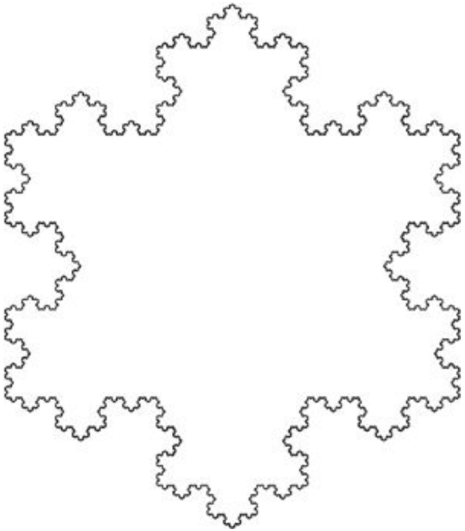
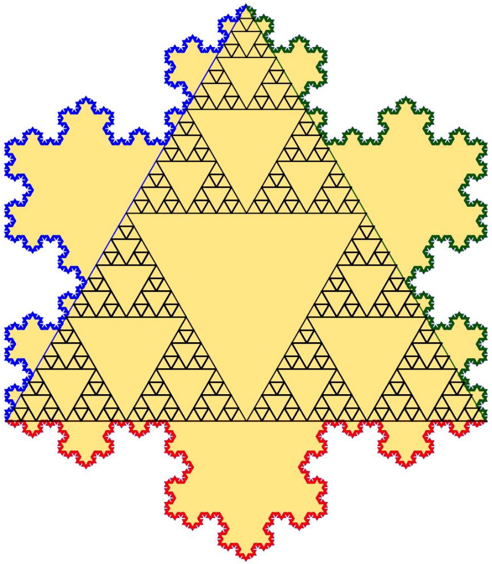


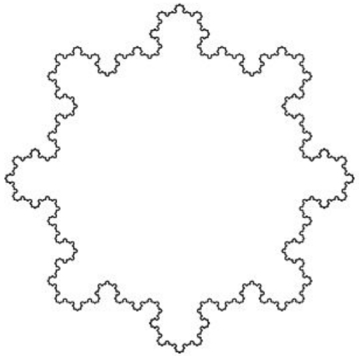
VON KOCH SNOWFLAKE



CAN AN OBJECT HAVE BOTH FINITE AND INFINITE PROPERTIES ?

