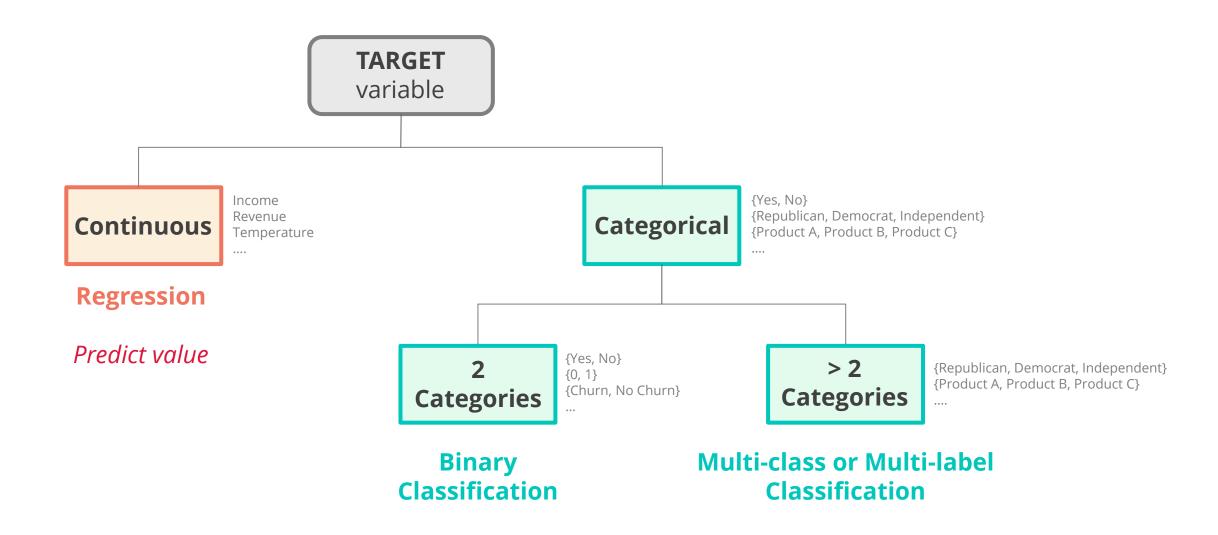
Supervised Learning: Classification

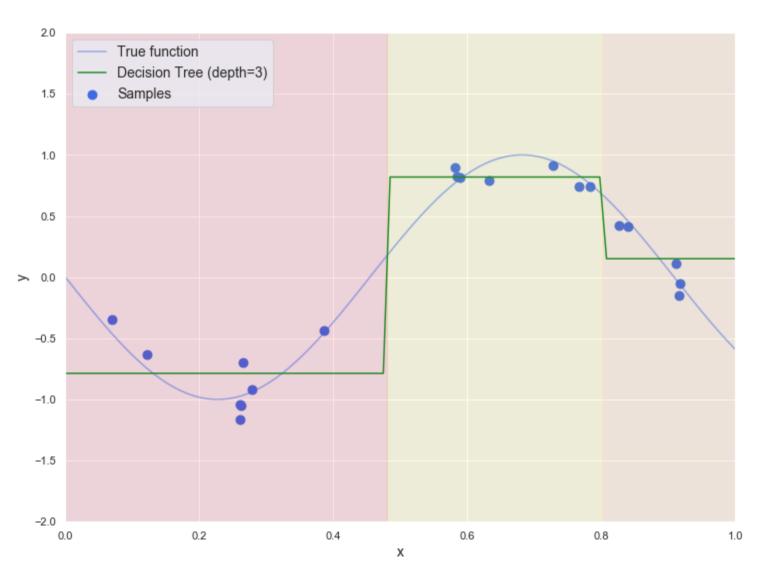
Spring 2025

- 1. Introduction
- 2. The Data Science Process
- 3. Supervised Learning: Classification
- 4. Unsupervised Learning
- 5. The Grunt Work
- 6. Wrap Up

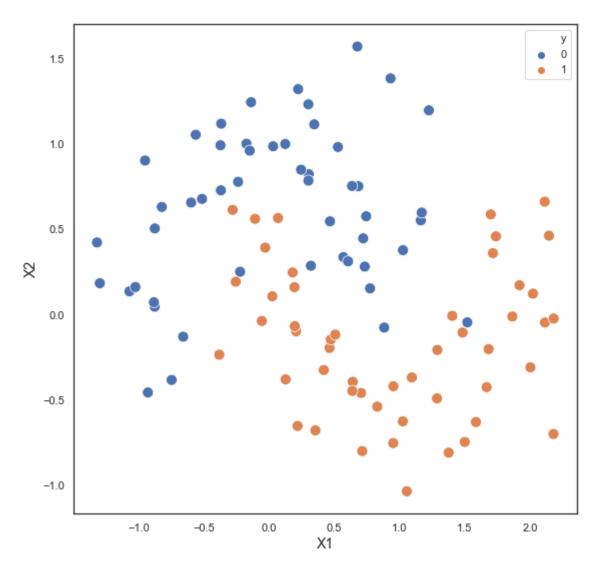


Predict class

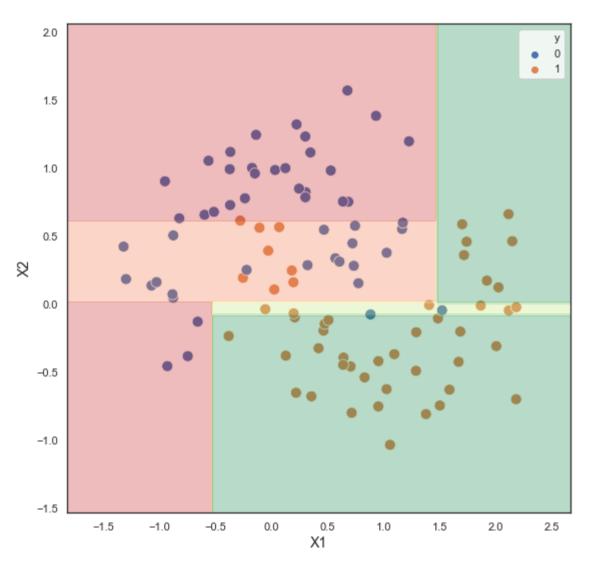
Classification Trees



Recursive partitioning on continuous data (outcome)



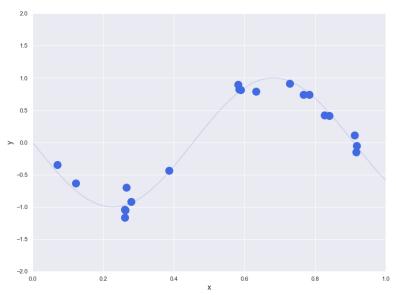
Binary outcome (y)



Recursive partitioning

1 How to partition the data?

2 When to stop?





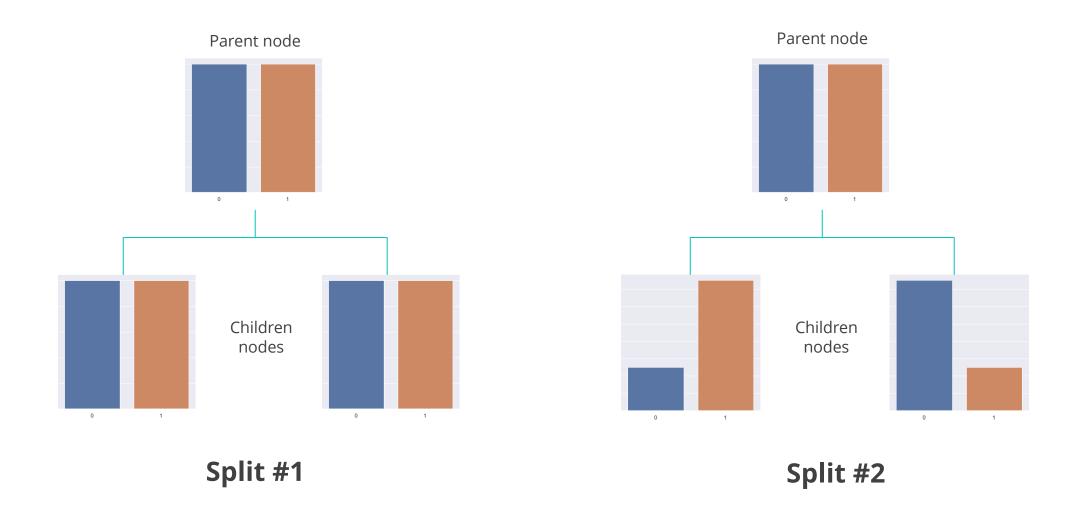
REGRESSION TREES

$$MSE = \frac{1}{n} \sum_{j=1}^{J} \sum_{i \in R_j} (y_i - \hat{y}_{R_j})^2$$

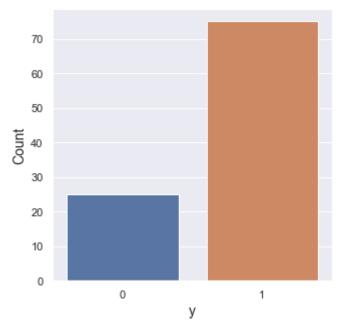
CLASSIFICATION TREES



Measure of Impurity

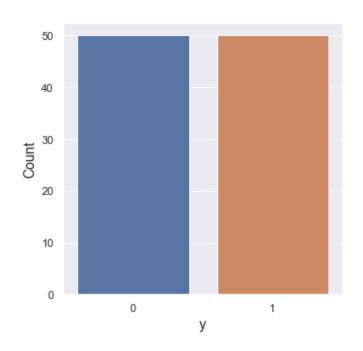


How to determine which split is better?



