Acoustic Source Localization Techniques

Team Spectrum
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Problem statement

With the proliferation of marine vessels for trade and military applications,the safety of territorial waters poses new challenges to the nation's navy.Tracking foreign vessels without being exposed and alerting them is a critical challenge.
Although active SONAR techniques exist which help in monitoring the surroundings, it has a major drawback of exposing the tracker to the tracked. Passive SONAR technique is employed to achieve the objective of stealth tracking. We wish to explore how this technology works, understand the limitations and build a better design in the future.

Objective

Our objective is to build a simulation tool to test acoustic source localization algorithms in terrestrial and underwater environments.

Two algorithms are described in the project which locate an audio source. One of the algorithms works in 2-D while the other is built for 3-D. The results of the algorithms are compared with the actual values and the efficiency and limitation of the algorithms are presented. The simulation tool built as part of this project can be used during the design phase of a passive SONAR system.

One of the most used technique to locate a sound source involves the usage of an array of sensors (microphones/hydrophones)[1]. One way to locate a source using an array of sensors is to find out the time difference of arrival (TDoA) the signal between two sensors.

An array of minimum 3 microphones can localize a source in 2-D and an array of minimum four microphones can localize a source in 3-D.

[1] Electrical transducer which converts sound into electrical signal

- **Localization in 2-D**
  Right angled triangle configuration: At each vertex of a right angled triangle, a microphone is placed. Let the vertices be - P, Q, R. We fix the coordinate system with point R as the origin and S is the audio source.

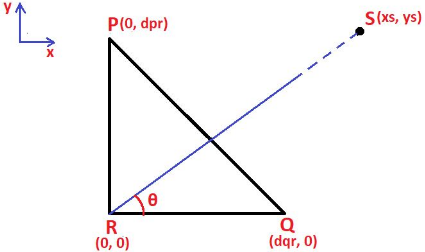


Figure 1: Configuration of three microphones

- **Localization in 3-D**
  Square configuration: Four microphones A, B, C, D are placed on the vertices of a square, with C being the origin.

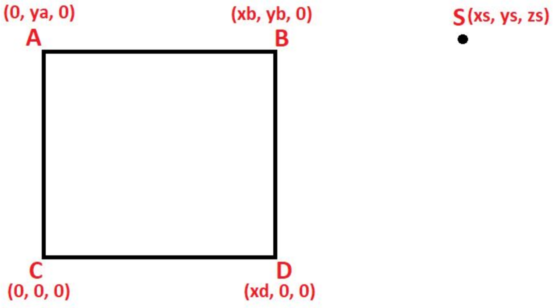


Figure 2: Configuration of the four microphones

Let $x_1[n]$ and $x_2[n]$ be the samples received by two spatially separated sensors at an instant nT_s, where T_s is the inverse of sampling rate, F_s.

$$x_1[n] = \alpha_1 s[n] + n_1[n] \quad (1)$$
$$x_2[n] = \alpha_2 s[n - \tau] + n_2[n] \quad (2)$$

where $n_1(n)$, and $n_2[n]$ is the noise at each of the sensors which is uncorrelated with the signal, τ is the time delay, and α_1, α_2 are the attenuation parameters. The total length of each sequence x_1, x_2 is equal to N.

We have tried out four different methods to calculate TDoA between signals.

1. Normalized frequency based TDoA

Let F be the source frequency which is known and F_s be the sampling frequency of the DT sequences $x_1[n]$, $x_2[n]$ given above. Consider samples within a window of size N. Now taking DFT of both the signals.

$$x_1[n] \underset{DFT}{\Longleftrightarrow} X_1[k]$$
$$x_2[n] \underset{DFT}{\Longleftrightarrow} X_2[k]$$

Now calculate $\frac{F}{F_s}$ and round it to the nearest integer m.
The time difference estimate w.r.t. the reference signal x_1, $\hat{\tau}^{DS}$, is given by:

$$\hat{\tau}^{DS} = \frac{\angle X_2[m] - \angle X_1[m]}{2\pi F}$$

The following figure shows a plot between actual time difference and estimated difference when the source is moved around the sensor array by $360°$, assuming no noise.

