EE23010 NCERT Exemplar

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Let X_1, X_2, X_3....,X_n be a random sample of size $n\,(\geq 2)$ from a population having probability density function

$$f\,(x;\theta) = \begin{cases} \frac{2}{\theta x}\,(\log_e x)\,e^{-\frac{(\log_e x)^2}{\theta}} & ,0 < x < 1 \\ 0 & ,otherwise \end{cases}$$

where $\theta > 0$ is an unknown parameter. The maximum likelihood estimator of θ is,
Solution:
The likelihood function of θ in $0 < x < 1$ is given by,

$$L\,(\theta; X_1, X_2, ..., X_n) = \prod_{i=1}^{n} \frac{2}{(\theta x_i)}(-\log_e x_i)e^{\frac{-(\log_e x_i)^2}{\theta}} \tag{1}$$

Taking the natural log on both sides,

$$\ln\,[L(\theta; X_1, X_2, ..., X_n)] = n\ln 2 - n\ln\theta + \sum_{i=1}^{n}\ln\,(-\ln x_i) - \sum_{i=1}^{n}(\ln X_i) - \sum_{i=1}^{n}\frac{(\log_e X_i)^2}{\theta} \tag{2}$$

To find the maximum likelihood estimator of θ, we need to maximize the likelihood function. Therefore,

$$\frac{\partial L\,(\theta; X_1, X_2, ..., X_n)}{\partial\theta} = 0 \tag{3}$$

$$\frac{-n}{\theta} + \frac{1}{(\theta)^2}\sum_{i=1}^{n}(\log_e X_i)^2 = 0 \tag{4}$$

$$\theta = \frac{1}{n}\sum_{i=1}^{n}(\log_e X_i)^2 \tag{5}$$