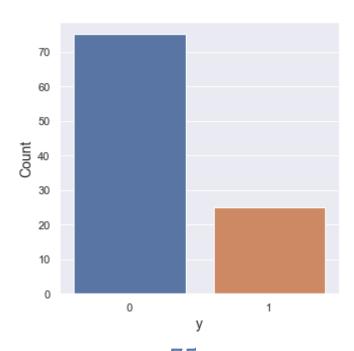
$$p_{y=0} = \frac{25}{100} = \mathbf{0.25}$$

$$p_{y=1} = \frac{75}{100} = \mathbf{0.75}$$



$$p_{y=0} = \frac{50}{100} = \mathbf{0.50}$$

$$p_{y=1} = \frac{50}{100} = \mathbf{0.50}$$



$$p_{y=0} = \frac{75}{100} = \mathbf{0.75}$$

$$p_{y=1} = \frac{25}{100} = \mathbf{0.25}$$

0.375

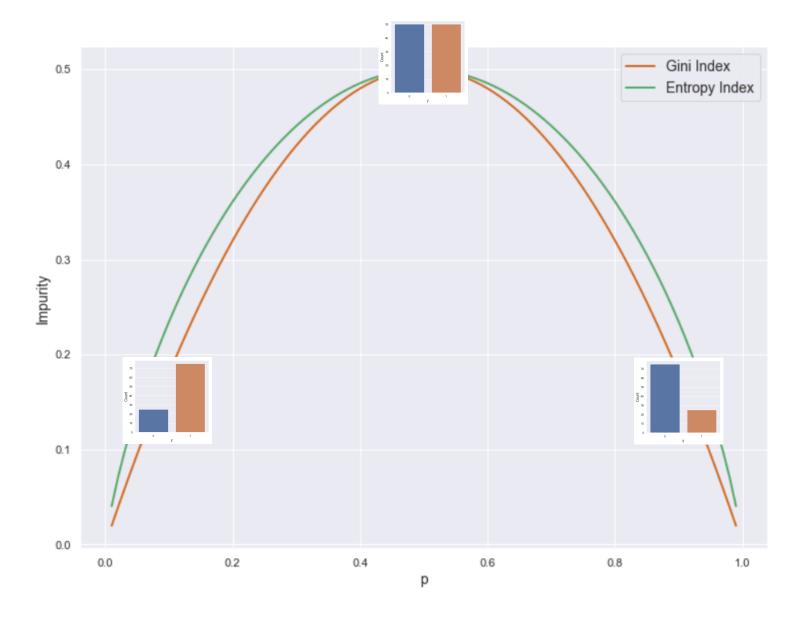
0.50

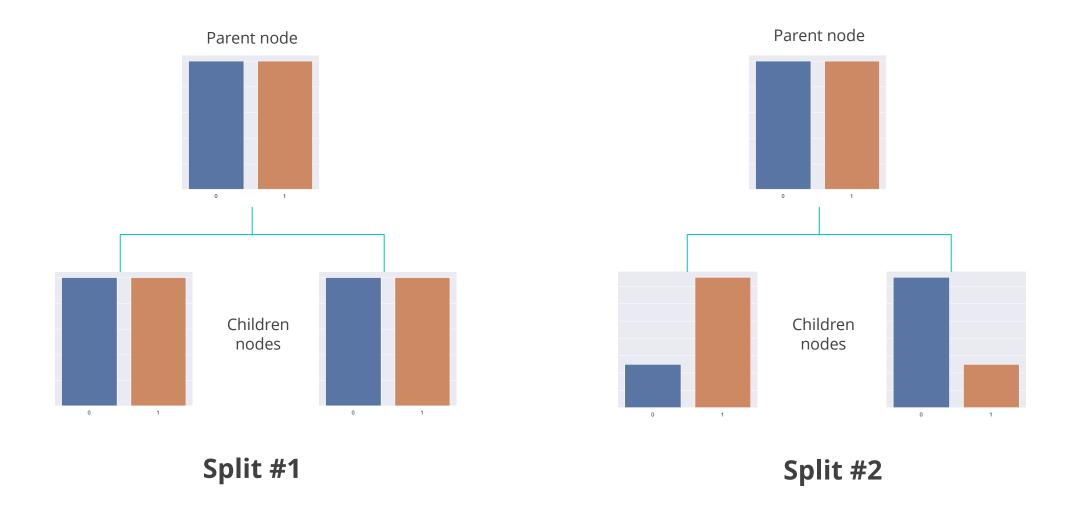
0.375

$$p_{y=1} * (1 - p_{y=1}) + p_{y=0} * (1 - p_{y=0})$$

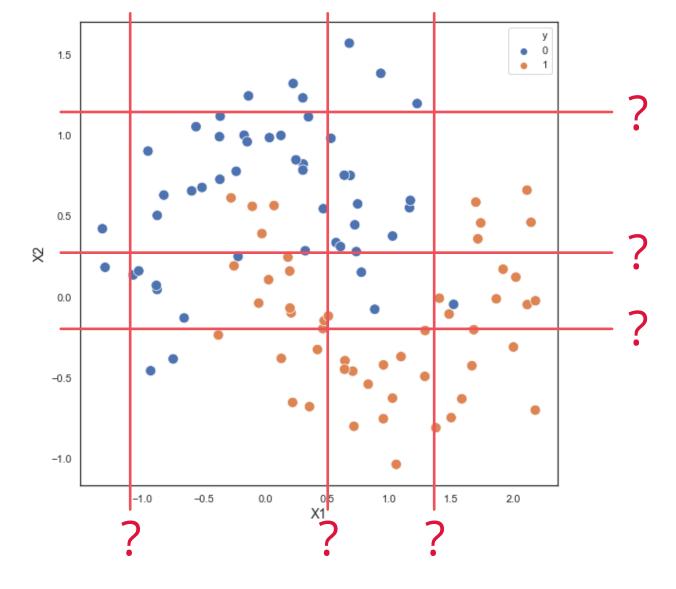
$$Entropy = -\sum_{k} p_k \log_2 p_k$$

$$Gini = \sum_{k} p_k (1 - p_k)$$

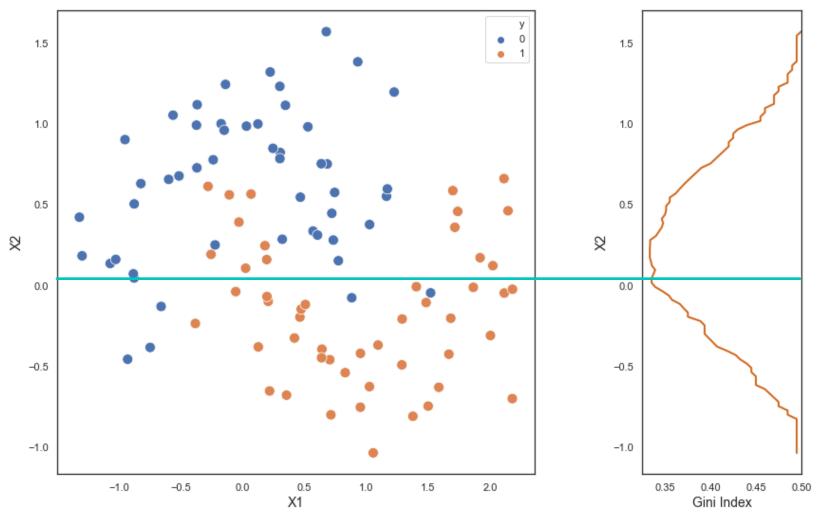




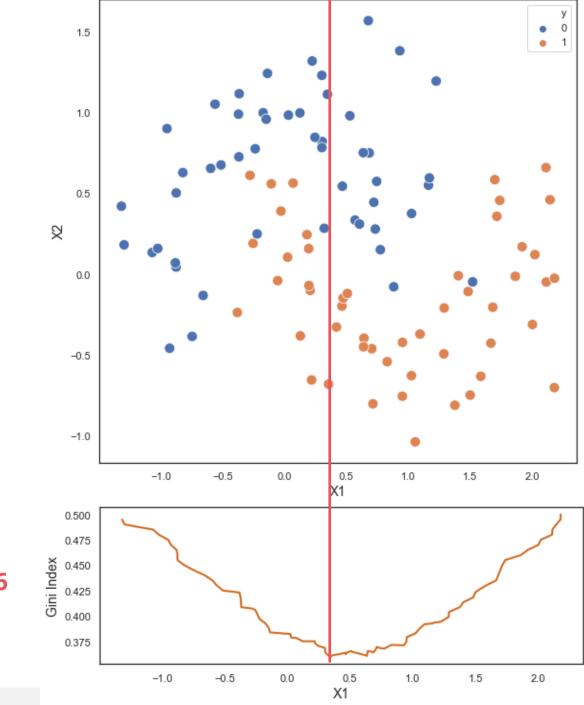
Split #2 is better because the children node have low impurity.



