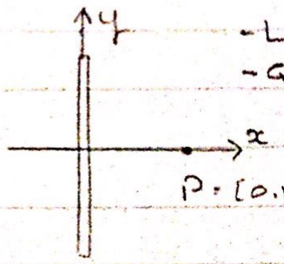
Mon, Jan 29, 18

SYDE 283

Computational Assignment 3

Exercise 1: Computing Electric Field Analytically



- L = 1m
- Q: ±1 μC

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2+(L/2)^2}}$$

* $\frac{1}{4\pi\epsilon_0} \approx 9\times10^9\ N\cdot m^2/C^2$

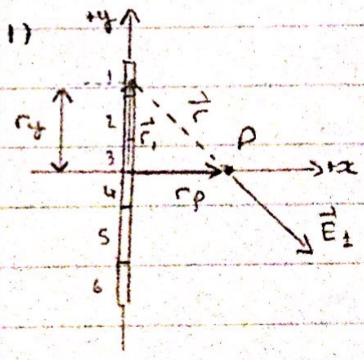
$$= (9\times10^9)\frac{(1\times10^{-6})}{0.1\sqrt{0.1^2+(1/2)^2}}$$

$$= 1.765\times10^5\ N/C$$

$$\therefore \vec{E}_{(0.1,0,0)} = [1.765\times10^5, 0, 0]\ N/C$$

Exercise 2: Computing E-Field due to Uniformly Charged Rod Numerically

1)



- $\vec{r} = \vec{r}_p - \vec{r}_1$
- +Q = 1 μC
- l = 1m
- rod = 6 pieces
- $E_{<0.1,0,0>}$?

2) Piece 1: $x = 0$

$y = \frac{1}{6}+\frac{1}{6}+\frac{1}{2}(\frac{1}{6}) = 0.41666$

$z = 0$

$\Rightarrow \langle 0, 0.41\bar{6}, 0\rangle$

3) Position of P relative to center of first piece ($\vec{r}$)

* $\vec{r} = \vec{r}_p - \vec{r}_1$

$\Rightarrow \vec{r} = [\vec{r}_p, -\vec{r}_1, 0]$

$\therefore \vec{r} = [0.1, -0.41\bar{6}, 0]$

$\|\vec{r}\| = 0.1^2 + 0.41\bar{6}^2 = 0.1836 \Rightarrow \hat{r} = \left[\frac{0.1}{\sqrt{0.1836}}, \frac{-0.41\bar{6}}{\sqrt{0.1836}}, 0\right]$

$\hat{r} = [0.23, -0.97, 0]$

4) $\vec{E}_{(p)} = k\frac{Q}{r^2}\hat{r}$

$k\frac{(\frac{1}{6}(1)\times10^{-6}\ C)}{0.1836}\hat{r}$

$= [1911.8, -7957.5, 0]\ N/C$

* due to first piece.

Piece 2: $\vec{r}_2 : [0.1, -0.25, 0]$ $\hat{r} : [0.371, -0.928, 0]$

$\|\vec{r}_2\| : \sqrt{0.1^2 + 0.25^2}$

$: 0.2693$

$\Rightarrow \vec{E}_2 : k \frac{\left(\frac{1}{6} \times 10^{-6}\right)}{0.0725} \hat{r} : [7675.9, -19200, 0]$

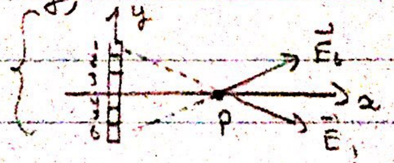
Piece 3: $\vec{r}_3 : [0.1, -0.083, 0]$ $\hat{r} : [0.769, -0.641, 0]$

$\|\vec{r}_3\| : 0.130$ $\Rightarrow E_3 : [68254.4, -56893.5, 0]$

5. With these 3 (above), and by symmetry, we can see that:

$E_{1x} : E_{6x}$ $E_{2x} : E_{5x}$ $E_{3x} : E_{4x}$

$E_{1y} : -E_{6y}$ $E_{2y} : -E_{5y}$ $E_{3y} : -E_{4y}$



6. $\therefore E : 2(1911.8) + 2(7675.9) + 2(68254.4)$

≈ 155684.2

$= 1.55 \times 10^5$ N/C. * $\vec{E}_{net}$

This value is less than the value calculated analytically in part 1 as a more numeric approach is taken - thoroughly breaking down individual components + forces to calculate the $\vec{E}_{net}$ (values cancel out, considering x-y, etc). Also, it is limited by n-6.

To get an even more accurate result, the rod must be divided into even more segments, observing $\vec{E}_{net}$ as $n \rightarrow \infty$

Exercise 3 Computing E-Field due to a Uniformly Charged Rod Numerically (with a computer program)

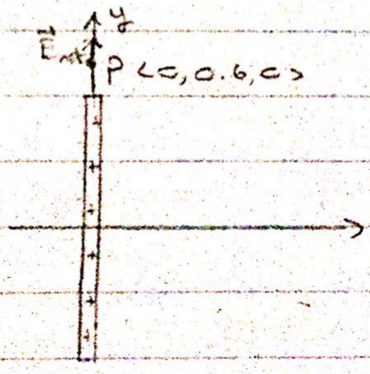
~ Same answer as Ex 2 w/ $n=6 \Rightarrow 1.55 \times 10^5$ N/C (code attached)

Exercise 4: Investigating the Effect of a Smaller Piece size.

N	$\vec{E}_{net}$ (N/C)
6	155193.891
10	174309.696
50	176505.826
100	176504.848
500	176504.535
1000	176504.525

The value of N sufficient to give a result that agrees within 1% of the analytic result from Exercise 1 is actually about $\underline{N=11}$ (177776, 0.72% ...)

Exercise 5. Computing the Electric Field at a Point on the Axis of the Rod.



Analytically:

Using...

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{(Ch. 21)}$$

$$= \frac{(1 \times 10^{-6})}{4\pi\epsilon_0 (0.6)^2}$$

$$= 24.9 \times 10^4 \text{ N/C}$$

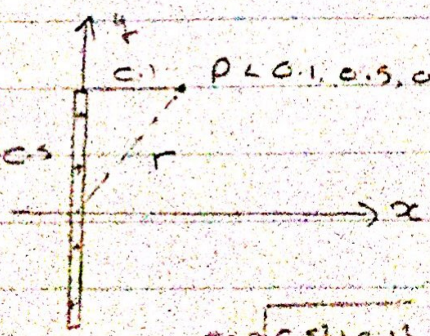
Program:

For $\langle 0, 0.6, 0\rangle$,

$\vec{E} = [0,\ 25 \times 10^4,\ 0]$ N/C

∴ A similar result is achieved using $N=1$ (analytical answer)

Exercise 6: Computing the Electric Field at any Point in Space



$r = \sqrt{0.5^2 + 0.1^2}$

$= 0.5099$

Assuming same parameters * (e.g. $L=1$, $Q=+1\mu C$)

$\Rightarrow$ with $n=6$ (in program)

$$\vec{E}_{net} = [6788.635,\ 33943.177,\ 0] \text{ N/C}$$

$$= [6.79 \times 10^3,\ 3.39 \times 10^4,\ 0] \text{ N/C}$$