AMME5520 – Advanced Control & Optimization

# Motion Planning

Project 1 - Report

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# Contents

# Abstract

The following report involves the details surrounding a plan for a time-optimal path utilized by an *Unmanned Aerial Vehicle* (UAV). The path of the UAV is assumed to travel point-to-point within a complex wind field. This is represented by a *vector field* at points within the 2D space denoted in *figure 1*. Each point represents a 2D vector representing the winds which are assumed to be known within the system design. Furthermore, this report includes an analysis of the system and its planning with each path.

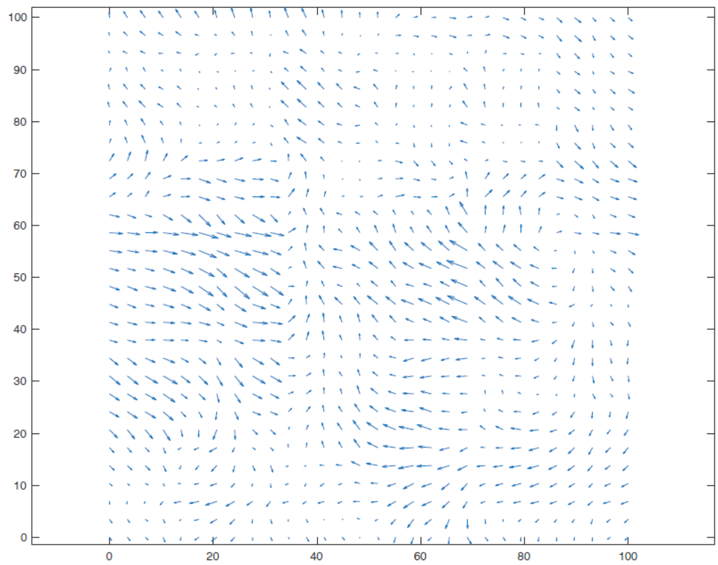



*Figure 1 - Vector Field*

# Introduction

Within the 2D space, the maximum wind-speed is defined by default as 1. The UAV can move with an *air-speed* $|V_a|$ of 1 – the calculation of the actual velocity at any point (relative to the ground) is defined by *Equation 1*.

$$V = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = V_a + V_w(x, y)$$

*Equation 1 – Actual Velocity of UAV Formula*

Note:

- where $|V_a| = 1$
- it is assumed that traveling in an equal direction to the wind will result in a maximum speed of 2.
- It is assumed that traveling against the wind at its strongest point, the UAV may not move at all $|V_a| = 0$.

## Assumptions

The following is a summation of all the assumptions made when designing the system:

- The wind vectors are assumed to be known – modelled by the randomly generated 2D vector field in *figure 1*.

- The velocity vectors that are part of the wind velocity at each point on the 2D space are also known, (the $x$ and $y$ elements of the $V_w$).

- **Speed:**
  - The wind speed is assumed to be of $|V_a| = 1$ with wind at estimated or approximate perpendicular directions to UAV path
    - Therefore, $V = 1\ to\ 2$ with a $V_w$ being the velocity of the wind being additional to the $|V_a|$ const 1 (also highlighted in *Equation 1*.
  - The wind speed is assumed to be of a maximum value $V = 2$ with wind at a perfect parallel direction to the UAV path.
    - Since $V = |V_a| + V_w(x, y)$, where $|V_a| = 1$ (given) and $V_w(x, y) = 1$ in a situation of perfect tail wind.
  - The wind speed is assumed to be of a minimum value of $V = 0$ with wind at a perfect parallel and opposing direction travelling against the path of the UAV.
    - Since $V = |V_a| + V_w(x, y)$, where $|V_a| = 1$ (given) and $V_w(x, y) = -1$ in a situation of perfect head wind.
    - The negative value of $V_w(x, y)$ signifies the direction of wind being against the UAV.
  - The UAV PRM must have at least two points to allow for at least one path to be illustrated onto the 2D space.
  - The points are to be randomly sampled from a range (as mentioned with a lower limit of two).
  - The first point that is randomly sampled will be defined as a start position for the UAV.
    - An end or desired position can be requested by the user.

# Probabilistic Roadmap

## System Design Outline

A *Probabilistic Road-Map* (PRM) was designed and implemented by overlaying points and paths to each point on the 2D space. These points are randomly generated onto the map by sampling a range of values – as mentioned in the assumptions, there will be a lower limit to the sampling range of at least two points (hence, at least one path for the UAV to travel at all times). The upper limit of the sampling range has been limited to currently 10 points but further testing with higher range values have been performed with varying degrees of success (discussed below within this section).

The PRM containing a random set of values between 2-10 points is illustrated in *figure 2*. This exemplar displays a five-point PRM that was randomly sampled. Furthermore, all points have a connecting path to their neighbours and therefore, a cost of travel between each path was calculated utilizing velocity, distance, time of the wind and path.

To allow for a great amount of data to be translated to the participant of the system, the output and calculations made surrounding the PRM include; the direction in-terms of a compass direction, distance between each points, speed of wind, calculation of velocity of wind (integrating direction and speed) and calculation of cost of travel (utilizing *Equation 1*).

To summarise all calculations performed and highlight the specifications according to which the UAV flight has been modelled:

$$V_w(x, y) = \sqrt{{V_w}_x^{\,2} + {V_w}_y^{\,2}}$$

Furthermore, the time can also be calculated as a factor of the cost to travel hence, allowing the cost to travel to be a value incorporating distance, velocity and time:

$$t = \frac{d}{[V_a + V_w(x,y)]}$$

Assumptions would be required upon the units of each variable. For simplicity, it would be reasonable (through a realistic depiction) that distance is a value of metres while speed is a value of metres per second (meaning the average speed of the UAV would range from anywhere between 0 – 7.2 kilometres per hour – seemingly realistic to the performance of an entry level UAV).

An additional comparison of gradients between the various wind vectors as well as the paths and the comparison between their directions has been employed. This allows for the cost to travel function to be influenced by the direction of wind, more specifically the velocity of the UAV calculation considers this comparison and respectively either, sums the wind speed onto the UAV speed ($|V_a| = 1, where\ V_w(x,y) > 0$) or subtracts this speed from the UAV speed ($|V_a| = 1, where\ V_w(x,y) < 0$), along each of the paths the UAV travels, point-to-point.

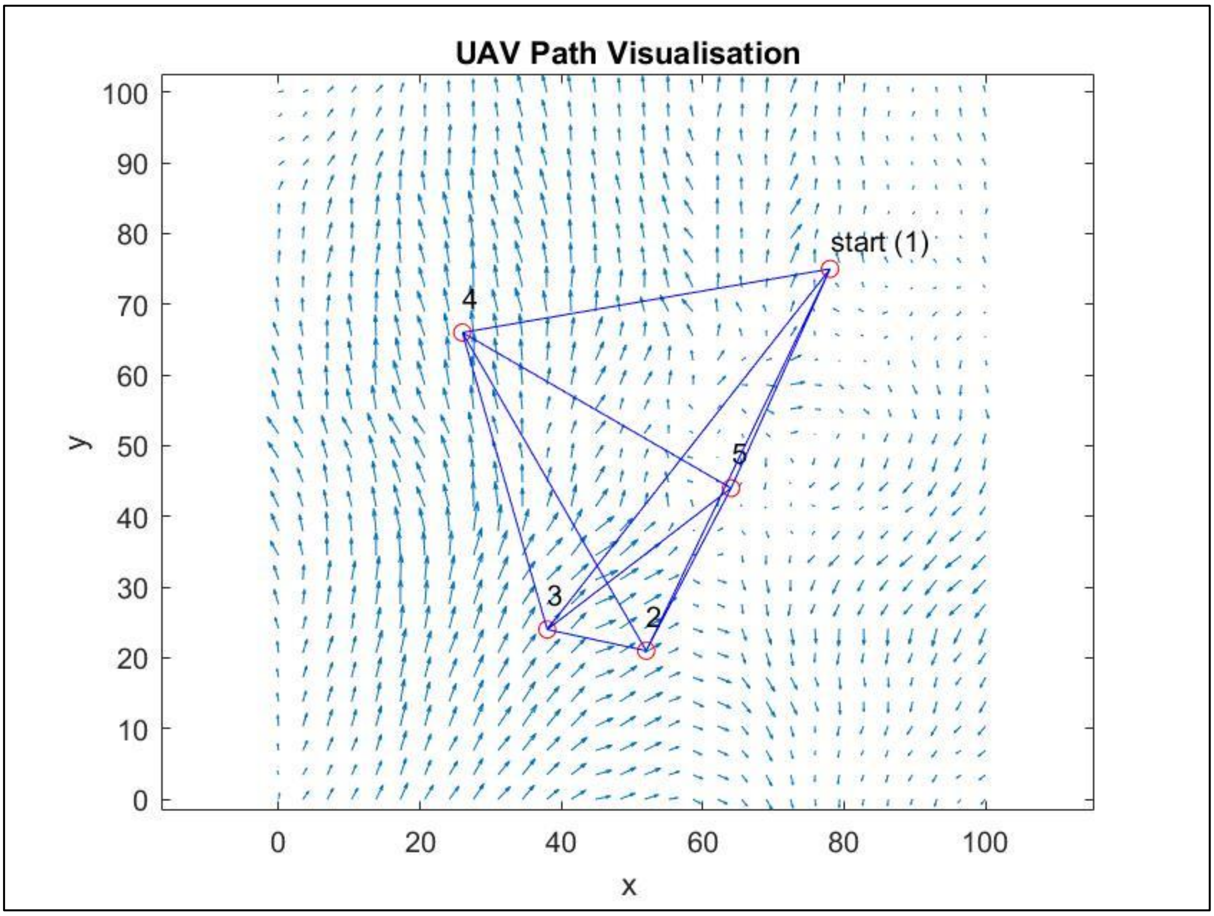



*Figure 2 - 5 Point PRM*

**MATLAB Command Window**

| | | |
|---|---|---|
| point 1 | wind speed = 0.35 | with a direction NW |
| point 2 | wind speed = 0.64 | with a direction NE |
| point 3 | wind speed = 0.70 | with a direction NE |
| point 4 | wind speed = 0.83 | with a direction NW |
| point 5 | wind speed = 0.05 | with a direction NW |

| | | | |
|---|---|---|---|
| 1 - 2 = 59.93 | with gradient = 2.08 | with direction SW | and cost to travel = 44.26 |
| 1 - 3 = 64.82 | with gradient = 1.27 | with direction SW | and cost to travel = 47.87 |
| 1 - 4 = 52.77 | with gradient = 0.17 | with direction SW | and cost to travel = 38.97 |
| 1 - 5 = 34.01 | with gradient = 2.21 | with direction SW | and cost to travel = 25.12 |
| 2 - 3 = 14.32 | with gradient = -0.21 | with direction NW | and cost to travel = 8.73 |
| 2 - 4 = 51.97 | with gradient = -1.73 | with direction NW | and cost to travel = 31.68 |
| 2 - 5 = 25.94 | with gradient = 1.92 | with direction NE | and cost to travel = 15.81 |
| 3 - 4 = 43.68 | with gradient = -3.50 | with direction NW | and cost to travel = 25.71 |
| 3 - 5 = 32.80 | with gradient = 0.77 | with direction NE | and cost to travel = 19.31 |
| 4 - 5 = 43.91 | with gradient = -0.58 | with direction SE | and cost to travel = 255.26 |

point 1 wind velocity = 0.35
point 2 wind velocity = 0.64
point 3 wind velocity = 0.70
point 4 wind velocity = -0.83
point 5 wind velocity = 0.05
>>

By inspecting the above information that is outputted by the system, the user will have a comprehensive understanding of the cost to travel of each path in the PRM.