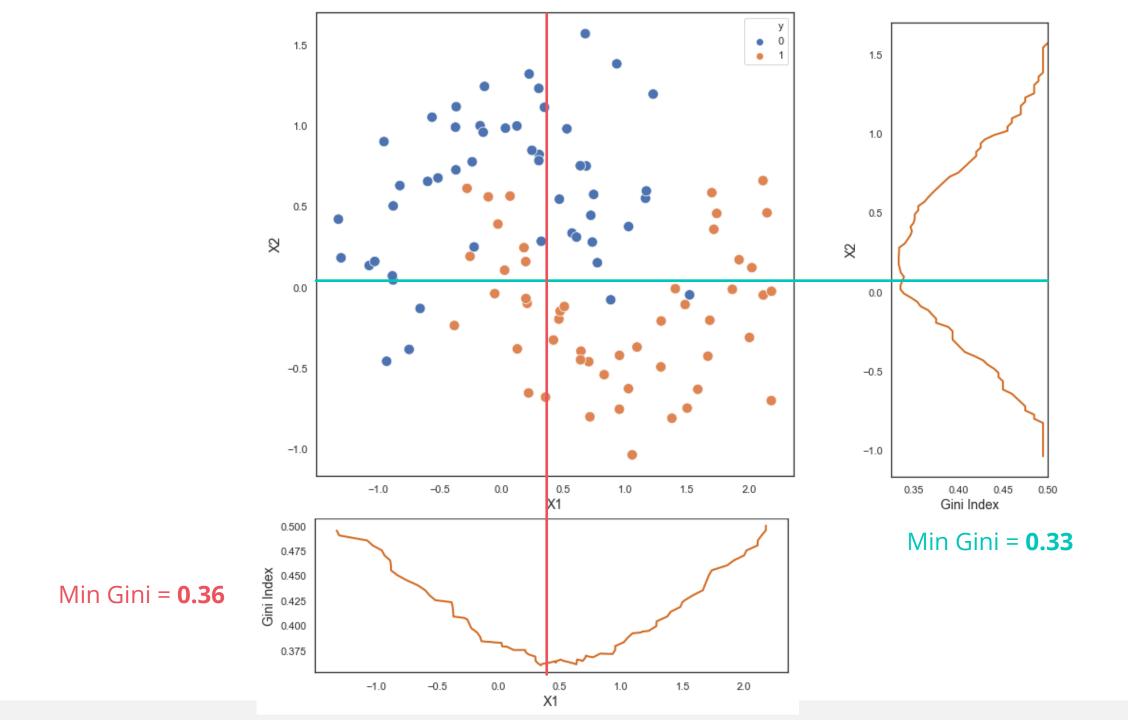
Where is the optimal cut-off (partition) that would yield the lowest impurity?