By Aarjav, Aanya, Anang and Mahek

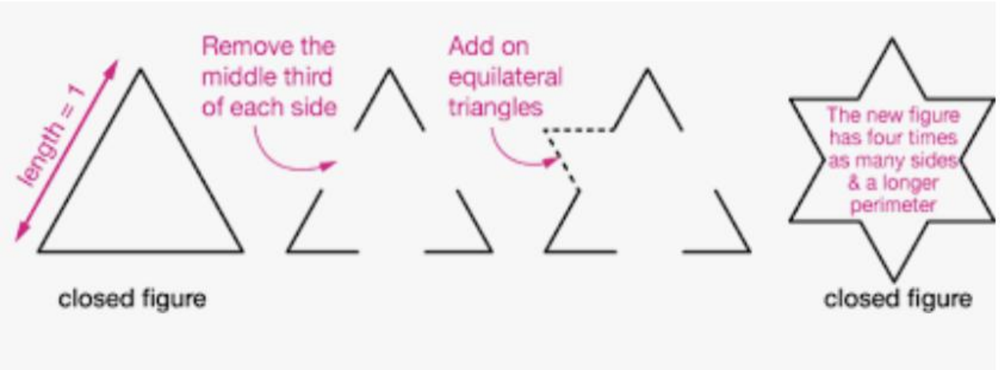
HISTORY OF VON KOCH SNOWFLAKE

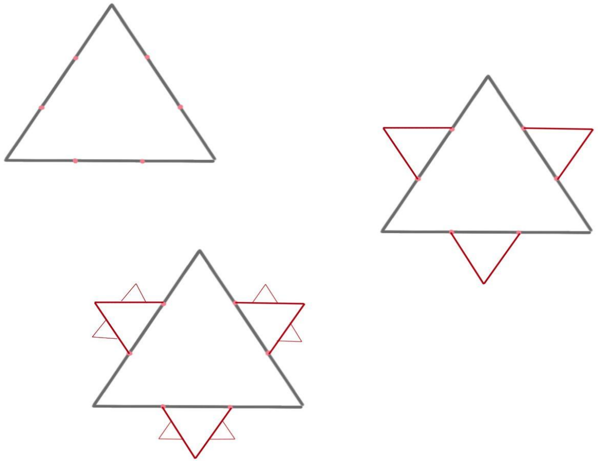


- The Koch Snowflake was created by the Swedish mathematician Niels Fabian Helge von Koch.
- In his 1904 paper he used the Koch Snowflake to show that it is possible to have figures that are continuous everywhere but differentiable nowhere.
- His contributions are still a major part in the study of fractal geometry

THE PHASES OF A SNOWFLAKE

1. Start with an equilateral triangle
2. Divide one side into thrice parts and remove the middle part.
3. Add an equilateral triangle there.
4. Continue this with more iterations.





STEPS TO CREATE A SNOWFLAKE

NTH TERM OF SNOWFLAKE :

3 12 48 192 768

X4 X4 X4 X4

$$a_n = a \cdot r^{n-1}$$

Note: $a = 3$

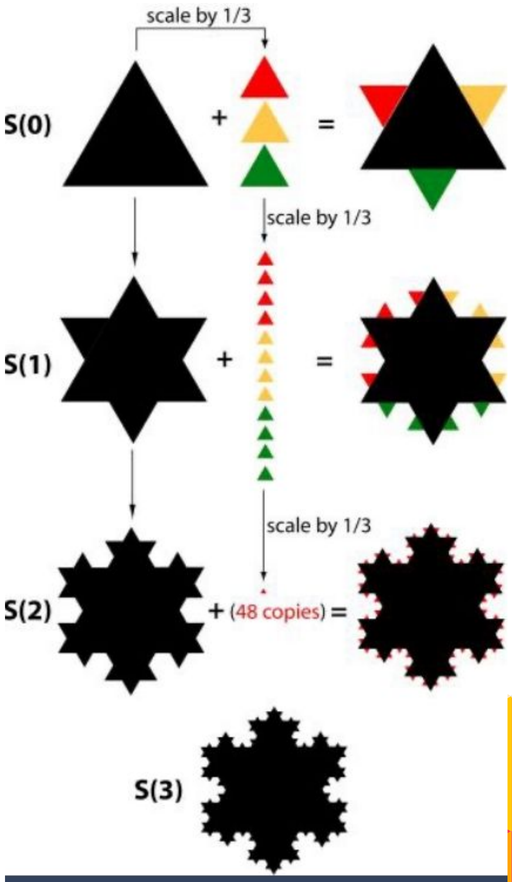
$r = 4$

$$= \frac{a \times r^n}{r}$$

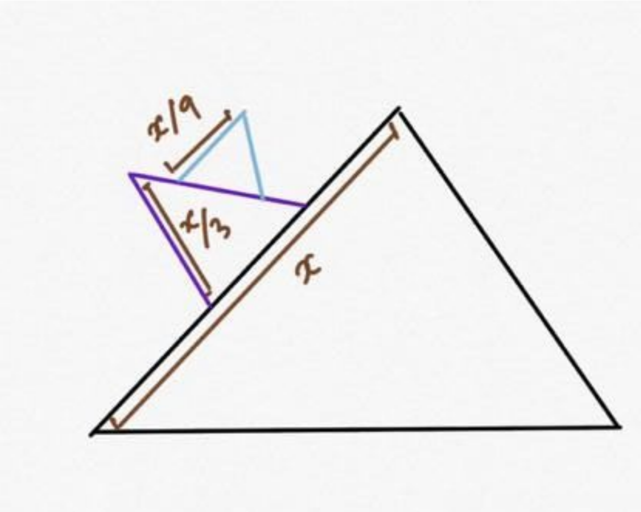
$$= \frac{3 \cdot 4^n}{4}$$

= n^{th} term for number of sides is:

$$\boxed{\left(\frac{3}{4}\right) \times (4)^n}$$



LENGTH



- In every iteration, the length of a side is 1/3 the length of a side from the preceding stage. If we begin with an equilateral triangle with side length x, then the length of a side in iteration " a " is
- length = $x*3^{-a}$
- For iterations 0 to 3, length = a, a/3, a/9 and a/27.

