

① Solⁿ: Here $n = 20$, $\bar{x} = 42$, $S = 5$

$\mu = 45$, $\sigma = ?$ (unknown)

let H_0 : The sample is drawn from a population with mean $\mu = 45$

$\therefore H_1: \mu \neq 45$

$$\text{under } H_0, \quad Z = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{42 - 45}{5/\sqrt{20}}$$

$$Z = \frac{-3}{5/\sqrt{20}} = -0.1341$$

$\therefore |Z| = 0.1341 < 1.96$ the significant value of Z at 5% level of significance, H_0 is accepted \therefore the sample is drawn from the population with mean 45 Ans

(2) solⁿ:- H_0 : The variances are equal i.e. $\sigma_1^2 = \sigma_2^2$
i.e. the samples have been drawn from normal
populations with same variance

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

Under null hypothesis, the

$$\text{test statistic } F = \frac{S_1^2}{S_2^2} \quad (S_1^2 > S_2^2)$$

$$\text{and } S_1^2 = \frac{\sum (x_1 - \bar{x}_1)^2}{n_1 - 1}$$

$$S_2^2 = \frac{\sum (x_2 - \bar{x}_2)^2}{n_2 - 1}$$

Computation for S_1^2 and S_2^2

$$\text{Here } n_1 = 6$$

$$n_2 = 7$$

x_1	$(x_1 - \bar{x}_1)$	$(x_1 - \bar{x}_1)^2$	x_2	$(x_2 - \bar{x}_2)$	$(x_2 - \bar{x}_2)^2$
29	0.84	0.7056	28	-3.14	9.8596
30	1.84	3.3856	30	-1.14	1.2996
31	2.14	4.5956	33	+1.86	3.4596
24	-4.16	17.3056	32	0.86	0.7396
27	-1.16	1.3456	32	0.86	0.7396
28	-0.16	0.0256	29	-2.14	4.5796
			34	2.86	8.1796
total		30.9136	total		16.098

$$\bar{x}_1 = \frac{169}{6} = 28.16$$

$$s_2^2 = \frac{\sum (x_2 - \bar{x}_2)^2}{n_2 - 1}$$

$$s_1^2 = \frac{\sum (x_1 - \bar{x}_1)^2}{n_1 - 1}$$

$$= \frac{16.098}{6}$$

$$= \frac{30.9136}{5} = 6.1827$$

$$= 2.683$$

$$\therefore F = \frac{s_1^2}{s_2^2} = \left(\frac{6.1827}{2.683} \right) = 2.30$$

$$\therefore s_1^2 > s_2^2$$

Conclusion: The tabulated value of F at

$n_1 = 6 - 1$ and $n_2 = 7 - 1$ at 5% level of significance is 4.39

Since the tabulated value of F is greater than the calculated value, H_0 is accepted i.e. there is no significant difference the variance i.e. the samples have been drawn from the normal population with same variance.

③ Solⁿ:- We have $\bar{x}_1 = 67.5$ and $\bar{x}_2 = 68.0$
 $n_1 = 1000$ and $n_2 = 2000$

Or the hypothesis, that the samples are drawn from the same population of S.D = 5.5

we get

$$Z = \frac{(\bar{x}_2 - \bar{x}_1)}{\sigma \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \left(\frac{0.5}{9.5 \times 0.0387} \right)$$
$$= 1.3599$$

$$\therefore |Z| = 1.3599$$

Conclusion. As the calculated value of $|Z| < 1.96$ the significant value of Z at 5% level of significance, H_0 is accepted i.e. there is no significant difference between mean scores. Hence samples are regarded to have been drawn from the same population.

④ Soln:- Here $n = 200$

$$\bar{x} = 5 = 5.5, \sigma = 5$$

$$\text{let } \alpha = 5\%$$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{5.5 - 5}{5/\sqrt{200}} = 2$$

Table value $Z_{\alpha} = 1.96$

Since $Z = 2 > 1.96$, Reject H_0

\therefore The given sample was not taken from the population. Ans.

⑤ Soln:- let \bar{X} be the sample mean of a random sample of size n , then

$$\bar{X} \sim N(\mu, 4^2/n)$$

$$\text{and } \mu = 30$$

$$P(25 < \bar{X} - \mu < 35) = 0.98$$

$$\Rightarrow P\left(\frac{25-30}{4/\sqrt{n}} < \frac{\bar{X}-\mu}{\frac{4}{\sqrt{n}}} < \frac{35-30}{4/\sqrt{n}}\right) = 0.98$$

$$\Rightarrow P\left(\frac{25-30}{4/\sqrt{n}} < Z < \frac{35-30}{4/\sqrt{n}}\right) = 0.98$$

$$\Rightarrow P\left(\frac{-5}{4/\sqrt{n}} < Z < \frac{5}{4/\sqrt{n}}\right) = 0.98 \Rightarrow P\left(0 < Z < \frac{5}{4/\sqrt{n}}\right) = 0.49$$

$$\frac{5}{4/\sqrt{n}} = 2.32 \Rightarrow n = \left(\frac{2.32 \times 4}{5}\right)^2 = 3.44 \approx 4 \text{ Ans}$$