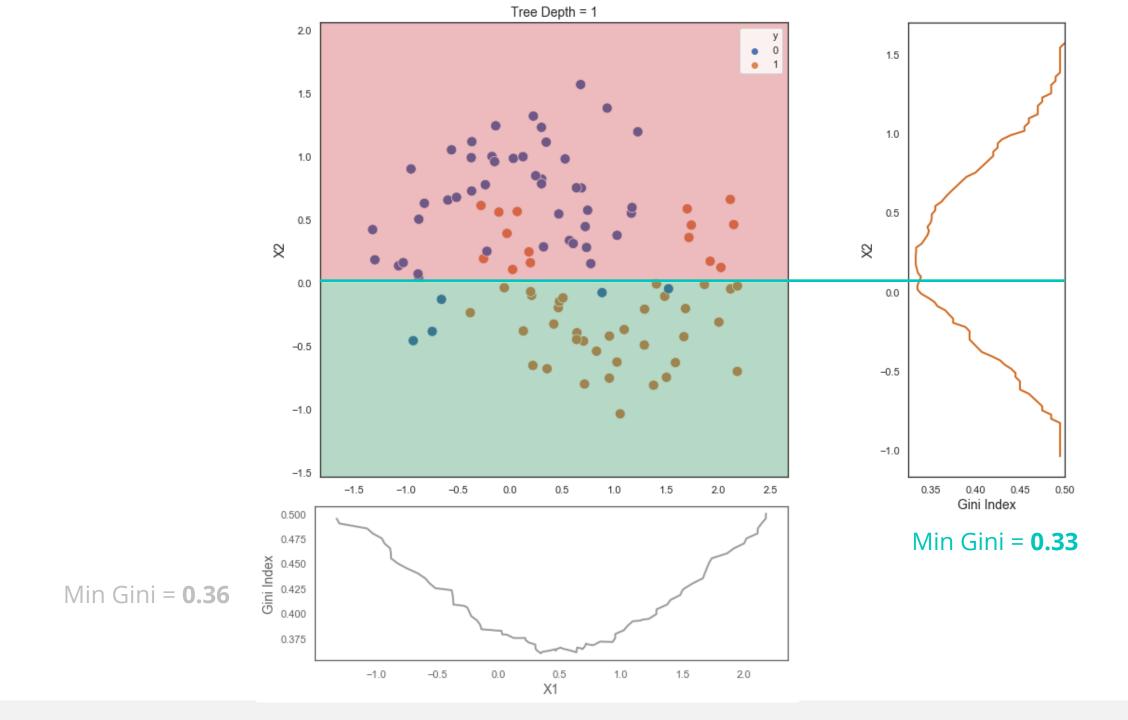


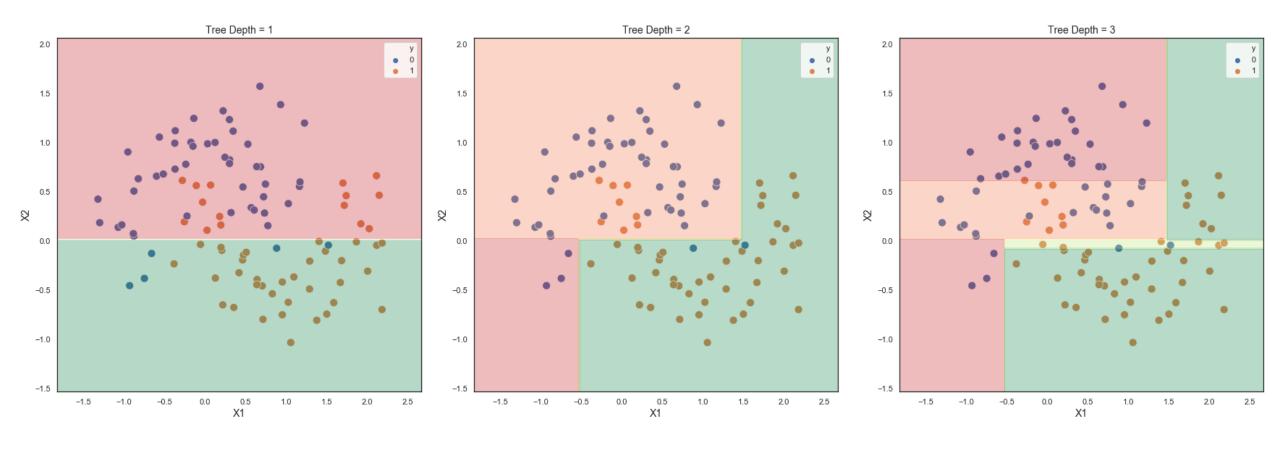
Min Gini = **0.33**



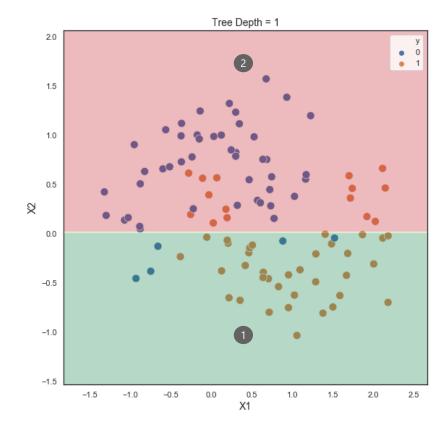
Min Gini = **0.36**

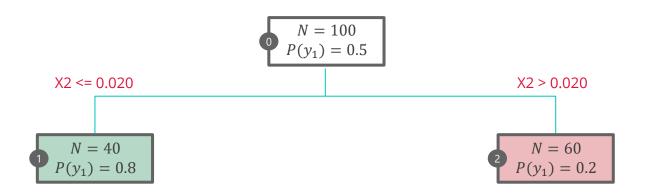


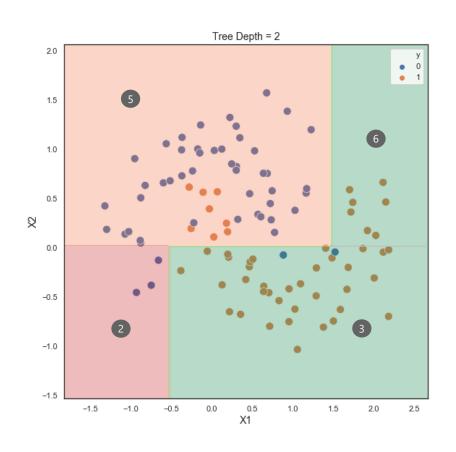


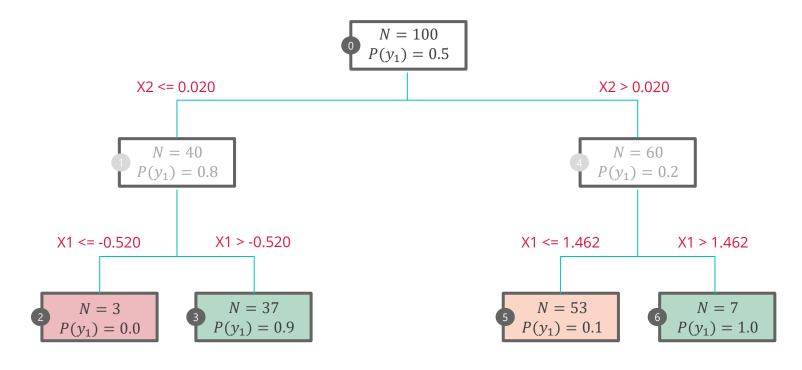


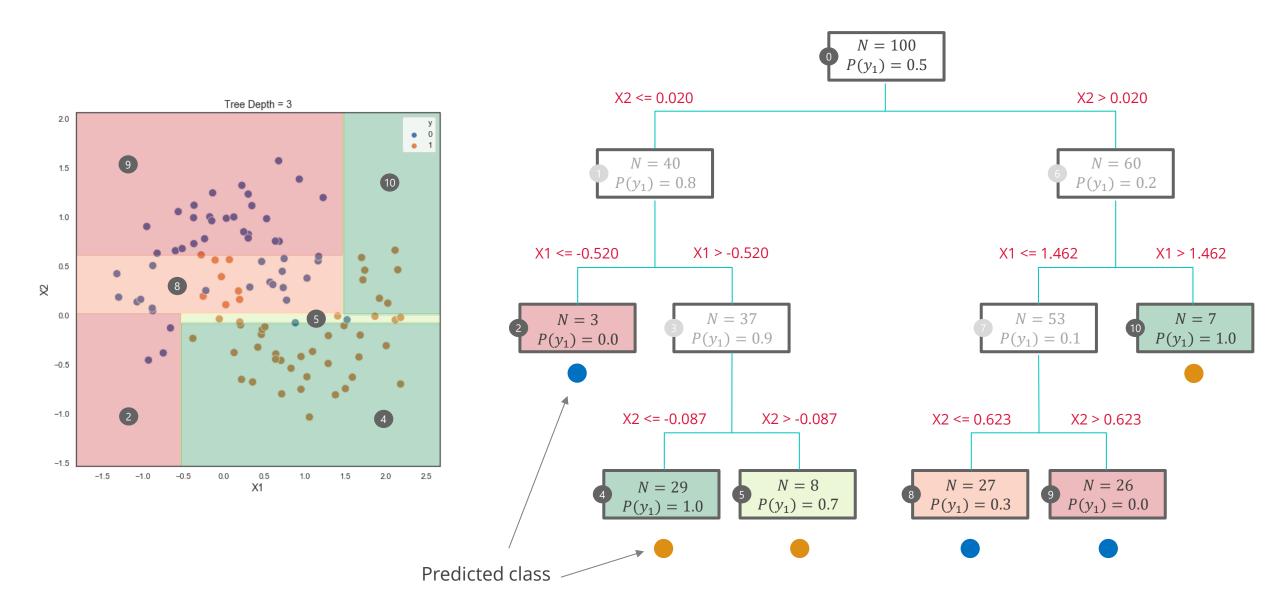
Reduction in impurity → "Information Gain"



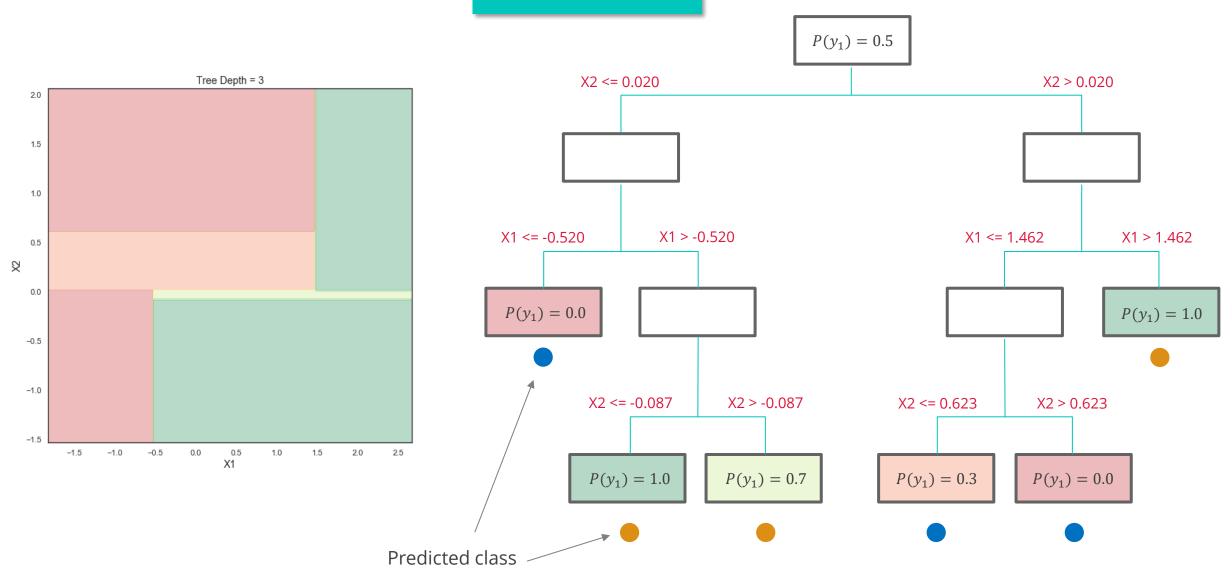


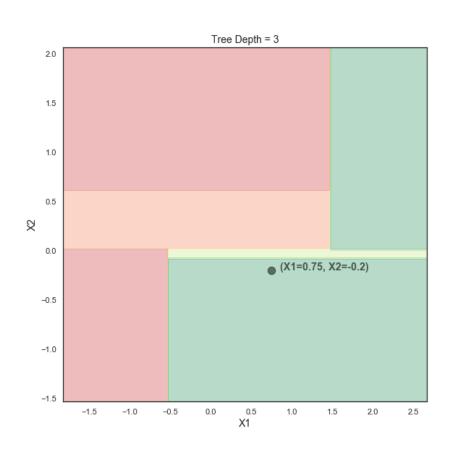


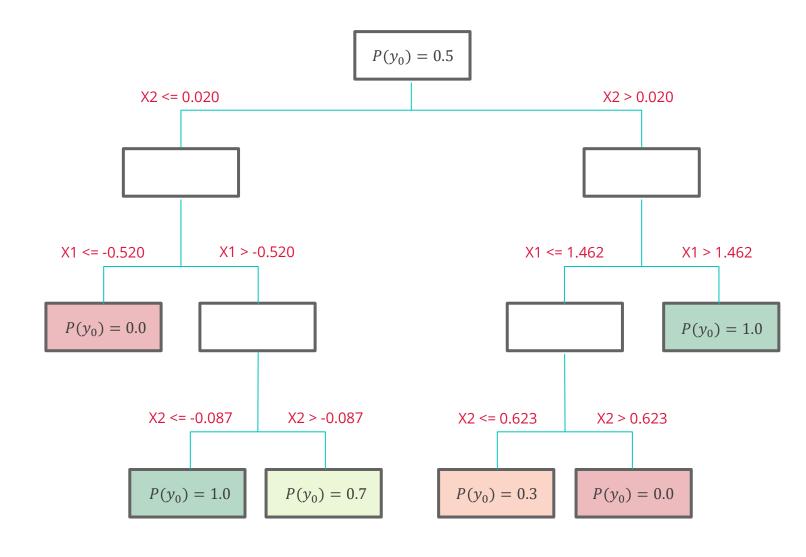




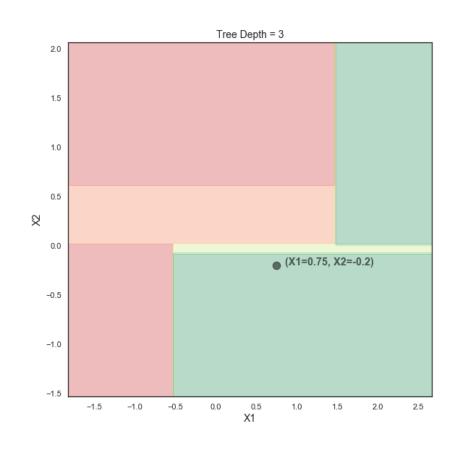
The Model

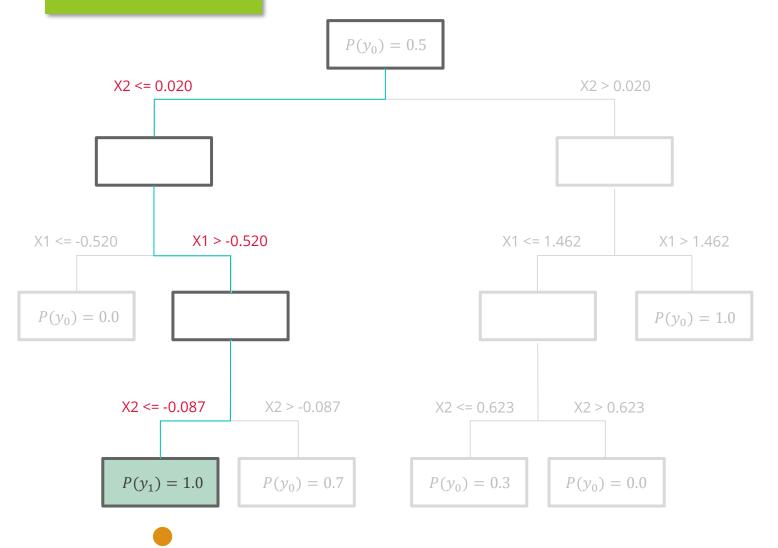






Prediction





Decision Trees for Classification: Summary

Impurity measure	Formula
Entropy (Cross-entropy, deviance)	$-\sum_{k}p_{k}\log_{2}p_{k}$
Gini index	$\sum_{k} p_k (1 - p_k)$
Mis-classification error	$\sum_{k} (1 - p_k)$
Chi-Square	$\sum_{i} \frac{(x_i - m_i)^2}{m_i}$

Entropy
$$H = -\sum_{k} p_k \log_2 p_k$$

Low Entropy

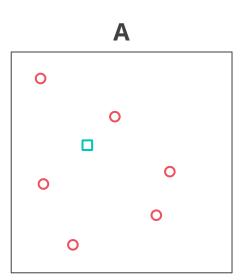
One class is more likely than the other.



