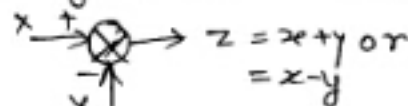


⑦  
Block diagram :- A block diagram of the system is a pictorial representation of the entire system. It represents the relationship between the input and the output of the entire system. Different blocks are interconnected to each other as per the sequence of operation. By means of block diagram, we can be easily represent of complicated system.

Block diagram :- It is a pictorial representation of the cause and effect relationship between input and output of the system.

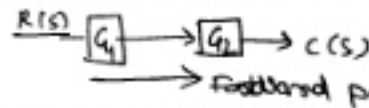
Output :-  $\text{output} = \text{Gain} \times \text{Input}$ . The value at the input is multiplied to the value of block gain to get output.

Summing point :- More than one signal can be added or subtracted at summing point.

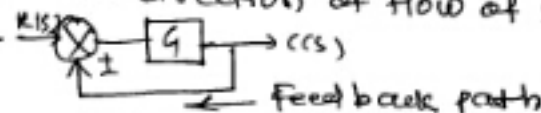


Take off point :- The point from which a signal is taken for the feedback purpose is called take off point.

Forward path :- The direction of flow of signal is from input to output.

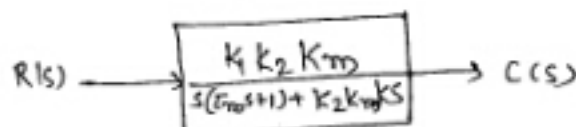
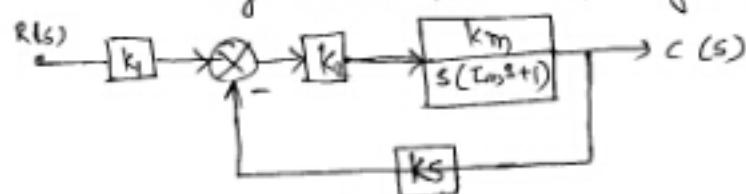


Feedback path :- The direction of flow of signal is from output to input.

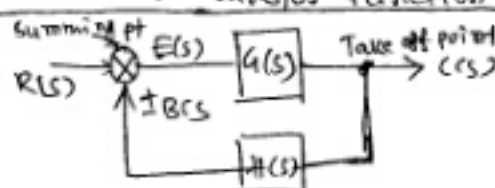


\* Block diagram Reduction :- It is necessary to simplify the block diagram to a single block or equivalently, to find the overall transfer function, so that the system behaviour can be studied. Reducing a block diagram to a single block representing the transfer function of the entire control system is known as block diagram.

Example :- considering the simple block diagram of s/m.



\* closed loop Transfer function :- consider



$R(s)$  = Laplace of input signal  $r(t)$

$E(s)$  = Laplace of error signal

$B(s)$  = Laplace of feedback

$G(s)$  = forward transfer function

$H(s)$  = Feedback transfer function

$$\therefore C(s) = E(s) G(s)$$

$$\therefore E(s) = R(s) \pm B(s)$$

$$\therefore C(s) = [R(s) \pm B(s)] G(s)$$

$$\therefore C(s) = R(s) G(s) \pm B(s) G(s)$$

$$\text{Now } B(s) = C(s) \cdot H(s)$$

$$\therefore C(s) = R(s) G(s) \pm C(s) \cdot H(s) G(s)$$

$$\therefore C(s) \mp C(s) \cdot H(s) \cdot G(s) = R(s) G(s)$$

$$\therefore C(s) [1 \mp G(s) H(s)] = R(s) G(s)$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s) H(s)}$$

If the system is a negative feedback

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)}$$

If the system is a positive feedback

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s) H(s)}$$

open loop transfer function is given by

$$G(s) \cdot H(s)$$



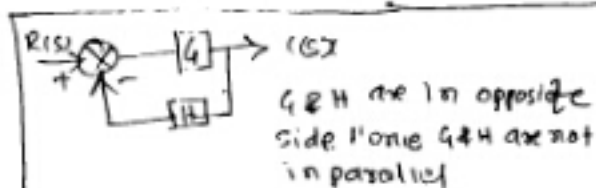
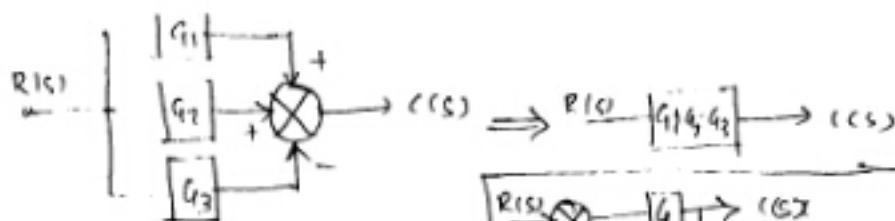
\* Rules for block diagram reduction :

(8)

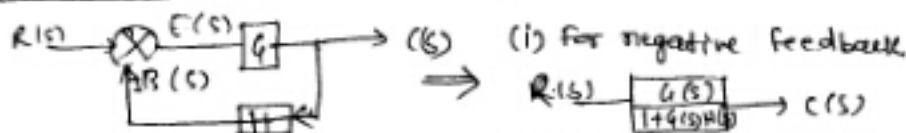
1) Blocks in series or cascade

$$R(s) \xrightarrow{G_1} \xrightarrow{G_2} \dots \xrightarrow{G_n} C(s) \Rightarrow R(s) \xrightarrow{G_1 G_2 \dots G_n} C(s)$$

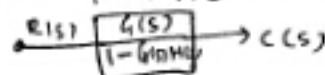
2) Blocks in parallel :-



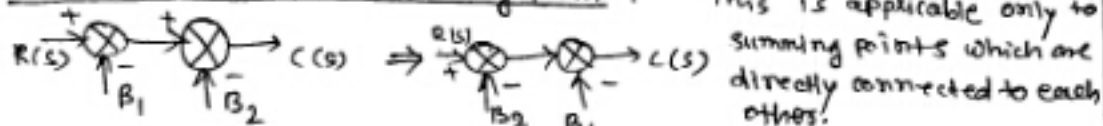
3) Eliminate feedback loop :-



(ii) for positive feedback

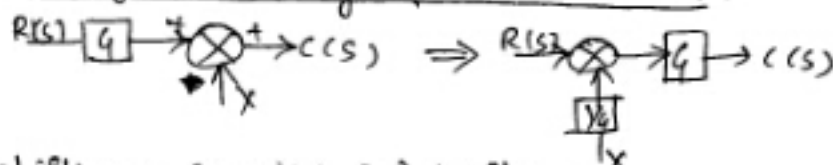


4) Associative law for summing point :-

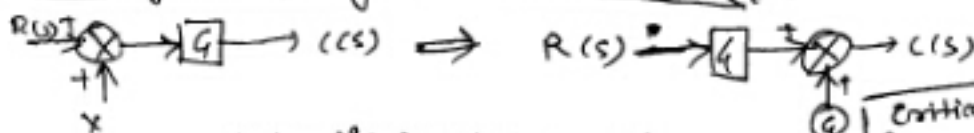


This is applicable only to summing points which are directly connected to each other.

