EE23010 NCERT Exemplar

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Question 63.2022
Consider a channel over which either symbol x_A or symbol x_B is transmitted. Let the output of the channel Y be the input to a maximum likelihood (ML) detector at the receiver. The conditional probability density for Y given x_A and x_B are:

$$f_{Y|x_A}(y) = e^{-(y+1)}u(y+1)$$
$$f_{Y|x_B}(y) = e^{(y-1)}(1-u(y-1))$$

where $u(.)$ is the standard unit step function. the probability of symbol error for this system is

Solution:
Decision in favour of x_A when

$$f_{Y|x_A}(y) > f_{Y|x_B}(y) \tag{1}$$

Decision in favour of x_B when

$$f_{Y|x_A}(y) < f_{Y|x_B}(y) \tag{2}$$

From the figure,

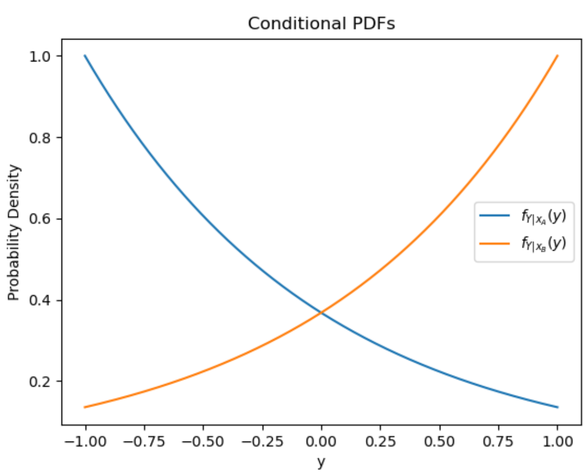

Fig. 1. Conditional pdf

$$\begin{cases} f_{Y|x_A}(y) < f_{Y|x_B}(y) & , y < -1 \\ f_{Y|x_A}(y) > f_{Y|x_B}(y) & , -1 < y < 0 \\ f_{Y|x_A}(y) < f_{Y|x_B}(y) & , 0 < y < 1 \\ f_{Y|x_A}(y) > f_{Y|x_B}(y) & , y > 1 \end{cases} \tag{3}$$

Probability of error when x_A is transmitted:
When x_A is transmitted, error occurs in $0 < y < 1$ because, in this interval the likelihood of observing Y given x_A is lower than the likelihood of observing Y given x_B, Therefore,

$$P_{e_{x_A}} = \int_0^1 f_{Y|x_A}(y)\,dy \tag{4}$$
$$= \int_0^1 e^{-(y+1)}u(y+1)\,dy \tag{5}$$
$$= \int_0^1 e^{-(y+1)}dy \tag{6}$$
$$= e^{-1} - e^{-2} \tag{7}$$
$$P_{e_{x_A}} = 0.23 \tag{8}$$

Probability of error when x_B is transmitted:
When x_B is transmitted, error occurs in $-1 < y < 0$ because, in this interval the likelihood of observing Y given x_A is higher than the likelihood of observing Y given x_B, Therefore,

$$P_{e_{x_B}} = \int_{-1}^0 f_{Y|x_B}(y)\,dy \tag{9}$$
$$= \int_{-1}^0 e^{(y-1)}(1-u(y-1)) \tag{10}$$
$$= \int_{-1}^0 e^{(y-1)} \tag{11}$$
$$= e^{-1} - e^{-2} \tag{12}$$
$$P_{e_{x_B}} = 0.23 \tag{13}$$

Outside of these intervals, the ML detector can more reliably make a decision.
The probability of error can be given as

$$P_e = \Pr(x_A)P_{e_{x_A}} + \Pr(x_B)P_{e_{x_B}} \tag{14}$$
$$= 0.23 \times (\Pr(x_A) + \Pr(x_B)) \tag{15}$$
$$= 0.23 \tag{16}$$