High Entropy

Both classes are nearly equally likely.





$$p_{\square} = \frac{1}{7} \qquad p_{\bigcirc} = \frac{6}{7}$$

$$H = -\sum_{k} p_k \log_2 p_k$$

$$H = -p_{\square} \log_2 p_{\square} - p_{\bigcirc} \log_2 p_{\bigcirc}$$

Low Entropy

$$H_A = 0.59$$

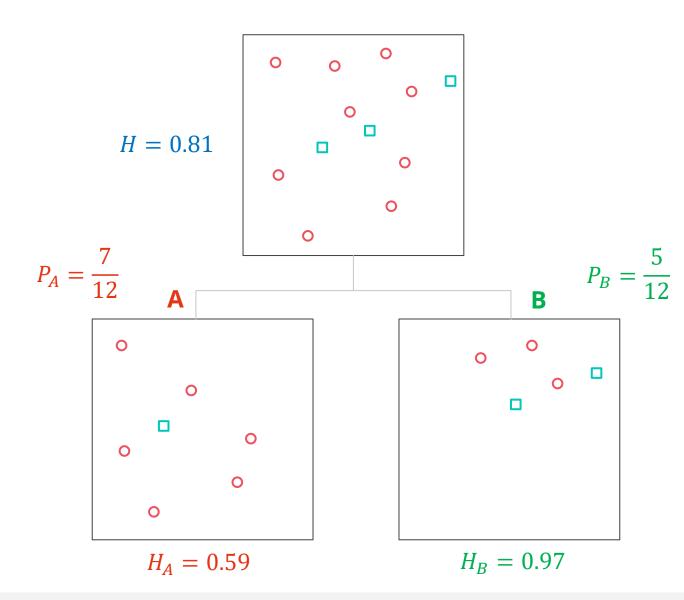
High purity

$$p_{\square} = \frac{2}{5} \qquad p_{\bigcirc} = \frac{3}{5}$$

$$H_B = 0.97$$
 High Entropy

Low purity

Information Gain



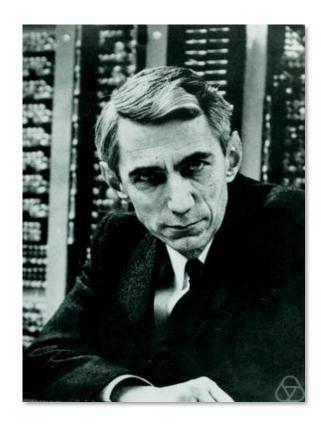
$$IG = H - (H_A * P_A + H_B * P_B)$$

$$IG = 0.81 - \left(0.59 * \frac{7}{12} + 0.97 * \frac{5}{12}\right)$$

$$IG = 0.81 - 0.75 = 0.06$$

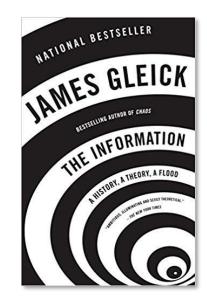
- Information Gain: The amount by which the ambiguity (entropy) decreases due to the split.
- The goal is to find the split that maximizes the information gain.

Information Entropy



Claude Shannon (1916 –2001) The father of Information Theory

$$H = -\sum_{k} p_k \log_2 p_k$$





Information Entropy (7-minute <u>video</u>)



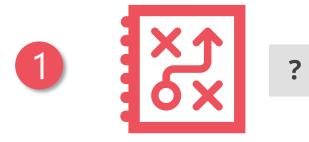
$$P_A = 0.25$$
 $P_B = 0.25$
 $P_C = 0.25$
 $P_D = 0.25$



$$P_A = 0.50$$
 $P_B = 0.125$
 $P_C = 0.125$
 $P_D = 0.25$

Which machine is producing more **information**?

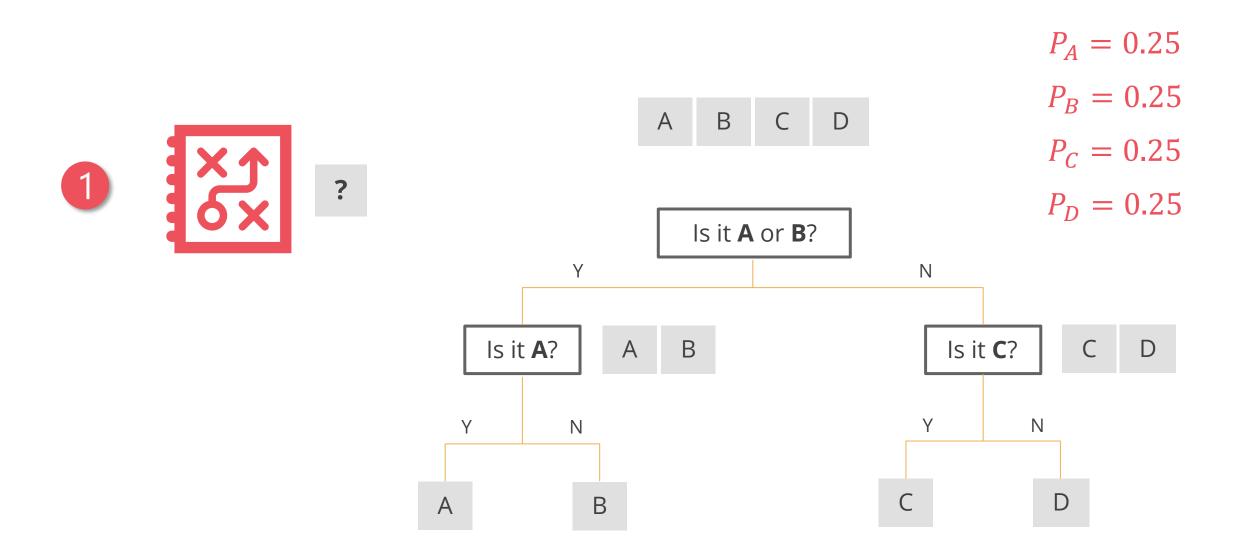




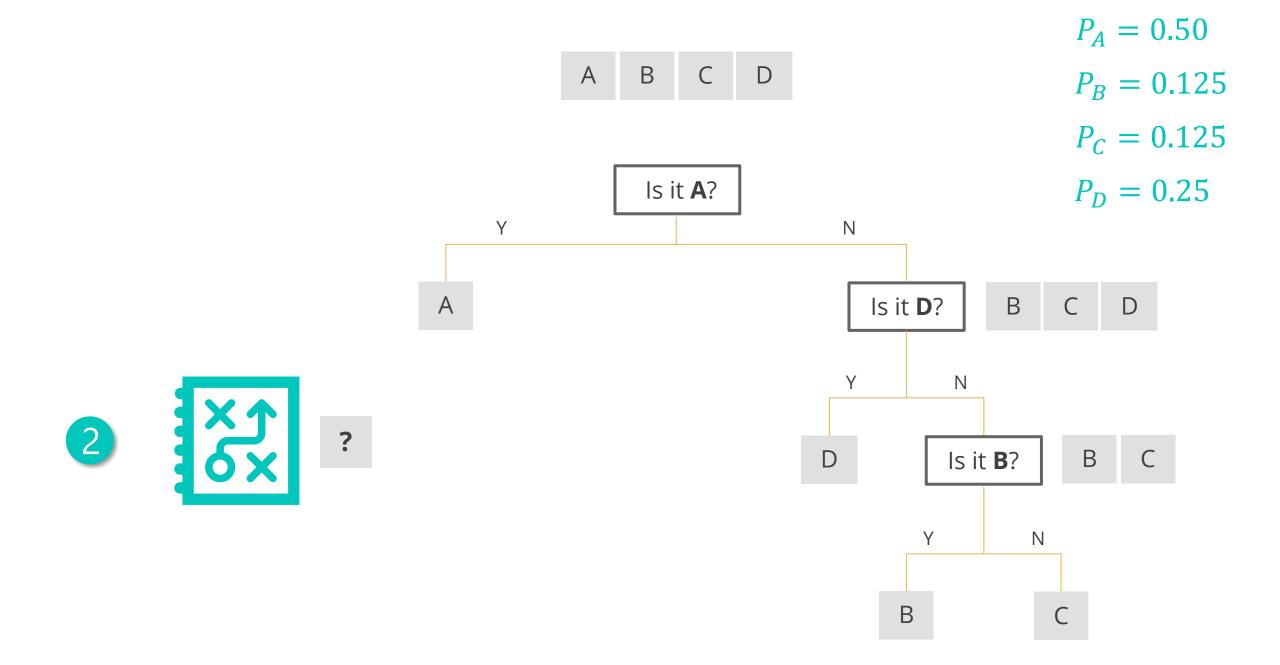
Which machine is producing more **information**?

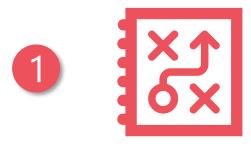


If you had to predict the **next symbol** from a machine, **how many (yes/no) questions** you would have to ask?



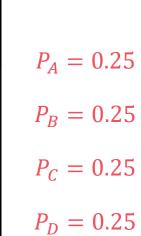
The uncertainty in **machine 1** is **two questions** per symbol.

