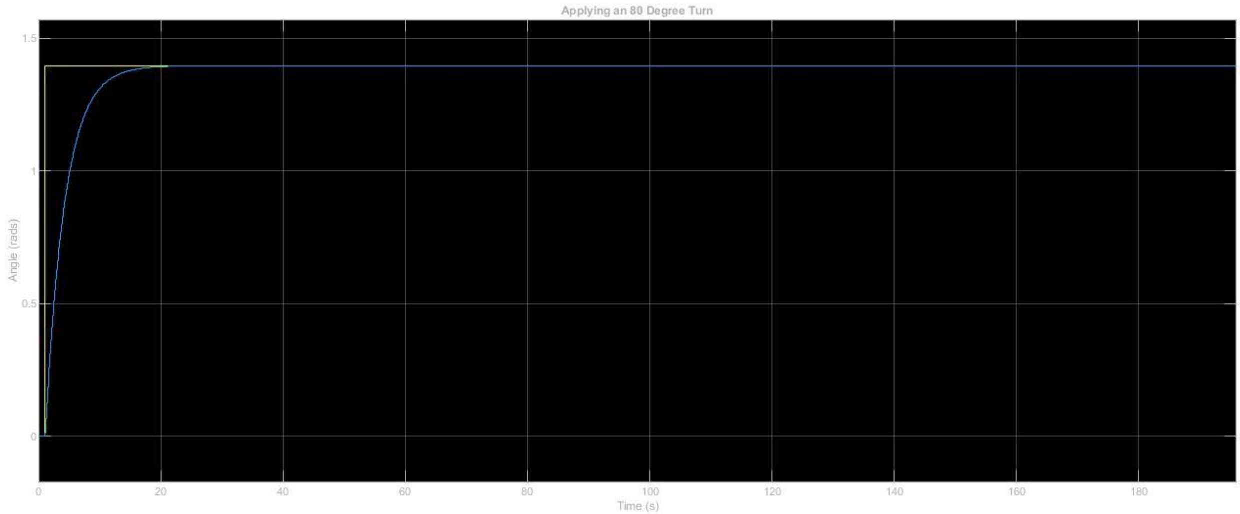


Figure 29: 80 Degree Turn Angle Response

System Zero

The addition of the zero of transfer function results in a step response (faster decrease, rise time and peak time). Most significantly, within this control module, the overshoot increases. In the case of a vehicle that is reversing and hence, experiencing a negative velocity, the trace will undershoot and stabilize to a steady state.

The initial shoot is in the opposite direction to the steady state response previously plotted. As denoted, the step response shoots in one direction then to the other.