# Dijkstra's Algorithm Implementation

## Cost to Travel

Dijkstra's Algorithm was implemented and displayed through the system in the most comprehensive manner possible to the user of the system – this was a breakdown of the cost of each path being compared to the desired path that the user of the system is requested to input. This input is error checked affirming that the input is within the specifications of the randomly generated sample range. This aspect of the system can be improved in future by employing a character check to confirm input is a valid integer and have improved, more specific error messages.

Once the user has inputted their desired travel point, the system returns to the user a cost of travel between the points on the PRM. This is displayed in *figure 3* below, tested against a larger range of sampled points initially.

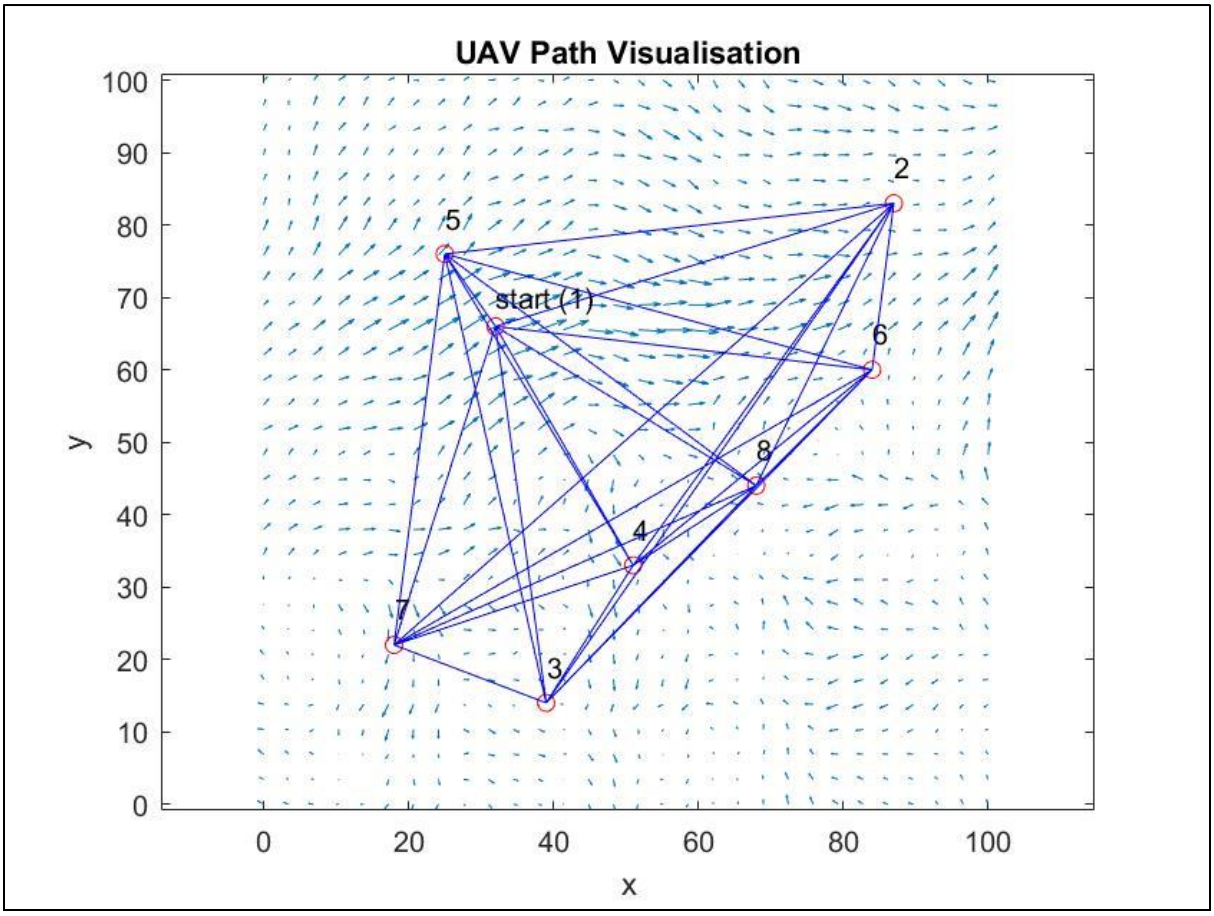



*Figure 3 – 8 Point PRM Cost to Travel*

**MATLAB Command Window**

total points in the system are 8
What is the desired point to travel? 2

| | | |
|---|---|---|
| point 1 | wind speed = 0.92 | with a direction NE |
| point 2 | wind speed = 0.18 | with a direction SE |
| point 3 | wind speed = 0.04 | with a direction SW |
| point 4 | wind speed = 0.26 | with a direction SE |
| point 5 | wind speed = 0.52 | with a direction NE |
| point 6 | wind speed = 0.17 | with a direction NE |
| point 7 | wind speed = 0.31 | with a direction SW |
| point 8 | wind speed = 0.26 | with a direction NW |

| | | | |
|---|---|---|---|
| 1 - 2 = 57.57 | with gradient = 0.31 | with direction NE | and cost to travel = 29.94 |
| 1 - 3 = 52.47 | with gradient = -7.43 | with direction SE | and cost to travel = 27.29 |
| 1 - 4 = 38.08 | with gradient = -1.74 | with direction SE | and cost to travel = 19.81 |
| 1 - 5 = 12.21 | with gradient = -1.43 | with direction NW | and cost to travel = 6.35 |
| 1 - 6 = 52.35 | with gradient = -0.12 | with direction SE | and cost to travel = 27.23 |
| 1 - 7 = 46.17 | with gradient = 3.14 | with direction SW | and cost to travel = 595.84 |
| 1 - 8 = 42.19 | with gradient = -0.61 | with direction SE | and cost to travel = 544.43 |
| 2 - 3 = 84.05 | with gradient = 1.44 | with direction SW | and cost to travel = 71.14 |
| 2 - 4 = 61.61 | with gradient = 1.39 | with direction SW | and cost to travel = 52.15 |
| 2 - 5 = 62.39 | with gradient = 0.11 | with direction SW | and cost to travel = 52.81 |
| 2 - 6 = 23.19 | with gradient = 7.67 | with direction SW | and cost to travel = 19.63 |
| 2 - 7 = 92.10 | with gradient = 0.88 | with direction SW | and cost to travel = 77.95 |

| | | | |
|---|---|---|---|
| 2 - 8 = 43.38 | with gradient = 2.05 | with direction SW | and cost to travel = 36.72 |
| 3 - 4 = 22.47 | with gradient = 1.58 | with direction NE | and cost to travel = 23.33 |
| 3 - 5 = 63.56 | with gradient = -4.43 | with direction NW | and cost to travel = 65.98 |
| 3 - 6 = 64.35 | with gradient = 1.02 | with direction NE | and cost to travel = 62.08 |
| 3 - 7 = 22.47 | with gradient = -0.38 | with direction NW | and cost to travel = 21.68 |
| 3 - 8 = 41.73 | with gradient = 1.03 | with direction NE | and cost to travel = 43.31 |
| 4 - 5 = 50.25 | with gradient = -1.65 | with direction NW | and cost to travel = 67.96 |
| 4 - 6 = 42.64 | with gradient = 0.82 | with direction NE | and cost to travel = 57.66 |
| 4 - 7 = 34.79 | with gradient = 0.33 | with direction SW | and cost to travel = 47.04 |
| 4 - 8 = 20.25 | with gradient = 0.65 | with direction NE | and cost to travel = 27.38 |
| 5 - 6 = 61.13 | with gradient = -0.27 | with direction SE | and cost to travel = 40.20 |
| 5 - 7 = 54.45 | with gradient = 7.71 | with direction SW | and cost to travel = 113.57 |
| 5 - 8 = 53.60 | with gradient = -0.74 | with direction SE | and cost to travel = 111.79 |
| 6 - 7 = 76.16 | with gradient = 0.58 | with direction SW | and cost to travel = 91.80 |
| 6 - 8 = 22.63 | with gradient = 1.00 | with direction SW | and cost to travel = 19.33 |
| 7 - 8 = 54.63 | with gradient = 0.44 | with direction NE | and cost to travel = 79.34 |

| | |
|---|---|
| point 1 | wind velocity = -0.92 |
| point 2 | wind velocity = 0.18 |
| point 3 | wind velocity = -0.04 |
| point 4 | wind velocity = -0.26 |
| point 5 | wind velocity = -0.52 |
| point 6 | wind velocity = 0.17 |
| point 7 | wind velocity = -0.31 |
| point 8 | wind velocity = 0.26 |

the minimum cost to travel to point 2 is 29.94
\>>

## Method for Optimal Motion Planning

With the system design, there is a limitation to its illustration of the optimal path plan. The current version of the system denotes the lowest cost to travel to any point from a starting position or point to a user defined end point, however, does not output the breakdown of the path or cost. With future development of this system, an output outlining the full path and breakdown of each path's costs would be ideal.

Currently, however, the user has all information required, outputted to the screen, that in a manner denotes the ideal steps involved in the UAV's optimal path and cost breakdown – simply a coloured illustration on the 2D space that depicts this path and a final printed message would be needed for further development of the system.

The method of implementing this in order to fully complete Dijkstra's algorithm within the system would firstly require a manner of tracking all paths from each point similar to how currently the listing is achieved.

## Motion Planning to Other Points

In order to incorporate motion planning with other arbitrary points along paths or in the 2D space, the Dijkstra's algorithm with the system can partially be modified to cater to motion between these new points. The calculations of wind velocities at this point can be made by using an outward sampling of surrounding points near a path, using already designed calculations. The system, however, would require a Dijkstra algorithmic modification to obtain, within the surrounding area, a concluding optimal point (or set of points) with minimal counter acting wind velocities.

Similar to what is already employed in the system design, the information relating to wind velocities at set points is most important due to its capability to increase the UAV's speed and reduce travel time – to travel to arbitrary points not initially on the PRM can therefore, be achieved by sampling all points that line between the start position and the user inputted end position. The sampled points that denote an increasing effect on the velocity of the UAV would, therefore, be more desirable for travel and a path then can be formed by the system to the user inputted endpoint.

# Testing & Development

## Altering Sample Space

The system displays a very reliable and accurate design when the number of sample points *K*, are altered. The system has been designed so that this value can be quickly altered by the participant. As part of future development of the system, the sample points should be enquired to the user allowing for greater flexibility of the system. Furthermore, this additional user input would require sufficient and reliable error checking, for example, checking validity of inputted characters and applying limitations to the input.

The following *figure 4* and *figure 5* denotes an example of the system being tested with a differing value of *K* or sample points. Throughout development, it was clear that memory can have an effect on the ability to operate the system on greater values of sample points or ranges of values. Furthermore, increasing the sample points would increase the requirements of performance of the system, since greater number of calculations are made, and more wind velocity points are sampled as part of the optimal path calculations.