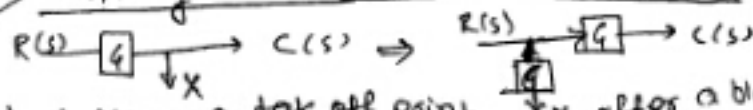
5) Shifting a summing pt before a block :-



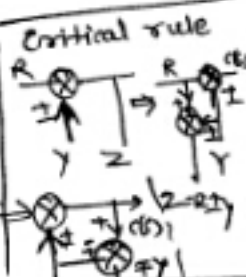
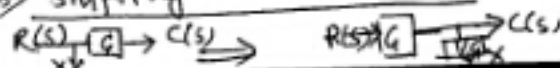
6) Shifting a summing point after a block :-



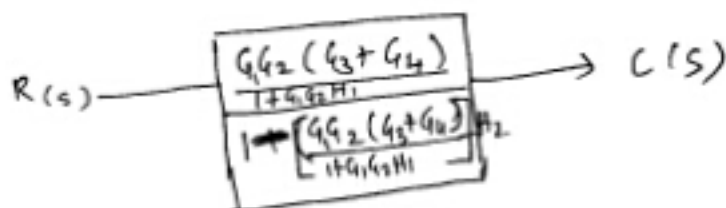
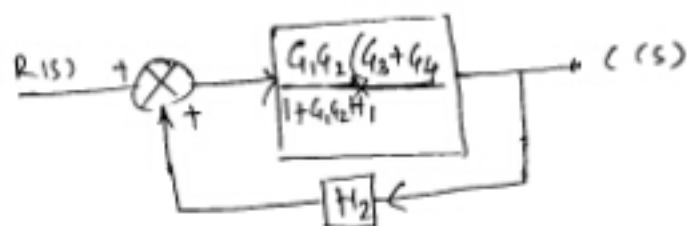
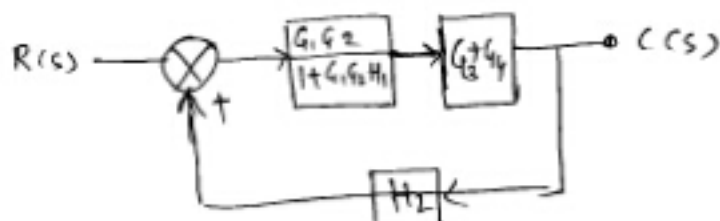
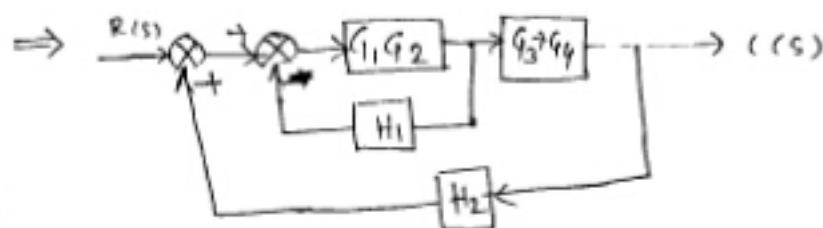
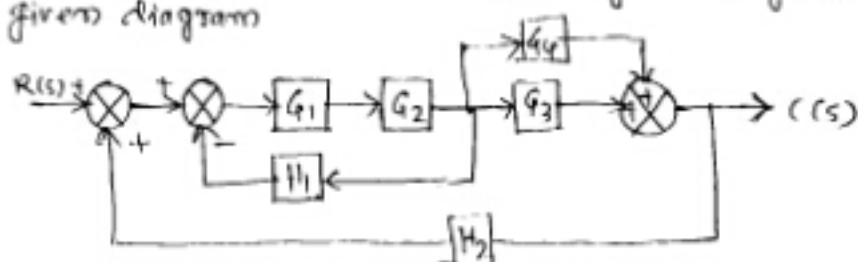
7) Shifting a take off point before a block :-