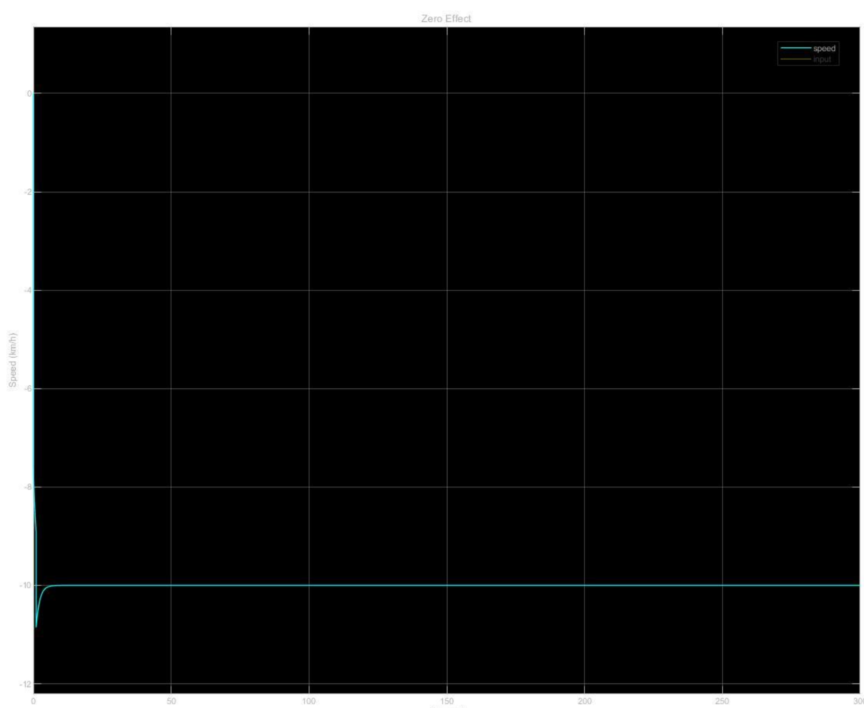


Figure 30: Zero Effect - Speed Response

Discussion

Overview

Overall, the project had achieved its aim through deriving equations for a linearized engine model for a cruise control PI control system design. The project See appendix for a summary of the project and results for the longitudinal PI controller design (improved version with calculations below accounted for in the controller design). The lateral position and angular controller (PID) analysis involved the investigation of the linearized and nonlinearized derivations with plots of a high angle lane change manoeuvre, target speeds and the angles of lane changing along with the discussion of the system zero or reversing of the vehicle.

Understandings and Discoveries

In summary, the longitudinal design involved the creation of a plant alongside the calculation k_i and k_p values to create a PI controller and derive a valid transfer function. The disturbance was modelled as a constant input in either state 1 or 0 however, later it was found a step function (input) gives a more realistic perspective of a force disturbance. The target speeds of $40km/h, 60km/h$ and $80km/h$ were successfully responded to by the system and overshot as expected by a small percentage or value on the trace. The increase of mass was modelled through the change in the calculated mass which affected the k values and the transfer function. The incline slope of the vehicle being modelled to travel up a 20% grad denoted a slower response with a greater overshoot, overcompensating for the incline/force and hence, has a longer time to reach steady state. This was partially expected however, it was discovered that the response also plateaued above the desired input speed, which was concluded to be the overshot speed of the system responding to the incline. The mass was responded by the control system as expected with the slower time to reach the desired speed, a slight fluctuation/instability and a longer time to reach steady state (with increasing weight or mass).

The lateral vehicle design of the PID controller that provides autonomous steering demonstrates that at high angles of lane changing, the speed is affected with instability, along with the zero system or reverse scenario. The increasing speeds also resulted in the angles to be more steep angular response; this would require improvements to allow for better speed stability.

- At higher speeds, the controller should decrease the steepness of the angle when changing lanes and take more time to reach stability.

Improvements

The most significant discoveries and understandings includes the differing plots for the longitudinal controller from the linearized and nonlinearized models as well as the errors of calculating the k_i, k_d and k_p values under the *calculations* section. The following is an improvement to the longitudinal derivations:

$$linearized\ equation: m\dot{v} + \frac{1}{2}A\rho c_D f(v) = u$$

$$through\ Taylor\ Expansion: m\dot{\delta_v} + \frac{1}{2}A\rho c_D = u_0 + \delta_u$$

$$equilibrium\ at\ \delta = 0:$$

$$\therefore m\ddot{v} + \frac{1}{2}A\rho c_D v_0 \dot{v} = \dot{u}$$

$$\dot{u}(t) = k_p\big(\dot{r}(t) - \dot{v}(t)\big) + k_i\big(r(t) - v(t)\big)$$

$$\rightarrow \ddot{v} + \left(\frac{k_p + A\rho c_D v_0}{m}\right)\dot{v} + \left(\frac{k_i}{m}\right)v = \frac{k_i \dot{r}}{m}$$

$$\therefore G(s) = \frac{\left(\frac{k_i}{m}\right)}{s^2 + \left(\frac{k_p + A\rho c_D v_0}{m}\right)s + \frac{k_i}{m}}$$

$$k_i = m \times {\omega_n}^2$$

(Rachid Attia, 2014)

See appendix for improved results for the longitudinal controller.

The lateral controller requires an improvement on the angle estimations that have been made with the PI controller components with the speed response. The previously mentioned angular controller improvement; at higher speeds, the controller should decrease the steepness of the angle when changing lanes and take more time to reach stability, can be improved with the employment of a renewed transfer function alongside a more clear set of k values.

Future Directions

For the future of this project, it is recommended to use this improved PI controller design. Furthermore, future research and progress on the project includes integration of the cruise control system onto autonomous distance detection, allowing for developments in autonomous vehicle systems. Finally, lateral controller with the PID can be employed for future development and researched employment with lane keeping systems in vehicles and automated safe lane changing systems. These systems would require the employment of sensors and vehicle distance systems.