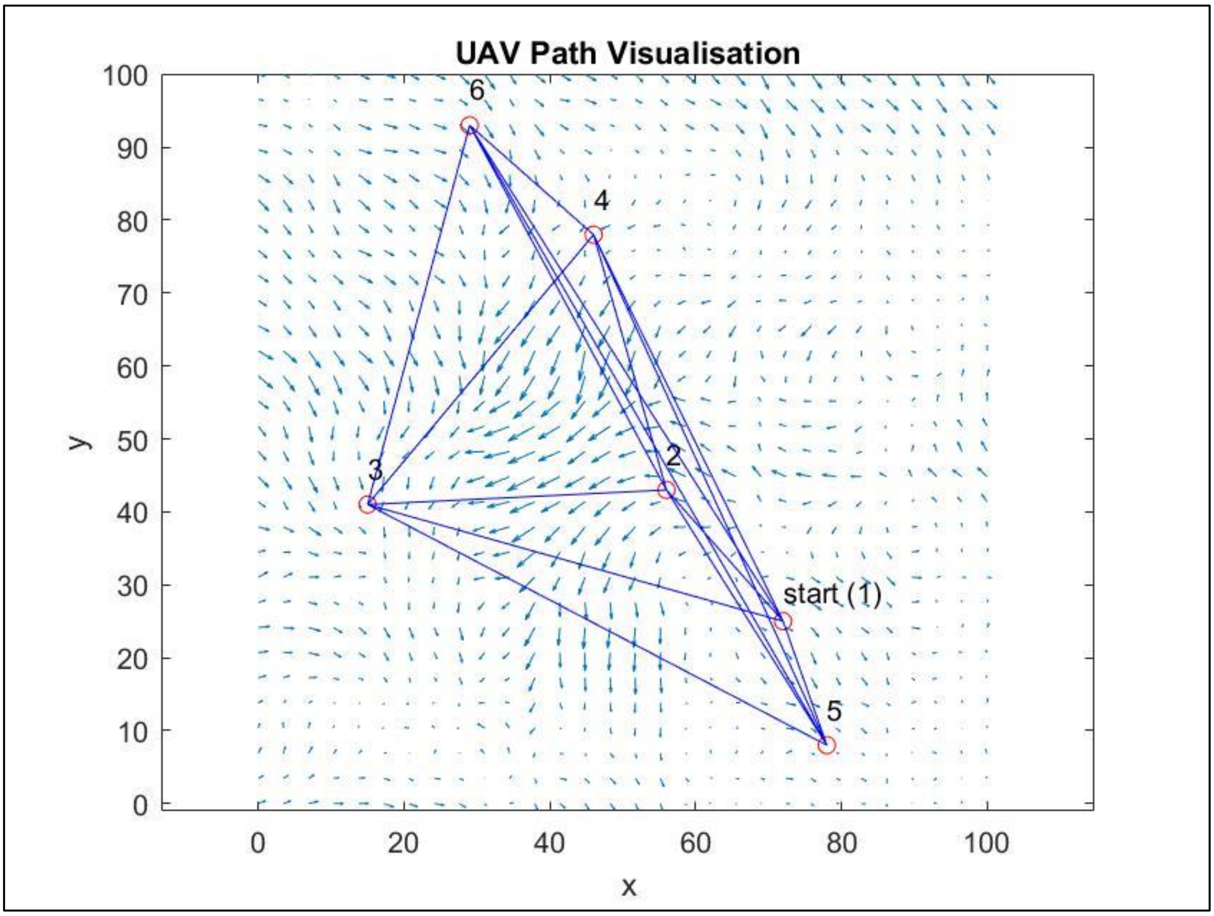



*Figure 4 - 6 Point PRM*

total points in the system are 6
What is the desired point to travel? 3

| | | |
|---|---|---|
| point 1 | wind speed = 0.26 | with a direction SE |
| point 2 | wind speed = 0.43 | with a direction NW |
| point 3 | wind speed = 0.25 | with a direction SE |
| point 4 | wind speed = 0.24 | with a direction SW |
| point 5 | wind speed = 0.12 | with a direction SE |
| point 6 | wind speed = 0.51 | with a direction SE |

| | | | |
|---|---|---|---|
| 1 - 2 = 24.08 | with gradient = -1.13 | with direction NW | and cost to travel = 32.59 |
| 1 - 3 = 59.20 | with gradient = -0.28 | with direction NW | and cost to travel = 46.95 |
| 1 - 4 = 59.03 | with gradient = -2.04 | with direction NW | and cost to travel = 79.89 |
| 1 - 5 = 18.03 | with gradient = -2.83 | with direction SE | and cost to travel = 24.40 |
| 1 - 6 = 80.45 | with gradient = -1.58 | with direction NW | and cost to travel = 63.80 |
| 2 - 3 = 41.05 | with gradient = 0.05 | with direction SW | and cost to travel = 28.62 |
| 2 - 4 = 36.40 | with gradient = -3.50 | with direction NW | and cost to travel = 25.38 |
| 2 - 5 = 41.34 | with gradient = -1.59 | with direction SE | and cost to travel = 73.06 |
| 2 - 6 = 56.82 | with gradient = -1.85 | with direction NW | and cost to travel = 100.43 |
| 3 - 4 = 48.27 | with gradient = 1.19 | with direction NE | and cost to travel = 38.56 |
| 3 - 5 = 71.12 | with gradient = -0.52 | with direction SE | and cost to travel = 56.82 |
| 3 - 6 = 53.85 | with gradient = 3.71 | with direction NE | and cost to travel = 43.02 |
| 4 - 5 = 76.97 | with gradient = -2.19 | with direction SE | and cost to travel = 62.20 |
| 4 - 6 = 22.67 | with gradient = -0.88 | with direction NW | and cost to travel = 18.32 |
| 5 - 6 = 98.11 | with gradient = -1.73 | with direction NW | and cost to travel = 111.93 |

| | |
|---|---|
| point 1 | wind velocity = 0.26 |
| point 2 | wind velocity = -0.43 |
| point 3 | wind velocity = 0.25 |
| point 4 | wind velocity = 0.24 |
| point 5 | wind velocity = -0.12 |
| point 6 | wind velocity = 0.51 |

the minimum cost to travel to point 3 is 46.95
>>

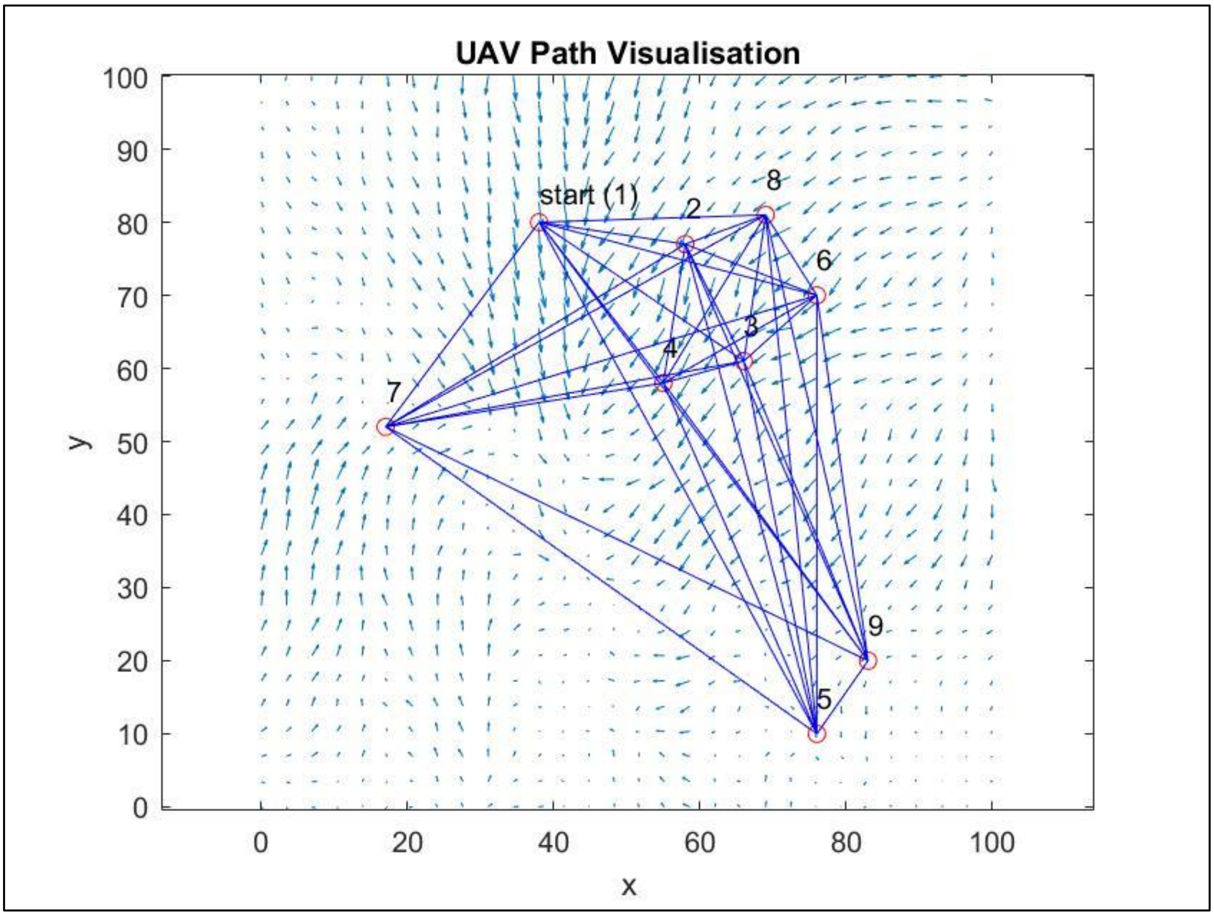



*Figure 5 - 9 Point PRM*

**MATLAB Command Window**

total points in the system are 9
What is the desired point to travel? 3

| | | |
|---|---|---|
| point 1 | wind speed = 0.85 | with a direction SE |
| point 2 | wind speed = 0.76 | with a direction SW |
| point 3 | wind speed = 0.48 | with a direction SW |
| point 4 | wind speed = 0.68 | with a direction SW |
| point 5 | wind speed = 0.20 | with a direction SE |
| point 6 | wind speed = 0.63 | with a direction SW |
| point 7 | wind speed = 0.22 | with a direction NE |
| point 8 | wind speed = 0.57 | with a direction SW |
| point 9 | wind speed = 0.23 | with a direction SW |

| | | | |
|---|---|---|---|
| 1 - 2 = 20.22 | with gradient = -0.15 | with direction SE | and cost to travel = 10.93 |
| 1 - 3 = 33.84 | with gradient = -0.68 | with direction SE | and cost to travel = 18.28 |
| 1 - 4 = 27.80 | with gradient = -1.29 | with direction SE | and cost to travel = 15.02 |
| 1 - 5 = 79.65 | with gradient = -1.84 | with direction SE | and cost to travel = 43.03 |
| 1 - 6 = 39.29 | with gradient = -0.26 | with direction SE | and cost to travel = 21.23 |
| 1 - 7 = 35.00 | with gradient = 1.33 | with direction SW | and cost to travel = 18.91 |
| 1 - 8 = 31.02 | with gradient = 0.03 | with direction NE | and cost to travel = 16.76 |
| 1 - 9 = 75.00 | with gradient = -1.33 | with direction SE | and cost to travel = 40.52 |
| 2 - 3 = 17.89 | with gradient = -2.00 | with direction SE | and cost to travel = 10.16 |
| 2 - 4 = 19.24 | with gradient = 6.33 | with direction SW | and cost to travel = 10.92 |
| 2 - 5 = 69.38 | with gradient = -3.72 | with direction SE | and cost to travel = 39.39 |

