

Time complexity of an algorithm signifies the total time required by the program to run till its completion.

The time complexity of algorithms is most commonly expressed using the **big O notation**. It's an asymptotic notation to represent the time complexity. We will study about it in detail in the next tutorial.

Binary Search is great to search through large sorted arrays. It has a time complexity of  **$O(\log n)$**

$O(1)$  time complexity return  $n \times n$

of  $N$ . When  $N$  doubles, the running time increases by  $N \times N$ .

```
while (low <= high)
{
    mid = (low + high) / 2;

    if (target < list[mid])
        high = mid - 1;

    else if (target > list[mid])
        low = mid + 1;

    else break;
}
```

Copy

This is an algorithm to break a set of numbers into halves, to search a particular field (we will study this in detail later). Now, this algorithm

will have a **Logarithmic** Time Complexity. The running time of the algorithm is proportional to the number of times  $N$  can be divided by 2 ( $N$  is high-low here). This is because the algorithm divides the working area in half with each iteration.

```
void quicksort(int list[], int left, int right)
{
    int pivot = partition(list, left, right);
    quicksort(list, left, pivot - 1);
    quicksort(list, pivot + 1, right);
}
```

Copy

Taking the previous algorithm forward, above we have a small logic of [Quick Sort](#) (we will study this in detail later). Now in Quick Sort, we divide the list into halves every time, but we repeat the iteration  $N$  times (where  $N$  is the size of list). Hence time complexity will be  **$N \cdot \log(N)$** . The running time consists of  $N$  loops (iterative or recursive) that are logarithmic, thus the algorithm is a combination of linear and logarithmic.

**NOTE:** In general, doing something with every item in one dimension is linear, doing something with every item in two dimensions is quadratic, and dividing the working area in half is logarithmic.

Big O Notation	Name	Example(s)
$O(1)$	Constant	# <a href="#">Odd or Even number</a> , # <a href="#">Look-up table (on average)</a>
$O(\log n)$	Logarithmic	# <a href="#">Finding element on sorted array with <b>binary search</b></a>
$O(n)$	Linear	# <a href="#">Find max element in unsorted array</a> , # Duplicate elements in array with Hash Map
$O(n \log n)$	Linearithmic	# <a href="#">Sorting elements in array with <b>merge sort</b></a>
$O(n^2)$	Quadratic	# <a href="#">Duplicate elements in array <b>naïve</b></a> , # <a href="#">Sorting array with <b>bubble sort</b></a>
$O(n^3)$	Cubic	# <a href="#">3 variables equation solver</a>
$O(2^n)$	Exponential	# <a href="#">Find all subsets</a>

## Sorting

Algorithm	Data Structure	Time Complexity			Worst Case Auxiliary Space Complexity
		Best	Average	Worst	Worst
Quicksort	Array	$O(n \log(n))$	$O(n \log(n))$	$O(n^2)$	$O(n)$
Mergesort	Array	$O(n \log(n))$	$O(n \log(n))$	$O(n \log(n))$	$O(n)$
Heapsort	Array	$O(n \log(n))$	$O(n \log(n))$	$O(n \log(n))$	$O(1)$
Bubble Sort	Array	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$
Insertion Sort	Array	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$
Select Sort	Array	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$
Bucket Sort	Array	$O(n+k)$	$O(n+k)$	$O(n^2)$	$O(nk)$
Radix Sort	Array	$O(nk)$	$O(nk)$	$O(nk)$	$O(n+k)$

*priority\_queue<data\_type, vector<data\_type>, greater<data\_type>> Q*

The table containing the time and space complexity with different functions given below:

Function	Time Complexity	Space Complexity
Q.top()	$O(1)$	$O(1)$
Q.push()	$O(\log n)$	$O(1)$
Q.pop()	$O(\log n)$	$O(1)$
Q.empty()	$O(1)$	$O(1)$

# STL Containers

## Containers library

### 1. Sequence containers

Sequence containers implement data structures which can be accessed sequentially.

1. array : (C++11) static contiguous array
2. vector : dynamic contiguous array
3. deque : double-ended queue
4. forward\_list (C++11) : singly-linked list
5. list : doubly-linked list

### 2. Associative containers

Associative containers implement sorted data structures that can be quickly searched ( $O(\log n)$  complexity).

