Exercise Sheet 3 - Fan-Beam Reconstruction

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In this exercise, we implement a fan-beam filtered back-projection algorithm and test it with the phantom that we created in Exercise 1.

1. **Fan-Beam Projection:** Implement a ray-driven fan-beam projector for a linear-detector and use it to create a "fan-o-gram", $g(t, \beta)$, of your phantom. Be aware that in this case the angular spacing is not equidistant. The opening angle of the fan is determined by the maximum detector size $t_{\max}$ and the source-to-detector distance $\mathrm{d_{SD}}$. $\mathrm{d_{SI}}$ is the source-to-isocenter distance. Remember that the projections have to be generated using a certain angular range that is provided to the projection function.

   Task: Create and implement the method
   `create_fanogram(phantom,number_of_projections, detector_spacing,detector_sizeInPixels,angular_increment, d_si,d_sd)`

   In contrast to parallel geometry, the last two parameters have a methodical impact on the result. Ensure that during rotation the source and the detector will not hit the phantom.

   Further, note that the angular scan range is defined by your geometry! Determine the minimum fan-beam scan range and create a minimum "fan-o-gram" from your phantom. Recall that **each point** of your phantom needs to have at least 180° coverage. This is known as **Short Scan**.

2. **Rebinning:** Recall the rebinning formulation presented during the lecture. Implement the rebinning and use it to convert your "fan-o-gram", $g(t, \beta)$, into a parallel sinogram, $p(s, \theta)$. Remember that interpolation is required in order to read out the values from the "fan-o-gram".

   Hint: In a full 360° "fan-o-gram" each ray has been sampled twice, building a pair of two values that need to be the same. If your lookup in the short-scan "fan-o-gram" goes out of range, try to redirect the lookup to the corresponding sample in the range.

   Task: Create and implement the method
   `rebinning(fanogram,d_si,d_sd)`

3. **Filtered Back-Projection (FBP):** Reconstruct your phantom from the parallel sinogram (= rebinned fanogram) that you generated in task 2. Use the filtered back-projection algorithm that you implemented in Exercise 2.

◇ **Fan-Beam Reconstruction:** Implement a fan-beam back-projector and cosine weights for a fan-beam filtered back-projection.
Which differences do you observe compared to the rebinning approach?