Commercial Aircraft Pitch Control System

Control System Design – MATLAB & Simulink

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***Abstract* — The following document will highlight the investigation and overall control system design of a commercial aircraft altitude controller. The following report will establish a simple Simulink control system model with reference to equations of motion for a corresponding aircraft.**

Keywords – pitch control, airplane, aircraft, longitudinal, control system.

Fig. 1. Axis of Motions in Flight. (*Zaman, 2010*)

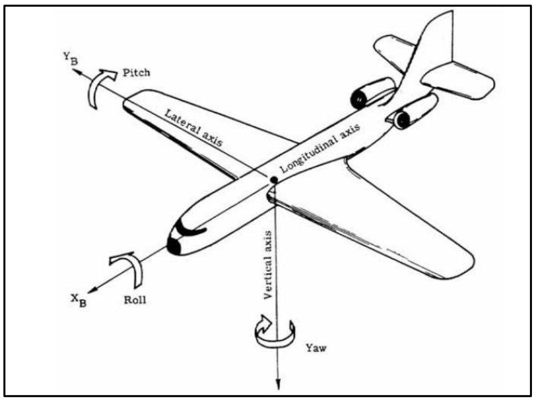

Figure 1: Axis of Airplane Motion

Fig. 2. Axis of Motions in Flight. (*Dowling, Kundu and Cohen, 2012*)

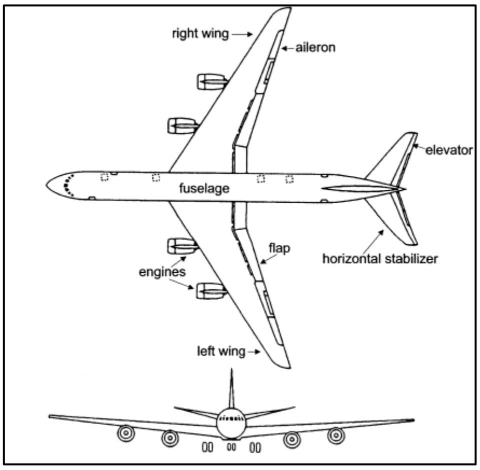

Figure 2: Elevator – Rear Component

I. Introduction: Pitch Control System

The air travel industry, spreading from commercial, military and private flying, as well as elements of space travel and high altitude flight, has over the current decades been advancing with a greater influence of automation and control systems.

The aid of control systems has benefited flight crews with navigation, management and stability characteristics of the aircraft. The overall pitch control and autopilot systems has reduced manual operations, workload and in turn minimized extended hours of direct control for pilots, over the aircraft.

The following investigation more specifically is the design of the control system focusing on the pitch motion of an aircraft (refer to *Figure 1)*, particularly an airplane. In context of the real-world problem that this report aims to focus on, the described issue can be defined as;

> *the need for the aircraft to maintain a specified altitude which can be adjusted in the controller design. The output of the model, $y(t)$ should track a reference, r inputted along with angles on axis of interest.*

The pitching motion is the primary concern of this controller where the airplane will have this pitch controller design capable of processing *pitch rate* and *pitch angle.* This can be defined as control variables within this system.

The pitch control on modern aircrafts are controlled by the tail component of the airplane, more specifically the horizontal stabilizer.

II. Background Information

Overall, the control system design is simplified to allow for modelling in accommodation for simulation constraints. These constraints more specifically involve that of the tools at hand, the software capabilities and current knowledge with the hardware and software. Furthermore, the time constraints on this investigation accommodates the unrealistic control simulation with assumptions highlighted in *Section III*.

The pitch controller aims to create adjust angle of the aircraft and hence, the rear generates lift force. The controller finally will, therefore, stabilize with movements of the nose up and down. The rear horizontal stabilizer that is on an airplanes tail is what creates an elevator.

This elevator will be the **input** of the system in particularly expressed as the ***elevator deflection angle, δ*** and the **output** of the system will be the ***pitch angle, θ*** of the airplane.

A. *Longitudinal Control*

With reference to *Figure 1,* the coordinate system highlights the focus on the airplanes pitch motion with reference to the appropriate angles. The Free-Body Diagram *(FBD)* in *Figure 3,* visualizes the forces acting on the axis and is concerned with the appropriate equations.