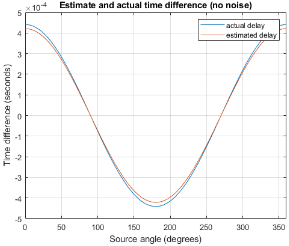


Figure 3: Actual and estimated time difference with no noise

When noise of varying levels is added and the actual and estimated time differences are plotted, we observe the following graphs:

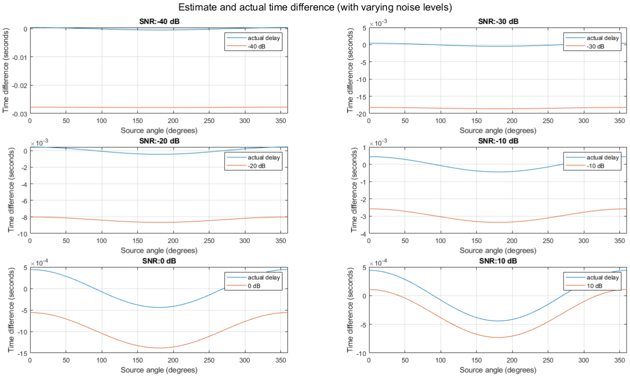


Figure 4: Actual and estimated time difference with varying noise - Normalized frequency method

We tried 2 other methods for calculating TDoA: Cross-correlation and GCC-PHAT, the results of which are shown in the final report. But finally we decide to go with Hilbert transform based TDoA.

2. Hilbert Transform based TDoA

The Hilbert Transform of a signal $r(t)$ is defined as:

$$\hat{r}(t) = \mathcal{H}\{r(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{r(\tau)}{t-\tau}d\tau = \frac{1}{\pi t} \circledast r(t)$$

where the integral is a Cauchy Principal Value (CPV) and $\circledast$ denotes convolution.
The following method of calculating TDoA using hilbert transform is applicable only for **narrowband signals**[2].

Let the energy of the narrowband signal be concentrated around $\pm f_0$. Let column vector of length N - $\hat{x}_1[n]$ - represent the Hilbert transform of the sampled version of the reference signal ($x_1[n]$) and column vector of same length, $x_2[n]$, represent the sampled version of the delayed signal. The time difference estimate, $\hat{\tau}^{HT}$, is given by

$$\hat{\tau}^{HT} = \frac{1}{2\pi f_0} \text{arcsin} \left(-\frac{x_2^T \hat{x_1}}{x_1^T x_1} \right)$$

The following figure shows a plot between actual time difference and estimated time difference when the source is moved around the sensor array from 0° to 360°, assuming no noise.

Estimating TDoA VII

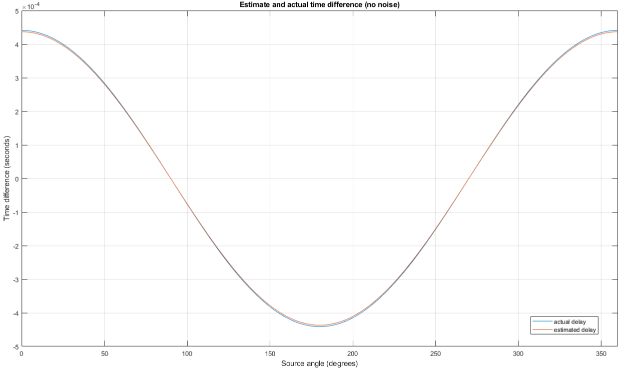


Figure 5: Actual and estimated time difference with no noise

When noise of varying levels is added and the actual and estimated time differences plotted, without the use of a pre-filter, the observations are:

