Arithmetical Increase or Arithmetical Mean Method

Where Arithmetical Increase Method is used

The arithmetical Increase Method is mainly adopted for **old and developed** towns, where the rate of population growth is nearly constant. Therefore, it is assumed that the rate of growth of the population is constant. It is similar to simple interest calculations. The population predicted by this method is the lowest of all.

Arithmetical Increase Method Derivation

$dP/dt = K$ (say), where, dP/dt represents rate of growth of population.

Integrating the above equation over P1 to P2 over a time period of t1 to t2,
$\int dP = K\int dt$
$[P2 - P1] = k * [t2 - t1]$
$P2 = P1 + K * \Delta t$
$P2 = P1 + \bar{x} * n$
$P2 = P1 + n\bar{x}$

Arithmetical Increase Method Formula

$Pn = Po + n\bar{x},$
where,
Po - last known population
Pn - population (predicted) after 'n' number of decades,
n - number of decades between Po and Pn and,
x̄ - the rate of population growth.

Arithmetical Increase Method Example Problem

The following data (common data) will be used in the example problems for all other methods to be discussed.

Year	Population
1930	25000
1940	28000
1950	34000
1960	42000
1970	47000

Question: With the help of the common data find the population for the year 2020 using the arithmetic increase method.
Solution:
Step 1: Find the increase in population each decade.

Year	Population	Increase
1930	25000	-
1940	28000	3000
1950	34000	6000
1960	42000	8000
1970	47000	5000

Step 2: Find the average rate of increase of population ($\bar{x}$)
$\bar{x} = (3000+6000+8000+5000)/4$
$\bar{x} = 22000/4$
$\bar{x} = 5500$

Step 3: Find the number of decades (n) between the last known year and the required year

$n = 5$ (5 decades elapsed between 1970 and 2020)

Step 4: Apply the formula $\boldsymbol{P_n = P_o + n\bar{x}}$,
$P_{[2020]} = P_{[1970]} + (5 * 5500)$
$P_{[2020]} = 47000 + 27500$
$\boldsymbol{P_{[2020]} = 74{,}500}$. Therefore, population at 2020 will be 74,500.

Geometrical Increase Method

Where Geometrical Increase Method is used

This method is adopted for **young and developing towns**, where the rate of growth of population is proportional to the population at present (i.e., $dP/dt \propto P$). Therefore, it is assumed that the percentage increase in population is constant. It is similar to compound interest calculations. The population predicted by this method is the highest of all.

Geometrical Increase Method Derivation

Let's say, for the 0th-year population is ***P***
For 1st year/decade, according to this method, the population would become,
$P + (r/100)P$, where r is the growth rate.
For 2nd year/decade, according to this method, population would become,
$[P + (r/100)P] + (r/100)[P + (r/100)P]$
$= P[1+(r/100)]^2$
Generalizing the above equation, we get,
$P_n = P_o[1 + (r/100)]^n$

Geometrical Increase Method Formula

$P_n = P_o[1 + (r/100)]^n$,
where,
P_o - last known population,
P_n - population (predicted) after 'n' number of decades,
n - number of decades between P_o and P_n and,
r - growth rate = (increase in population/initial population) * 100 (%).
r could be found as arithmetic mean (i.e., $(r_1 + r_2 + r_3 \ldots r_n)/n$) or as a geometric mean (i.e., nth root of $(r_1 * r_2 * r_3 \ldots r_n)$), for the given data. According to Indian standards r should be calculates using geometric mean method.

Geometrical Increase Method Example Problem

Question: With the help of the common data find the population for the year 2020 using the Geometrical increase method.
Solution:
Step 1: Find the increase in population each decade.
Step 2: Find the growth rate.

Year	Population	Increase in population	Growth rate
1930	25000	-	-
1940	28000	3000	(3000/25000) * 100 = 12%
1950	34000	6000	(6000/28000) * 100 = 21.4%
1960	42000	8000	(8000/34000) * 100 = 23.5%
1970	47000	5000	(5000/42000) * 100 = 11.9%

Step 3: Find the average growth rate (r) using geometrical mean.
$r = \sqrt[4]{(12 * 21.4 * 23.5 * 11.9)}$
r = 16.37 %

Step 4: Find the number of decades (n) between the last known year and the required year
n = 5 (5 decades elapsed between 1970 and 2020)

Step 5: Apply the formula $P_n = P_o[1 + (r/100)]^n$
$P_{[2020]} = P_{[1970]}[1 + (16.37/100)]^5$
$P_{[2020]} = 47000[1.1637]^5$
$P_{[2020]}$ = 1,00,300. Therefore, population at 2020 will be 1,00,300.

