Classical Mechanics(H1) (SC1.102)

IIIT-H, Semester Winter 24, Quiz 1

Full Marks: 30, Duration: 45min, January 30 2024

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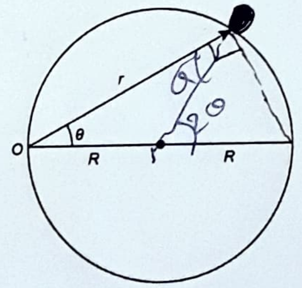
1. A simple pendulum of mass m and massless wire of length ℓ is hanged from a block of mass M. The block M is kept on an horizontal frictionless surface. How many degrees of freedom do the system have? Write down the Lagrangian of the system assuming gravitational acceleration g. Show that for small amplitude oscillation (in vertical plane) the time period of the pendulum is [10]

$$T = 2\pi\sqrt{\frac{\ell}{g}}\sqrt{\frac{M}{M+m}}.$$

2. A particle of mass m moves in a circular orbit of radius R (as shown in the figure below) under the influence of an attractive central force that is directed to a fixed point O situated on the circle. Show that the force varies as the inverse fifth power of the distance r from O. [10]



3. An object is in motion under a central force

$$F(r) = -\frac{3a}{r^4},$$

Draw the effective potential $V'(r)$ as a function of r and indicate the $r_{\min}$ and $r_{\max}$ of the motion. Draw the trajectories of the particle (in $r-\theta$ plane) in the following scenarios *i)* the motion starts at a distance $r > r_{\max}$, and *ii)* it starts at a distance $r < r_{\min}$. Is motion allowed for $r_{\min} < r < r_{\max}$? [10]

$\frac{1}{2} m \dot{r}^2 + \frac{1}{2}\frac{l^2}{m r^2}$

$\frac{1}{2} m r^2 \dot{\theta}^4 + \frac{1}{2}\frac{l^2}{m r^2}\dot{\theta}^2$

$m \dot{r}^2 \ddot{\theta} + \frac{l^2}{m r^2}\dot{\theta}$

Classical Mechanics(H1) (SC1.102)

IIIT-H, Semester Winter 24, Midterm Exam

10 questions, Full Marks: 50, Duration: 120 minutes

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1. There is a system of three masses m_1, m_2 and m_3. The distance between m_1 and m_2 is a constant ℓ_{12}, and the distance between m_2 and m_3 is a constant ℓ_{23}. The distance between m_1 and m_3 can vary.

   (a) How many constraints does the system have?

   (b) How many degrees of freedom will the system have if it *i)* moves in a two-dimensional flat surface, *ii)* in three dimensional space, and *iii)* in a hypothetical N dimensional space. [1+1+1+1]

2. Consider a system with the following Lagrangian

$$L = \frac{1}{2}m(\dot{x}_1^2 + \dot{x}_2^2) + m\dot{x}_1\dot{x}_2 - \frac{1}{2}k(x_1^2 + x_2^2) - kx_1x_2 .$$

   where $\dot{x}_1$ indicates time derivative of x_1. How many degrees of freedom does the system have ? [2]

3. Using calculus of variation show that the shortest distance between two points in a flat surface is a straight line. *Hint: Consider the flat surface as the $x-y$ plane. Write the infinitesimal distance between two points $d\ell = \sqrt{dx^2 + dy^2}$. You may take x as the independent variable. Integrating $d\ell$ gives the total distance ℓ. $\delta\ell = 0$ correspond to Euler-Lagrange equation which you need to use.* [7]

4. A stone of mass m is sliding along the x axis such that a force acts on the particle which is proportional to its displacement from the origin ($x = 0$) and is directed towards the origin. Write down the Lagrangian of the system and obtain the equation of motion. *Hint: Obtain the potential from the force.* [6]

5. A tennis ball of mass m is thrown in the sky in a direction making some angle with the vertical direction y. The ball travels in the vertical $x-y$ plane. Using Cartesian coordinate, write down the Lagrangian of the system. Obtain the equations of motion. Show that in the horizontal x-direction the speed is constant. [7]

6. There is a mass point m that is constrained to move on the surface of cylinder of radius R whose axis is along the z direction. The Lagrangian of the system in cylindrical coordinate is

$$L = \frac{1}{2}m(R^2\dot{\theta}^2 + \dot{z}^2) - \frac{1}{2}k(R^2 + z^2),$$

   where k is a constant. Obtain the generalized momentum corresponding to θ and z. Write down the Hamiltonian of the system and obtain the Hamilton's equations of motion. Show that the angular momentum about the z-axis is a constant of motion. Also show that it executes simple harmonic motion along the z direction. [8]

7. A tennis ball is dropped on a surface from a height h. The ball makes a couple of bounces before it comes to a halt. Draw the phase space diagram (with height in the horizontal axis and momentum in the vertical axis.). *Hint: When the ball hits the floor it loses some energy, and the direction of the momentum changes.* [5]

8. Consider the Hamiltonian $H = \frac{1}{2}(p^2 + q^2)$. Determine using Poisson bracket if the transformation $Q = \frac{1}{2}(q^2 + p^2)$ and $P = -\tan^{-1}(q/p)$ is canonical.

[5]

9. Prove that $[F, G + K] = [F, G] + [F, K]$ where F, G, K are functions of generalized coordinates and momentum, and $[.., ..]$ indicates Poisson bracket.

[3]

10. You are sitting on a couch watching Netflix with your cat on your lap. The cat starts walking with a constant velocity in the $+X$ direction with respect to you. Draw a spacetime diagram in your rest frame and show your worldline, and that of the cat. Draw the world line of the cat if it is running with uniform acceleration.

[3]