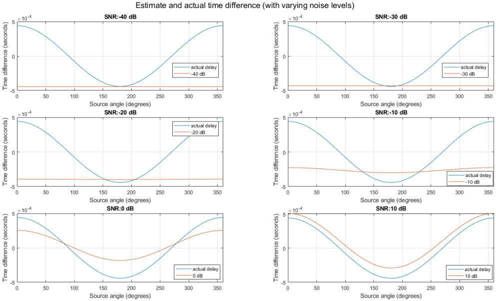


Figure 6: Actual and estimated time difference with varying noise - Hilbert transform based TDoA estimation

We have chosen to go ahead with this estimator due to several reasons:

- The estimator has been specially derived for narrowband signals and we are dealing with narrowband signal in our project.
- The no noise graph is approximately same as the actual graph and it is difficult to distinguish both of them. Hence, by using a filter we can reduce the noise and use this estimator.
- The computation cost can be reduced by using FFT algorithm such that overall time complexity of the estimator is approximately $\mathcal{O}(N\,log\,N)$.

[2] the energy of the signal is not distributed between multiple frequencies but concentrated around a centre frequency.

Filter design I

The design of a filter block prior to TDoA estimation requires a separate research and base altogether. In this project, we do not aim to present the best filtering method, but use a convinient filter to achieve our goal of localization.
The filtering performed here is not in a conventional sense. It is based on the fact that DFT magnitude plot of a noisy sequence gives a peak at frequencies present within the signal. Since in this project we use only a single frequency signal, this algorithm works well. The steps performed are:

1. Take DFT of the sequence
2. Identify the peaks
3. Replace all frequencies except the one's at which peak exist with zero
4. Perform IDFT to obtain the filtered signal.

Same noisy signal was passed through the custom filter and a Butterworth lowpass filter.

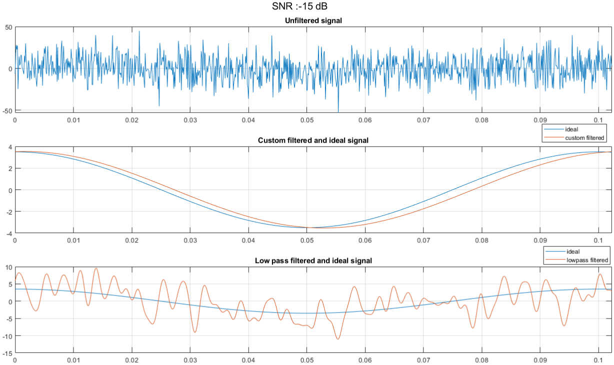


Figure 7: Output of custom filter

2-D Localization: Right angled triangle configuration

We aim to obtain the azimuth angle of arrival(AoA) of the source signal w.r.t x-axis at origin.

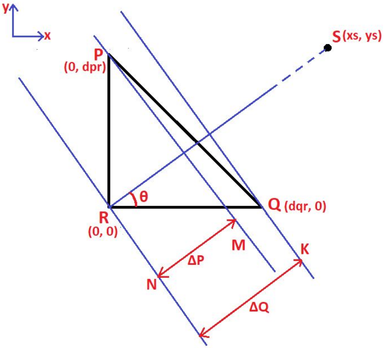


Figure 8: Estimating AoA

Let t_{pr} t_{qr} be the Time difference of arrival between microphones P&R, Q&R respectively and v is the speed of sound. Then the estimated AoA is:

$$\theta_{est} = \frac{\theta_{pr} + \theta_{qr}}{2}$$

where,

$$\theta_{pr} = arcsin(\frac{vt_{pr}}{d_{pr}})$$

$$\theta_{qr} = arccos(\frac{vt_{qr}}{d_{qr}})$$

The derivation of the above results is shown in the final report.

3-D Localization: Square configuration

We aim to obtain the 3-D coordinates of the source using the time differences of arrival.

