

EE23010 NCERT Exemplar

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Question 63.2022

Consider a channel over which either symbol x_A or symbol x_B is transmitted. Let the output of the channel Y be the input to a maximum likelihood (ML) detector at the receiver. The conditional probability density for Y given x_A and x_B are:

$$f_{Y|x_A}(y) = e^{-(y+1)} u(y+1)$$

$$f_{Y|x_B}(y) = e^{(y-1)} (1 - u(y-1))$$

where $u(\cdot)$ is the standard unit step function. the probability of symbol error for this system is

Solution:

Decision in favour of x_A when

$$f_{Y|x_A}(y) > f_{Y|x_B}(y) \quad (1)$$

Decision in favour of x_B when

$$f_{Y|x_A}(y) < f_{Y|x_B}(y) \quad (2)$$

From the figure,

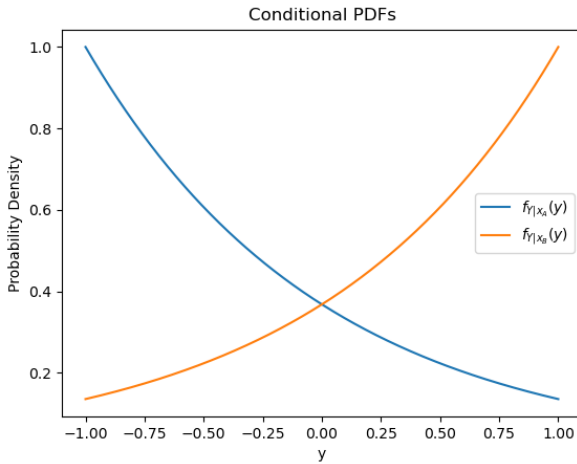


Fig. 1. Conditional pdf

$$\begin{cases} f_{Y|x_A}(y) < f_{Y|x_B}(y) & , y < -1 \\ f_{Y|x_A}(y) > f_{Y|x_B}(y) & , -1 < y < 0 \\ f_{Y|x_A}(y) < f_{Y|x_B}(y) & , 0 < y < 1 \\ f_{Y|x_A}(y) > f_{Y|x_B}(y) & , y > 1 \end{cases} \quad (3)$$

1) When $0 < y < 1$.

In this interval, when x_A is transmitted, error occurs because the likelihood of observing Y given x_A is lower than the likelihood of observing Y given x_B . Therefore,

$$P_{e_{x_A}} = \int_0^1 f_{Y|x_A}(y) dy \quad (4)$$

$$= \int_0^1 e^{-(y+1)} u(y+1) dy \quad (5)$$

$$= \int_0^1 e^{-(y+1)} dy \quad (6)$$

$$= e^{-1} - e^{-2} \quad (7)$$

$$P_{e_{x_A}} = 0.23 \quad (8)$$

2) When $-1 < y < 0$.

In this interval, when x_B is transmitted, error occurs because the likelihood of observing Y given x_A is higher than the likelihood of observing Y given x_B . Therefore,

$$P_{e_{x_B}} = \int_{-1}^0 f_{Y|x_B}(y) dy \quad (9)$$

$$= \int_{-1}^0 e^{(y-1)} (1 - u(y-1)) dy \quad (10)$$

$$= \int_{-1}^0 e^{(y-1)} dy \quad (11)$$

$$= e^{-1} - e^{-2} \quad (12)$$

$$P_{e_{x_B}} = 0.23 \quad (13)$$

3) When $y < -1$ and $y > 1$.

There are no errors in these intervals as the ML detectors can more reliably to make a decision. Therefore,

$$P_e = 0 \quad (14)$$

Hence, the total probability of error for this system can be given as,

$$P_e = \Pr(x_A) P_{e_{x_A}} + \Pr(x_B) P_{e_{x_B}} \quad (15)$$

$$= 0.23 \times (\Pr(x_A) + \Pr(x_B)) \quad (16)$$

$$= 0.23 \quad (17)$$