PERIMETER

Perimeter = no. of sides × length

$$P = (3 * 4^n) * (X * 3^{-n})$$

Iteration 1 $\longrightarrow$ $3x$

Iteration 2 $\longrightarrow$ $4x$

Iteration 3 $\longrightarrow$ $\frac{16x}{3}$

IN THE ABOVE SERIES , THE PERIMETER IS INCREASING BY (4/3) TIMES EACH ITERATION

Let p be perimeter

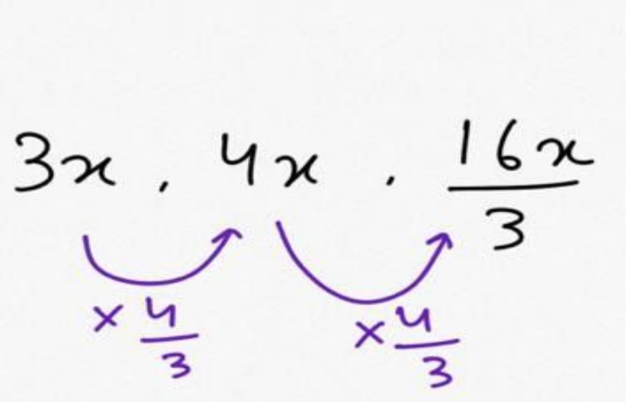
n - iteration

$p = n * \text{length}$

$p = (3*4^{n})*(x*3^{-n})$

IN THE IMAGE GIVEN ALONGSIDE, WE CAN CLEARLY INFER THAT A GEOMETRIC PROGRESSION IS FORMED.

PERIMETER



OBSERVATION:

The perimeter increases by 4/3 with each iteration. Why?

1. This is because the length of each side decreases by ⅓ times each iteration .
2. On the other hand , the number of sides is being increased by 4 times the previous value each iteration.

Also , here n is an infinite number , thus making the perimeter of the snowflake an infinite quantity.

AREA

At each iteration , side becomes ⅓ times the original side and the number of sides

AREA OF EQUILATERAL TRIANGLE

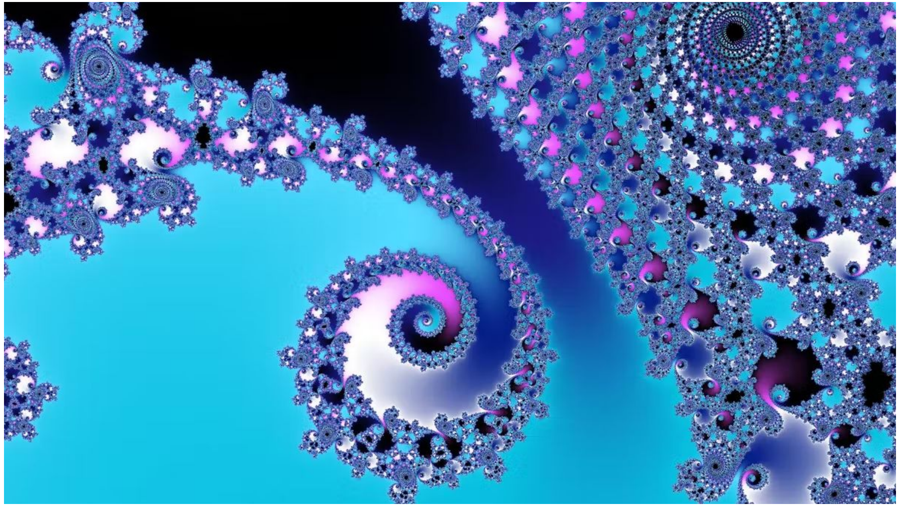
Formula $= \frac{\sqrt{3}}{4} a^2$ [a is side]