Incremental Increase Method

Where Incremental Increase Method is used

This method is adopted for **average-sized towns under normal conditions**, where the rate of population growth is not constant i.e., either increasing or decreasing. It is a combination of the arithmetic increase method and geometrical increase method. Population predicted by this method lies between the arithmetical increase method and the geometrical increase method.

Incremental Increase Method Formula

$\boldsymbol{P_n = (P_o + n\bar{x}) + ((n(n+1))/2)^* \bar{y}}$,
where,
P_o - last known population,
P_n - population (predicted) after 'n' number of decades,
n - number of decades between P_o and P_n,
$\bar{x}$ - mean or average of increase in population and,
$\bar{y}$ - algebraic mean of incremental increase (an increase of increase) of population.

Incremental Increase Method Example Problem

Question: With the help of the common data find the population for the year 2020 using the Incremental Increase method.
Solution:
Step 1: Find the increase in population in each decade.
Step 2: Find the incremental increase i.e., increase of increase.

Year	Population	Increase in population	Incremental Increase
1930	25000	-	-
1940	28000	3000	-
1950	34000	6000	6000 - 3000 = 3000
1960	42000	8000	8000 - 6000 = 2000
1970	47000	5000	5000 - 8000 = -3000

Step 3: Find $\bar{x}$ and $\bar{y}$ as average of Increase in population and Incremental increase values respectively.
$\bar{x} = (3000+6000+8000+5000)/4$
$\bar{x} = 5500$
$\bar{y} = (3000+2000-3000)/3$
$\bar{y} = 2000/3$

Step 4: Find the number of decades (n) between the last known year and the required year
n = 5 (5 decades elapsed between 1970 and 2020)

Step 5: Apply the formula $\boldsymbol{P_n = (P_o + n\bar{x}) + ((n(n+1))/2)^* \bar{y}}$,
$P_{[2020]} = (P_{[1970]} + n\bar{x}) + ((n(n+1))/2)^* \bar{y}$
$P_{[2020]} = 47000 + (5 * 5500) + (((5 * 6)/2) * (2000/3))$
$\boldsymbol{P_{[2020]} = 84{,}500.}$ Therefore, population at 2020 will be 84,500.

Decreasing Rate of Growth Method

Where Decreasing Rate of Growth Method is used

This method is adopted for a town which is reaching **saturation population**, where the rate of population growth is decreasing. In this method, an average decrease in growth rate (S) is considered.

Decreasing Rate of Growth Method Formula

$P_n = P(n\text{-}1) + ((r(n\text{-}1) - S)/100) * P(n\text{-}1),$
where,
Pn - population at required decade,
P(n-1) - population at previous decade (predicted or available),
r(n-1) - growth rate at previous decade and,
S - average decrease in growth rate.

Due to the very nature of the formula, which requires population data at the previous decade i.e., P(n-1), this method requires the calculation of population at each successive decade (from the last known decade) instead of directly calculating population at the required decade.

Decreasing Rate of Growth Method Example Problem

Question: With the help of the common data find the population for the year 2020 using the decreasing rate of growth method.
Solution:
Step 1: Find the increase in population.
Step 2: Find the growth rate (r) as in the geometrical increase method.
Step 3: Find the decrease in the growth rate.

Year	Population	Increase in population	Growth rate (r)	Decrease in growth rate
1930	25000	-	-	-
1940	28000	3000	12%	-
1950	34000	6000	21.4%	12 - 21.4 = -9.4%

Year	Population	Increase in population	Growth rate (r)	Decrease in growth rate
1960	42000	8000	23.5%	21.4 - 23.5 = -2.1%
1970	47000	5000	11.9%	23.5 - 11.9 = 11.6%

Step 4: Find the average of decrease in growth rate(s).
S = (-9.4-2.1+11.6)/3
S = 0.1/3
S = 0.03%

Step 5: Apply the formula ***Pn = P(n-1) + ((r(n-1) - S)/100) * P(n-1),*** and find the population at successive decade till the population at required data is arrived.
$P_{[1980]} = P_{[1970]} + ((r_{[1970]} - S)/100) * P_{[1970]}$
$P_{[1980]}$ = 47000 + ((11.9 - 0.03)/100) * 47000
$P_{[1980]}$ = 52579
$P_{[1990]} = P_{[1980]} + ((r_{[1980]} - S)/100) * P_{[1980]}$
$P_{[1990]}$ = 52579 + ((11.87 - 0.03)/100) * 52579, here $r_{[1980]}$ is directly found as 11.9 - 0.03 i.e., $r_{[1970]}$ - S, which equals to 11.87.
$P_{[1990]}$ = 58,804
Similarly, $P_{[2020]}$ could be found.

Graphical Method

In this method, the population vs time graph is plotted and is extended accordingly to find the future population. It is to be done by an experienced person and is almost always prone to error.