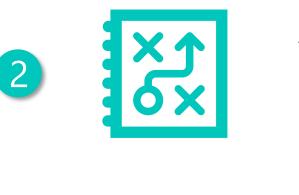




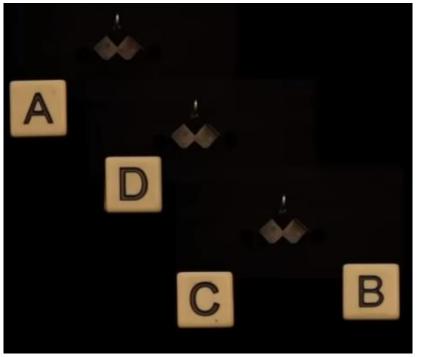
2 questions











$$P_A = 0.50$$

$$P_B=0.125$$

$$P_C = 0.125$$

$$P_D = 0.25$$

Expected # of bounces = $P_A * 1 + P_B * 3 + P_c * 3 + P_D * 2$

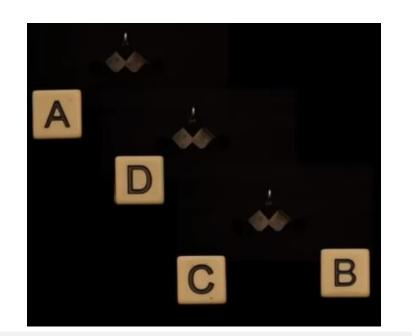
$$= 0.5 * 1 + 0.125 * 3 + 0.125 * 3 + 0.25 * 2$$

= 1.75

= *Expected* # *of questions*







$$P_A = 0.50$$

$$P_B=0.125$$

$$P_C = 0.125$$

$$P_D=0.25$$





2 questions





1.75 questions

Machine 2 is producing

less information, because
there's less uncertainty
(or surprise)
about its output.

A Mathematical Theory of Communication

By C. E. SHANNON

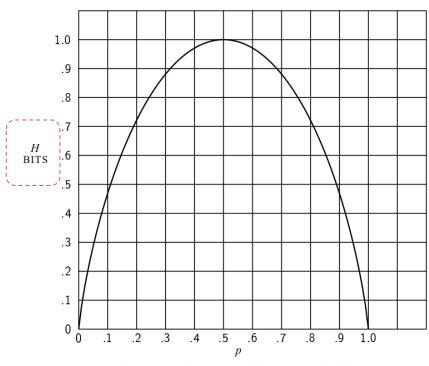


Fig. 7—Entropy in the case of two possibilities with probabilities p and (1-p).

$$H = -\sum_{k} p_k \log_2 p_k$$

$$H = \sum_{k} p_k * \# of bounces_k$$

$$\# of bounces = log_2(\# of outcomes)$$

of outcomes =
$$\frac{1}{p}$$

of bounces =
$$log_2\left(\frac{1}{p}\right) = -log_2p$$



Classification Trees in scikit-learn

```
class sklearn.tree.DecisionTreeClassifier(
   criterion='gini',
   splitter='best',
   max_depth=None,
   min_samples_split=2,
   min_samples_leaf=1,
   min_weight_fraction_leaf=0.0,
   max_features=None,
   random_state=None,
   max_leaf_nodes=None,
   min_impurity_decrease=0.0,
   min_impurity_split=None,
   class_weight=None,
   ccp_alpha=0.0)
```

```
# Import from sklearn.tree import DecisionTreeClassifier
```

```
# Define clf = DecisionTreeClassifier()
```

```
# Fit clf.fit(x_train, y_class)
```

```
# Predict clf.predict(x_test)
```

class sklearn.tree.DecisionTreeClassifier(

```
criterion='gini',
splitter='best',
max_depth=None,
min_samples_split=2,
min_samples_leaf=1,
min_weight_fraction_leaf=0.0,
max_features=None,
random_state=None,
max_leaf_nodes=None,
min_impurity_decrease=0.0,
min_impurity_split=None,
class_weight=None,
ccp_alpha=0.0)
```

The function to measure the quality of a split.

Supported criteria are "gini" for the Gini impurity and "entropy" for the information gain.

```
class sklearn.tree.DecisionTreeClassifier(
   criterion='gini',
   splitter='best',
   max_depth=None,
   min_samples_split=2,
   min_samples_leaf=1,
   min_weight_fraction_leaf=0.0,
   max_features=None,
   random_state=None,
   max_leaf_nodes=None,
   min_impurity_decrease=0.0,
   min_impurity_split=None,
   class_weight=None,
   ccp_alpha=0.0)
```

The maximum depth of the tree.

If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples.

Recommendation: max_depth between 6 and 10

```
class sklearn.tree.DecisionTreeClassifier(
   criterion='gini',
   splitter='best',
   max_depth=None,
   min_samples_split=2,
   min_samples_leaf=1,
   min_weight_fraction_leaf=0.0,
   max_features=None,
   random_state=None,
   max_leaf_nodes=None,
   min_impurity_decrease=0.0,
   min_impurity_split=None,
   class_weight=None,
   ccp_alpha=0.0)
```

The minimum number of samples required to split an internal node:

If int, then consider min_samples_split as the minimum number.

ceil(min_samples_split * n_samples)
 are the minimum number of
 samples for each split.

Recommendation: min_samples_split = 0.05

```
class sklearn.tree.DecisionTreeClassifier(
   criterion='gini',
   splitter='best',
   max_depth=None,
   min_samples_split=2,
   min_samples_leaf=1,
   min_weight_fraction_leaf=0.0,
   max_features=None,
   random_state=None,
   max_leaf_nodes=None,
   min_impurity_decrease=0.0,
   min_impurity_split=None,
   class_weight=None,
   presort=False)
```

The minimum number of samples required to be at a leaf node.

A split point at any depth will only be considered if it leaves at least

min_samples_leaf training samples in each of the left and right branches.

Recommendation: min_samples_leaf = 0.02

```
class sklearn.tree.DecisionTreeClassifier(
   criterion='gini',
   splitter='best',
   max_depth=None,
   min_samples_split=2,
   min_samples_leaf=1,
   min_weight_fraction_leaf=0.0,
   max_features=None,
   random_state=None,
   max_leaf_nodes=None,
   min_impurity_decrease=0.0,
   min_impurity_split=None,
   class_weight=None,
   ccp_alpha=0.0)
```

Set a user-defined seed for reproducible results.

If int, random_state is the seed used by the random number generator.

Recommendation: Always set a seed (e.g., 314) to ensure reproducible results.

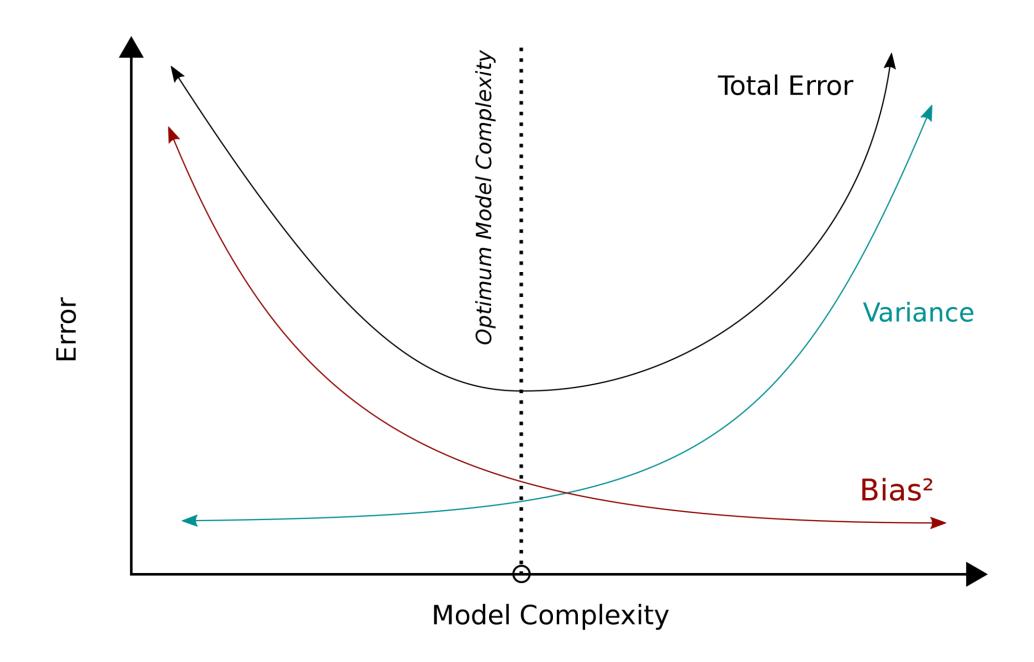
```
class sklearn.tree.DecisionTreeClassifier(
   criterion='gini',
   splitter='best',
   max_depth=None,
   min_samples_split=2,
   min_samples_leaf=1,
   min_weight_fraction_leaf=0.0,
   max_features=None,
   random_state=None,
   max_leaf_nodes=None,
   min_impurity_decrease=0.0,
   min_impurity_split=None,
   class_weight=None,
   ccp_alpha=0.0)
```

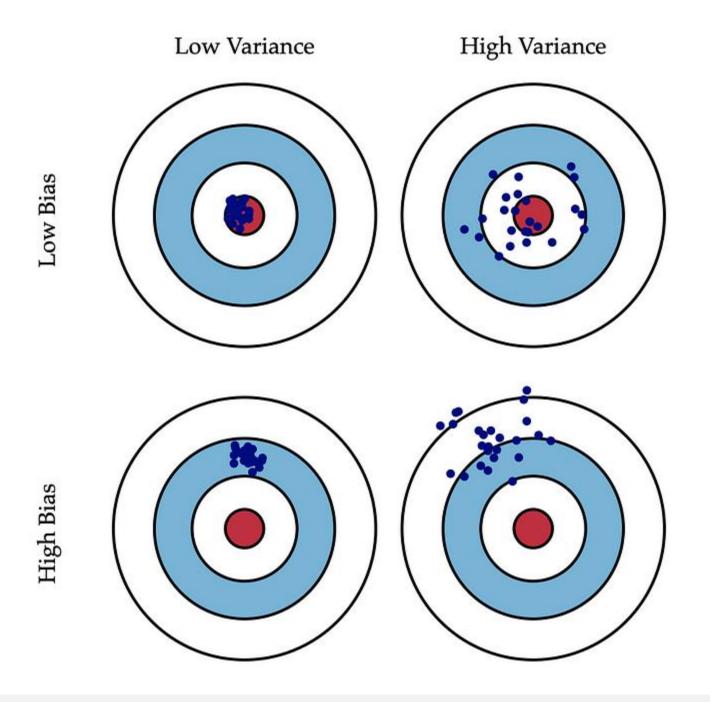
Weights associated with classes in the form {class_label: weight}.