Currently, research is constantly being made, as evident in the reference and citation below, with lane keeping and predictive vehicle movement through motor racing to improve performance and gain higher speeds through difficult corners which are at challenging angles. Therefore, algorithms are aiming to work alongside speed controllers and wheelbase controllers (such as in four-wheel drive and four-wheel steering) to assist with the efficient movement of the vehicle through the most effective racing lines.

(Oliver, Tamas, & Szilard, 2016)

Conclusion

In summation, the project successfully modelled the longitudinal and lateral control system designed within simulation and modelling software *(MATLAB 2018b – Simulink)*. For the longitudinal control, the responses are as expected for the change in vehicle target speed, inclined slope response and differing masses (see appendix for a summarised PI controller design and outputs). The most evident flaw within the control system is the transfer function equation and selection of coefficients (and ultimately parameters) which evident from the output traces has had a minimal effect on the input and output responses. Therefore, the control system for longitudinal design operates as required from the design specifications and parameters.

Bibliography

AMME3500 Systems Dynamics and Control: Design Project 01. (2020). Retrieved from Canvas: https://canvas.sydney.edu.au/courses/21204/assignments/180692

L, G., & A, S. (2007). *Vehicle Propulsion Systems: Introduction to Modeling and Optimization.*

Oliver, T., Tamas, B., & Szilard, A. (2016). *Design of Lane Keeping Algorithm of Autonomous Vehicle.*

Rachid Attia, R. O. (2014). *Combined Longitudinal and Lateral Control for Automated Vehicle Guidance, Vehicle System Dynamics.*

