Vibration Isolation: 20/3/19

Laboratory Report

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Submission Date: 03/4/19

AMME2500

ENGINEERING DYNAMICS

Contents

Vibrations Lab

AMME2500 Experiment Report

Introduction

Vibrations can be divided into two different categories, forced and free vibrations. Forced vibrations are the result of external forces acting on a system. Free vibrations are oscillations in which the total energy is constant over time. During the laboratory session, three main investigations took place within the experiment:

- Calculating the Spring Constant
- Free Vibrations
- Forced Vibrations

Aims

To demonstrate the concepts of forced and free vibrations and compare each concept mathematically and experimentally.

Sketches

Apparatus

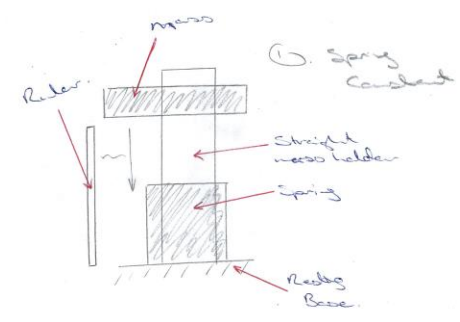


Figure 1: Equipment Setup Spring Constant

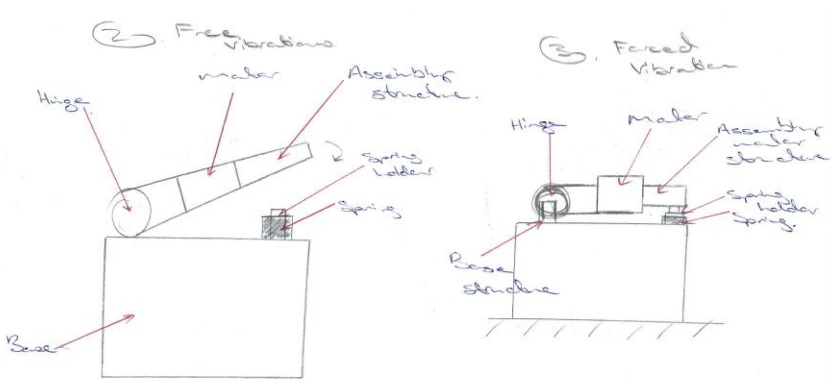


Figure 2: Equipment Setup Force and Free Vibrations

Roles

In *figure 1* the weight holder is designed to be the centre point of the masses that are lowered down towards the holder's base and hence, causing the spring to compress with even distribution and allowing for the setup to be balanced when operated upon.

In *figure 2* the motor assembly is to sit on the spring and is supported by the base mass, hinge and surrounding structure to allow safe operation when powered.

Method

Spring Constant

1. Setup equipment as displayed in *figure 1*.
2. Record the mass of the first weight.
3. Record the initial spring height using a ruler.
4. Place the weight onto the spring.
5. Measure the spring height using a ruler.
6. Repeat steps 4-5 for additional weights.
7. Calculate the elastic force using the formula: $F_{elastic} = mg$.
8. Record the total weight and spring heights in the table below to calculate the spring constant using the formula: $F_{elastic} = k\Delta y, where\ \Delta y = change\ in\ spring\ height$.

Free Vibration

1. Set up the equipment as displayed in *figure 2*.
2. Lift the motor assembly approximately 30cm upwards.
3. Reset the vibration recording software and initiate the recording.
4. Drop the motor assembly and allow it to drop onto the spring where vibrations will commence.
5. Stop the recording once the vibration trace is flatlined (vibrations have stopped).

Forced Vibrations

1. Set up the equipment as displayed in *figure 2*.
2. Start the rotation of the motor.
3. Measure the RPM of the motor using a laser device.
4. Record the peak waveform on vibration recording software.
5. Record the frequency (resonant frequency) at that peak point.
6. Increase the rotational speed of the motor in increments.
7. Repeat steps 3-6 till maximum RPM is reached on the assembly.