Fig. 3. Free Body Diagram of Airplane Motion. (*Pitch Motion*)

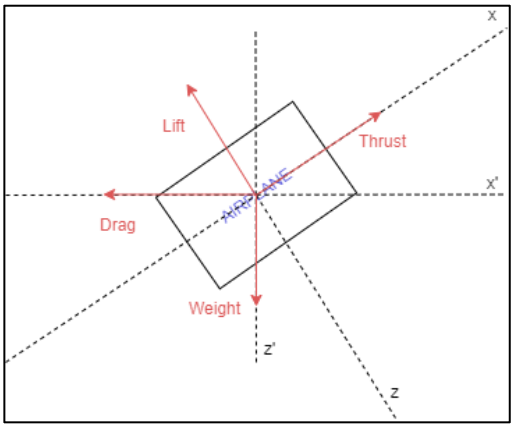

Figure 3:FBD

Fig. 4. Angular Component Diagram. (*Longitudinal Angles*)

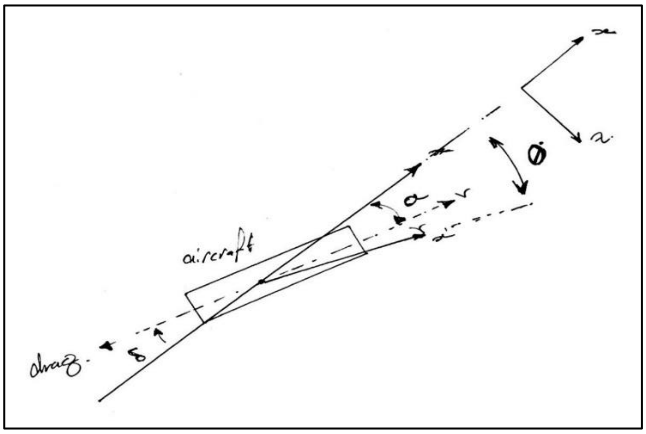

Figure 4: Angular Constants - Diagram

B. *Equations of Motion – Pitch*

The first – order derivative equations *(ODE)* of the aircraft in its longitudinal motion can be written as:

$$\dot{\alpha} = \mu\Omega\sigma\left[-(C_L + C_D)\alpha + \frac{1}{(\mu - C_L)}q - (C_w \sin\gamma)\theta + C_L\right]$$

$$\dot{q} = \frac{\mu\Omega}{2i_{yy}}\{[C_M - \eta(C_L + C_D)]\alpha + [C_M + \sigma C_M(1 - \mu C_L)]q + (\eta C_w \sin\gamma)\delta\}$$

$$\dot{\theta} = \Omega q$$

(Aircraft Longitudinal Dynamics, 2004)

Note: the equations refer to the *rate of the angle of attack*, *proportion of pitch rate* and *rate of pitch angle* respectively.

The following equations have been referenced from established calculations and analysis of aircraft with steady constant velocity and altitude.

III. Assumptions & Formulas

With the following assumptions, the analysis of motion of the airplane is primarily in focus of the *angle of attack* and the *pitch rate*:

1. The airplane is travelling with constant velocity.
2. The airplane is travelling on a constant altitude.
3. The changes in pitch does not affect the speed of the airplane.

Note: The thrust and drag forces are balanced.

With these assumptions for the investigation and reference to *Figure 3* and *Figure 4*, the forces acting on the airplane in the *x* and *y* directions are balanced (lift and weight are in balance). Furthermore, the assumption of the pitch not affecting the speed of the airplane allows for a simpler ODE analysis.

A. *Equations of Motion – Variables*

1) *Angular Variables*

$\alpha = angle\ of\ attack$
$q = pitch\ rate$
$\theta = pitch\ angle\ (\alpha + \gamma)$
$\delta = deflection\ angle$

2) *Coefficient Variables*

$C_L: Coefficient\ of\ Lift$
$C_M: Coefficient\ of\ Pitch\ Moment$
$C_D: Coefficient\ of\ Drag$
$C_W: Coefficient\ of\ Weight$

3) *Constants*

$\sigma = \frac{1}{1+\mu C_L}$ *(Constant Sigma)*
$\eta = \mu\sigma C_K$ *(Constant Nu)*
i_{yy} *(Normalized Moment of Inertia)*

4) *Formulations*

$$\Omega = \frac{2U}{c}$$

$U =\ Equilibrium\ Flight\ Speed$

$$\mu = \frac{\rho Sc}{4m}$$

$\rho = Density\ of\ Surrounding\ Air$
$S = Platform\ Area\ of\ Wing$
$c = Average\ Chord\ Length$
$m = Mass\ of\ Aircraft$

Note: Further constant values and processing of formulas is highlighted in the appendix (heading *A* and *B*).