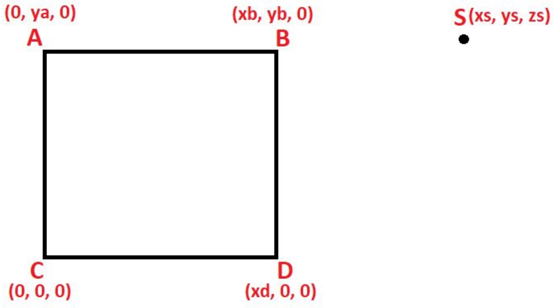


Figure 9: Configuration of the four microphones

Localization of Source IV

Let t_{ac}, t_{bc}, t_{dc} be the Time difference of arrivals of the sound wave between microphones $A\&C$, $B\&C$, $D\&C$ respectively and v be the speed of sound. Then the estimated coordinates of the source are:

$$x = \frac{D_{ba}(D_{dc} - D_{ac})(d^2 + D_{dc}D_{ac}) - D_{dc}(D_{ba} - D_{bd})(d^2 + D_{ba}D_{bd})}{2d(D_{bd}D_{dc} - D_{ac}D_{ba})}$$

$$y = \frac{(D_{dc} - D_{ac})(d^2 + D_{dc}D_{ac}) + 2dD_{ac}}{2dD_{dc}}$$

$$z = \sqrt{(\frac{d^2 - 2dx_s - D_{dc}^2}{2D_{dc}})^2 - x^2 - y^2}$$

where,

$$
\begin{aligned}
D_{ba} &= v(t_{bc} - t_{ac}) \\
D_{bd} &= v(t_{bc} - t_{dc}) \\
D_{ac} &= = vt_{dc} \\
D_{dc} &= vt_{dc}
\end{aligned}
$$

The derivation of the above results is shown in the final report.

2-D Localization

The following plot is obtained when a source is moved around the sensors from 0° to 360°, without noise:

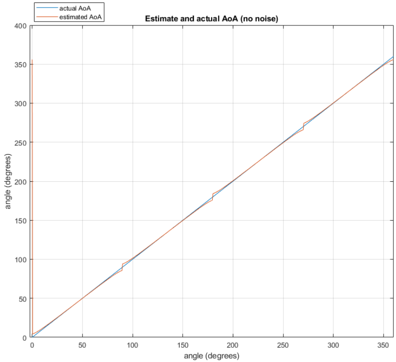


Figure 10: Actual and estimated angle of arrival, without noise

The following plot shows the estimated and actual angle of arrival when a noisy signal with different SNR is emanated from the source using custom filter.

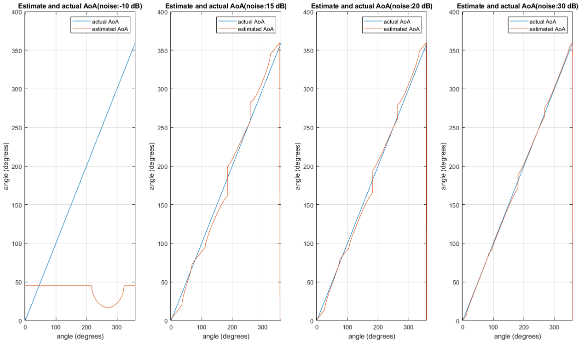
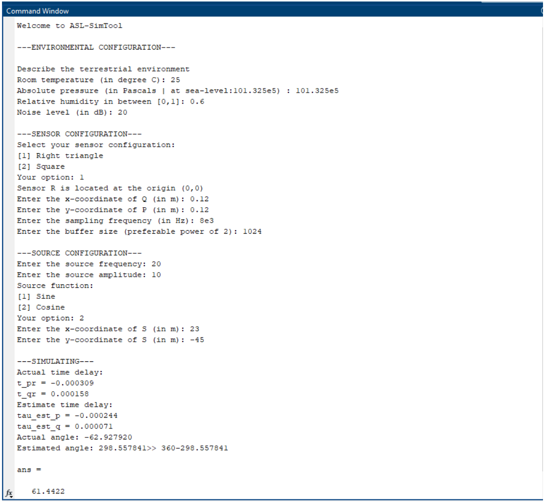