8) Shifting a take off point

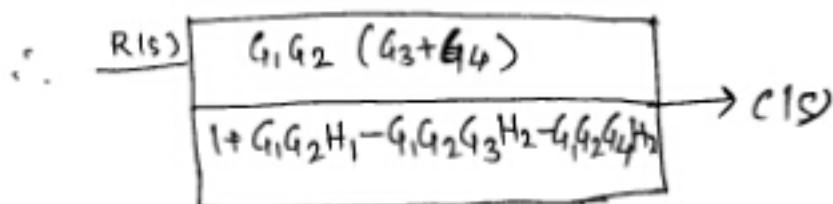


\* Find the single block equivalent by block diagram reduction for given diagram

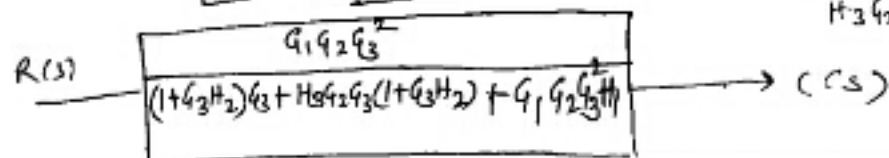
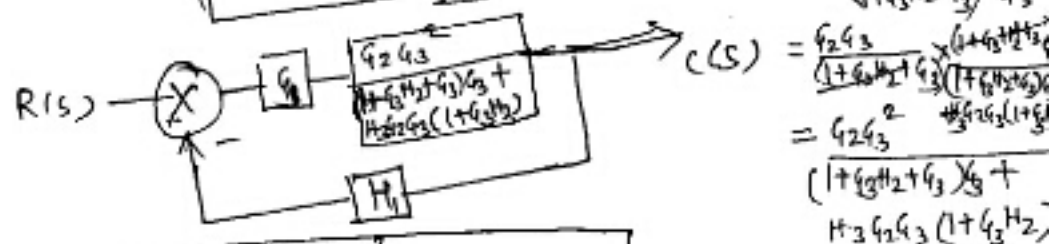
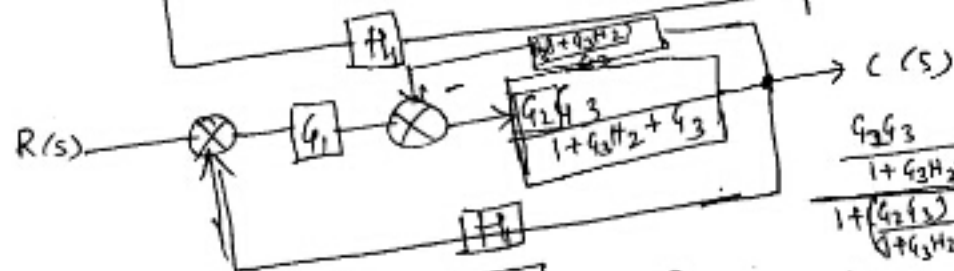
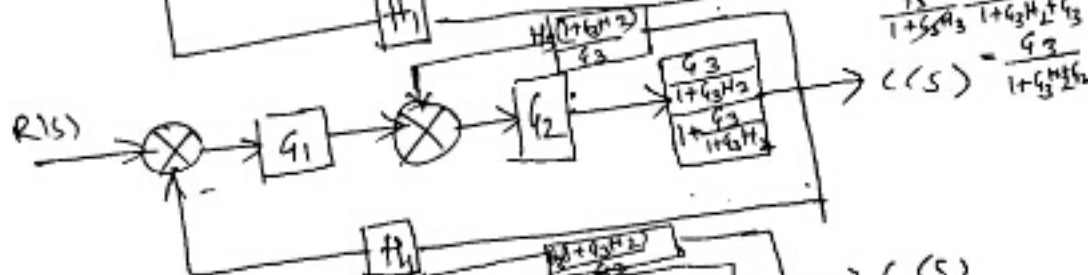
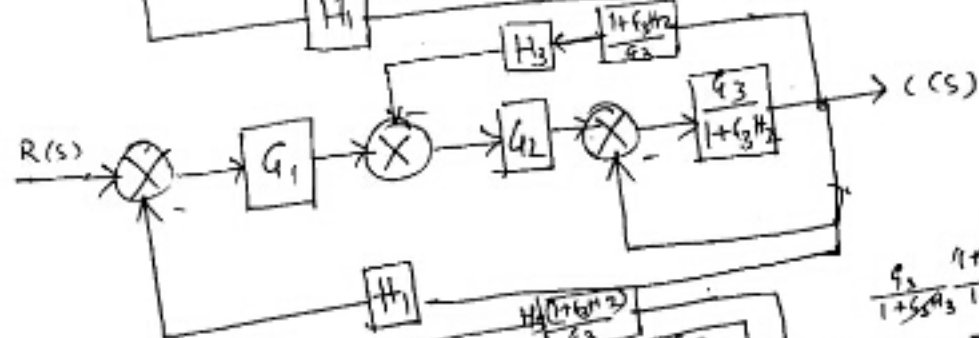
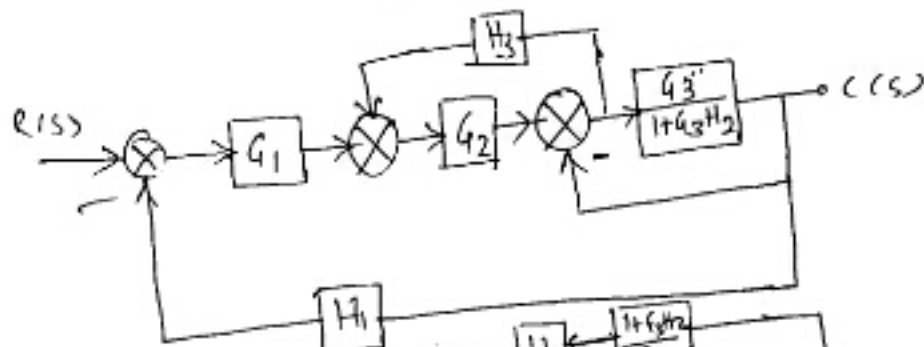
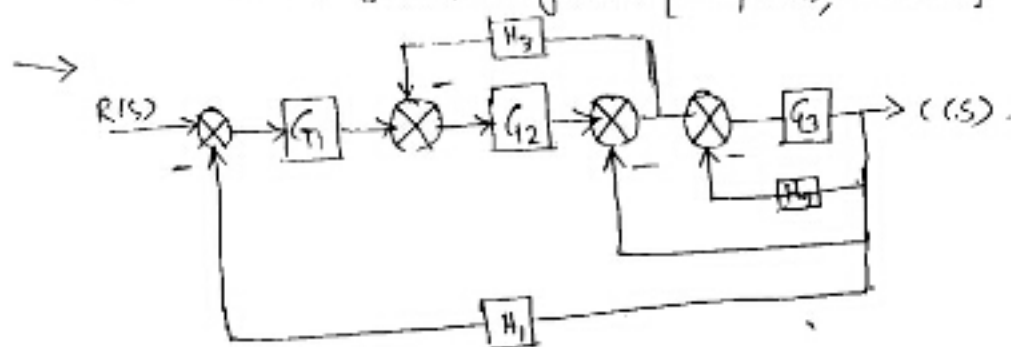


$$= \frac{G_1 G_2 (G_3 + G_4)}{1 + G_1 G_2 H_1 - G_1 G_2 (G_3 + G_4) H_2}$$

$$= \frac{G_1 G_2 (G_3 + G_4)}{1 + G_1 G_2 H_1 - G_1 G_2 G_3 H_2 - G_1 G_2 G_4 H_2}$$



\* Find the single transfer function by block diagram reduction of given diagram. [may 2013, marks 10]