| | | | |
|---|---|---|---|
| 2 - 6 = 19.31 | with gradient = -0.39 | with direction SE | and cost to travel = 10.97 |
| 2 - 7 = 48.02 | with gradient = 0.61 | with direction SW | and cost to travel = 27.26 |
| 2 - 8 = 11.70 | with gradient = 0.36 | with direction NE | and cost to travel = 49.04 |
| 2 - 9 = 62.24 | with gradient = -2.28 | with direction SE | and cost to travel = 260.77 |
| 3 - 4 = 11.40 | with gradient = 0.27 | with direction SW | and cost to travel = 7.70 |
| 3 - 5 = 51.97 | with gradient = -5.10 | with direction SE | and cost to travel = 35.08 |
| 3 - 6 = 13.45 | with gradient = 0.90 | with direction NE | and cost to travel = 25.94 |
| 3 - 7 = 49.82 | with gradient = 0.18 | with direction SW | and cost to travel = 96.06 |
| 3 - 8 = 20.22 | with gradient = 6.67 | with direction NE | and cost to travel = 13.65 |
| 3 - 9 = 44.38 | with gradient = -2.41 | with direction SE | and cost to travel = 29.96 |
| 4 - 5 = 52.39 | with gradient = -2.29 | with direction SE | and cost to travel = 31.20 |
| 4 - 6 = 24.19 | with gradient = 0.57 | with direction NE | and cost to travel = 75.39 |
| 4 - 7 = 38.47 | with gradient = 0.16 | with direction SW | and cost to travel = 119.91 |
| 4 - 8 = 26.93 | with gradient = 1.64 | with direction NE | and cost to travel = 16.04 |
| 4 - 9 = 47.20 | with gradient = -1.36 | with direction SE | and cost to travel = 28.11 |
| 5 - 6 = 60.00 | with gradient = 0.00 | with direction S | and cost to travel = 50.17 |
| 5 - 7 = 72.42 | with gradient = -0.71 | with direction NW | and cost to travel = 90.06 |
| 5 - 8 = 71.34 | with gradient = -10.14 | with direction NW | and cost to travel = 59.66 |
| 5 - 9 = 12.21 | with gradient = 1.43 | with direction NE | and cost to travel = 10.21 |
| 6 - 7 = 61.68 | with gradient = 0.31 | with direction SW | and cost to travel = 37.82 |
| 6 - 8 = 13.04 | with gradient = -1.57 | with direction NW | and cost to travel = 7.99 |
| 6 - 9 = 50.49 | with gradient = -7.14 | with direction SE | and cost to travel = 30.95 |
| 7 - 8 = 59.54 | with gradient = 0.56 | with direction NE | and cost to travel = 48.85 |
| 7 - 9 = 73.35 | with gradient = -0.48 | with direction SE | and cost to travel = 60.18 |
| 8 - 9 = 62.59 | with gradient = -4.36 | with direction SE | and cost to travel = 39.77 |

point 1 wind velocity = 0.85
point 2 wind velocity = -0.76
point 3 wind velocity = 0.48
point 4 wind velocity = 0.68
point 5 wind velocity = 0.20
point 6 wind velocity = 0.63
point 7 wind velocity = 0.22
point 8 wind velocity = 0.57
point 9 wind velocity = 0.23

the minimum cost to travel to point 3 is 10.16
>>

The system also denotes arising issues with formatting in outputs when much larger amounts of sample points, for example 17 points on a PRM in the 2D space (this is noted in the *Appendix figure 1*). Furthermore, there would be a limit to the stored calculations and data in relation to the sampled points – possibly due to the limitations of MATLAB and stock settings configured with RAM and memory allocation of the system. Similarly, the value function looses its accuracy in terms of calculated values and visualisation.

The system currently contains a visualisation and method of depicting the connection of points on the PRM or graph within the 2D space. The number of points to connect can be further optimized by eliminating values that are not viable or have a large cost to travel which would not be traversed by the UAV. This can be accomplished with further development with the Dijkstra algorithm upon which only cost to travel and cost to go viable paths can be included into the visualisation. Furthermore, a second visualisation can be included with this more viable paths for the UAV, which can be utilized to reduce the number of points denoted within the system – only the necessary points could be potentially visualised onto a second figure in the system.

The system loses its accuracy to denote the most optimal path for the UAV to traverse when a large amount of sample points is added into the system. This is due to the cost of travel that is outputted by the system and the paths being displayed not precisely showing the exact breakdown or steps involved in the most efficient travel to the user defined end point. This can be improved with future development, by employing a manner to better display the paths very specifically and outlining to the screen or user exactly each path involved with getting to the user defined endpoint.

The design of the system is moderately robust with handling the computational complexity of increasing sampling points since, the system utilizes a minimal amount of pre-defined factors or variables (aside from the assumptions) – allows for the computations to not be affected by changes and hence, is reliable with sample point alterations. In contrast however, the timing and processes of the system increases resulting in a greater use of memory and time to calculate and output the data as information for the user. This was quantified by investigating the time of execution of the program – when the sampling points was increase by factor of two, the execution time increase by almost 1.6 seconds approximately. Some examples of this increase in processing time by the program is highlighted in *Appendix figure 3*.

### System Limitations & Timing

The model employed to store the values of wind velocities (speed and direction) and the paths information (gradient, direction, distance) as well as the computations involving cost to travel (time and values) are all limited within the space of the program due to memory allocation – there is a finite amount of sample points for which this system can operate and with higher sample points, the output becomes less accurate and the time of execution increases. Furthermore, with increased execution time and a computational delay when a large sample range is employed (or larger sample points are requested by the participant) the calculations denote a time delay hence, computations maybe less accurate. This could result in values involved in calculations being inaccurate (delayed values being used in functions) and in turn can return unreliable computations when these inaccurate values are employed in computations.

## Visualisation of Arrival Cost

The arrival cost was partially visualised through the outputted information that is communicated to the user, however, was not able to be properly implemented into the system with accurate and precise illustration. The cost to travel presently in the system has a total value and therefore, the path illustrated is highlighted to be only a small path to the end point or a partial path rather than all the steps involved with the UAV optimal travel.

With relation to the wind field the path that is depicted and visualised with the figure would ideally be parallel (approximately) between the direction of the UAV path and the direction of wind speeds.

The visualisation of the arrival cost or value function is denoted in the appendix *figure 5*. The overlaying visualisation of the value function is partially inaccurate with the exemplar in the appendix *figure 5* denoting a mirrored inaccurate gradient shade.

## Heuristic Model

### Proposed Heuristic

A *Heuristic Algorithm* that may be realistically viable for a UAV optimal path selection would be an evolutionary algorithm where distance and smoothing can also be applied to enhance the building of paths. This is a preferred algorithm over optimal algorithms since there is high computational complexity with the latter however, heuristic algorithms do include a A* algorithm and hence, exist in a random probability.

The system displays only the first path that it detects to the desired user inputted end point if an incorrect Heuristic is implemented. When this Heuristic is implemented the computational complexity is reduced however, the optimality of the system and the identification of the ideal path of the UAV is in accurate.

With this heuristic the path cost estimation is lower than the true cost as the number of wind vectors that are considered are less than those existent along the path, therefore, the heuristic employed is **admissible**. Furthermore, this is true for all paths and each path is illustrated to travel to all points in the PRM however, the paths do not display equality in the degenerate case – heuristic is not **consistent**.

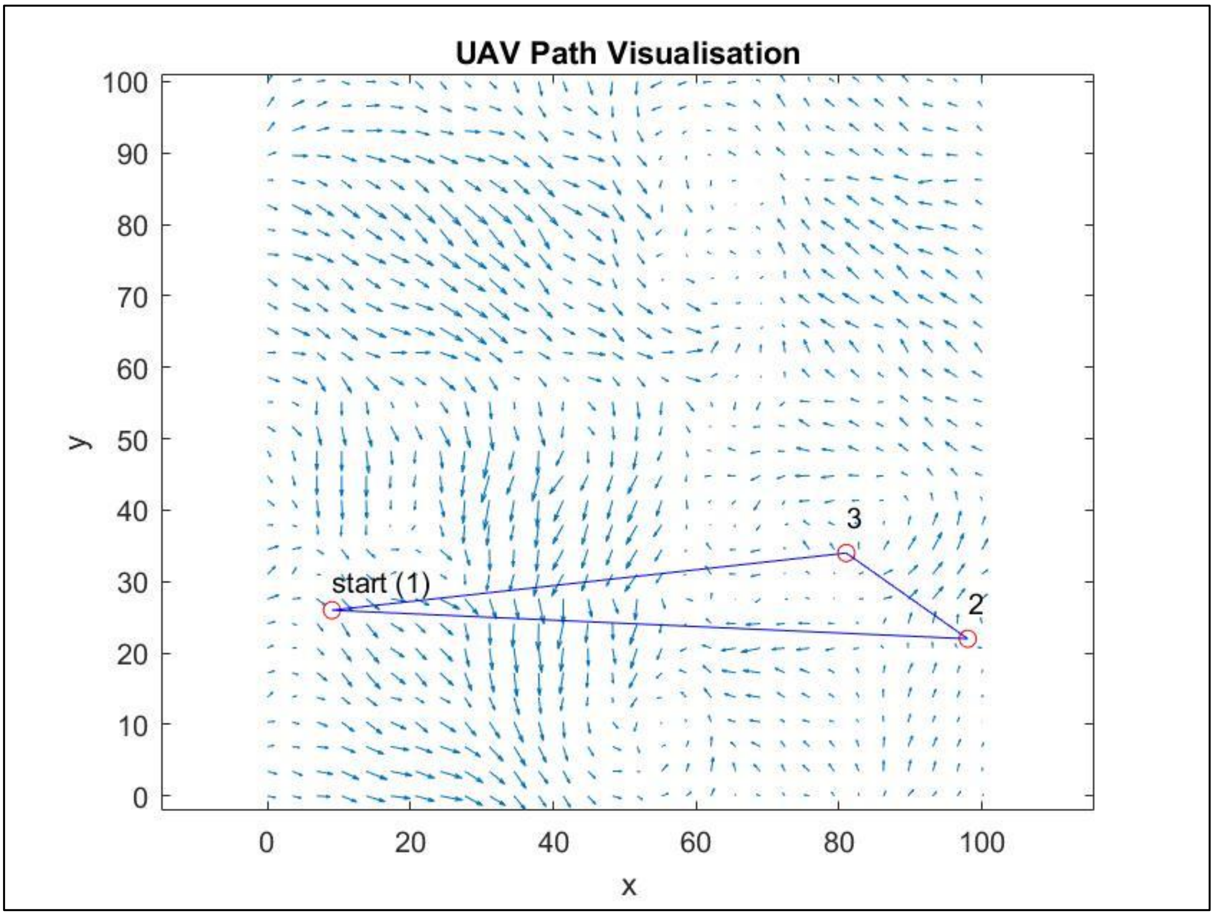



*Figure 6 - Degenerate Case*

## Comparison to Dijkstra's Algorithm

In comparison with optimal path algorithm, both aim to minimize the cost of UAV flight. This purposed heuristic algorithm utilizes only the points (or number of turns and distances). The heuristic algorithm is planned to be employed in future development by starting at the end point and stating the minimal cost path from this point moving in an outward surrounding fashion of analysis. This is tested and depicted in *figure 7*.

