Data Science Assignment 2

Vishaal Udandarao
(2016119)

1) Given, $n = 200$
$\bar{x} = 16$
$\sigma = 2.9$
Confidence level $= 95\%$

Since, $n > 30$, we will use the Z-distribution to find a 95% confidence interval.

We know, to find the confidence interval $\Rightarrow$

$$CI = \bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

<u>To find z value</u>

Given, confidence level $= 95\%$

$$\Rightarrow \alpha = \left(\frac{100-95}{100}\right) / 2$$

$$= 0.05/2$$

$$= \underline{\underline{0.025}}$$

By looking up the z table, we see that z value for $\alpha = 0.025$ is

$$\Rightarrow z = \underline{\underline{1.96}}$$

$$\therefore CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

$$= 16 \pm \frac{1.96 \times 2.9}{\sqrt{200}}$$

$$\Rightarrow CI = \left[16 - \frac{1.96 \times 2.9}{\sqrt{200}}, 16 + \frac{1.96 \times 2.9}{\sqrt{200}}\right]$$

$$\Rightarrow CI = [16 - 0.401, 16 + 0.401]$$

$$CI = \underline{\underline{[15.599, 16.401]}}$$

2) Given, population std ⇒

$\sigma = 4.6$

a) $n = 220$ (As $n > 30$, we use z table)

$\bar{x} = 16.2$

Confidence level = 92%

We know, confidence interval ⇒

$$CI = \bar{x} \pm z \frac{\sigma}{\sqrt{n}}$$

<u>To find z value</u>

Given, confidence level = 92%

$$\alpha = \left(\frac{100-92}{100}\right)/2$$

$$= \underline{\underline{0.04}}$$

From z table value ⇒

$$z = \underline{\underline{1.75}}$$

$\therefore CI = \bar{x} \pm \frac{Z\sigma}{\sqrt{n}}$

$= 16.2 \pm \frac{1.75 \times 4.6}{\sqrt{220}}$

$= 16.2 \pm 0.542$

$\Rightarrow CI = [15.658, 16.742]$

b) Here, given value of $\frac{Z\sigma}{\sqrt{n}}$

Therefore, we have $\Rightarrow$

$\frac{Z\sigma}{\sqrt{n}} = 10$ seconds

$= \frac{10}{60}$ minutes

Now, for 92% confidence level, $Z = 1.75$

$\Rightarrow \frac{1.75 \times 4.6}{\sqrt{n}} = \frac{1}{6}$

$$\sqrt{n} = 1.75 \times 4.6 \times 6$$

$$= 48.3$$

$$\Rightarrow n = \underline{\underline{2332.89}}$$

Since, we cannot have n as a floating value, we take the ceil of this value since n has to be atleast greater than this particular value. If n is less than this value, then the condition required won't be satisfied.

Therefore,

$$\boxed{n = 2333}$$

3) Given, proportion of consumers who bought newest generation of smart phone that are happy are => p.

a) We know margin of error =>

$$\text{margin of error} = \frac{\sigma Z}{\sqrt{n}}$$

=> Given, margin of error = 2% = 0.02

$$\Rightarrow 0.02 = \frac{\sigma Z}{\sqrt{n}}$$

$$\Rightarrow \frac{\sqrt{n} \times 0.02}{Z} = \sigma$$

For 80% confidence, Z-value from table is => 1.29

$$\Rightarrow \boxed{\frac{\sqrt{n} \times 0.02}{1.29} = \sigma}$$

Scanned by CamScanner

Now, we know that at the worst case, for 0.5 mean, to get max std deviation, half the points will lie on 0 and half of the points will lie on 1.