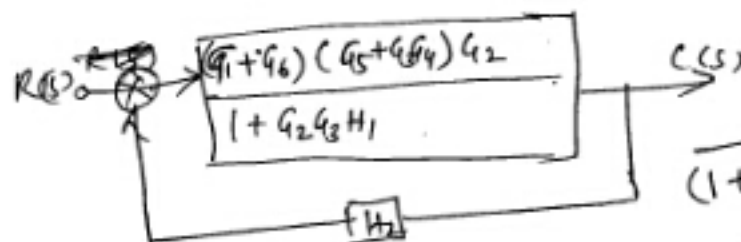
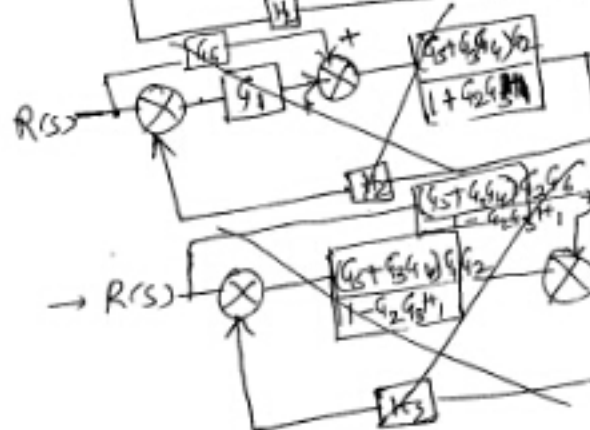
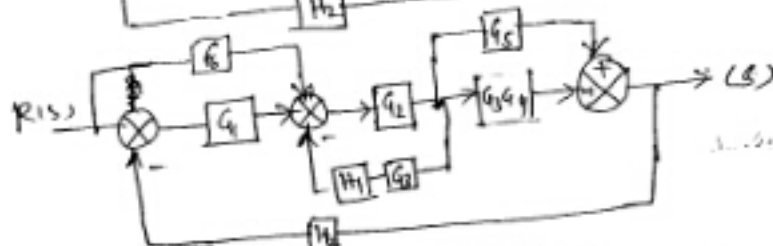
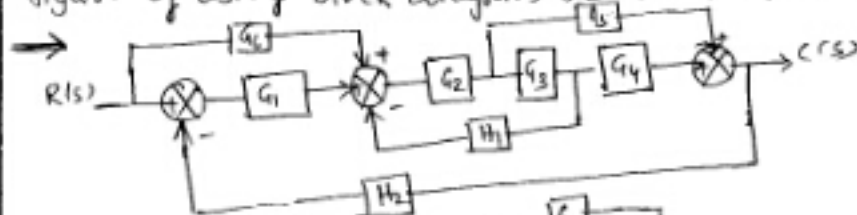


$$\frac{G_2 G_3}{1 + G_3 H_2} \times \frac{1 + G_2 H_3}{1 + G_3 H_2 + G_3 H_1 + G_3 H_2 H_3} = \frac{G_2 G_3}{1 + G_3 H_2 + G_3 H_1 + G_3 H_2 H_3}$$

$$\begin{aligned} \frac{G_2 G_3}{1 + G_3 H_2 + G_3 H_1 + G_3 H_2 H_3} &= \frac{G_2 G_3}{(1 + G_3 H_2)G_3 + H_1 G_2 G_3 (1 + G_3 H_2) + G_1 G_2 G_3 H_1} \\ &= \frac{G_2 G_3}{(1 + G_3 H_2)G_3 + H_1 G_2 G_3 (1 + G_3 H_2) + G_1 G_2 G_3 H_1} \end{aligned}$$

Find the transfer function of the block diagram shown in figure by using block diagram reduction method.

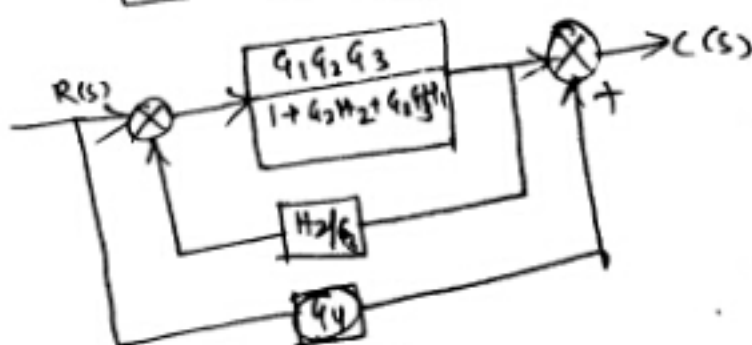
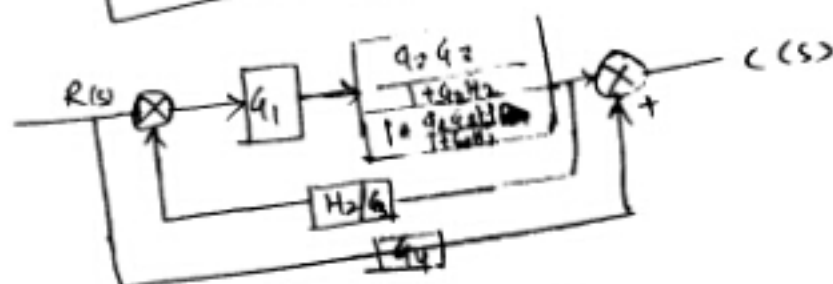
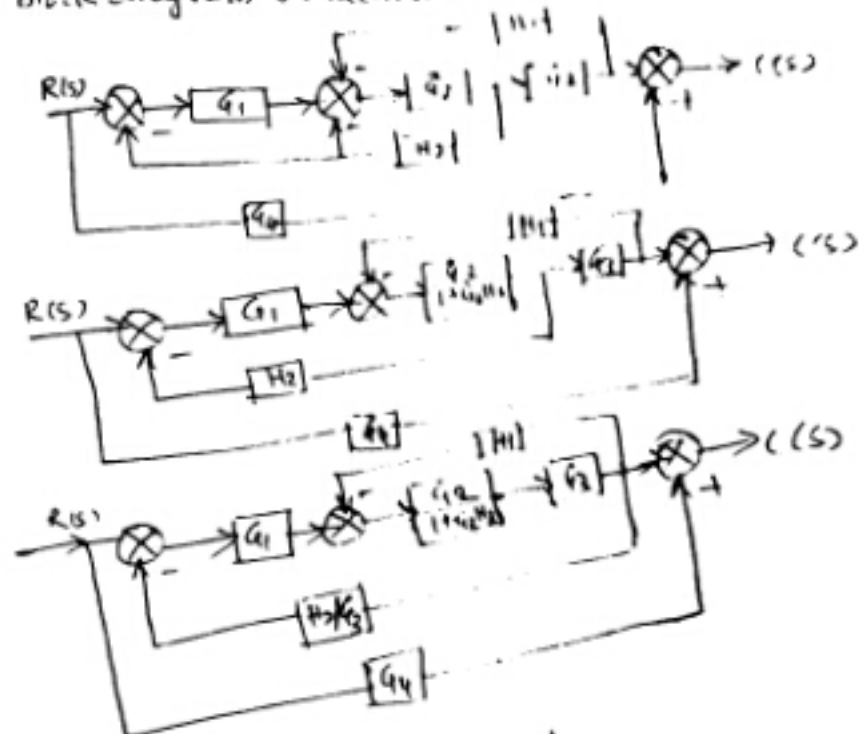
May/June - 2015  
(10 marks)



$$R(s) \rightarrow \frac{(G_1 + G_6)G_2G_3G_4}{1 + G_2G_3H_1 + G_1G_2G_3H_2 + G_2G_3G_4G_6H_2} C(s)$$