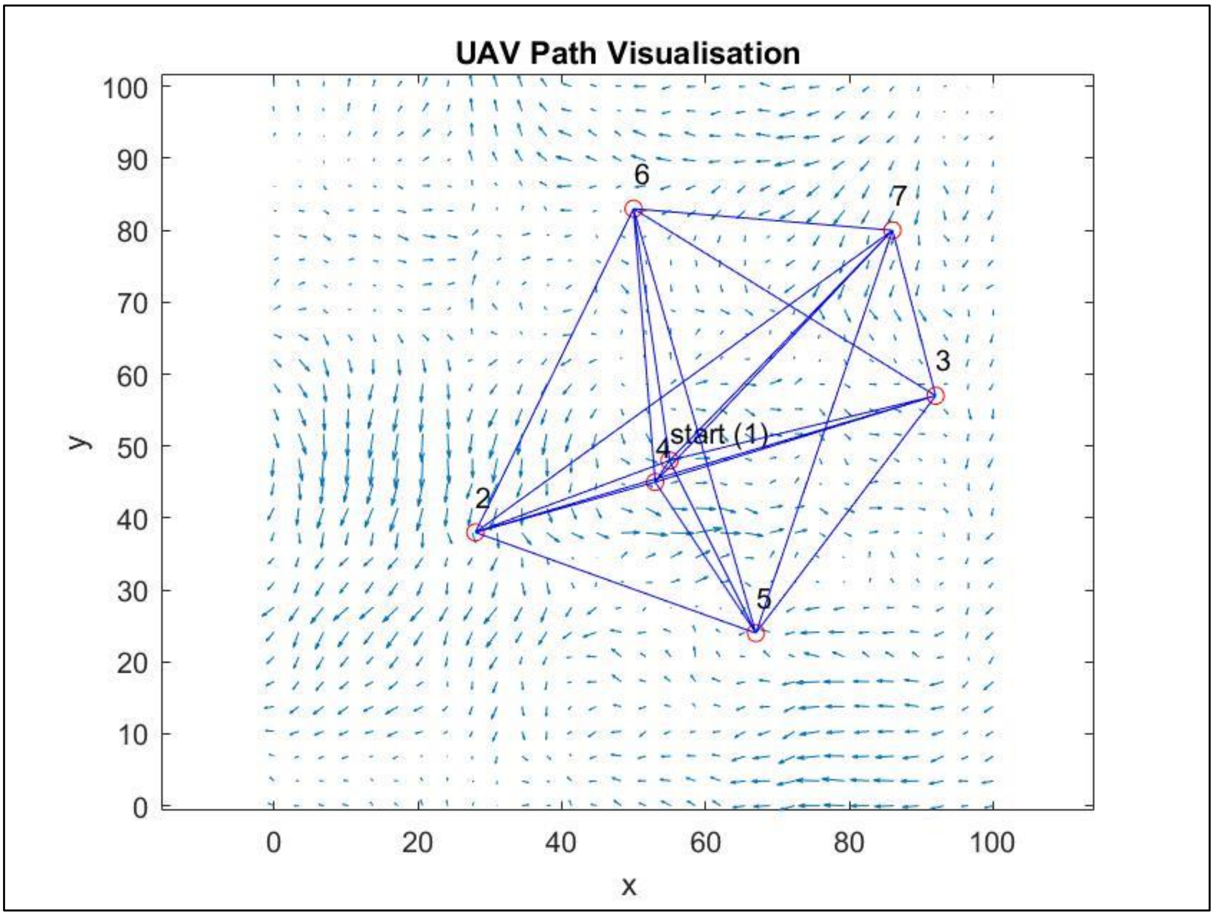



*Figure 7 - Heuristic Paths to Every Point*

**MATLAB Command Window:**

total points in the system are 7
What is the desired point to travel? 2

| | | |
|---|---|---|
| point 1 | wind speed = 0.33 | with a direction SE |
| point 2 | wind speed = 0.37 | with a direction SE |
| point 3 | wind speed = 0.18 | with a direction SE |
| point 4 | wind speed = 0.24 | with a direction SE |
| point 5 | wind speed = 0.25 | with a direction SW |
| point 6 | wind speed = 0.16 | with a direction SW |
| point 7 | wind speed = 0.35 | with a direction SW |

| | | | |
|---|---|---|---|
| 1 - 2 = 28.79 | with gradient = 0.37 | with direction SE | and cost to travel = 21.64 |
| 1 - 3 = 38.08 | with gradient = 0.24 | with direction NE | and cost to travel = 28.62 |
| 1 - 4 = 3.61 | with gradient = 1.50 | with direction SE | and cost to travel = 2.71 |
| 1 - 5 = 26.83 | with gradient = -2.00 | with direction SW | and cost to travel = 20.17 |
| 1 - 6 = 35.36 | with gradient = -7.00 | with direction NW | and cost to travel = 52.80 |
| 1 - 7 = 44.55 | with gradient = 1.03 | with direction NE | and cost to travel = 66.53 |
| 2 - 3 = 66.76 | with gradient = 0.30 | with direction NE | and cost to travel = 48.76 |
| 2 - 4 = 25.96 | with gradient = 0.28 | with direction NE | and cost to travel = 18.96 |
| 2 - 5 = 41.44 | with gradient = -0.36 | with direction SW | and cost to travel = 30.26 |
| 2 - 6 = 50.09 | with gradient = 2.05 | with direction NE | and cost to travel = 36.58 |
| 2 - 7 = 71.61 | with gradient = 0.72 | with direction NE | and cost to travel = 52.30 |
| 3 - 4 = 40.80 | with gradient = 0.31 | with direction SE | and cost to travel = 34.59 |
| 3 - 5 = 41.40 | with gradient = 1.32 | with direction SE | and cost to travel = 35.10 |

| | | | |
|---|---|---|---|
| 3 - 6 = 49.40 | with gradient = -0.62 | with direction NW | and cost to travel = 60.21 |
| 3 - 7 = 23.77 | with gradient = -3.83 | with direction NW | and cost to travel = 20.15 |
| 4 - 5 = 25.24 | with gradient = -1.50 | with direction SW | and cost to travel = 20.33 |
| 4 - 6 = 38.12 | with gradient = -12.67 | with direction NW | and cost to travel = 50.25 |
| 4 - 7 = 48.10 | with gradient = 1.06 | with direction NE | and cost to travel = 63.42 |
| 5 - 6 = 61.40 | with gradient = -3.47 | with direction NW | and cost to travel = 49.13 |
| 5 - 7 = 59.14 | with gradient = 2.95 | with direction NE | and cost to travel = 78.82 |
| 6 - 7 = 36.12 | with gradient = -0.08 | with direction SW | and cost to travel = 31.27 |

The heuristic algorithm has a strong path search ability and is suitable for the path illustrations made within this system. The Dijkstra's algorithm locates the typical shortest path as denoted in prior sections and extends from the starting point. The advantage of this design is that the shortest path, as displayed denotes a high success rate and good robustness with varying sample ranges. The disadvantage of this algorithm is that all nodes need to be traversed as denoted in the above sections and present system design.

### Testing Heuristic

In comparison with A* algorithm, the Dijkstra's algorithm, heuristic algorithm has poor efficiency with complex PRM networks, furthermore, the algorithm denoted issues with negative edge computations. The A* algorithm has the advantage of few extension nodes and is a preferrable implemented algorithm with real-time systems. The size of the UAV however is not considered within this system hence, the 2D space and points effecting the vehicle would not be regarded when calculating distance from node-to-node.

## Testing Speeds of UAV

The altering of UAV speed and the maximum wind speed would have a varying effect on the design of the system which can be partially tested in the program and modelled for further effects. When the speed of the UAV is increased the influence of the wind velocity would be reduced, more specifically with head wind – increasing the possible optimal paths of the system which also making direct path travel to an end point more viable. When the speed of the UAV is decreased the overall time of travel across all paths would be increased and the cost to travel with the wind velocities having an increased effect on the UAV.

## Smoothening of UAV Motion & Path

### Proposed Method of Smoothing

In order to increase the realistic path aspects of the system and depict a more accurate path that the UAV may traverse from point-to-point, the employed of cubic spline data interpolation would return a smoother path between points. Furthermore, testing with the such functionality shows that this can be better implemented through use of query points that are existent already within the system. The below *figure 8* denotes the points that can be added to a corner or turn to allow for smoothness – comparison between linear and interpolation for turns that will exist on PRM.

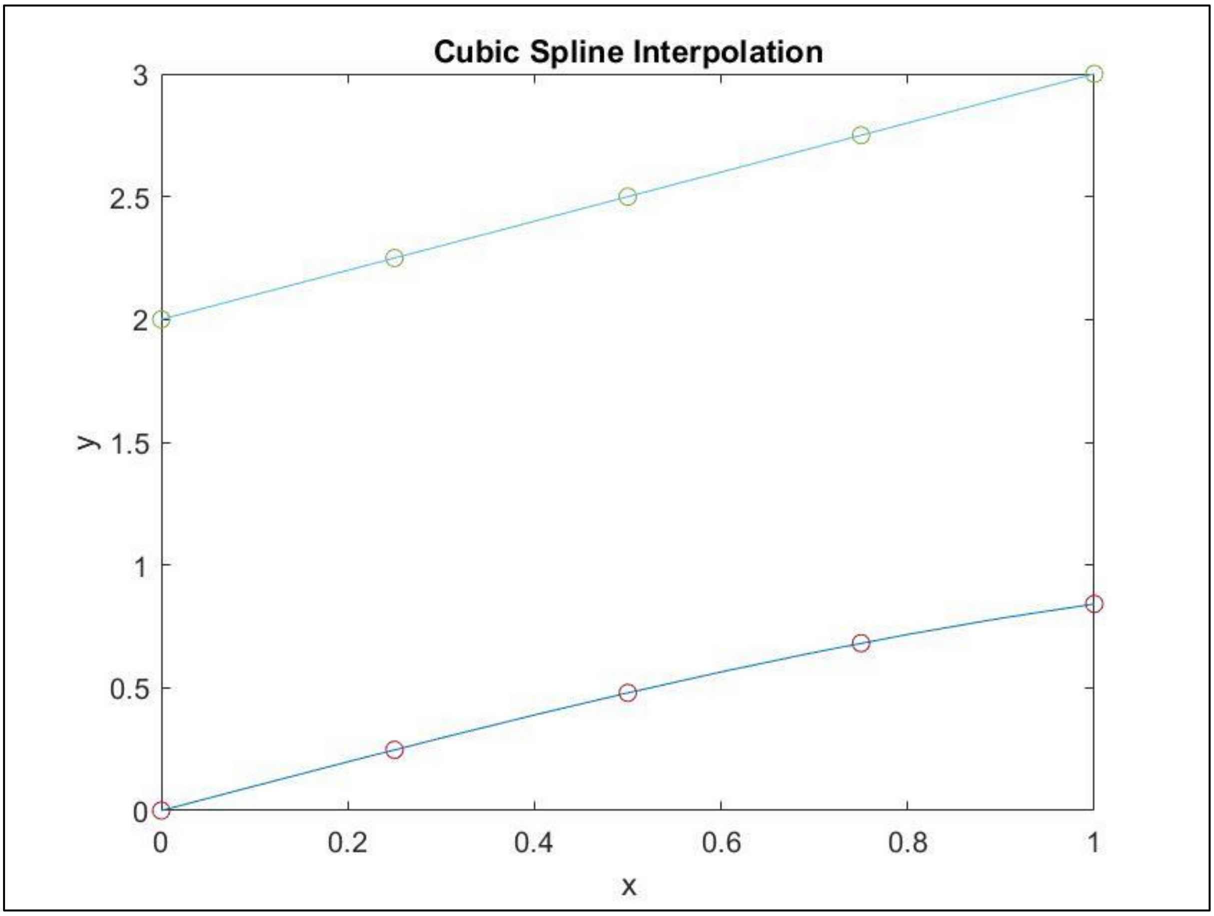
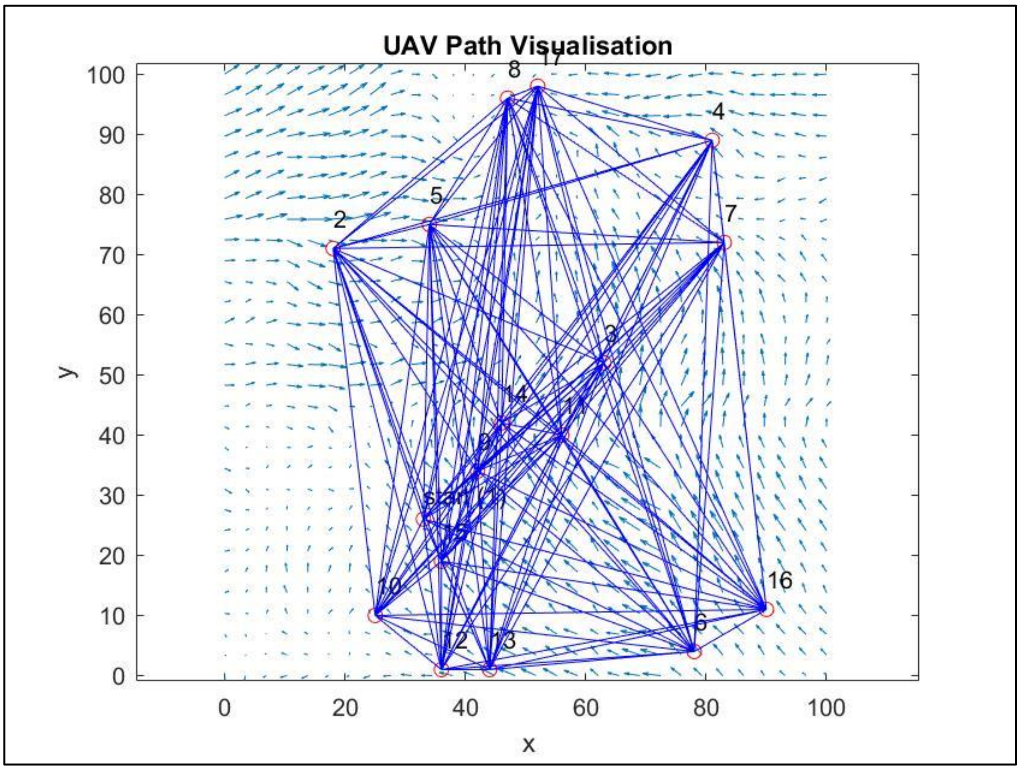