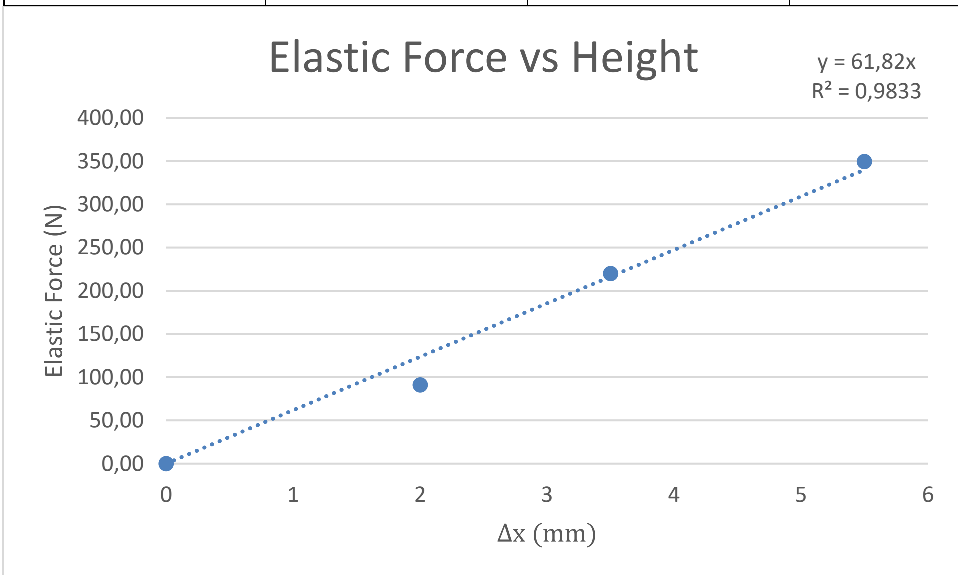
Results and Plots

Finding the Spring Constant

The spring motion was modeled by Hooke's Law as mentioned below with the equations and hence, the spring constant was found. Interestingly, since the values recorded where minute, the force could be modeled using weight force and could allow for a correct plot and result.

Table 1: Spring Constant Results

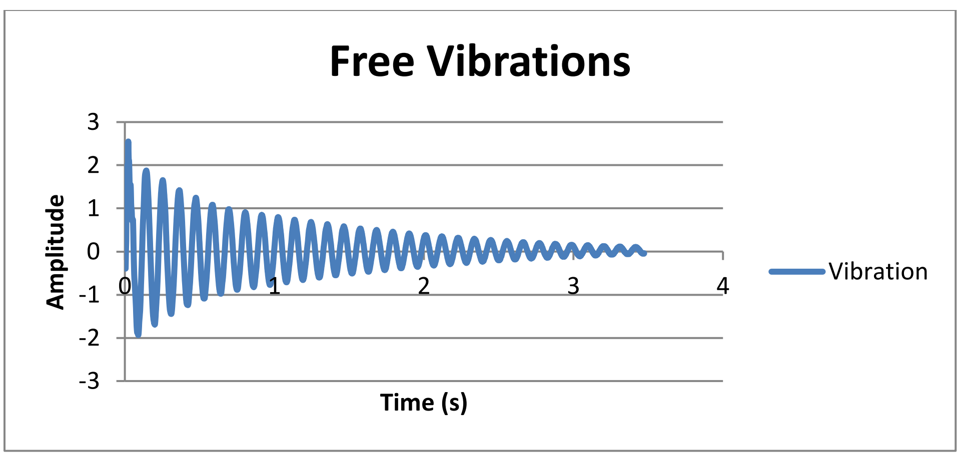
$x\ (mm)$	$\Delta x\ (mm)$	$total\ m\ (kg)$	$\Delta m\ (kg)$	$F_{elastic}\ (N)$
80	0	0	0	0
78	2	9.28	9.28	91.04
76.5	3.5	22.43	13.15	220.04
74.5	5.5	35.63	13.20	349.53



$$Using\ the\ equation: F_{elastic} = k\Delta x \ \rightarrow k = \frac{F_{elastic}}{\Delta x}\ where, F_{elastic} = W = mg$$

$$\therefore k = 61.82kN/m$$

Free Vibration



$$\therefore 9\ cycles\ (n)\ in\ 1\ second \rightarrow f_d = \frac{n}{time\ (s)} = 8Hz\ (damped\ frequency)$$

$$\therefore \omega_d = 2\pi f_d = 2\pi(8) = 56.55rads$$

$$\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right) = \frac{1}{9}\ln\left(\frac{2}{0.8}\right) \approx 0.10181$$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = \frac{0.10181}{\sqrt{4\pi^2 + 0.10181^2}} \approx 0.0162$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{56.55}{\sqrt{1-0.0162^2}} = 56.56rads$$

$$m = \frac{k}{\omega^2{}_n} = \frac{65820}{56.56^2} = 20.57kg$$

$$c = 2\zeta m\omega_n = 2(0.0162)(20.57)(56.56) = 37.70$$

$$x(t) = 2e^{-0.0162(56.56)t}\left[\cos(56.56) + \frac{(0.0162)(56.56)}{56.55}\sin(56.55t)\right]$$

Therefore, this homogenous differential equation is equal to the above graph.

