Discrete Mathematics

Practice Questions

2 marks

1. Find the GCD of 115,304.
2. Find one solution of $5x \equiv 7 \ (mod \ 4)$.
3. Find inverse of 11 modulo 13.
4. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $R$ be a relation on $A$ defined as $R = \{(a, b)|a^2 = b\}$. Find the elements, domain and range of the relation R.
5. Show that the relation $R = \{(a, b)|a - b = an \ even \ integer, \forall a, b \in \mathbb{Z}\}$ is an equivalence relation.
6. Let $R$ be a relation defined on a set of positive integers such that $\forall x, y \in Z^+$,
$xRy \ if \ and \ only \ if \ x - y \ is \ divisible \ by \ 3$. Prove that $R$ is equivalence relation.
7. Is (Z,>) a POset? Justify your answer.
8. Write the dual of the statement $(a \cap b) \cup a = a \cap (b \cup a)$
9. Define Independence number of a graph.
10. Define matching number of a graph with example.
11. Define perfect matching with example.
12. Write the chromatic polynomial of null graph with 7 vertices.
13. Find the chromatic polynomial of the following graph
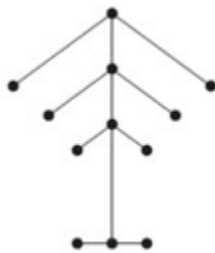


14. Is the following polynomial a chromatic polynomial? Justify your answer
$X^2$+2x-3.
15. Find the clique number of the graph
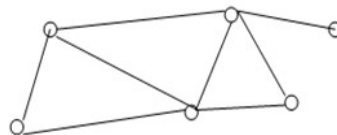


**16.** Is the graph $K_3$ perfect? Justify your answer.

5 marks

**17.** Show that $11^{34} \equiv 2 \ (mod \ 17)$
18. When n is a positive integer, show that $3^{2n+1} \equiv 3.2^n (mod \ 7)$.
19. Find the remainder when the sum $1! + 2! + 3! + \cdots \ldots + 100!$ is divided by 5.

20. Draw the Hasse diagram for the Poset (A,/) where / stands for divisibility and A={ 2,3,6,12,24,36}

21. Consider D= {1,2,3,4,5,6,10,12,15,20,30,60} the factors of 60. Prove that (D,/) is a lattice. Find complements of 2 and 10.

22. Consider the lattice L={1,2,3,4,6,12} ordered by divisibility . Find the lower and upper bound of L. Is L a complemented Lattice?

23. A new flag is to be designed with 6 vertical stripes using 4 colours. In how many ways can this be done so that no two adjacent stripes have the same colour?

10 marks

24. Draw the Hasse diagram of the poset $(D_{20}, |)$ and find its least and greatest elements, where $D_{20}$ is the set of positive divisors of 20.

25. Draw the Hasse diagram of $(\{2, 3, 4, 9, 12, 18, 36\}, |)$, where | is divides. Find the least,
   greatest, maximal and minimal elements of the subsets of the following
   a) $A_1 = \{12, 18\}$
   b) $A_2 = \{4, 9\}$
   c) $A_3 = \{2, 3, 12, 18\}$

26. Determine whether the relation $R$ on the set of all Web pages is reflexive, symmetric, anti-symmetric, and/or transitive, where (a, b) $\in R$ if and only if
   a) Everyone who has visited Web page a has also visited Web page b.
   b) There are no common links found on both Webpage a and Web page b.
   c) There is at least one common link on Web page a and Web page b.

27. Check whether the following Posets are Lattices or not
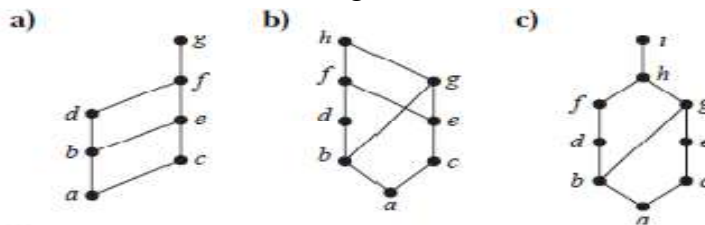


28. Find the chromatic polynomial for the following graph by applying decomposition theorem.
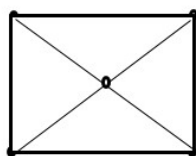


29. Determine the chromatic polynomial of the following graphs by decomposition theorem