*Figure 8 - Example of Smoothening with Points*

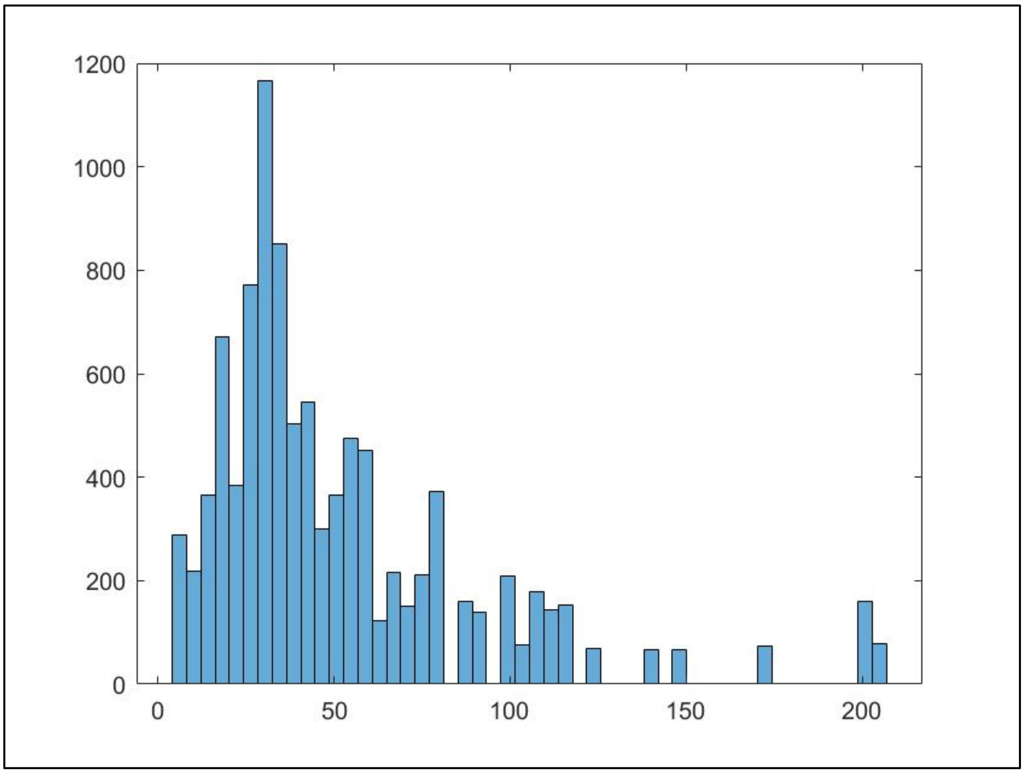
## Model of Smoothening UAV Motion

When the cubic spline data interpolation is employed into the system, the output is evident to have an impact on the point-to-point path and connecting paths however, the implementation requires tuning to output more desirable and realistic path traces. Furthermore, it was investigated that the path now travels over areas that may not be sampled and hence, is not included in the computation of wind velocities or rather the path is slightly offset from the true wind velocities that are sampled by the system. For future development, the implementation of cubic spline data interpolation can be employed with relationship to wind directions, therefore, the paths that are interpole are influenced by the values of wind velocities at points (partially implemented as part of the system design and sampling of wind velocities at points).

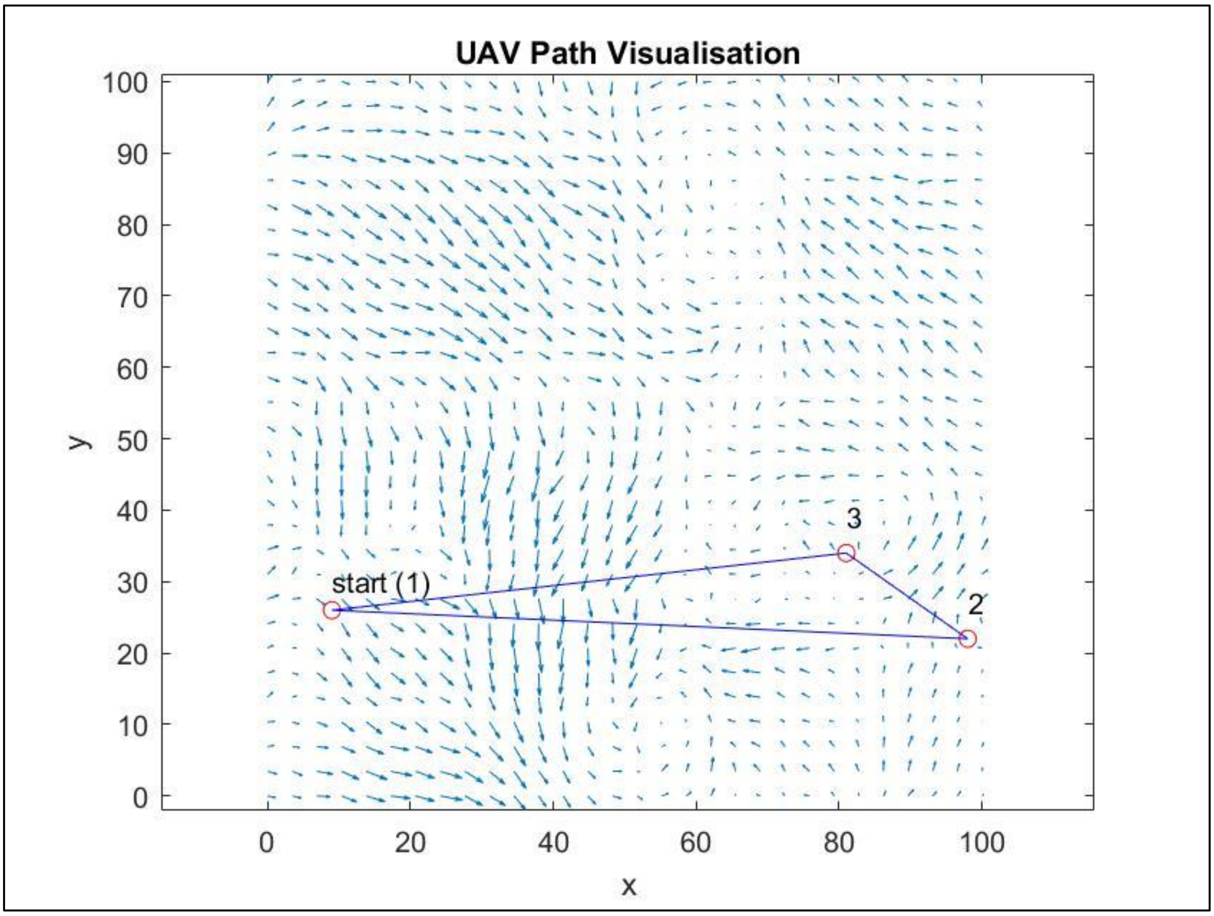
# Appendix




*Appendix Figure 1 - 17 Point PRM & Command Window*



*Appendix Figure 2 - Value Function for 17 Point PRM*




*Appendix Figure 3 - 3 Point Timing Example*

**MATLAB Command Window:**

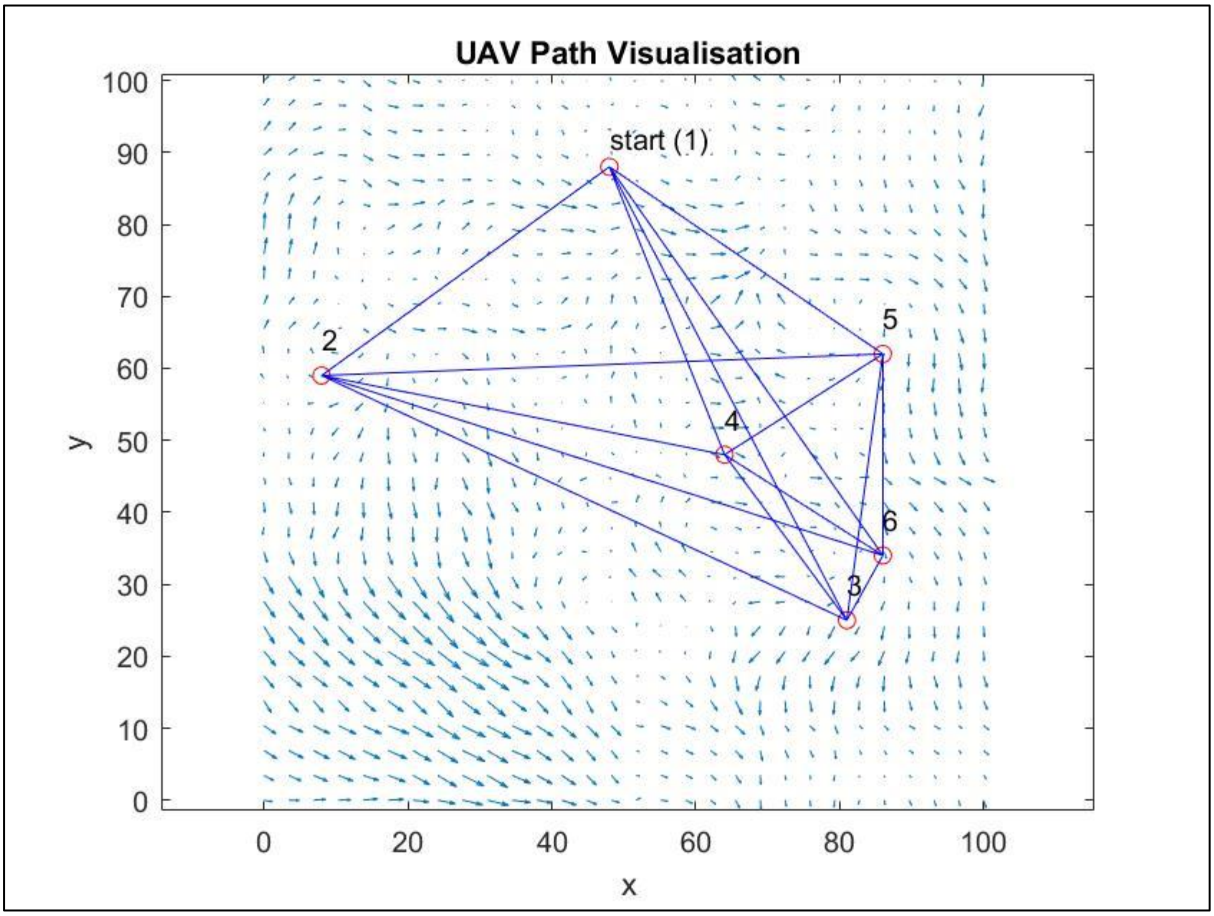
total points in the system are 3
What is the desired point to travel? 2

point 1 wind speed = 0.45 with a direction SE
point 2 wind speed = 0.18 with a direction NW
point 3 wind speed = 0.22 with a direction NW