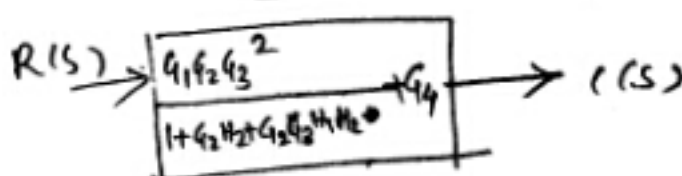
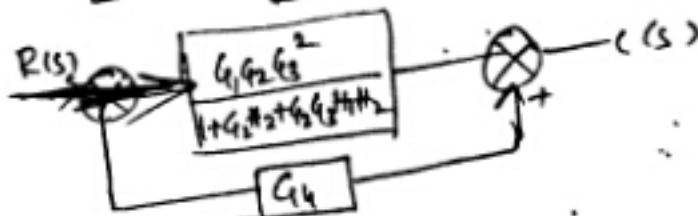
$$\begin{aligned} & \frac{(G_1 + G_6)(G_2G_3 + G_2G_3G_4)}{(1 + G_2G_3H_1) + (G_1 + G_6)(G_2G_3H_2)} \\ & = \frac{G_1G_2G_3G_4 + G_1G_2G_3H_2 + G_2G_3G_4G_6H_2}{1 + G_2G_3H_1 + G_1G_2G_3H_2 + G_2G_3G_4G_6H_2} \end{aligned}$$

Find the transfer function from the block diagram using block diagram reduction rules:



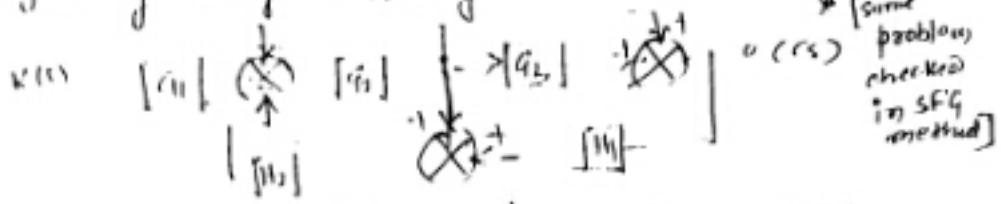
$$\frac{G_1 G_2 G_3}{1 + G_2 H_2 + G_2 G_3 H_1}$$

$$= \frac{G_1 G_2 G_3}{1 + G_2 H_2 + G_2 G_3 H_1}$$

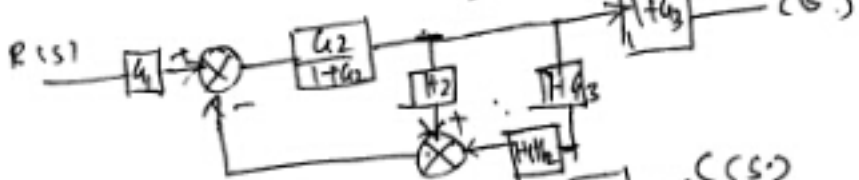
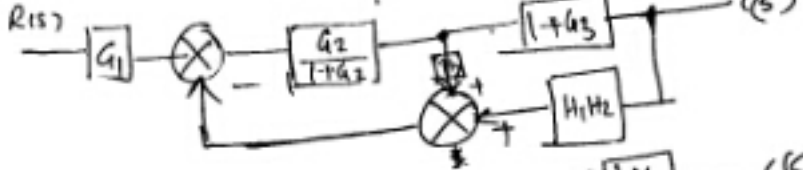
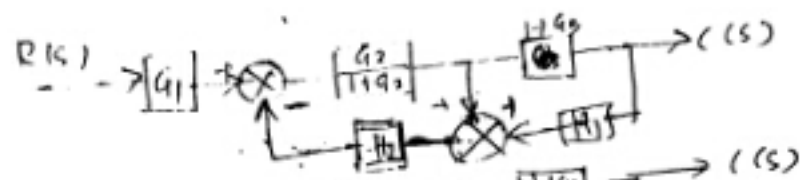
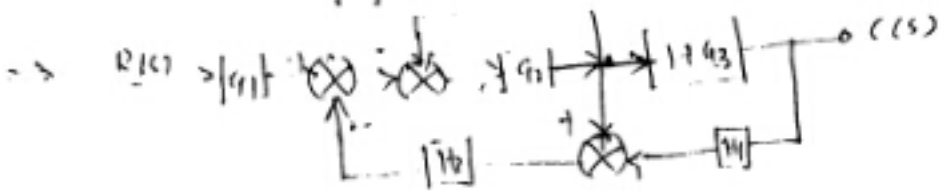


$$\frac{G_1 G_2 G_3^2}{1 + G_2 H_2 + G_2 G_3 H_1 H_2} + G_4$$

\* determine the transfer function of block diagram shown in figure by using block diagram reduction method.



\* [some problem checked in SFQ method]



$$C(s) = \frac{G_2}{1 + G_2 + (G_2 + G_3)(H_1 H_2) + H_2} R(s)$$

$$= \frac{G_2}{1 + G_2 + (G_2 + G_3)(H_1 H_2) + H_2} R(s)$$

$$C(s) = \frac{G_2}{1 + G_2 + G_2 H_1 H_2 + G_3 H_1 H_2 + H_2} R(s)$$

$$C(s) = \frac{G_1 G_2 + G_1 G_2 G_3}{1 + G_2 + G_2 H_1 H_2 + G_3 H_1 H_2 + H_2} R(s)$$

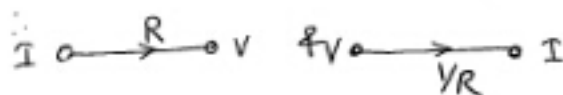


## \* Signal flow graphs :-

It is an alternative to block diagram. Signal flow graph consists only of branches unlike block diagram which consist of blocks, signals, summing points and take off points. It is a graphical representation of the relationship between variables of a set of linear algebraic equations. Every linear algebraic equation consists of dependent and independent variables. These variables are represented by small circles called as nodes. The relationship between nodes is represented by drawing a line between two nodes. Such lines are called branches.