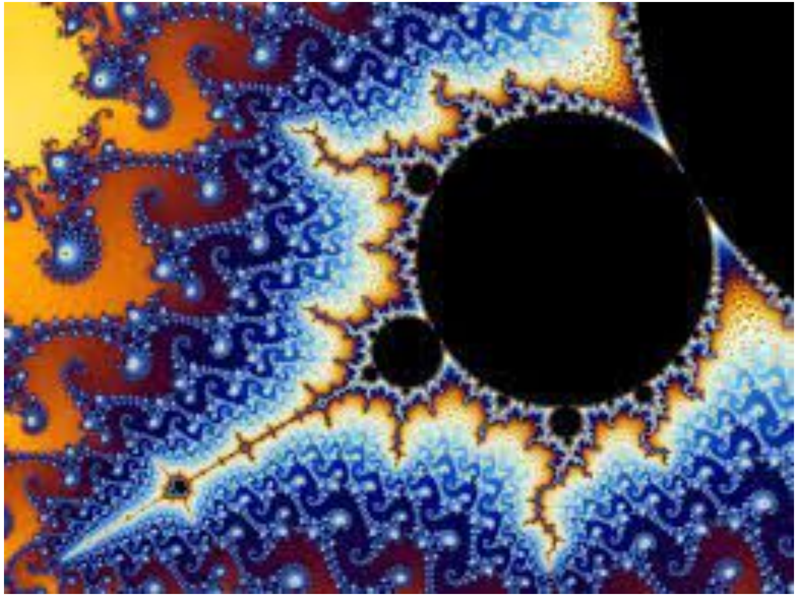
$$\therefore \text{Area} : \underbrace{\frac{\sqrt{3}}{4} a^2}_{1^{st}\text{ iteration}} + \underbrace{\frac{3\sqrt{3}\left(\frac{a}{3}\right)^2}{4}}_{2^{nd}\text{ iteration}} + \underbrace{\frac{12\sqrt{3}\left(\frac{a}{9}\right)^2}{4}}_{3^{rd}\text{ iteration}} \dots\dots$$

$$\therefore \text{Area} : \frac{1}{4} \frac{\sqrt{3}a^2}{4} \left(4 + 3(4)\left(\frac{1}{3}\right)^2 + 3(4)^2\left(\frac{1}{3^2}\right)^2 \dots \right)$$

WHAT IS A FRACTAL ?

- A never ending pattern
- Scaling symmetry
- Might be derived from specification or set

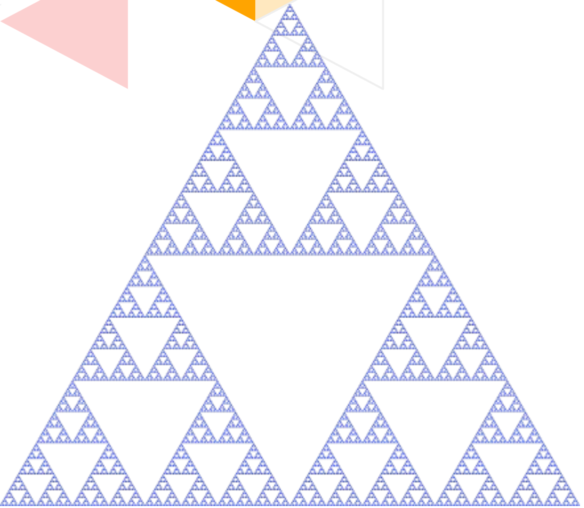


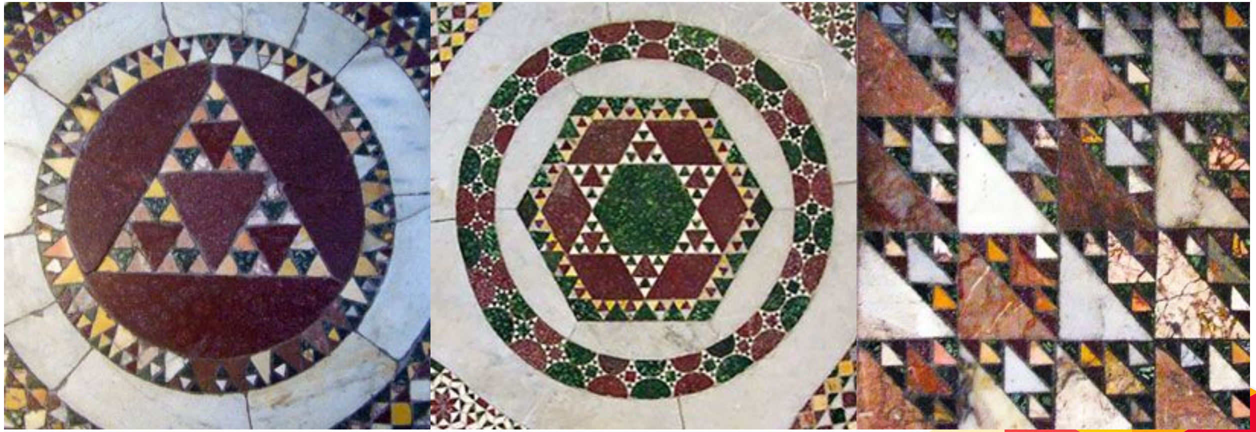


SIERPINSKI TRIANGLE :