IV. Equations & Calculations

A. *Derivations for the Equations of Motion*

- The following sub-section incorporates required known parameters for stable level flight.
- The following *Table 1*, highlights the dimensional derivatives required for modelling and system analysis:

TABLE I. Constant Values for Stable Level Flight

Parameter	Specifications
Q	$19.06lb/ft^2$
S	$144.9ft^2$
QS	$3400lb$
$QS\bar{c}$	$38596ft\cdot lb$
$\frac{\bar{c}}{2u_0}$	$0.04207s$

Table 1:Parameters for Stable Flight

a. Table 1 – [4]: (*Gudmundsson, 2014*)

- With the following dynamic equations, parameters and constants sourced from flight stability textbooks and papers as referenced, [1], [3] & [4], the following mathematical derivations can be determined:

$$X - mgS_\theta = m(\dot{u} - qv - rv) \quad (1)$$

$$Z + mgC_\theta C_\Phi = m(\dot{w} + pv - qu) \quad (2)$$

$$M = I_y\dot{q} + rq(I_x - I_z) + I_{zz}(p^2 - r^2) \quad (3)$$

(Dynamical Equations for Flight Vehicles, n.d.)

From the three equations above, linearization denotes the following with ***disturbance theory*** as denoted in [1]:

$$\delta = \delta_0 + \Delta\delta \quad (4)$$

- In order to perform the above linearization and derive (4), note the following assumptions with the *pitch axis*:

$$u = u_0 + \Delta u \qquad v = v_0 + \Delta v \qquad w = w_0 + \Delta w$$

$$p = p_0 + \Delta p \qquad q = q_0 + \Delta q \qquad r = r_0 + \Delta r$$

$$X = X_0 + \Delta X \qquad M = M_0 + MY \qquad Z = Z_0 + \Delta Z$$

Therefore, it is evident that most values are constants and can be processed into a transfer function.

Note: All equations and unknowns are referenced from [1]: (*Dynamical Equations for Flight Vehicles, n.d.)*.

- Referring to the *Equations of Motion – Pitch* section, the equations prior to its first derivative, can be assumed as following for allowing the solving of the unknowns:

$$\dot{\theta} = qC_\Phi - rS_\theta \qquad \textit{(Pitching Angle)}$$

$$r = \dot{\psi}C_\theta C_\Phi - \dot{\theta}S_\Phi \qquad \textit{(Pitching Rate)}$$

Refer to the appendix (heading *A & C*) for all other unknown variables and equations that have been introduced.

(Aircraft Longitudinal Dynamics, 2004) & (*Dynamical Equations for Flight Vehicles, n.d.).*

B. *Derivation of the Transfer Function*

With the known constant formulations, known equations and the stability variables, the equations can be simplified into the following equations (referring to the equations in *Section II, Part B: Equations of Motion – Pitch)*:

$$\dot{\alpha} = -0.313\alpha + 56.7q + 0.232\delta \quad (5)$$

$$\dot{q} = -0.0139\alpha - 0.426q + 0.0203\delta \quad (6)$$

$$\dot{\theta} = 56.7q \quad (7)$$

Note: All calculations and constant values that have processed in mathematical equations are correct to *three significant figures*.

Finally, all values that are substituted into the equation have been sourced from Boeings 747 aircraft [4].