1 - 2 = 89.09 with gradient = -0.04 with direction SW and cost to travel = 61.40
1 - 3 = 72.44 with gradient = 0.11 with direction NE and cost to travel = 49.93
2 - 3 = 20.81 with gradient = -0.71 with direction NW and cost to travel = 17.65

point 1 wind velocity = 0.45
point 2 wind velocity = 0.18
point 3 wind velocity = 0.22

the minimum cost to travel to point 2 is 61.40
2.34 is the total time of execution.
>>




*Appendix Figure 4 - Increased Points & Timing Effects*

**MATLAB Command Window:**

total points in the system are 6
What is the desired point to travel? 3

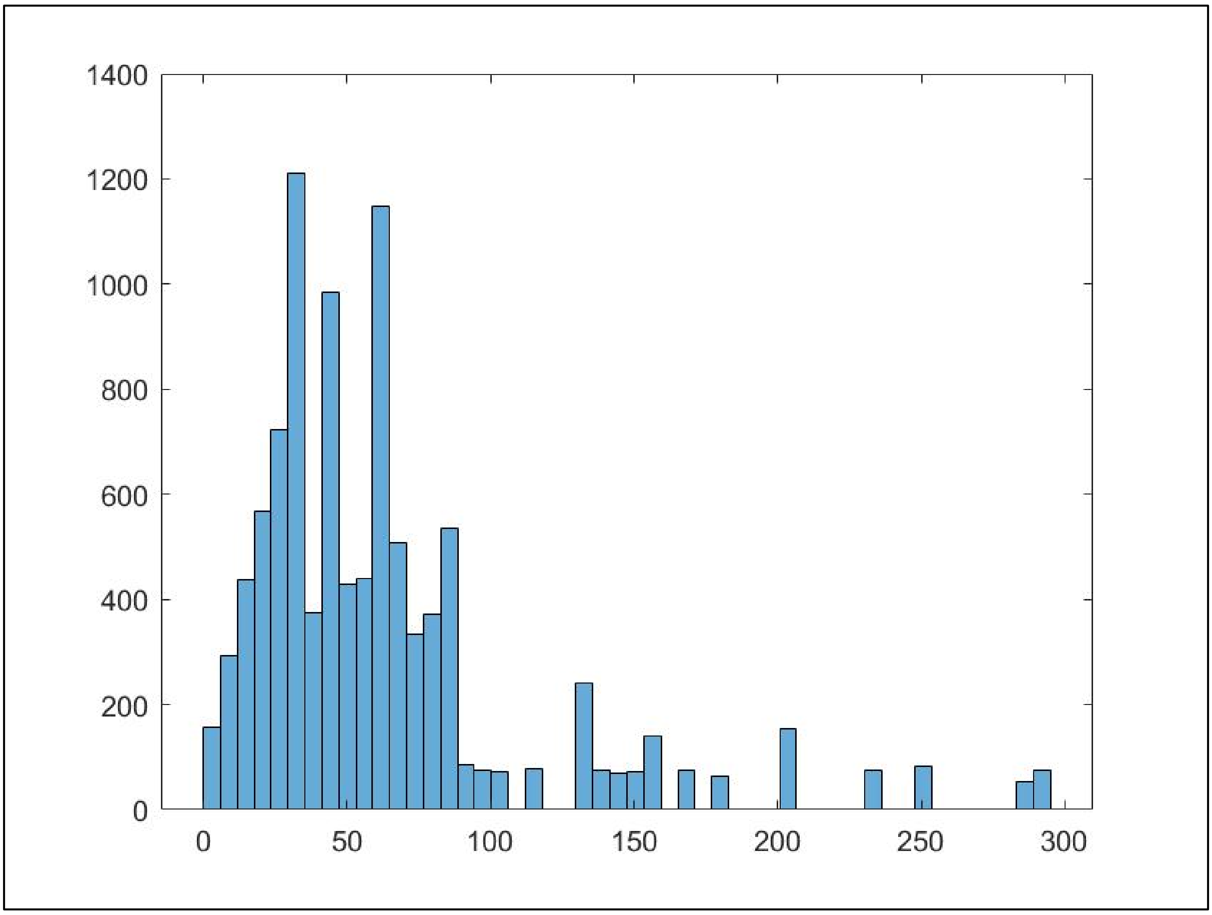
point 1 wind speed = 0.08 with a direction NE
point 2 wind speed = 0.20 with a direction NW
point 3 wind speed = 0.33 with a direction SW
point 4 wind speed = 0.35 with a direction SE
point 5 wind speed = 0.37 with a direction SW
point 6 wind speed = 0.21 with a direction SE

1 - 2 = 49.41 with gradient = 0.72 with direction SE and cost to travel = 53.87
1 - 3 = 71.12 with gradient = -1.91 with direction SW and cost to travel = 77.54
1 - 4 = 43.08 with gradient = -2.50 with direction SW and cost to travel = 46.97
1 - 5 = 46.04 with gradient = -0.68 with direction SW and cost to travel = 50.20
1 - 6 = 66.03 with gradient = -1.42 with direction SW and cost to travel = 71.99
2 - 3 = 80.53 with gradient = -0.47 with direction SW and cost to travel = 101.03
2 - 4 = 57.07 with gradient = -0.20 with direction SW and cost to travel = 47.44
2 - 5 = 78.06 with gradient = 0.04 with direction NE and cost to travel = 64.89
2 - 6 = 81.91 with gradient = -0.32 with direction SW and cost to travel = 102.76
3 - 4 = 28.60 with gradient = -1.35 with direction NW and cost to travel = 21.46
3 - 5 = 37.34 with gradient = 7.40 with direction NE and cost to travel = 55.98
3 - 6 = 10.30 with gradient = 1.80 with direction NE and cost to travel = 7.72
4 - 5 = 26.08 with gradient = 0.64 with direction NE and cost to travel = 19.32
4 - 6 = 26.08 with gradient = -0.64 with direction SW and cost to travel = 19.32

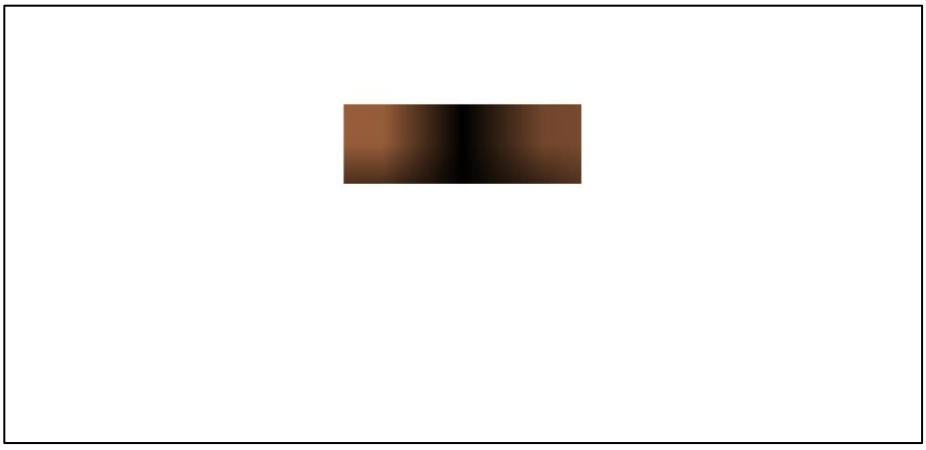
5 - 6 = 28.00 with gradient = -Inf with direction N and cost to travel = 20.37

point 1 wind velocity = -0.08
point 2 wind velocity = -0.20
point 3 wind velocity = 0.33
point 4 wind velocity = 0.35
point 5 wind velocity = 0.37
point 6 wind velocity = 0.21

the minimum cost to travel to point 3 is 77.54
3.97 is the total time of execution.
>>