The Sierpinski triangle may be constructed from an equilateral triangle by repeated removal of triangular subsets:

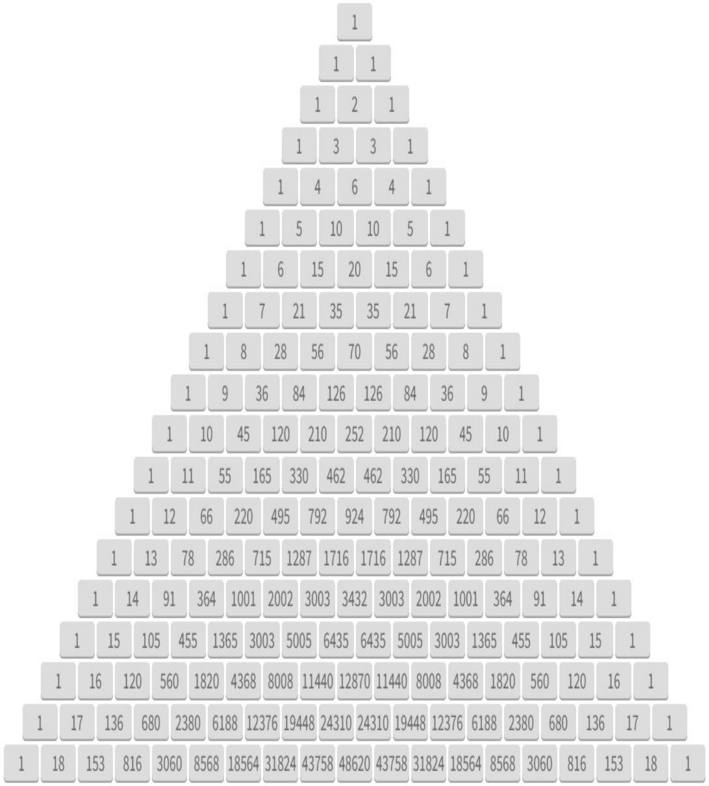
1. Start with an equilateral triangle.
2. Subdivide it into four smaller congruent equilateral triangles and remove the central triangle.
3. Repeat step 2 with each of the remaining smaller triangles infinitely.