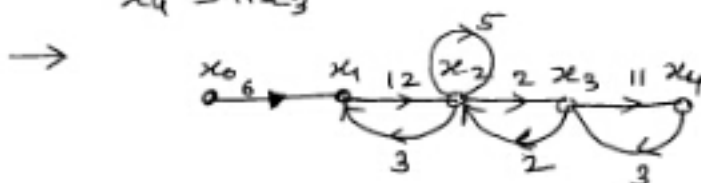
Considering two equations -

$$V = IR \quad \& \quad I = V/R$$



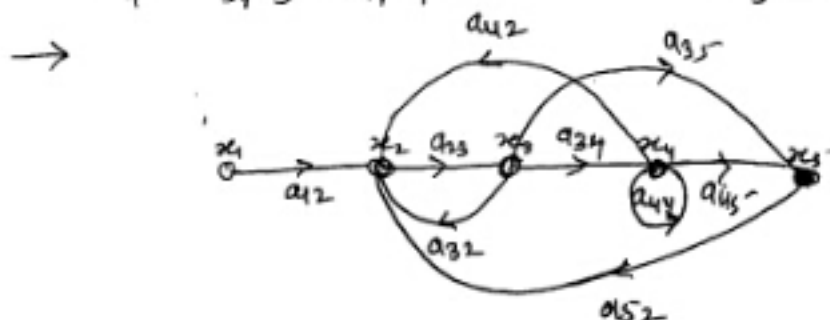
\* Consider a system represented by the following equations.  
Draw the signal flow graph of the system.

$$x_1 = 6x_0 + 3x_2, \quad x_2 = 12x_1 + 5x_2 + 2x_3, \quad x_3 = 2x_2 + 3x_4, \\ x_4 = 11x_3$$



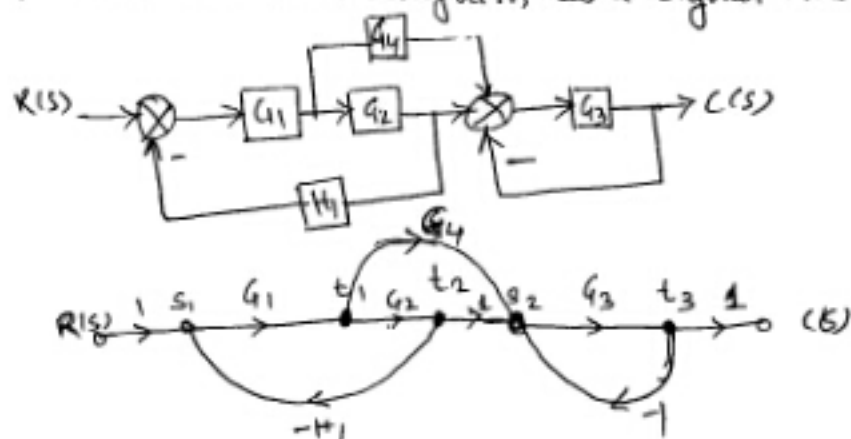
\* Consider a system represented by the given set of equations.  
Draw the SFG.

$$x_2 = a_{12}x_1 + a_{32}x_3 + a_{42}x_4 + a_{52}x_5, \quad x_3 = a_{23}x_2, \\ x_4 = a_{34}x_3 + a_{44}x_4, \quad x_5 = a_{35}x_3 + a_{45}x_4$$



\* Method to draw SFG from block diagrams: - The block diagrams are useful in representing control systems. For complicated systems, block diagram reduction is time consuming. Signal flow graph offers a much easier way to obtain the transfer function of the entire system.

\* Represent the block diagram as a signal flow graph



- \* Mason's Gain formula: - The objective of signal flow graph is to find the overall transfer function of the system with single formula (Mason's gain formula) to achieve the same result which is getting in block reduction method. It is a much easier method to obtain the transfer function of the entire control system. Once the signal flow graph is obtained, the T.F of the entire system is calculated using the Mason's gain formula -

$$\text{Overall Transfer function} = T.F = \frac{C(s)}{R(s)} = \frac{\sum F_i \Delta_i}{\Delta}$$

where  $i$  = no. of forward paths,

$F_i$  = Gain of  $i^{\text{th}}$  forward path

$\Delta$  = sfd determinant which is calculated as follows

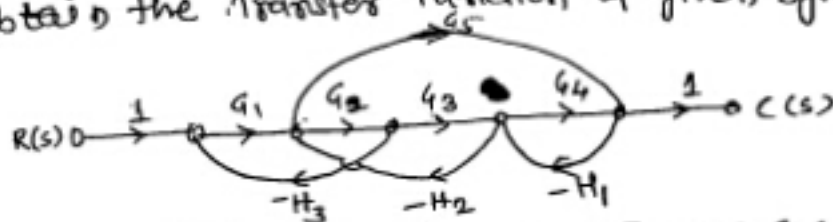
$\Delta = 1 - (\text{sum of all individual loop gains including self gains})$   
 $+ (\text{sum of all gain products of two non-touching loops})$   
 $- (\text{sum of all gain products of three non-touching loops})$   
 $+ \dots$

$$\therefore \Delta = 1 - (L_1 + L_2 + \dots) + (L_{12} + L_{21} + L_{32}) - (L_{13} + L_{23} + L_{31}) + \dots$$

$\therefore \Delta_i$  = the value of  $\Delta$  for the part of graph not touching to the  $i^{\text{th}}$  forward path.

$\therefore \Delta_i = 1 - \text{All the loops that do not touch the } i^{\text{th}} \text{ forward path.}$

- \* obtain the Transfer function of given signal flow graph



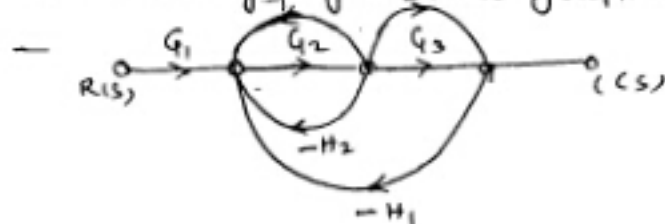
$\Rightarrow$  Total No. of forward paths = 2,  $\therefore F_1 = G_1 G_2 G_3 G_4$ ,  $F_2 = G_1 G_2$   
 Single touching loops: -  $L_{11} = -G_1 G_2 H_3$ ,  $L_{21} = -G_2 G_3 H_2$ ,  $L_{31} = -G_4 H_1$   
 $L_{41} = +H_1 H_2 G_3$   
 Two non-touching loops: -  $L_{12} = (-G_1 G_2 H_3)(-G_4 H_1) = G_1 G_2 G_4 H_1 H_3$   
 Three non-touching loops = Nil  
 $\Delta = 1 - (L_{11} + L_{21} + L_{31} + L_{41}) + L_{12}$   
 $= 1 - (-G_1 G_2 H_3 - G_2 G_3 H_2 - G_4 H_1 + G_1 H_1 H_2) + (G_1 G_2 G_4 H_1 H_3)$   
 $= 1 + G_1 G_2 H_3 + G_2 G_3 H_2 + G_4 H_1 - G_1 H_1 H_2 + G_1 G_2 G_4 H_1 H_3$