Figure 11: Actual and estimated angle of arrival, with noise



```
Command Window
Welcome to ASL-SimTool

---ENVIRONMENTAL CONFIGURATION---

Describe the terrestrial environment
Room temperature (in degree C): 25
Absolute pressure (in Pascals | at sea-level:101.325e5) : 101.325e5
Relative humidity in between [0,1]: 0.6
Noise level (in dB): 20

---SENSOR CONFIGURATION---
Select your sensor configuration:
[1] Right triangle
[2] Square
Your option: 1
Sensor R is located at the origin (0,0)
Enter the x-coordinate of Q (in m): 0.12
Enter the y-coordinate of P (in m): 0.12
Enter the sampling frequency (in Hz): 8e3
Enter the buffer size (preferable power of 2): 1024

---SOURCE CONFIGURATION---
Enter the source frequency: 20
Enter the source amplitude: 10
Source function:
[1] Sine
[2] Cosine
Your option: 2
Enter the x-coordinate of S (in m): 23
Enter the y-coordinate of S (in m): -45

---SIMULATING---
Actual time delay:
t_pr = -0.000309
t_qr = 0.000158
Estimate time delay:
tau_est_p = -0.000244
tau_est_q = 0.000071
Actual angle: -62.927920
Estimated angle: 298.557841>> 360-298.557841

ans =

   61.4422
```

Figure 12: Simulating right triangle configuration

3-D Localization

```
Command Window
Welcome to ASL-SimTool

---ENVIRONMENTAL CONFIGURATION---

Describe the terrestrial environment
Room temperature (in degree C): 25
Absolute pressure (in Pascals | at sea-level:101.325e5) : 101.325e5
Relative humidity in between [0,1]: 0.5
Noise level (in dB): 3000

---SENSOR CONFIGURATION---
Select your sensor configuration:
[1] Right triangle
[2] Square
Your option: 2
Sensor C is located at the origin (0,0)
Since it is a square configuration distance between adjacent sensors is same
*A(0,d,0)   *B(d,d,0)

*C(0,0,0)   *D(d,0,0)
Enter distance between any two sensors, d(in m): 0.1
Enter the sampling frequency (in Hz): 10e3
Enter the buffer size (preferable power of 2): 1024

---SOURCE CONFIGURATION---
Enter the source frequency: 10
Enter the source amplitude: 8
Source function:
[1] Sine
[2] Cosine
Your option: 2
Enter the x-coordinate of S (in m): 3
Enter the y-coordinate of S (in m): 4
Enter the z-coordinate of S (in m): 5

---SIMULATING---
Actual time delay:
t_ac = -0.000162
t_bc = -0.000284
t_dc = -0.000121
Estimate time delay:
tau_est_ac = -0.000158
tau_est_bc = -0.000277
tau_est_dc = -0.000118
```

Implementation Results V

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Estimate time delay:
tau_est_ac = -0.000158
tau_est_bc = -0.000277
tau_est_dc = -0.000118
x: 2.866151
y: 3.820779
z: 4.994270>>
```

Figure 13: Simulating square configuration

In this project, we have discussed multiple methods of TDoA estimation, source localization, and also provided two methods to perform the same. However, these methods are ad-hoc in nature. The localization algorithms heavily depend on how good the noise is filtered. The filter must be designed to eliminate phase distortion introduced due noise. We have presented a modular simulation toolkit - **ASL-SimTool** which can be used during the design phase of the hardware for acoustic source localization.

We plan to implement the methods discussed in the report on hardware.

- For better estimation, the signals must be filtered to a SNR level of 20 dB. Work needs to be done on the filtering part. The filter can be of analog and/or digital nature.
- Better TDoA estimation methods which provide good results in noisy environments can be explored.