Appendix

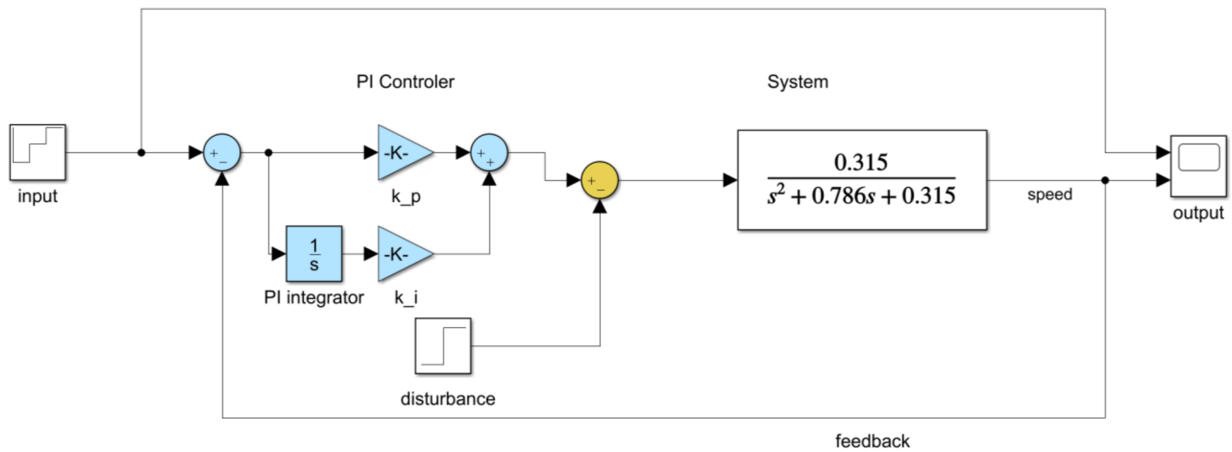


Figure 31: Improved Longitudinal PI Controller Design

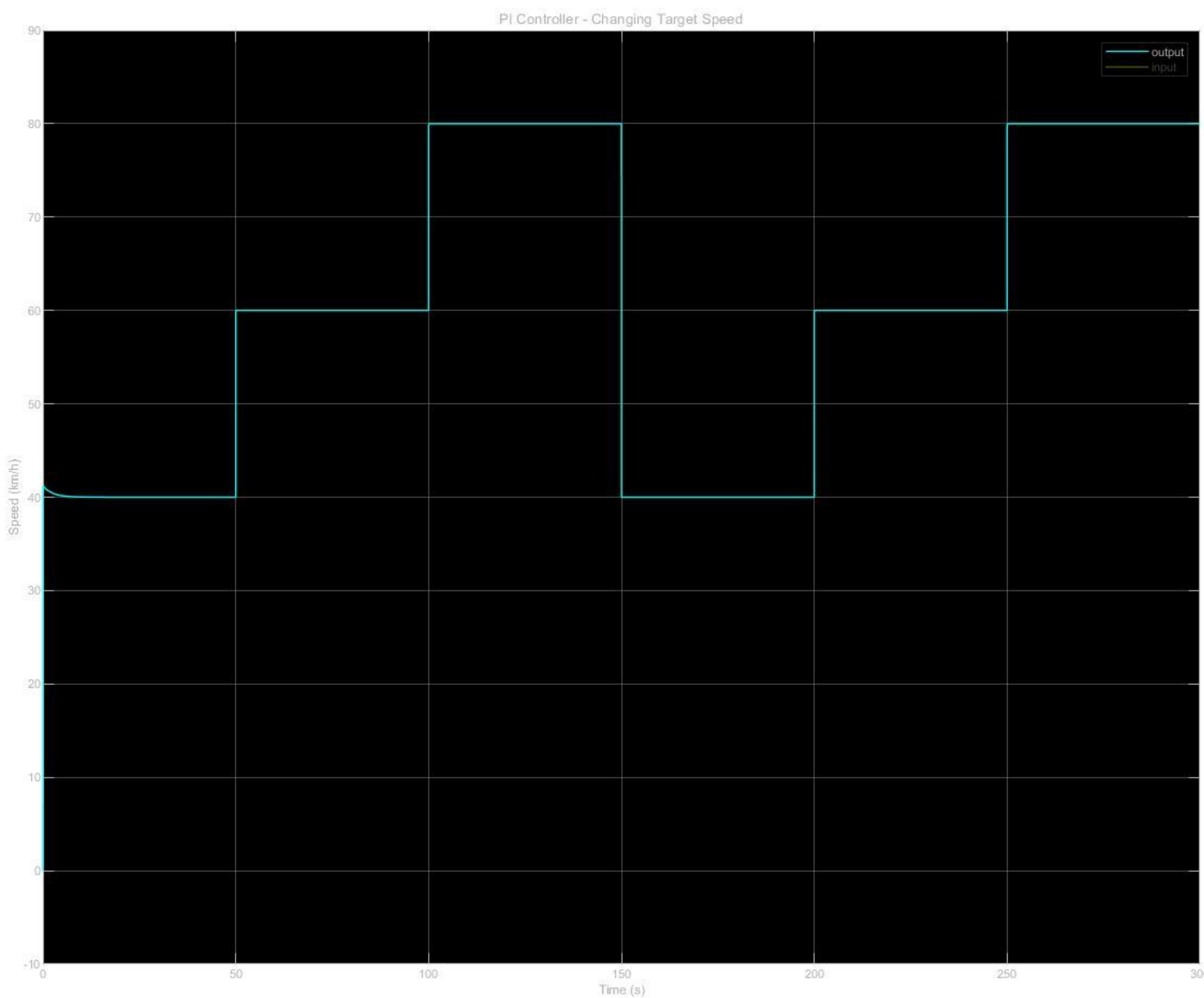


Figure 32: Target Speeds PI Controller Response