Take  $F_1$ , loops  $L_{11}, L_{21}, L_{31}, L_{41}$   
 $\Delta_1 = 1 - 0 = 1$

Take  $F_2$ , all loops above touch path  $F_2$ ,  
 $\therefore \Delta_2 = 1$

$$T.F = \frac{F_1 \Delta_1 + F_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{1 + G_1 G_2 H_3 + G_2 G_3 H_2 + G_4 H_1 - G_3 H_1 H_2 + G_2 G_3 H_1 H_2}$$

2. Obtain transfer function using Mason's gain formula for following signal flow graph. (May 2007, 10 marks)



$\Rightarrow$  No. of forward paths = 2,  $F_1 = G_1 G_2 G_3$  &  $F_2 = G_1 G_4$

No. of single loops :-  $L_{11} = -G_2$ ,  $L_{21} = -G_2 H_2$ ,  $L_{31} = -G_2 G_3 H_1$   
 $L_{41} = -G_2 H_1$

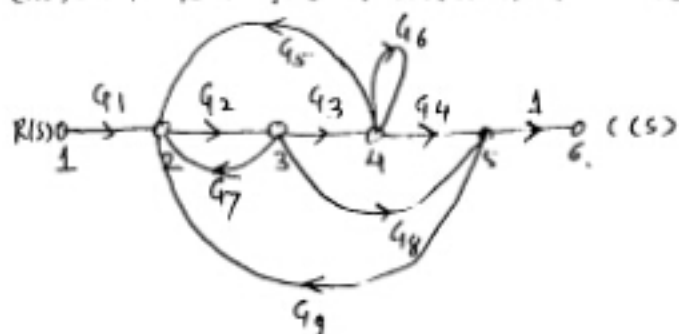
No. of 2 non-touching loops = 0

$$\therefore \Delta = 1 - (L_{11} + L_{21} + L_{31} + L_{41}) = 1 + G_2 + G_2 H_2 + G_2 G_3 H_1 + G_2 H_1$$

$$\Delta_1 = 1, \Delta_2 = 1$$

$$\therefore T.F = \frac{F_1 \Delta_1 + F_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_2 + G_2 H_2 + G_2 G_3 H_1 + G_2 H_1}$$

\* obtain transfer function for the following signal flow graph  
(Dec-2011, 10 marks)



$\Rightarrow$  No of forward path  $\Rightarrow F_1 = G_1 G_2 G_3 G_4$ ,  $F_2 = G_1 G_2 G_8$

Total No of loops  $\Rightarrow L_1 = G_2 G_3 G_5$ ,  $L_2 = G_2 G_7$ ,  $L_3 = G_6$ ,  
 $L_4 = G_2 G_8 G_9$ ,  $L_5 = G_2 G_3 G_4 G_9$

No of 2 non-touching  $\Rightarrow L_2 \times L_3 = G_2 G_7 G_6$   
loops

No of 3 non-touching  $\Rightarrow 0$  (Nil)  
loops

$$\therefore \Delta = 1 - (G_2 G_3 G_5 + G_1 G_2 G_8 + G_6 + G_2 G_8 G_9 + G_2 G_3 G_4 G_9 + G_2 G_7 G_6)$$

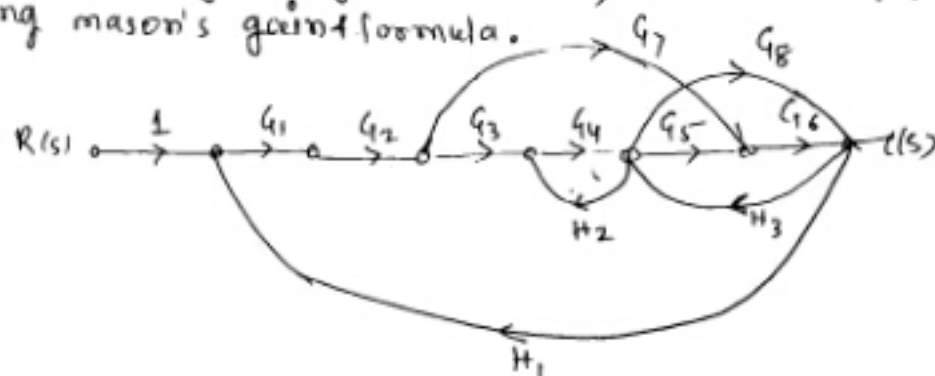
$$\therefore \Delta_1 = 1 - 0 = 1$$

Loop  $L_3$  does not touch the forward path  $F_2$

$$\therefore \Delta_2 = 1 - L_3 = (1 - G_6)$$

$$\therefore T.F = \frac{F_1 \Delta_1 + F_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_8 (1 - G_6)}{1 - (G_2 G_3 G_5 + G_1 G_2 G_8 + G_6 + G_2 G_8 G_9 + G_2 G_3 G_4 G_9 + G_2 G_7 G_6)}$$

\* For the diagram given below, obtain transfer function, using mason's gain formula.



$$\Rightarrow \text{No. of forward path, } F_1 = G_1 G_2 G_3 G_4 G_5 G_6$$

$$F_2 = G_1 G_2 G_7 G_6$$

$$F_3 = G_1 G_2 G_3 G_4 G_8$$

$$\text{Total single loops, } L_{11} = G_1 G_2 G_3 G_4 G_5 G_6 H_1, L_{21} = G_4 H_2$$

$$L_{31} = G_5 G_6 H_3, L_{41} = G_8 H_3, L_{51} = G_1 G_2 G_3 G_4 G_6 H_1$$

$$L_{61} = G_1 G_2 G_3 G_4 G_8 H_1$$

$$\text{No. of 2 no-touching loops} \Rightarrow L_{12} = L_{21} \times L_{61} = G_4 H_2 G_1 G_2 G_3 G_4 G_6 H_1$$

$$\text{No. of 3 non-touching loops } L_{13} = 0$$

$$\therefore \Delta = 1 - G_1 G_2 G_3 G_4 G_5 G_6 H_1 - G_4 H_2 - G_5 G_6 H_3$$

$$- G_8 H_3 - G_1 G_2 G_3 G_4 G_6 H_1 - G_1 G_2 G_3 G_4 G_8 H_1$$

$$+ G_4 H_2 G_1 G_2 G_3 G_4 G_6 H_1$$

$$\text{Take } F_1; \Delta_1 = 1 - 0 = 1,$$

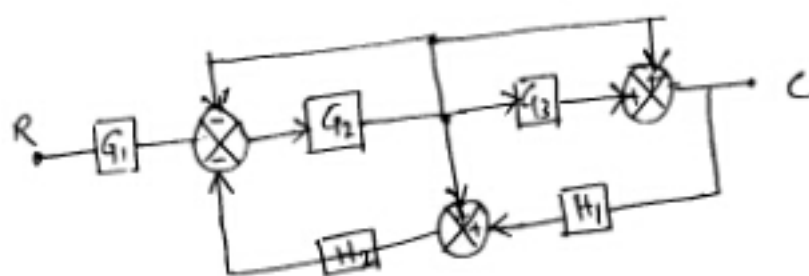
$$\text{Take } F_2; \Delta_2 = 1 - L_{21} = 1 - G_4 H_2$$