Therefore, the standard deviation in that case is $\Rightarrow$

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{(\bar{x} - x_i)^2}{n}}$$

$$= \sqrt{\frac{(0.5-0)^2 \cdot n/2 + (0.5-1)^2 \cdot n/2}{n}}$$

$$= \sqrt{\frac{(0.5)^2}{2} + \frac{(0.5)^2}{2}}$$

$$= \sqrt{(0.5/2)}$$

$$= \sqrt{0.25}$$

$$= 0.5$$

Scanned by CamScanner

Therefore $\Rightarrow$

$$\sqrt{n} \times \frac{0.02}{1.29} = 6$$

For worst case, n should be greater than equal to this σ value $\Rightarrow$

$$\sqrt{n} \times \frac{0.02}{1.29} \geq 0.5$$

$$\Rightarrow \sqrt{n} \geq \frac{0.5 \times 1.29}{0.02}$$

$$\Rightarrow \boxed{n \geq 1111.11}$$

$$\therefore n = 1112$$

~~b) Total no. of customers given = 10000~~
~~Proportion of customers that are happy = 400/10000 = 0.04~~

3.b) Given, $n = 10000$

We know Proportion of people happy $= 400$

Then, $p = \frac{400}{10000} = \underline{\underline{0.04}}$

<u>To calculate mean</u>

$$\bar{x} = \frac{400 \times 1 + (10000 - 4000) \times 0}{10000}$$

$$\bar{x} = \frac{400}{10000} = \underline{\underline{0.04}}$$

<u>To calculate standard deviation</u>

$$\sigma = \sqrt{\frac{400(1-0.04)^2 + (10000-400)(0-0.04)^2}{10000}}$$

$$= \sqrt{\frac{400 \times (0.96)^2 + 9600 \times (0.04)^2}{10000}}$$

$$= \underline{\underline{0.195}}$$

At 95% confidence interval, from Z-table, Z value is 1.96.

$\Rightarrow Z = 1.96$

$\therefore$ Confidence interval $\Rightarrow$

$$CI = \bar{x} \pm Z \frac{\sigma}{\sqrt{n}}$$

$$= 0.04 \pm \frac{1.96 \times 0.196}{\sqrt{10000}}$$

$$= 0.04 \pm 0.00384$$

$$\Rightarrow CI = [0.03616, 0.0438]$$

$\therefore$ The final confidence interval is $\Rightarrow$ $[0.03616, 0.0438]$

$\Rightarrow$ $[3.616\%, 4.38\%]$

4) We know that the maximum likelihood for the given data will be the parameter that maximizes the following ⟹

$$P(3,0,2,1,3,2,1,0,2,1,3,0,2,1,3,2,1,0,2,1 \mid \theta)$$

Therefore, let us call the above value ⟹ $f = p(x \mid \theta)$ [likelihood function]

Therefore, value of f will be ⟹

$$f = p(x \mid \theta) = \left(\frac{2\theta}{3}\right)^4 \times \left(\frac{\theta}{3}\right)^6 \times \left(\frac{2}{3}(1-\theta)\right)^6 \times \left(\frac{1-\theta}{3}\right)^4$$

Now taking log of this likelihood ⟹

$$\log f = \log p(x \mid \theta)$$
$$= 4[\log 2\theta - \log 3] + 6[\log\theta - \log 3] + 6[\log(2-2\theta) - \log 3] + 4[\log(1-\theta) - \log 3]$$

Differentiating wrt θ and equating to $0 \Rightarrow$

$$\frac{df}{d\theta} = 4\left[\frac{1}{2\theta} \times 2\right] + 6\left[\frac{1}{\theta}\right]$$

$$+ 6\left[\frac{1}{2-2\theta} \times -2\right]$$

$$+ 4 \times \left[\frac{1}{1-\theta} \times -1\right]$$

$$= \frac{4}{\theta} + \frac{6}{\theta} - \frac{6}{1-\theta} - \frac{4}{1-\theta}$$