If not given, all classes are supposed to have weight one.

Recommendation: class_weight = 'balanced'

when class imbalance is high





Decision Tree is too small

Underfitting

High bias

BOOSTING

Decision Tree is too large

Overfitting

High variance

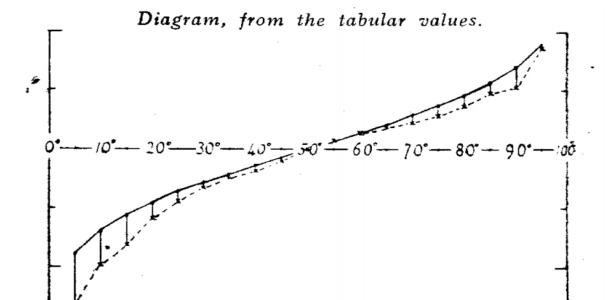


Random Forests

Distribution of the estimates of the dressed weight of a particular living ox, made by 787 different persons.

Degrees of the length of Array o -100	Estimates in lbs.	* Centiles		1
		Observed deviates from 1207 lbs.	Normal p.e =37	- Excess of Observed over Normal
5	1074	- 133	- ġo	+43
10	1109	- 98	- 70	+ 28
15	1126	- 81	- 57	+24
20	1148	- 59	- 46	+13
91 25	1162	- 45	- 37	+ \$
30	1174	- 33	- 29	+ 4
35	1181	- 26	- 21	+ 5
40	1188	- 19	- 14	+ 5
45	1197	- 10	- 7	+ 3
m 50	1207	0	0	. 0
55 :	1214	+ 7	÷ 7	0
60	1219	+ 12	+14	- 2
65	1225	+ 18	+21	- 3 - 6
70	1230	+ 23	+ 29	
93 75	1236	+ 29	+ 37	8
80	1243	+ 36	+ 46	- 10
85	1254	+ 47	+ 57	- 10
90	1267	+ 52	+70	- 18
95	1293	. + 86	+90	- 4

q1, q3, the first and third quartiles, stand at 25° and 75° respectively. m, the median or middlemost value, stands at 50°. The dressed weight proved to be 1198 lbs.

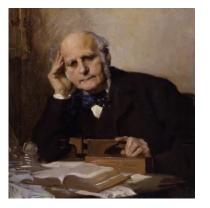


The continuous line is the normal curve with p.e. = 37. The broken line is drawn from the observations.

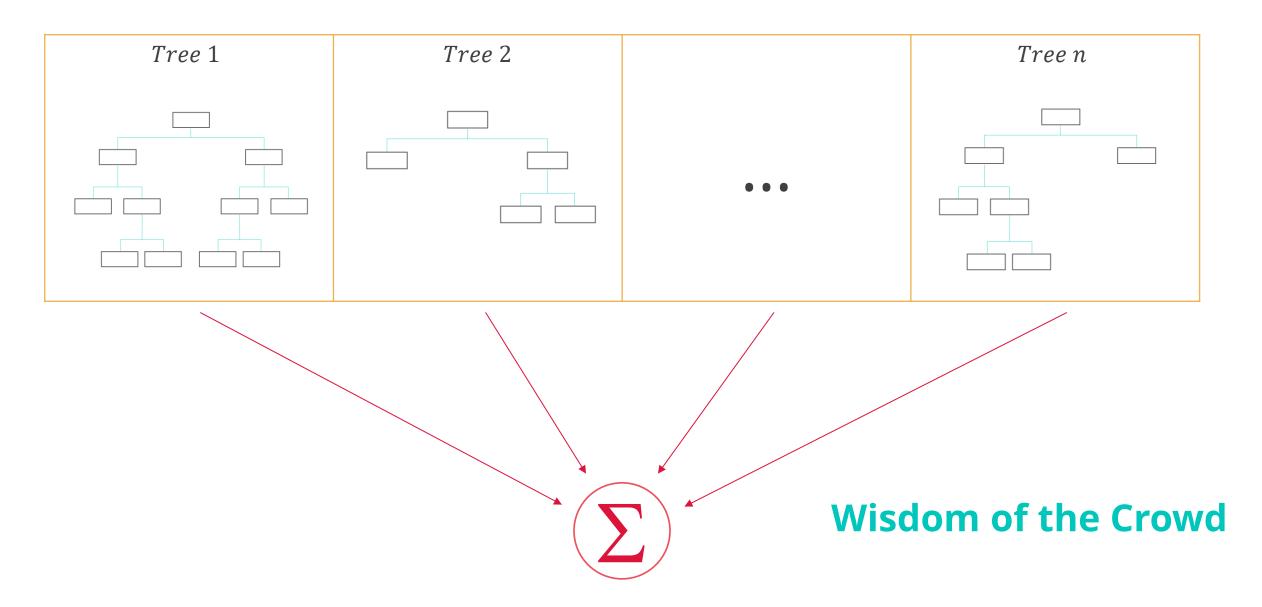
The lines connecting them show the differences between the observed and the normal.

Wisdom of the Crowd

Vox populi



Sir Francis Galton



The final prediction (of the ensemble) is the averaged prediction of the individual classifiers.

A random forest is a meta estimator

that fits a number of decision tree classifiers

on various sub-samples of the dataset

and uses averaging

to improve the predictive accuracy

and control over-fitting.

[scikit-learn]

Random Forests in scikit-learn

```
class sklearn.ensemble.RandomForestClassifier (
    n_estimators='warn',
    criterion='gini',
    max_depth=None,
    min_samples_split=2,
    min_samples_leaf=1,
    min_weight_fraction_leaf=0.0,
    max_features='auto',
    max_leaf_nodes=None,
    min_impurity_decrease=0.0,
    min_impurity_split=None,
    bootstrap=True,
    oob_score=False,
    n_jobs=None,
    random_state=None,
    verbose=0.
    warm_start=False,
    class_weight=None)
```

```
# Import from sklearn.ensemble import RandomForestClassifier
```

```
# Define
clf = RandomForestClassifier()
```

```
# Fit clf.fit(x_train, y_class)
```

```
# Predict clf.predict(x_test)
```

```
class sklearn.ensemble.RandomForestClassifier (
    n_estimators='warn',
    criterion='gini',
    max_depth=None,
    min_samples_split=2,
    min_samples_leaf=1,
    min_weight_fraction_leaf=0.0,
    max_features='auto',
    max_leaf_nodes=None,
    min_impurity_decrease=0.0,
    min_impurity_split=None,
    bootstrap=True,
    oob_score=False,
    n_jobs=None,
    random_state=None,
    verbose=0,
    warm_start=False,
    class_weight=None)
```

The function to measure the quality of a split.

Supported criteria are "gini" for the Gini impurity and "entropy" for the information gain.

```
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    n_estimators='warn',
    criterion='gini',
    max_depth=None,
    min_samples_split=2,
    min_samples_leaf=1,
    min_weight_fraction_leaf=0.0,
    max_features='auto',
    max_leaf_nodes=None,
    min_impurity_decrease=0.0,
    min_impurity_split=None,
    bootstrap=True,
    oob_score=False,
    n_jobs=None,
    random_state=None,
    verbose=0,
    warm_start=False,
    class_weight=None)
```

The maximum depth of the tree.

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Recommendation: max_depth between 6 and 10

```
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    n_estimators='warn',
    criterion='gini',
    max_depth=None,
    min_samples_split=2,
    min_samples_leaf=1,
    min_weight_fraction_leaf=0.0,
    max_features='auto',
    max_leaf_nodes=None,
    min_impurity_decrease=0.0,
    min_impurity_split=None,
    bootstrap=True,
    oob_score=False,
    n_jobs=None,
    random_state=None,
    verbose=0,
    warm_start=False,
    class_weight=None)
```

The minimum number of samples required to split an internal node:

If int, then consider min_samples_split as the minimum number.

Recommendation: min_samples_split = 0.05

```
class sklearn.ensemble.RandomForestClassifier (
    n_estimators='warn',
    criterion='gini',
    max_depth=None,
    min_samples_split=2,
    min_samples_leaf=1,
    min_weight_fraction_leaf=0.0,
    max_features='auto',
    max_leaf_nodes=None,
    min_impurity_decrease=0.0,
    min_impurity_split=None,
    bootstrap=True,
    oob_score=False,
    n_jobs=None,
    random_state=None,
    verbose=0,
    warm_start=False,
    class_weight=None)
```

The minimum number of samples required to be at a leaf node.

A split point at any depth will only be considered if it leaves at least min_samples_leaf training samples in each of the left and right branches.