*Appendix Figure 5 - Value Function Example for 17 Point PRM & Visualisation*



## References

Pearl, Judea (1984). *Heuristics: Intelligent Search Strategies for Computer Problem Solving*. Addison-Wesley. ISBN 978-0-201-05594-8.

```
total points in the system are 17
What is the desired point to travel? 6
point 1 wind speed = 0.37   with a direction NW
point 2 wind speed = 0.71   with a direction SE
point 3 wind speed = 0.52   with a direction NE
point 4 wind speed = 0.52   with a direction NW
point 5 wind speed = 0.58   with a direction SE
point 6 wind speed = 0.65   with a direction NW
point 7 wind speed = 0.43   with a direction NW
point 8 wind speed = 0.19   with a direction SW
point 9 wind speed = 0.50   with a direction NW
point 10 wind speed = 0.17  with a direction NW
point 11 wind speed = 0.41  with a direction NE
point 12 wind speed = 0.44  with a direction NW
point 13 wind speed = 0.56  with a direction NW
point 14 wind speed = 0.38  with a direction NW
point 15 wind speed = 0.45  with a direction NW
point 16 wind speed = 0.55  with a direction NW
point 17 wind speed = 0.16  with a direction SW

1 - 2 = 47.43   with gradient = -3.00   with direction NW   and cost to travel = 34.74
1 - 3 = 39.70   with gradient = 0.87    with direction NE   and cost to travel = 29.08
1 - 4 = 79.20   with gradient = 1.31    with direction NE   and cost to travel = 58.01
1 - 5 = 49.01   with gradient = 49.00   with direction NE   and cost to travel = 35.90
1 - 6 = 50.09   with gradient = -0.49   with direction SW   and cost to travel = 78.91
1 - 7 = 67.94   with gradient = 0.92    with direction NE   and cost to travel = 107.03
1 - 8 = 71.39   with gradient = 5.00    with direction NE   and cost to travel = 112.46
1 - 9 = 12.04   with gradient = 0.89    with direction NE   and cost to travel = 18.97
1 - 10 = 17.89  with gradient = 2.00    with direction SE   and cost to travel = 28.18
1 - 11 = 26.93  with gradient = 0.61    with direction NE   and cost to travel = 42.42
1 - 12 = 25.18  with gradient = -8.33   with direction SW   and cost to travel = 18.44
1 - 13 = 27.31  with gradient = -2.27   with direction SW   and cost to travel = 43.03
1 - 14 = 20.62  with gradient = 1.23    with direction NE   and cost to travel = 32.48
1 - 15 = 7.62   with gradient = -2.33   with direction SW   and cost to travel = 5.58
1 - 16 = 58.94  with gradient = -0.26   with direction SW   and cost to travel = 92.85
1 - 17 = 74.46  with gradient = 3.79    with direction NE   and cost to travel = 117.31
2 - 3 = 48.85   with gradient = -0.42   with direction SW   and cost to travel = 28.55
2 - 4 = 65.52   with gradient = 0.29    with direction NE   and cost to travel = 38.29
2 - 5 = 16.49   with gradient = 0.25    with direction NE   and cost to travel = 9.64
2 - 6 = 89.94   with gradient = -1.12   with direction SW   and cost to travel = 52.57
2 - 7 = 65.01   with gradient = 0.02    with direction NE   and cost to travel = 37.99
2 - 8 = 38.29   with gradient = 0.86    with direction NE   and cost to travel = 22.38
2 - 9 = 44.10   with gradient = -1.54   with direction SW   and cost to travel = 25.78
2 - 10 = 61.40  with gradient = -8.71   with direction SW   and cost to travel = 35.89
2 - 11 = 49.04  with gradient = -0.82   with direction SW   and cost to travel = 28.66
2 - 12 = 72.28  with gradient = -3.89   with direction SW   and cost to travel = 42.24
2 - 13 = 74.67  with gradient = -2.69   with direction SW   and cost to travel = 43.64
2 - 14 = 40.31  with gradient = -1.04   with direction SW   and cost to travel = 23.56
2 - 15 = 55.03  with gradient = -2.89   with direction SW   and cost to travel = 32.16
2 - 16 = 93.72  with gradient = -0.83   with direction SW   and cost to travel = 54.78
```

```
2 - 17 = 43.42  with gradient = 0.79    with direction NE   and cost to travel = 25.38
3 - 4 = 41.15   with gradient = 2.06    with direction NE   and cost to travel = 27.05
3 - 5 = 37.01   with gradient = -0.79   with direction NW   and cost to travel = 24.34
3 - 6 = 50.29   with gradient = -3.20   with direction SW   and cost to travel = 33.06
3 - 7 = 28.28   with gradient = 1.00    with direction NE   and cost to travel = 18.60
3 - 8 = 46.82   with gradient = -2.75   with direction NW   and cost to travel = 30.78
3 - 9 = 27.66   with gradient = 0.86    with direction SE   and cost to travel = 57.73
3 - 10 = 56.64  with gradient = 1.11    with direction SE   and cost to travel = 37.24
3 - 11 = 13.89  with gradient = 1.71    with direction SE   and cost to travel = 29.00
3 - 12 = 57.71  with gradient = 1.89    with direction SE   and cost to travel = 37.94
3 - 13 = 54.42  with gradient = 2.68    with direction SE   and cost to travel = 113.60
3 - 14 = 19.72  with gradient = 0.59    with direction SE   and cost to travel = 12.97
3 - 15 = 42.64  with gradient = 1.22    with direction SE   and cost to travel = 89.00
3 - 16 = 49.09  with gradient = -1.52   with direction SW   and cost to travel = 102.47
3 - 17 = 47.30  with gradient = -4.18   with direction NW   and cost to travel = 98.73
4 - 5 = 49.04   with gradient = 0.30    with direction SE   and cost to travel = 32.22
4 - 6 = 85.05   with gradient = 28.33   with direction SE   and cost to travel = 55.87
4 - 7 = 17.12   with gradient = -8.50   with direction SW   and cost to travel = 35.83
4 - 8 = 34.71   with gradient = -0.21   with direction NW   and cost to travel = 72.66
4 - 9 = 67.42   with gradient = 1.41    with direction SE   and cost to travel = 141.13
4 - 10 = 96.83  with gradient = 1.41    with direction SE   and cost to travel = 202.69
4 - 11 = 55.01  with gradient = 1.96    with direction SE   and cost to travel = 115.14
4 - 12 = 98.84  with gradient = 1.96    with direction SE   and cost to travel = 206.89
4 - 13 = 95.46  with gradient = 2.38    with direction SE   and cost to travel = 199.82
4 - 14 = 58.60  with gradient = 1.34    with direction SE   and cost to travel = 122.66
4 - 15 = 83.22  with gradient = 1.56    with direction SE   and cost to travel = 174.19
4 - 16 = 78.52  with gradient = -8.67   with direction SW   and cost to travel = 51.58
4 - 17 = 30.36  with gradient = -0.31   with direction NW   and cost to travel = 19.95
5 - 6 = 83.53   with gradient = -1.61   with direction SW   and cost to travel = 52.79
5 - 7 = 49.09   with gradient = -0.06   with direction SW   and cost to travel = 31.03
5 - 8 = 24.70   with gradient = 1.62    with direction NE   and cost to travel = 15.61
5 - 9 = 41.77   with gradient = -5.13   with direction SW   and cost to travel = 26.40
5 - 10 = 65.62  with gradient = 7.22    with direction SE   and cost to travel = 41.48
5 - 11 = 41.34  with gradient = -1.59   with direction SW   and cost to travel = 26.13
5 - 12 = 74.03  with gradient = -37.00  with direction SW   and cost to travel = 46.79
5 - 13 = 74.67  with gradient = -7.40   with direction SW   and cost to travel = 47.20
5 - 14 = 35.11  with gradient = -2.75   with direction SW   and cost to travel = 22.19
5 - 15 = 56.04  with gradient = -28.00  with direction SW   and cost to travel = 35.42
5 - 16 = 85.04  with gradient = -1.14   with direction SW   and cost to travel = 53.75
5 - 17 = 29.21  with gradient = 1.28    with direction NE   and cost to travel = 18.46
6 - 7 = 68.18   with gradient = 13.60   with direction NE   and cost to travel = 41.41
6 - 8 = 97.08   with gradient = -2.97   with direction NW   and cost to travel = 58.97
6 - 9 = 46.86   with gradient = -0.83   with direction NW   and cost to travel = 28.46
6 - 10 = 53.34  with gradient = -0.11   with direction NW   and cost to travel = 32.40
6 - 11 = 42.19  with gradient = -1.64   with direction NW   and cost to travel = 25.63
6 - 12 = 42.11  with gradient = 0.07    with direction SE   and cost to travel = 25.57
6 - 13 = 34.13  with gradient = 0.09    with direction SE   and cost to travel = 20.73
6 - 14 = 49.68  with gradient = -1.19   with direction NW   and cost to travel = 30.17
6 - 15 = 44.60  with gradient = -0.36   with direction NW   and cost to travel = 27.09
6 - 16 = 13.89  with gradient = 0.58    with direction NE   and cost to travel = 8.44
```

```
6 - 17 = 97.53  with gradient = -3.62   with direction NW   and cost to travel = 59.24
7 - 8 = 43.27   with gradient = -0.67   with direction NW   and cost to travel = 30.26
7 - 9 = 55.90   with gradient = 0.93    with direction SE   and cost to travel = 39.10
7 - 10 = 84.90  with gradient = 1.07    with direction SE   and cost to travel = 59.39
7 - 11 = 41.87  with gradient = 1.19    with direction SE   and cost to travel = 29.29
7 - 12 = 85.15  with gradient = 1.51    with direction SE   and cost to travel = 59.56
7 - 13 = 81.01  with gradient = 1.82    with direction SE   and cost to travel = 56.66
7 - 14 = 47.63  with gradient = 0.81    with direction SE   and cost to travel = 33.32
7 - 15 = 70.84  with gradient = 1.13    with direction SE   and cost to travel = 49.55
7 - 16 = 61.40  with gradient = -8.71   with direction SW   and cost to travel = 107.64
7 - 17 = 40.46  with gradient = -0.84   with direction NW   and cost to travel = 70.93
8 - 9 = 62.20   with gradient = 12.40   with direction SE   and cost to travel = 52.28
8 - 10 = 88.77  with gradient = 3.91    with direction SE   and cost to travel = 74.61
8 - 11 = 56.72  with gradient = -6.22   with direction SW   and cost to travel = 47.67
8 - 12 = 95.63  with gradient = 8.64    with direction SE   and cost to travel = 80.38
8 - 13 = 95.05  with gradient = 31.67   with direction SE   and cost to travel = 79.89
8 - 14 = 54.01  with gradient = 54.00   with direction SE   and cost to travel = 45.39
8 - 15 = 77.78  with gradient = 7.00    with direction SE   and cost to travel = 65.37
8 - 16 = 95.26  with gradient = -1.98   with direction SW   and cost to travel = 80.06
8 - 17 = 5.39   with gradient = 0.40    with direction NE   and cost to travel = 6.65
9 - 10 = 29.41  with gradient = 1.41    with direction SE   and cost to travel = 19.56
9 - 11 = 15.23  with gradient = 0.43    with direction NE   and cost to travel = 10.13
9 - 12 = 33.54  with gradient = 5.50    with direction SE   and cost to travel = 22.31
9 - 13 = 33.06  with gradient = -16.50  with direction SW   and cost to travel = 66.60
9 - 14 = 8.94   with gradient = 2.00    with direction NE   and cost to travel = 18.02
9 - 15 = 16.16  with gradient = 2.50    with direction SE   and cost to travel = 32.55
9 - 16 = 53.23  with gradient = -0.48   with direction SW   and cost to travel = 35.40
9 - 17 = 64.78  with gradient = 6.40    with direction NE   and cost to travel = 43.08
10 - 11 = 43.14 with gradient = 0.97    with direction NE   and cost to travel = 36.84
10 - 12 = 14.21 with gradient = -0.82   with direction SW   and cost to travel = 17.15
10 - 13 = 21.02 with gradient = -0.47   with direction SW   and cost to travel = 17.95
10 - 14 = 38.28 with gradient = 1.52    with direction NE   and cost to travel = 32.68
10 - 15 = 14.21 with gradient = 0.82    with direction NE   and cost to travel = 12.14
10 - 16 = 65.01 with gradient = 0.02    with direction NE   and cost to travel = 55.51
10 - 17 = 92.05 with gradient = 3.26    with direction NE   and cost to travel = 78.60
11 - 12 = 43.83 with gradient = 1.95    with direction SE   and cost to travel = 73.72
11 - 13 = 40.80 with gradient = 3.25    with direction SE   and cost to travel = 29.03
11 - 14 = 10.20 with gradient = -0.20   with direction NW   and cost to travel = 7.26
11 - 15 = 29.00 with gradient = 1.05    with direction SE   and cost to travel = 48.78
11 - 16 = 44.69 with gradient = -0.85   with direction SW   and cost to travel = 75.17
11 - 17 = 58.14 with gradient = -14.50  with direction NW   and cost to travel = 97.79
12 - 13 = 8.00  with gradient = 0.00    with direction E    and cost to travel = 5.54
12 - 14 = 42.20 with gradient = 4.10    with direction NE   and cost to travel = 29.23
12 - 15 = 18.00 with gradient = Inf with direction S    and cost to travel = 12.47
12 - 16 = 54.92 with gradient = 0.19    with direction NE   and cost to travel = 38.04
12 - 17 = 98.31 with gradient = 6.06    with direction NE   and cost to travel = 68.09
13 - 14 = 41.05 with gradient = 20.50   with direction NE   and cost to travel = 26.27
13 - 15 = 19.70 with gradient = -2.25   with direction NW   and cost to travel = 12.61
13 - 16 = 47.07 with gradient = 0.22    with direction NE   and cost to travel = 30.13
13 - 17 = 97.33 with gradient = 12.13   with direction NE   and cost to travel = 62.30
```

```
14 - 15 = 25.08 with gradient = 2.30    with direction SE   and cost to travel = 18.11
14 - 16 = 53.82 with gradient = -0.70   with direction SW   and cost to travel = 87.51
14 - 17 = 56.32 with gradient = 9.33    with direction NE   and cost to travel = 91.57
15 - 16 = 54.59 with gradient = -0.15   with direction SW   and cost to travel = 99.17
15 - 17 = 80.60 with gradient = 4.94    with direction NE   and cost to travel = 146.43
16 - 17 = 94.94 with gradient = -2.29   with direction NW   and cost to travel = 61.07

point 1 wind velocity = -0.37
point 2 wind velocity = 0.71
point 3 wind velocity = -0.52
point 4 wind velocity = 0.52
point 5 wind velocity = 0.58
point 6 wind velocity = 0.65
point 7 wind velocity = -0.43
point 8 wind velocity = -0.19
point 9 wind velocity = 0.50
point 10 wind velocity = 0.17
point 11 wind velocity = -0.41
point 12 wind velocity = 0.44
point 13 wind velocity = 0.56
point 14 wind velocity = -0.38
point 15 wind velocity = -0.45
point 16 wind velocity = 0.55
point 17 wind velocity = 0.16
the minimum cost to travel to point 6 is 33.06

6.29 is the total time of execution.
>>
```