- These equations where derived through the following:

TABLE II. PERTINENT COEFFICIENTS

Angle of Attack ($\boldsymbol{\alpha}$)		*Elevator Deflection Angle* ($\boldsymbol{\delta}$)	
Constant Parameter	**Value**	**Constant Parameter**	**Value**
C_D	0.0863	C_D	0.000175
C_L	4.80	C_L	0.355
C_M	-0.720	C_M	-0.923

Table 2:Coefficients for Derivations

a. Table 2 – [1]: (*Dynamical Equations for Flight Vehicles, n.d.*)

- To compute the final transfer function expression, the above system equations with all constants substituted (5), (6) and (7) are to undertake the *Laplace Transformation* which results in the following expression:

$$\Delta q = \Delta\dot{\theta} \rightarrow \Delta q(s) = s\Delta\theta(s):$$

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{1}{s}\cdot\frac{\Delta q(s)}{\Delta\theta(s)} \quad (8)$$

Note: The equation, (8) above, was more specifically derived from manipulation of the equations (1), (2) and (3):

$$\left(\frac{d}{dt} - X_U\right)\Delta u - X_W\Delta w + (g\cos\theta_0)\Delta\theta = X\delta_e\Delta\delta_e \quad (9)$$

$$-Z_U\Delta u + \left[(1 - Z_W)\frac{d}{dt} - Z_W\right]\Delta w - \left[(u_0 + Z_q)\frac{d}{dt} - g\sin\theta_0\right]\Delta\theta = Z_{\delta_e}\Delta\delta_e - M_U\Delta u \quad (10)$$

$$-\left(M_W\frac{d}{dt} + M_W\right)\Delta w + \left(\frac{d^2}{dt^2} - M_q\frac{d}{dt}\right)\Delta = M_{\delta_e}\Delta\delta_e \quad (11)$$

(Aircraft Longitudinal Dynamics, 2004)

Hence, the full expression of equation (8) with the manipulation of the equations above is as follows:

$$\frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{-\left(M_{\delta_e} + \frac{M_{\dot{\alpha}}Z_{\delta_e}}{u_0}\right)s - \left(\frac{M_\alpha Z_{\delta_e}}{u_0} - \frac{M_{\dot{\alpha}}Z_\alpha}{u_0}\right)}{s^2 - \left(M_q + M_{\dot{\alpha}} + \frac{Z_\alpha}{u_0}\right)s + \left(\frac{Z_\alpha M_q}{u_0} - M_\alpha\right)}$$

(Cvetković, Mitrović, Stojiljković and Vasov, 2009)

- The transfer function therefore can finally be expressed as:

$$P(s) = \frac{\theta(s)}{\delta(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s} \quad (12)$$

C. *State – Space Representation*

The following section denotes the state – space representation of the transfer function for processing in MATLAB and Simulink.

$$\begin{bmatrix}\dot{\alpha}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}-0.313 & 56.7 & 0\\ -0.0139 & -0.426 & 0\\ 0 & 56.7 & 0\end{bmatrix}\begin{bmatrix}\alpha\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}0.232\\ 0.0203\\ 0\end{bmatrix}[\delta]$$

- From the selected output and input signals, the model will be outputting pitch angle, the output equation utilized in the modelling therefore, is:

$$y = [0 \quad 0 \quad 1]\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\end{bmatrix} + [\theta]$$

Equations & Force Analysis Referenced from:
(Dynamical Equations for Flight Vehicles, n.d.) & (Caughey, 2011).

V. MATLAB & SIMULINK MODELLING

The state – space within this investigation is, within this investigation, employed to perform in a full state feedback controller design with the pitch angle outputted.

A. *Design Requirements*

The design criteria overall involves the feedback controller design with a list of requirements to allow for efficient performance:

1) *Rise Time:* $< 3s$
2) *Settling Time:* $< 10s$
3) $Steady - State\ Error$: $< 2\%$
4) *Overshoot:* $< 10\%$

These specifications will allow for an efficient response time from the input of the controller or the input signal (minimal delay), a low steady – state error with a settling time that corresponds to the error band. An overshoot of less than 10% also enables low discrepancies in the modelled system in correspondence to the desired input signal.

The steady – state error of less than 2% corresponds with a pitch angle of -0.4 & 0.4 $saturation\ limitations$, for the controller $(60 - 70\ degrees)$. This is discussed further in the fine tuning description, *Section V, Part C.*

The reference value, r is set to $r = 0.1\ radian$.

B. *Simulink Controller Model*

The following section displays the investigations controller design. The decision of utilizing a full state feedback is one that coincides with the air travel industry during the modern era [7].

