SYDE 283

Computational Assignment 6

Exercise 1: Motion of a Charged Particle in a Uniform Magnetic Field:

$\vec{B} = -10^{-4}\,T\,\hat{k}$ $(-z)$ $\alpha$-particle

$P(0.0, 0.0, 0.0)$ $\hookrightarrow \vec{v} = 3{,}000\ m/s\ \hat{i}$ $(x)$

In a unif. mag field, the charged $\alpha$ particle's trajectory will be shaped in a circular orbit - as magnetic field is perpendicular to velocity.

$\vec{F}_c = q\vec{v}\times\vec{B}$

$\frac{mv^2}{R} = qvB$ $(+\hat{j})$ $\longrightarrow \therefore R = \frac{mv}{qB}$

$\frac{mv}{R} = qB$

An alpha particle w/ $\vec{v}\perp\vec{B}$ will undergo circular motion. Starting position $(0,0,0)$ and velocity is in $\hat{i}$, $\therefore$ center of circular path is $\perp \vec{v}$ @ the starting point.

$$R = \frac{mv}{qB} \qquad \left\{ a = 0,\ b = R,\ (y-b)^2 + (x-a)^2 = R \right\}$$

$$\Rightarrow x^2 + (y-R)^2 = \left(\frac{mv}{qB}\right)^2$$

$$\therefore x^2 + y^2 - \frac{2mvy}{qB} + \cancel{\left(\frac{mv}{qB}\right)^2} = \cancel{\left(\frac{mv}{qB}\right)^2}$$

$$x^2 + y^2 = \frac{2mvy}{qB}$$

$$\hookrightarrow x = \sqrt{\frac{2mvx}{qB} - y^2}$$

The path of the particle is given by $\left(\sqrt{\frac{2mvy}{qB} - y^2},\ x,\ 0\right)$ in this $\vec{B}$

An $H^+$ ion that enters the $\vec{B}$ has a negative charge and half the mass of an alpha particle. I would expect the particle to spin in the opposite direction about $(0, -\frac{mv}{qB}, 0)$ with about half the radius of orbit of the alpha particle.

**Exercise 2** Motion of a Charged Particle in the Magnetic Field of a Magnetic Dipole

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\vec{r}(\vec{\mu}\cdot\vec{r})}{r^5} - \frac{\vec{\mu}}{r^3}\right), \quad |\vec{\mu}| = 10^4 \quad \Big| \quad \alpha: P(0.0, -8.0, 2.0), \ \vec{v}_0 = 100 \frac{m}{s} (\hat{j}$$

I think the trajectory of the $\alpha$-particle will be a complex trig. function w/ many turns, cyclically. This is due to the elliptical, varying field lines of the $\vec{B}$ prod. by a dipole.

After plot:

↳ The plot is seemingly random, as the particle moves in closed, circular paths. The plot in Exercise 1 was defined clearly, whereas this was not.

**Exercise 3**: Motion of a Charged Particle in a "Magnetic Bottle"

I hypothesize that the charged particle's trajectory will be circular, but also drifting around the position-time graph, based on the vector plot. It will not be confined, as there are circular loops, and movement would become more chaotic from an equation to another.

↳ Plot not showing correctly! So answering other questions based on online help and intuition *

The path seems similar to that of Exercise 2, but substantially different from Exercise 1. Based on a plot, the particle would be confined forever, as |amplitude| in $z(t)$ remains relatively constant.