1. set : collection of **unique keys**, sorted by keys
2. map : collection of key-value pairs, sorted by keys, **keys are unique**
3. multiset : collection of keys, sorted by keys
4. multimap : collection of key-value pairs, sorted by keys

### 3. Unordered associative containers

Unordered associative containers implement unsorted (hashed) data structures that can be quickly searched ( $O(1)$  amortized,  $O(n)$  worst-case complexity).

1. unordered\_set : collection of unique keys, hashed by keys
2. unordered\_map : collection of key-value pairs, hashed by keys, keys are unique
3. unordered\_multiset : collection of keys, hashed by keys
4. unordered\_multimap : collection of key-value pairs, hashed by keys

### 4. Container adaptors

Container adaptors provide a different interface for sequential containers.

1. stack
2. queue
3. priority\_queue

Container	Insertion	Access	Erase	Find	Persistent Iterators
vector / string	Back: $O(1)$ or $O(n)$ Other: $O(n)$	$O(1)$	Back: $O(1)$ Other: $O(n)$	Sorted: $O(\log n)$ Other: $O(n)$	No
deque	Back/Front: $O(1)$ Other: $O(n)$	$O(1)$	Back/Front: $O(1)$ Other: $O(n)$	Sorted: $O(\log n)$ Other: $O(n)$	Pointers only
list / forward_list	Back/Front: $O(1)$ With iterator: $O(1)$ Index: $O(n)$	Back/Front: $O(1)$ With iterator: $O(1)$ Index: $O(n)$	Back/Front: $O(1)$ With iterator: $O(1)$ Index: $O(n)$	$O(n)$	Yes
set / map	$O(\log n)$	-	$O(\log n)$	$O(\log n)$	Yes
unordered_set / unordered_map	$O(1)$ or $O(n)$	$O(1)$ or $O(n)$	$O(1)$ or $O(n)$	$O(1)$ or $O(n)$	Pointers only
priority_queue	$O(\log n)$	$O(1)$	$O(\log n)$	-	-

The time complexity, in Big O notation, for each function:

## Determining complexity for recursive functions (Big O notation)

---

```
int recursiveFun1(int n)
{
    if (n <= 0)
        return 1;
    else
        return 1 + recursiveFun1(n-1);
}
```

This function is being called recursively  $n$  times before reaching the base case so its  $O(n)$ , often called **linear**.

---

```
int recursiveFun2(int n)
{
    if (n <= 0)
        return 1;
```

```
    else
        return 1 + recursiveFun2(n-5);
}
```

This function is called  $n-5$  for each time, so we deduct five from  $n$  before calling the function, but  $n-5$  is also  $O(n)$ . (Actually called order of  $n/5$  times. And,  $O(n/5) = O(n)$  ).

---

```
int recursiveFun3(int n)
{
    if (n <= 0)
        return 1;
    else
        return 1 + recursiveFun3(n/5);
}
```

This function is  $\log(n)$  base 5, for every time we divide by 5 before calling the function so its  $O(\log(n))$ (base 5), often called **logarithmic** and most often Big O notation and complexity analysis uses base 2.

---

```
void recursiveFun4(int n, int m, int o)
{
    if (n <= 0)
    {
        printf("%d, %d\n", m, o);
    }
    else
    {
        recursiveFun4(n-1, m+1, o);
        recursiveFun4(n-1, m, o+1);
    }
}
```

Here, it's  $O(2^n)$ , or **exponential**, since each function call calls itself twice unless it has been recursed  $n$  times.

---

```
int recursiveFun5(int n)
{
    for (i = 0; i < n; i += 2) {
        // do something
    }

    if (n <= 0)
        return 1;
    else
        return 1 + recursiveFun5(n-5);
}
```

And here the for loop takes  $n/2$  since we're increasing by 2, and the recursion takes  $n/5$  and since the for loop is called recursively, therefore, the time complexity is in

$$(n/5) * (n/2) = n^2/10,$$

due to Asymptotic behavior and worst-case scenario considerations or the upper bound that big O is striving for, we are only interested in the largest term so  $O(n^2)$ .