$$\Rightarrow \frac{df}{d\theta} = 0$$

$$\Rightarrow \frac{6+4}{\theta} - \frac{6+4}{1-\theta} = 0$$

$$\Rightarrow \frac{10}{\theta} = \frac{10}{1-\theta}$$

$$\Rightarrow 1-\theta = \theta$$

$$\Rightarrow 2\theta = 1$$

$$\Rightarrow \boxed{\theta = \frac{1}{2}}$$

Scanned by CamScanner

Therefore, maximum likelihood estimate of $\theta \Rightarrow$

$$\boxed{\theta^{*} = \frac{1}{2}}$$

5) Given,

$$f(x|\sigma) = \frac{1}{2\sigma} e^{\left(\frac{-|x|}{\sigma}\right)}$$

Now, the likelihood ~~of~~ function is $\Rightarrow$

$$f = p(x|\theta) = \frac{1}{2\sigma} e^{\left(\frac{-|x_1|}{\sigma}\right)} \times \frac{1}{2\sigma} e^{\left(\frac{-|x_2|}{\sigma}\right)} \times \dots \times \frac{1}{2\sigma} e^{\left(\frac{-|x_n|}{\sigma}\right)}$$

$$= \left(\frac{1}{2\sigma}\right)^{n} e^{\left(\frac{-\sum_{i=1}^{n}|x_i|}{\sigma}\right)}$$

Taking log likelihood $\Rightarrow$

$$\log f = \log p(x \mid \theta)$$

$$= \log\left(\left(\frac{1}{2\sigma}\right)^n e^{\left(\frac{-\sum_{i=1}^{n} |x_i|}{\sigma}\right)}\right)$$

$$= \log\left(\frac{1}{2\sigma}\right)^n + \log e^{\left(\frac{-\sum_{i=1}^{n} |x_i|}{\sigma}\right)}$$

$$= n \log\left(\frac{1}{2\sigma}\right) - \frac{\sum_{i=1}^{n} |x_i|}{\sigma}$$

Differentiating wrt θ and equating to $0 \Rightarrow$

$$\frac{d(\log f)}{d\theta} = n \times \frac{1}{(1/2\sigma)} \times \frac{1}{2} \times \frac{-1}{\sigma^2}$$

$$- \sum_{i=1}^{n} |x_i| \times \frac{-1}{\sigma^2}$$

$$= n \times 2\sigma \times \frac{-1}{2\sigma^2} + \frac{\sum_{i=1}^{n} |x_i|}{\sigma^2}$$

$$= -\frac{n}{\sigma} + \frac{\sum_{i=1}^{n} |x_i|}{\sigma^2}$$

$$\frac{d(\log f)}{d\sigma} = 0$$

$$\Rightarrow -\frac{n}{\sigma} + \frac{\sum_{i=1}^{n} |x_i|}{\sigma^2} = 0$$

$$\Rightarrow \frac{n}{\sigma} = \frac{\sum_{i=1}^{n} |x_i|}{\sigma^2}$$

As σ cannot be 0 (otherwise divide by 0 error),

Therefore $\Rightarrow$

$$\frac{n}{\cancel{\sigma}} = \frac{\sum_{i=1}^{n} |x_i|}{\sigma^{\cancel{2}}}$$

$$\Rightarrow \sigma = \frac{\sum_{i=1}^{n} |x_i|}{n}$$

Therefore, max likelihood estimate for σ

$$\Rightarrow \boxed{\sigma^* = \frac{\sum_{i=1}^{n} |x_i|}{n}}$$

Scanned by CamScanner

6) Given, n data samples drawn from a uniform distribution $\Rightarrow$

$U(0, \theta)$

~~Now likelihood function~~.

We know the PDF for uniform distribution is $\Rightarrow$

$$f(x|\theta) = \frac{1}{\theta - 0} = \frac{1}{\theta} \quad \left(\text{for all } x \in [0, \theta]\right)$$

Therefore, likelihood function is $\Rightarrow$

$$f = p(x|\theta) = \underbrace{\frac{1}{\theta} \times \frac{1}{\theta} \times \cdots \frac{1}{\theta}}_{n \text{ terms}}$$

$$p(x|\theta) = \frac{1}{\theta^n} = \theta^{-n}$$

Taking log likelihood $\Rightarrow$

$$\log f = \log p(x|\theta) = \log \theta^{-n}$$

$$= -n \log \theta$$

Now differentiating w.r.t $\theta \Rightarrow$

$$\frac{d}{d\theta}(\log f) = -n \times \frac{1}{\theta} = \frac{-n}{\theta}$$

However, we notice that to maximize $p(x|\theta)$, we need all the terms to be non zero.