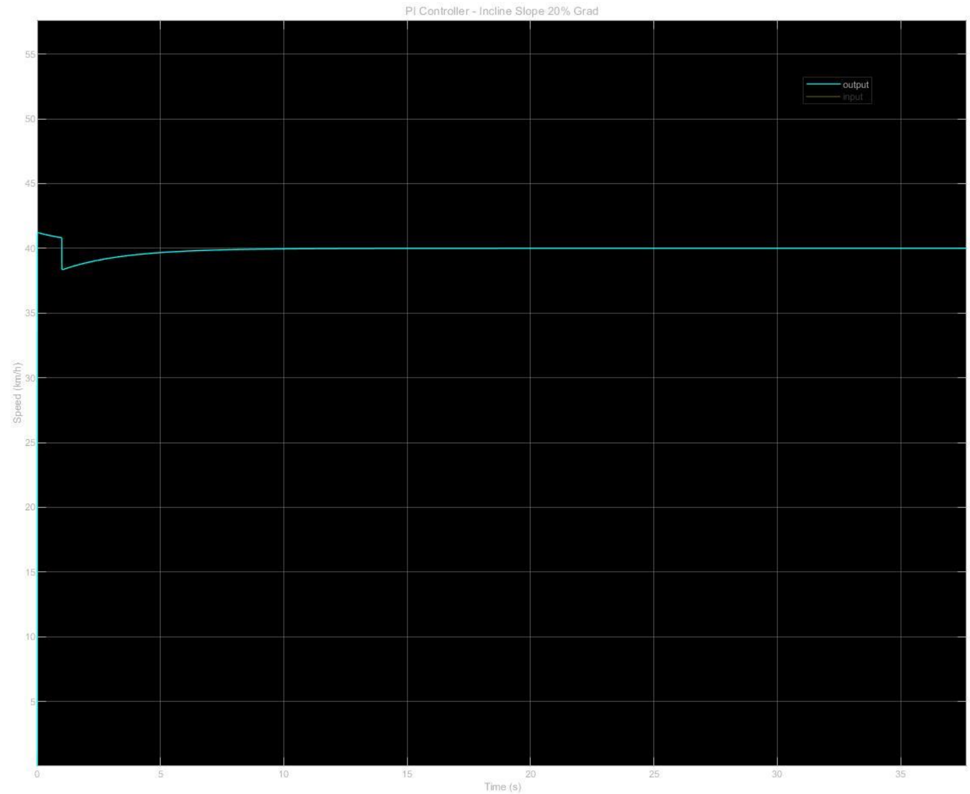


Figure 33: Hill Incline Speed Response - PI Controller

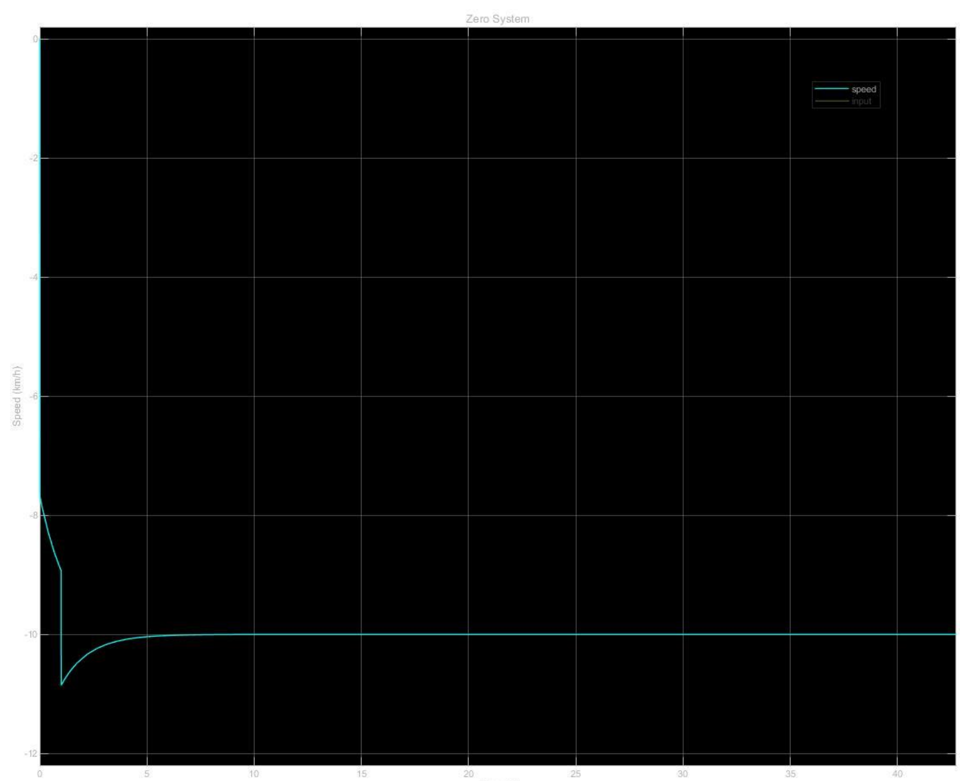


Figure 34: Zero System Zoomed In