(Zhu, 2006)

Furthermore, employing a state – space method allows for a simplified controller design that can be analyzed in a stabilized system.

Refer to *Figure 5*, which denotes the controller design utilized in the investigation:

Fig. 5. *Pitch Controller Simulated Design*. (*MATLAB & Simulink*)

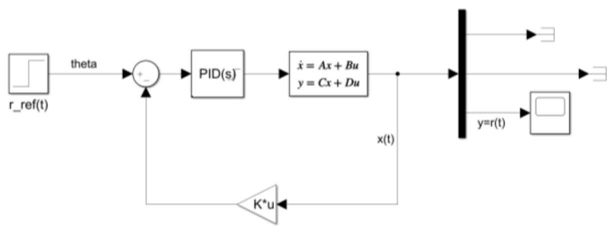

Figure 5:Full State Feedback Controller

From the design denoted in *Figure 5*, the specified output of interest is the pitch angle which is found using the de-multiplexer function.

C. *Pitch Angle Tuning*

The PID system allows for the minimization of the response time, since the requirement of the overall airplane pitch controller is to respond quickly to the pilots input.

The following steps were also taken to allow for fine tuning of the controller:

a) Pitch Angle Limitations $(-0.436\ to\ 0.436)$
b) PID Controller Tuning

With Simulink modeling, the PID controller can be saturated, allowing for a limitation on the expected output pitch angle. This limitation avoids the output being a large value which can cause:

1) *damage to the actuators of the airplane*
   a. *causing mechanical damage or,*
2) *the airplane to stall.*

The overall controller was therefore, prepared for final adjustments to meet the design criteria set out in the requirements of this investigation.

VI. RESULTS & DISCUSSION

Initially, the full state feedback controller was unable to satisfy all requirement criteria of the design set out in the investigation. It is a priority within the operation of pitch control in an airplane (realistically) to have overshoot reduced as low as possible to avoid mentioned issues from *Section V, Part C.*

The settling time, therefore, was revised with the initial design to be increased as less than 15 seconds which displays a trace that converges with minimal overshoot to the desired input signal.

TABLE III. CONSTANT VALUES FOR STABLE LEVEL FLIGHT

Performance Parameter	**Output *(Pitch Angle)***
Rising Time (T_r)	$0.998s$
Settling Time (T_s)	$14.7s$
Overshoot $(\%\ OS)$	5.88%
Steady State Error	0.06%

Table 3:Results of Control System Model

a. Table 3 – Results: (*Control System Performance Results.*)

The following plot displays the output of the pitch controller and denotes the signal converging to the inputted reference, as previously mentioned equal to $0.1\ radians$:

Fig. 6. Output Curve of Response. (*Pitch Controller*)

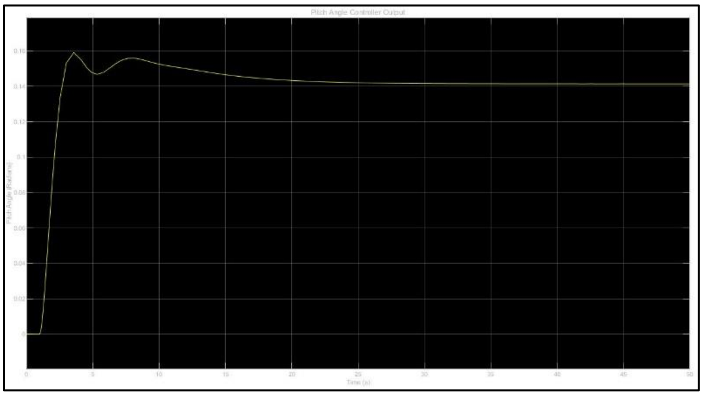

Figure 6:Full State Feedback Response

TABLE IV. TUNING PERFORMANCE OF PID

PID Component	**First Trial**	**Second Trial**	**Final Result**
Proportional	0.1172	0.52868	0.21946
Integral	0.093756	0.25346	0.14371
Derivative	0.029996	0.087391	0.06691
Settling Time	17.2s	9.45s	14.7s

Table 4: PID Tuning

a. Table 4 – PID Performance Results: (*Testing Performance*)

Fig. 7. First Trial. (*Pitch Controller*)