For this to be true, all the terms should be within the range $[0, \theta]$. This will only be true when θ is the max of all the terms and hence likelihood will be maximized.

Therefore, maximum likelihood estimate is $\Rightarrow$

$$\boxed{\theta^* = \max_{x} (x_i)}$$

$$\Rightarrow \boxed{\theta^* = \max_{x_i} (x_1, x_2, x_3, \dots, x_n)}$$

Scanned by CamScanner

7) Given, matrix $\Rightarrow$

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Now, $A^T \Rightarrow$

$$A^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Therefore,

$$A^TA = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 0 & 1 \\ 2 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$$

Now, singular values are the square roots of the eigen values of $A^T A$ in decreasing order.

Therefore, to find eigen values of $A^T A \Rightarrow$

Characteristic polynomial $\Rightarrow$

$$|A - \lambda I| = 0$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} 2-\lambda & 2 & 0 & 1 \\ 2 & 2-\lambda & 0 & 1 \\ 0 & 0 & -\lambda & 0 \\ 1 & 1 & 0 & 2-\lambda \end{bmatrix}$$

$\Rightarrow$ On solving $\Rightarrow$

$$\lambda^4 - 6\lambda^3 + 6\lambda^2 = 0$$

$$\Rightarrow \lambda = 0, 0, \sqrt{3}+3, 3-\sqrt{3}$$

Therefore, the singular values are $\Rightarrow$

$\sigma_1 = \sqrt{3+\sqrt{3}}$, $\sigma_2 = \sqrt{3-\sqrt{3}}$; $\sigma_3 = 0$

$\Rightarrow \sigma_1 = 2.175$

$\Rightarrow \sigma_2 = 1.126$

$\Rightarrow \sigma_3 = 0$

$\Rightarrow \sigma_4 = 0$

Therefore, singular values are $\Rightarrow$

2.175, 1.126, 0 and 0.

8) Given, matrix $\Rightarrow$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

To find SVD, we have to find singular values of A.

For that, we have to find eigen values of A^TA.

$$A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Therefore, to calculate eigenvalues of A^TA,

Characteristic equation $\Rightarrow$

$$|A^TA - \lambda I| = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 0 & 1 & 0 \\ 0 & 1-\lambda & 0 & 1 \\ 1 & 0 & 1-\lambda & 0 \\ 0 & 1 & 0 & 1-\lambda \end{bmatrix}$$

On solving characteristic equation,

$$\lambda^4 - 4\lambda^3 + 4\lambda^2 = 0$$

$\Rightarrow$ Eigen values are $\Rightarrow$

$\underline{\underline{\lambda = 0, 2}}$ [both have algebraic multiplicity = 2, hence total no of roots = 4]

Now, to get S, we need to find singular values of A.

Singular values of A are the square roots of the eigen values of $A^T A$ (only non zero)

Therefore, singular values will be

$\Rightarrow$ (in ~~increase~~ decreasing order)

$\Rightarrow \sqrt{2}, \sqrt{2}$

$\Rightarrow 1.414, 1.414$

$\therefore$ We know S will be an $m \times n$ matrix with diagonal elements as singular values $\Rightarrow$

$$\therefore \boxed{S = \begin{bmatrix} 1.414 & 0 & 0 & 0 \\ 0 & 1.414 & 0 & 0 \end{bmatrix}}$$

Now to find V, we need to find the orthogonal eigen vectors of A^TA.

Therefore, <u>for $\lambda = 2$</u>

$$|A^TA - \lambda I| = 0$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

To find RREF $\Rightarrow$

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now, we have $\Rightarrow$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

$\Rightarrow x_1 - x_3 = 0$

$\Rightarrow x_2 - x_4 = 0$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Therefore, the eigen vectors corresponding to $\lambda = 2$ are $\Rightarrow$

$$V_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Now, to normalize $\Rightarrow$

$$V_1 = \frac{1}{\sqrt{1+1}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$v_2 = \frac{1}{\sqrt{1+1}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

Therefore $\Rightarrow$

$$V = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

$$V^T = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\Rightarrow V^T = \begin{bmatrix} 0.707 & 0 & 0.707 & 0 \\ 0 & 0.707 & 0 & 0.707 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

Now to find U, we have to find the orthogonal eigen vectors of AA^T.