⑥ Soln :- Here $n = 64$, $\bar{x} = 32$

$$\mu = 38$$

$$s = 5.8$$

H_0 : The sample is drawn from a population with mean

$$\mu = 38$$

$$H_1 : \mu \neq 38$$

$$\text{Under } H_0, z = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{32 - 38}{\left(\frac{5.8}{\sqrt{64}}\right)}$$

$$= -\left(\frac{6 \times \sqrt{64}}{5.8} \times 10\right) = -68.20$$
$$= -8.27$$

$$\text{So, } |z| = 8.27$$

Conclusion : Since the calculated value of $|z| > 1.96$, the significant value of z at 5% level of significance H_0 is rejected i.e. the sample is not drawn from the population with mean 38 i.e. average late spacing is less than 40 Ans.

⑦ Soln :- $H_0 : \sigma^2 = (0.022)^2 = 4.84 \times 10^{-4}$

$$n = 18, s^2 = 0.000324 = 3.24 \times 10^{-4}$$

$$H_1 : \sigma \neq 0.022$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$= \left[\frac{(18-1)(3.24) \times 10^{-4}}{4.84 \times 10^{-4}} \right]$$

$$= \left(\frac{17 \times 3.24}{4.84} \right) = 11.38$$

$$\text{Table value } \chi^2_{17}(0.05) = 27.59 > 11.38$$

Hence H_0 is accept Ans

⑧ soln: Here $n = 15$

$$s = 7 \Rightarrow s^2 = (7)^2$$

$$\sigma = 7.6 \Rightarrow \sigma^2 = (7.6)^2$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$= \frac{(15-1)(7)^2}{(7.6)^2} = \frac{6 \times 7^2}{(7.6)^2} = 5.09$$

Table value of $\chi^2(0.05) = 12.6$

Since $12.6 > 5.09$ Hence H_0 is accepted.

⑨ soln: Here $n = 100$, $\bar{x} = 80$, $s = 10$

$$\mu = 78 \text{ cm}$$

H_0 : The sample is drawn from a population with mean 78

H_1 : $\mu \neq 78$

$$\text{Under } H_0 \quad z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{80 - 78}{10/\sqrt{100}} = -2$$

$$\text{i.e. } |z| = 2$$

Conclusion: Since $|z| = 2 > 1.96$, the significant value of z at 5% level of significance.

H_0 is rejected i.e. the sample is not drawn from the population with mean 78 Ans

⑩ Solⁿ:- Given $H_0: \sigma_1^2 = \sigma_2^2$
 $H_1: \sigma_1^2 \neq \sigma_2^2$ at $\alpha = 0.01$

$$n_1 = 8, n_2 = 8$$

$$s_1^2 = 3.89, s_2^2 = 4.02$$

$$\frac{n_1 s_1^2}{n_1 - 1} = \frac{8(3.89)}{7} = 4.44$$

$$\frac{n_2 s_2^2}{n_2 - 1} = \frac{8(4.02)}{7} = 4.59$$

$$\therefore \text{Test statistic } F = \frac{\left(\frac{n_1 s_1^2}{n_1 - 1}\right)}{\left(\frac{n_2 s_2^2}{n_2 - 1}\right)} = \frac{4.44}{4.59} = 0.967$$

$$\text{Table value } F_{\alpha=0.01} = 3.79$$

Since $3.79 > 0.967$, Accept H_0 Ans.

⑪ Solⁿ:- Here $n = 900$, $\bar{x} = 3.4$, $\mu = 3.2$, $\sigma = 2.3$

H_0 : Assume that the sample is drawn from large population with mean 3.2 & S.D = 2.3

H_1 : $\mu \neq 3.25$ (Apply two tailed test)

$$\text{under } H_0; z = \left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \right) = \left(\frac{3.4 - 3.2}{2.3/\sqrt{900}} \right) = 0.261$$

Conclusion: As the calculated value of $|z| = 0.261 < 1.96$ the significant value of z at 5% level of significance. H_0 is accepted i.e. the sample is drawn from the population with mean 3.2 & S.D = 2.3 Ans.

(12) Soln :- Here $n = 100$

$$S = 15$$

$$\bar{X} = 116, \mu = 120$$

H_0 : There is no significant difference between \bar{X} & μ

under H_0

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{116 - 120}{15/10} = \frac{-4 \times 10}{15} = -2.66$$

$$\therefore |Z| = 2.66$$

Conclusion : As the calculated value of $|Z| > 1.96$ at 5% level of significance. H_0 is rejected
 \therefore dictation at the rate of 120 words can not possible. Ans.