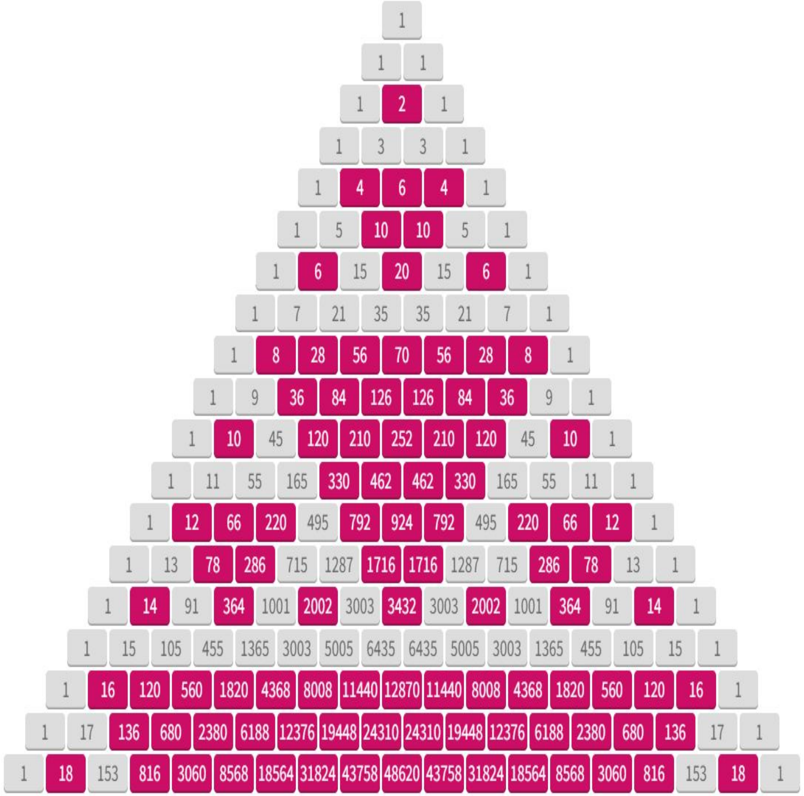




Floor tilings from different churches in Rome

PATTERNS IN THE SIERPINSKI TRIANGLE(AS A PASCAL'S TRIANGLE)

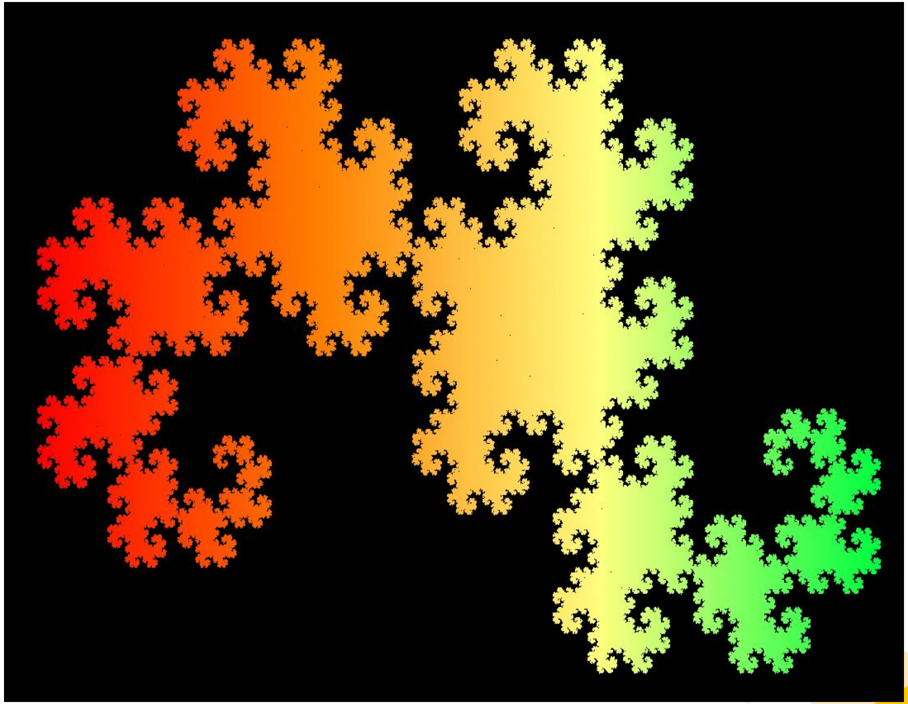




HEIGHWAY DRAGON

The Heighway dragon was discovered by John Heighway in 1966 and named by William Harter. It is recursive in nature.