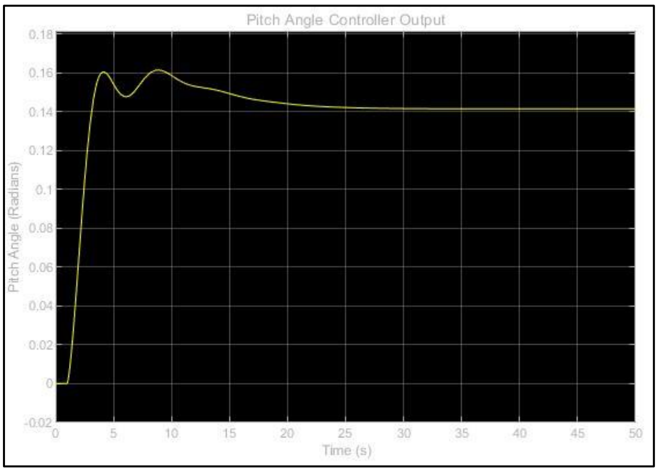

Figure 7: Pitch Angle Response - First Trial

Fig. 8. Second Trial. (*Pitch Controller*)

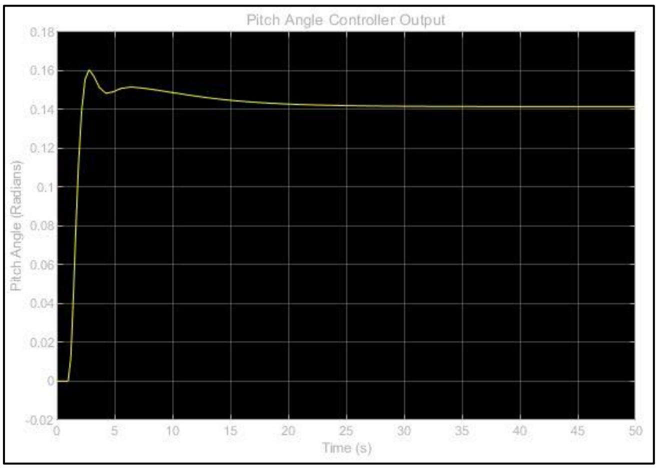

Figure 8: Pitch Angle Response - Second Trial

From the above PID controller performance specifications that resulted from tuning, it was concluded that the result should have a settling time in between the *First* and *Second Trial* since this is a low overshoot value close of 5%, as previously mentioned is important for the airplane's performance.

Furthermore, the oscillating signal property can be minimized by having a low derivative gain therefore, the result is a slight increase on this parameter which as seen in *Figure 6*, demonstrates a smoothening effect. The increase to integral and derivative grain also creates an underdamped system with a slightly lower settling time as evident in the *Second Trial*.

The derivative gain term in the *Final Result*, therefore, was altered to mitigate this and create a more damped system. To further allow for the signal to be closer to the reference value using saturation specified in *Section V, Part A*. Similarly, the integral gain term was increased which allowed for a closer output in comparison to the reference value, however the overshoot was increased. Therefore, the integral gain was altered to the *Final Result* value and furthermore, reduced the settling time slightly.

Finally, the results were close to an acceptable system design and the requirements were met as close as possible, however the settling time requirement had to be increased and is discussed in *Section VII*.

Attempts were made to reduce the proportional gain however; this would increase other gains. In summation the settling time was a parameter that could be sacrificed for an increase since its effects on the airplane performance would not be as detrimental as overshoot or high response time (which as mentioned before can mechanically destroy elevator actuators and stall the airplane).

VII. Conclusion

From the above results that is collected, the proposed control system requirements were not fully met, resulting in an increase to the settling time from < 10 seconds to < 15 seconds. Other than the incapability to satisfy the settling time requirement, the full state feedback design suffers from oscillations which can be reduced with a decrease in integrator gain however, results in an increased settling time.

In conclusion, the overall investigation successfully designs and models the operation of a pitch angle controller for a Boeing 747 however, in a perfect scenario with reduced disturbance and exterior factors such as, unbalanced forces and other axis of motion.

A. Future Applications & Recommendations

In future investigations or developments from these findings it is recommended to utilize this design for an autopilot system with further development on the mathematical derivations regarding further disturbances. From research [7], the root locus method design of a pitch controller will denote a better performance and a reduced settling time due to the addition of a compensator. Utilizing these modifications should improve the settling time and rise time of the system with a similar overshoot being maintained.