For this, we find $AA^T \Rightarrow$

$$AA^T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Therefore, on solving characteristic equation, we get $\Rightarrow$

$\underline{\underline{\lambda = 2}}$ (with multiplicity 2)

Therefore, the corresponding eigen vectors are $\Rightarrow$

$$(AA^T - \lambda I).V = 0$$

$$\Rightarrow \left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right) V = 0$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Therefore since both x_1 and x_2 are free variables $\Rightarrow$

$\Rightarrow x_1 = x_1 + 0x_2$

$\Rightarrow x_2 = 0x_1 + x_2$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\therefore$ The eigen vectors are $\Rightarrow$

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Hence, the normalized orthogonal eigen vectors are $\Rightarrow$

$$u_1 = \frac{1}{\sqrt{1+0}} v_1 = \frac{1}{\sqrt{1}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$u_2 = \frac{1}{\sqrt{1+0}} v_2 = \frac{1}{\sqrt{1}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boxed{U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

∴ The SVD is ⇒

$$A = U S V^T$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1.414 & 0 & 0 & 0 \\ 0 & 1.414 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.707 & 0 & 0.707 & 0 \\ 0 & 0.707 & 0 & 0.707 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

9) Given, matrix ⟹

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

The characteristic equation is ⟹

$$|A - \lambda I| = 0$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{bmatrix}$$

$$\Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow -9\lambda + (1-\lambda)(4-\lambda)(-\lambda-5) + 36 = 0$$

$$\Rightarrow -9\lambda + (4 - \lambda - 4\lambda + \lambda^2)(-\lambda - 5) + 36 = 0$$

$$\Rightarrow -9\lambda + (4 - 5\lambda + \lambda^2)(-\lambda - 5) + 36 = 0$$

$$\Rightarrow -9\lambda + (-4\lambda - 20 + 5\lambda^2 + 25\lambda - \lambda^3 - 5\lambda^2) + 36 = 0$$

$\Rightarrow -\lambda^3 + 12\lambda + 16 = 0$

On solving, we get $\Rightarrow$

$$\boxed{\lambda = 4, -2, -2}$$

Therefore let us consider each eigen value separately.

For the $\lambda = 4$

We know, $Av = \lambda v$

$\Rightarrow (A - \lambda I) \cdot v = 0$

$\Rightarrow$ We have to solve the system of equations.

$$\Rightarrow A - \lambda I = \begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix}$$

Performing Gauss Jordan elimination $\Rightarrow$

$$\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore,

$$(A-\lambda I)\,v=0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$\Rightarrow x_1 - \frac{1}{2}x_3 = 0 \Rightarrow x_1 = \frac{1}{2}x_3$

$\Rightarrow x_2 - \frac{1}{2}x_3 = 0 \Rightarrow x_2 = \frac{1}{2}x_3$

$\Rightarrow x_3 = x_3$

Therefore $\Rightarrow$

$$v = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3\begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

Take $x_3=1$, Then eigenvector $\Rightarrow$

$$v = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

For $\lambda = -2$

$$A - \lambda I = \begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix}$$

Finding RREF $\Rightarrow$

$$\begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore,

$$(A - \lambda I)v = 0$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow x_1 - x_2 + x_3 = 0$$

$$\Rightarrow x_1 = x_2 ~~+~~ - x_3$$

$$\Rightarrow x_2 = x_2 + 0x_3$$

$$\Rightarrow x_3 = 0x_2 + x_3$$

Therefore,

$$V = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Therefore, we will have two eigen vectors where $\Rightarrow$

$\Rightarrow$ when $x_2 = 0$ and $x_3 = 1$

$$V = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow$ when $x_2 = 1$ and $x_3 = 0$

$$V = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ The eigen values are $\lambda_1 = 4$, $\lambda_2 = -2$ and $\lambda_3 = -2$.

The corresponding eigen vectors are $\Rightarrow$

$$v_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$