Forced Vibration

Resonant frequency was found close to theoretical value using the plot below and hence, the result differs from the free vibration result. This is due to the uneven increments of the motor assembly acceleration caused by manual operation of the machine as well as gravity causing a recoil effect.

$\omega_n = 2\pi f_n \rightarrow f_n = \frac{56.56}{2\pi} = 9.00Hz.$

$\ddot{x} + 2\zeta m\omega_n\dot{x} + \omega_n{}^2 x = \frac{F_0 sin\omega t}{m}$

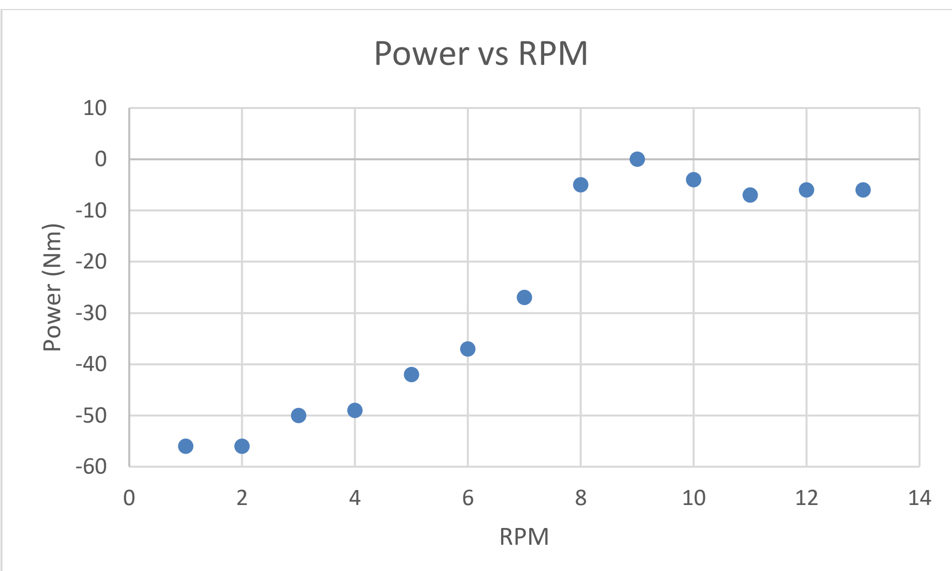


Table 2: Forced Vibration

RPM	Power
60.04	-56
75.94	-56
83.24	-50
112.5	-49
144.6	-42
218.3	-37
333.7	-27
506.4	-5
583.1	0
689.8	-4
796	-7
899.3	-6
924	-6

Discussion

The constant spring value displays a gradient relationship when graphed and allows for calculation for the experimental result with net force and the difference in height of the spring. The three categories of damped vibration include:

- The vibration is dissipated in the system. This is known as a damped free vibration with one degree of free dome such as the linear spring mass system.
    - $(\zeta > 1)$ refers to an overdamped system.
        - System decays to equilibrium exponentially with oscillation.
    - $(\zeta = 1)$ refers to a critically damped system.
        - System returns to equilibrium almost instantly.
    - $(\zeta < 1)$ refers to an underdamped system.
        - System returns to equilibrium at reduced frequency with amplitude gradually decreasing to zero.

Within the investigation the angular frequency ω_d was found first which allows for the calculation of natural angular frequency ω_n hence, displaying the oscillations related to the damping frequency denoted in the formulas above.

Errors of measurement in this experiment, specifically in the forced vibration section, is due to approximation for power as well as the limitations of accuracy and precision in the equipment such as the ruler and the laser. However, the results are accurate as they are observed to be close to the expected results.

The experiment displayed vibrations, both forced and free, with resulting calculated values defining oscillating systems motion. In conclusion, this experiment allowed for the observation of resonance frequency in free and forced vibration systems as well as further understanding on theory vibrations of one degree of freedom.