The incorporation of realistic disturbances with reference to other axis of movement of the airplane will allow for the development of a realistic autopilot controller or an improved pitch angle controller.

Finally, the changes mentioned in *Section VII, Part A* are not continued in this investigation due to time constraints and overall knowledge on the mathematical derivations and dynamic equations around realistic flight of a Boeing 747 style aircraft.

VIII. Appendix

A. *Equations used for Linerization*

The following section highlights equations utilized by other textbooks and research papers for the establishing of known stability equations specifically utilized for the linearized equation (4).

$p = \dot{\Phi} - \dot{\psi} S_{\theta}$ *(Rolling Rate)*

$q = \dot{\theta} C_{\Phi} + \dot{\psi} C_{\theta} S_{\Phi}$ *(Yawning Rate)*

$\dot{\Phi} = p + q S_{\Phi} T_{\theta} + r C_{\Phi} T_{\theta}$ *(Roll Angle)*

$\dot{\psi} = (q S_{\Phi} + r C_{\Phi}) \sec\theta$ *(Yaw Angle)*

(Aircraft Longitudinal Dynamics, 2004) & (*Dynamical Equations for Flight Vehicles, n.d.).*

B. *Longitudinal Derivative Constants*

The following highlights all longitudinal derivative stability constants utilized for achieving stability in the system and deriving the equations of longitudinal motion.

TABLE V. Other Pitch Stability Parameters

Longitudinal Derivatives	***Pitching Moment Variables***	***Values*** $(\boldsymbol{FT^{-1}})$
Angle of Attack	M_{δ_e}	-11.87
Pitching Rate	M_q	-2.05
Elevator Deflection	M_{α}	-8.8

Table 5: Other Parameters utilized in the Equations (1), (2) & (3) to maintain Stability.

b. Table 5 – [1] & [3]: (*Aircraft Longitudinal Dynamics, 2004 & Dynamical Equations for Flight Vehicles, n.d.*)

C. *Boeing 747 Constant Values*

TABLE VI. Boeing 747 Constant Values

Condition	2	5	7	9	10
h (ft)	SL	20,000	20,000	40,000	40,000
$\mathbf{M}_{\infty}$	0.25	0.500	0.800	0.800	0.900
α (degrees)	5.70	6.80	0.0	4.60	2.40
W (lbf)	564,032.	636,636.	636,636.	636,636.	636,636.
I_y (slug-ft^2)	32.3×10^6	33.1×10^6	33.1×10^6	33.1×10^6	33.1×10^6
$\mathbf{C}_L$	1.11	0.680	0.266	0.660	0.521
$\mathbf{C}_D$	0.102	0.0393	0.0174	0.0415	0.0415
$\mathbf{C}_{L_\alpha}$	5.70	4.67	4.24	4.92	5.57
$\mathbf{C}_{D\alpha}$	0.66	0.366	0.084	0.425	0.527
$\mathbf{C}_{m_\alpha}$	-1.26	-1.146	-.629	-1.033	-1.613
$\mathbf{C}_{L_{\dot{\alpha}}}$	6.7	6.53	5.99	5.91	5.53
$\mathbf{C}_{m_{\dot{\alpha}}}$	-3.2	-3.35	-5.40	-6.41	-8.82
$\mathbf{C}_{L_q}$	5.40	5.13	5.01	6.00	6.94
$\mathbf{C}_{m_q}$	-20.8	-20.7	-20.5	-24.0	-25.1
$\mathbf{C}_{L_\mathbf{M}}$	0.0	-.0875	0.105	0.205	-.278
$\mathbf{C}_{D_\mathbf{M}}$	0.0	0.0	0.008	0.0275	0.242
$\mathbf{C}_{m_\mathbf{M}}$	0.0	0.121	-.116	0.166	-.114
$\mathbf{C}_{L_{\delta_e}}$	0.338	0.356	0.270	0.367	0.300
$\mathbf{C}_{m_{\delta_e}}$	-1.34	-1.43	-1.06	-1.45	-1.20

Table 6: Values utilized in the Equations & Derivations

a. Table 6 – [4]

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