Here’s how the heighway dragon unfolds:
Simulation



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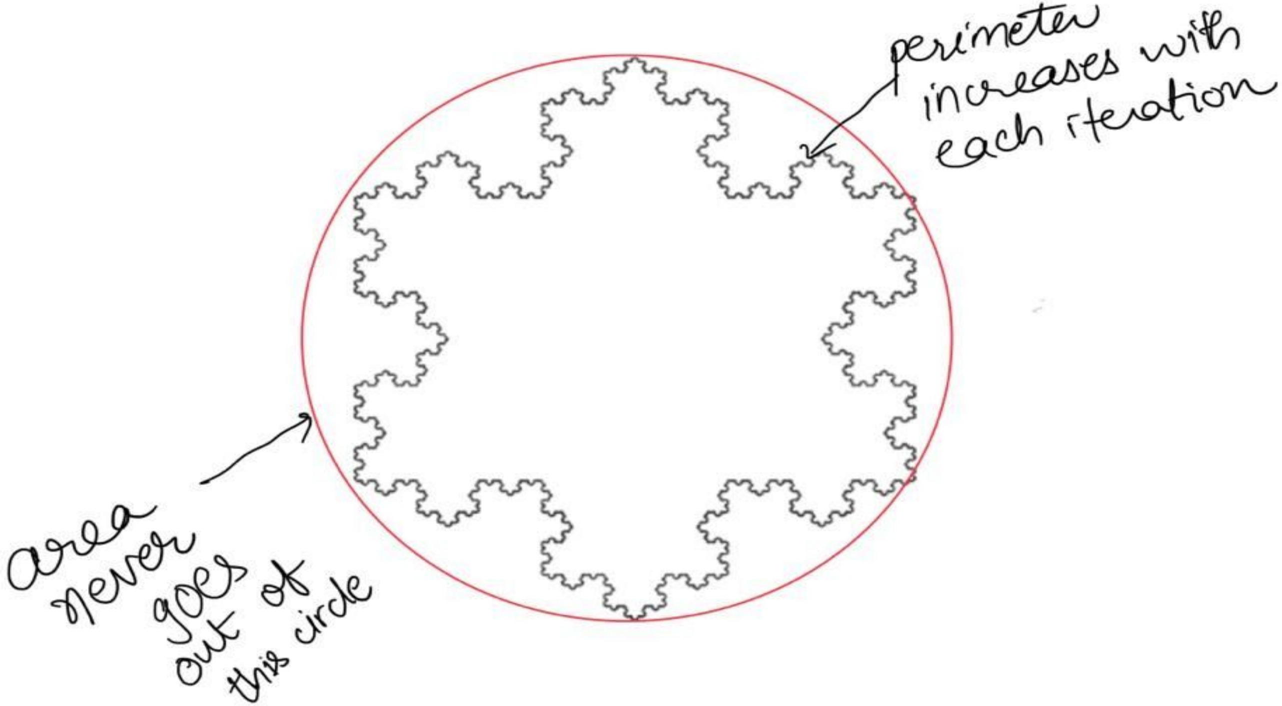
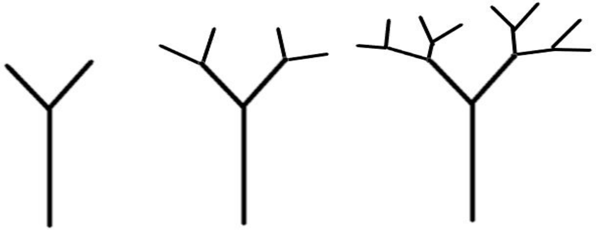
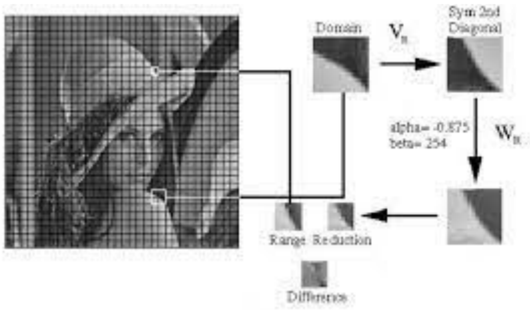


REAL-LIFE EXAMPLES

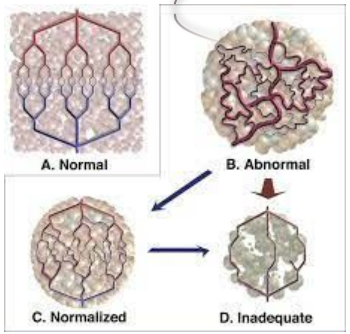


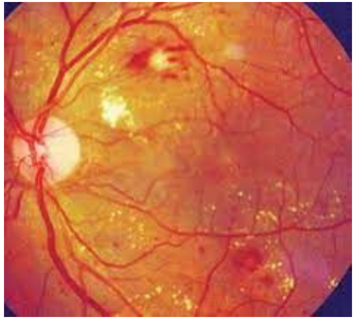
Fractals found in nature(leaves and trees)

Fractal cities



Fractals in photography





Fractal medicines

REFLECTION

Learner profiles addressed:

1. Thinker
2. Knowledgeable
3. Risk takers
4. Communicators
5. Reflective

“Study the science of art. Study the art of science.
Develop your senses – especially learn how to see.
Realize that everything connects to everything else.”

— Leonardo da Vinci

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THANK YOU