$$\text{Take } F_3; \Delta_3 = 1 - 0 = 1$$

$$T.F = \frac{F_1 \Delta_1 + F_2 \Delta_2 + F_3 \Delta_3}{\Delta}$$

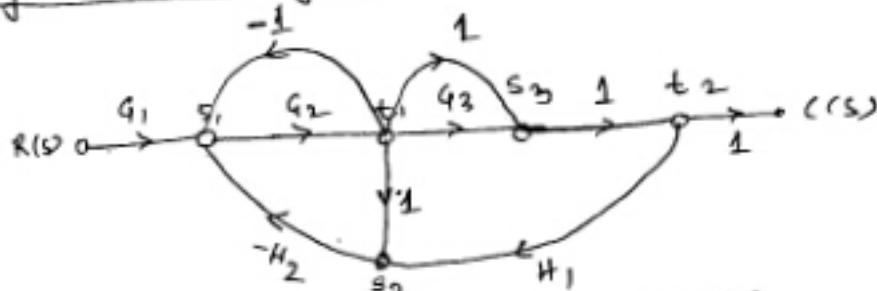
$$= \frac{G_1 G_2 G_3 G_4 G_5 G_6 + (G_1 G_2 G_3 G_4 G_6)(1 - G_4 H_2) + G_1 G_2 G_3 G_4 G_8}{1 - G_1 G_2 G_3 G_4 G_5 G_6 H_1 - G_4 H_2 - G_5 G_6 H_3 - G_8 H_3 - G_1 G_2 G_3 G_4 G_6 H_1 - G_1 G_2 G_3 G_4 G_8 H_1 + G_4 H_2 G_1 G_2 G_3 G_4 G_6 H_1}$$

\* Draw a signal flow graph for the system shown below and hence obtain the transfer function using Mason's gain formula.  
(Nov/Dec-2015, 10 marks)



[Some problem checked in block diagram reduction method]

→ Signal Flow diagram:-



No. of forward paths;  $F_1 = G_1 G_2 G_3$ ;  $F_2 = G_1 G_2$

No. of single loops;  $L_{11} = -G_2 H_2$ ,  $L_{21} = -G_2$ ,  $L_{31} = -G_2 G_3 H_1 H_2$

No. of 2 non-touching loops:- Nil  $L_{41} = -G_2 H_1 H_2$

No. of 3 non-touching loops:- Nil

$$\therefore \Delta = 1 + G_2 H_2 + G_2 + G_2 G_3 H_1 H_2 + G_2 H_1 H_2$$

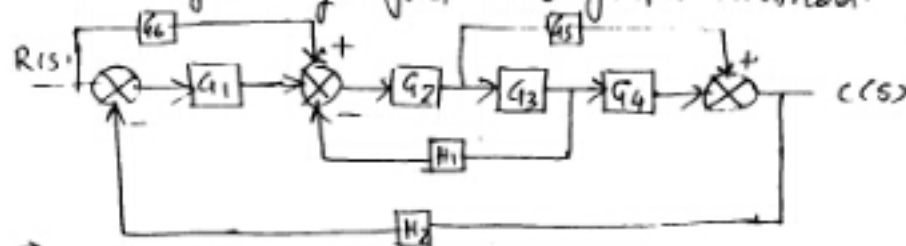
$$\therefore \Delta_1 = 1 - 0$$

$$\therefore \Delta_2 = 1 - 0$$

$$\therefore T.F = \frac{F_1 \Delta_1 + F_2 \Delta_2}{\Delta}$$

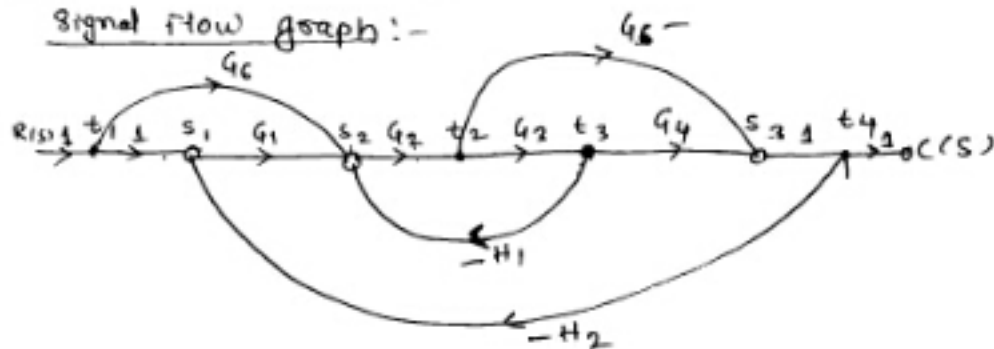
$$= \frac{G_1 G_2 G_3 + G_1 G_2}{1 + G_2 H_2 + G_2 + G_2 G_3 H_1 H_2 + G_2 H_1 H_2}$$

\* Find the transfer function of block diagram shown in figure by using signal flow graph method.



(Some problems checked by block reduction method)

Signal flow graph:-



No. of forward path;  $F_1 = G_1 G_2 G_3 G_4$ ,  $F_2 = G_6 G_2 G_3 G_4$   
 $F_3 = G_6 G_2 G_5$ ,  $F_4 = G_1 G_2 G_5$

No. of single loops;  $L_{11} = -G_2 G_3 H_1$ ,  $L_{21} = -G_1 G_2 G_3 G_4 H_2$   
 $L_{31} = -G_2 G_5 H_2 G_1$

No. of 2 non touching loops;  $L_{12} = \text{Nil}$

$$\Delta = 1 + G_2 G_3 H_1 + G_1 G_2 G_3 G_4 H_2 + G_1 G_2 G_5 H_2$$

$$\Delta_1 = 1 - 0$$

$$\Delta_2 = 1 - 0$$

$$\Delta_3 = 1 - 0$$

$$\Delta_4 = 1 - 0$$

$$\begin{aligned} \text{T.F} &= \frac{F_1 \Delta_1 + F_2 \Delta_2 + F_3 \Delta_3 + F_4 \Delta_4}{\Delta} \\ &= \frac{G_1 G_2 G_3 G_4 + G_2 G_3 G_4 G_6 + G_2 G_5 G_6 + G_1 G_2 G_5}{1 + G_2 G_3 H_1 + G_1 G_2 G_3 G_4 H_2 + G_1 G_2 G_5 H_2} \end{aligned}$$