Recommendation: min_samples_leaf = 0.02

```
class sklearn.ensemble.RandomForestClassifier (
    n_estimators='warn',
    criterion='gini',
    max_depth=None,
    min_samples_split=2,
    min_samples_leaf=1,
    min_weight_fraction_leaf=0.0,
    max_features='auto',
    max_leaf_nodes=None,
    min_impurity_decrease=0.0,
    min_impurity_split=None,
    bootstrap=True,
    oob_score=False,
    n_jobs=None,
    random_state=None,
    verbose=0,
    warm_start=False,
    class_weight=None)
```

The number of features to consider when looking for the best split:

If int, then consider max_features features at each split.

```
If "auto", then
max_features=sqrt(n_features).

If "log2", then
max_features=log2(n_features).

If None, then max_features=n_features.
```

Recommendation: max_features = 'auto'

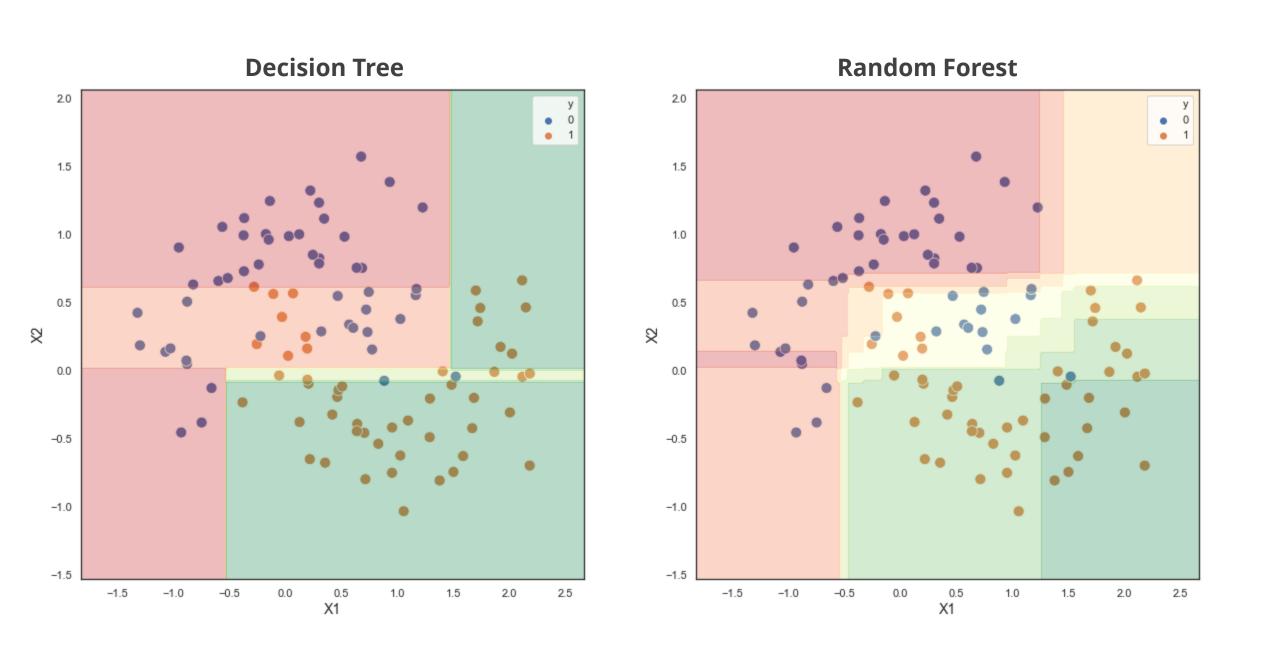
```
class sklearn.ensemble.RandomForestClassifier (
    n_estimators='warn',
    criterion='gini',
    max_depth=None,
    min_samples_split=2,
    min_samples_leaf=1,
    min_weight_fraction_leaf=0.0,
    max_features='auto',
    max_leaf_nodes=None,
    min_impurity_decrease=0.0,
    min_impurity_split=None,
    bootstrap=True,
    oob_score=False,
    n_jobs=None,
    random_state=None,
    verbose=0,
    warm_start=False,
    class_weight=None)
```

Whether bootstrap samples are used when building trees.

Recommendation: bootstrap = True

Bagging (Bootstrap Aggregation)

- O Random Forest: For each tree, it uses a random subset of data as well as a random subset of all available features.
- O The goal is to reduce the variance by doing this.



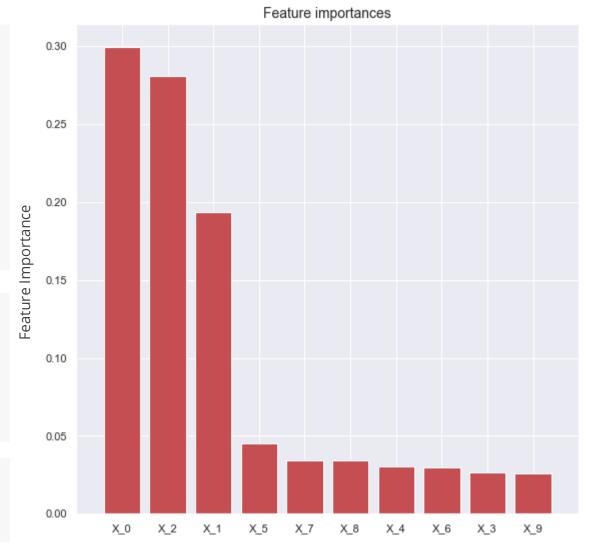
Feature Importance

```
# Build a forest and compute the feature importances
from sklearn.ensemble import RandomForestClassifier

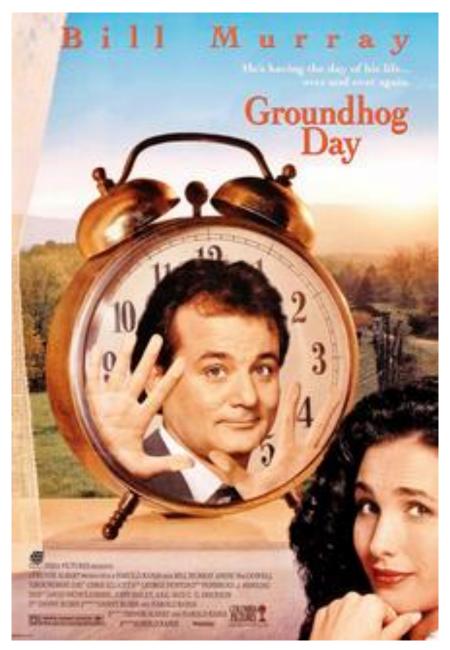
forest = RandomForestClassifier(random_state=314)

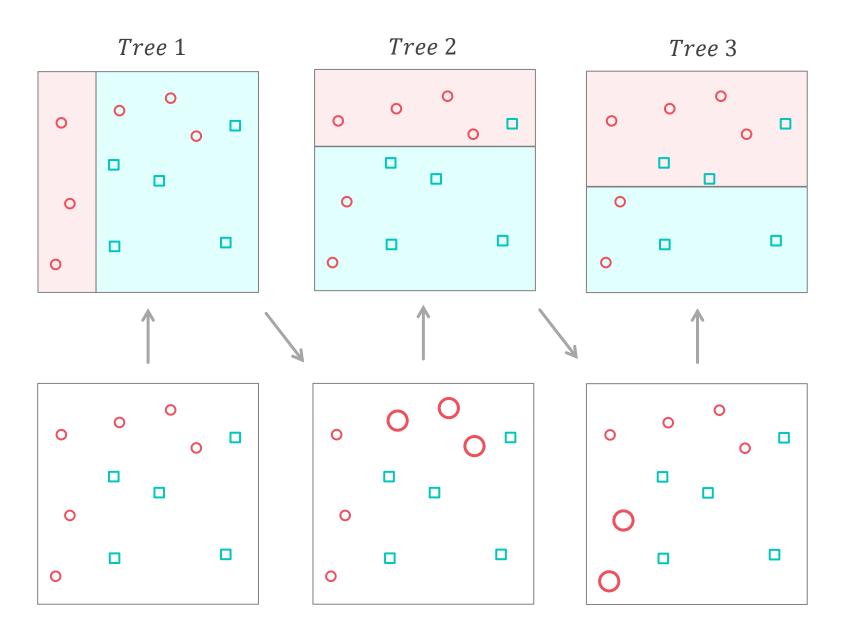
forest.fit(X, y)
```

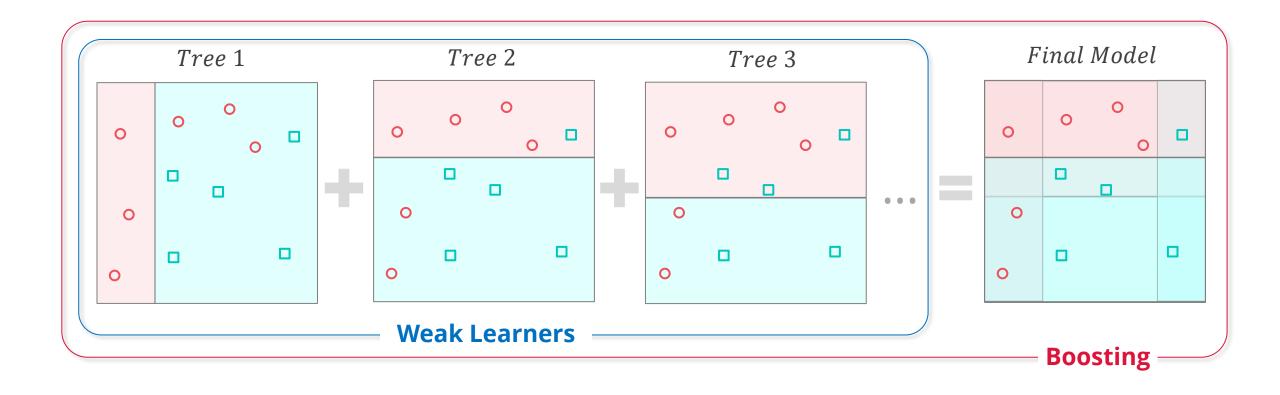
```
importances = forest.feature_importances_
indices = np.argsort(importances)[::-1]
```



Gradient Boosting Classifiers



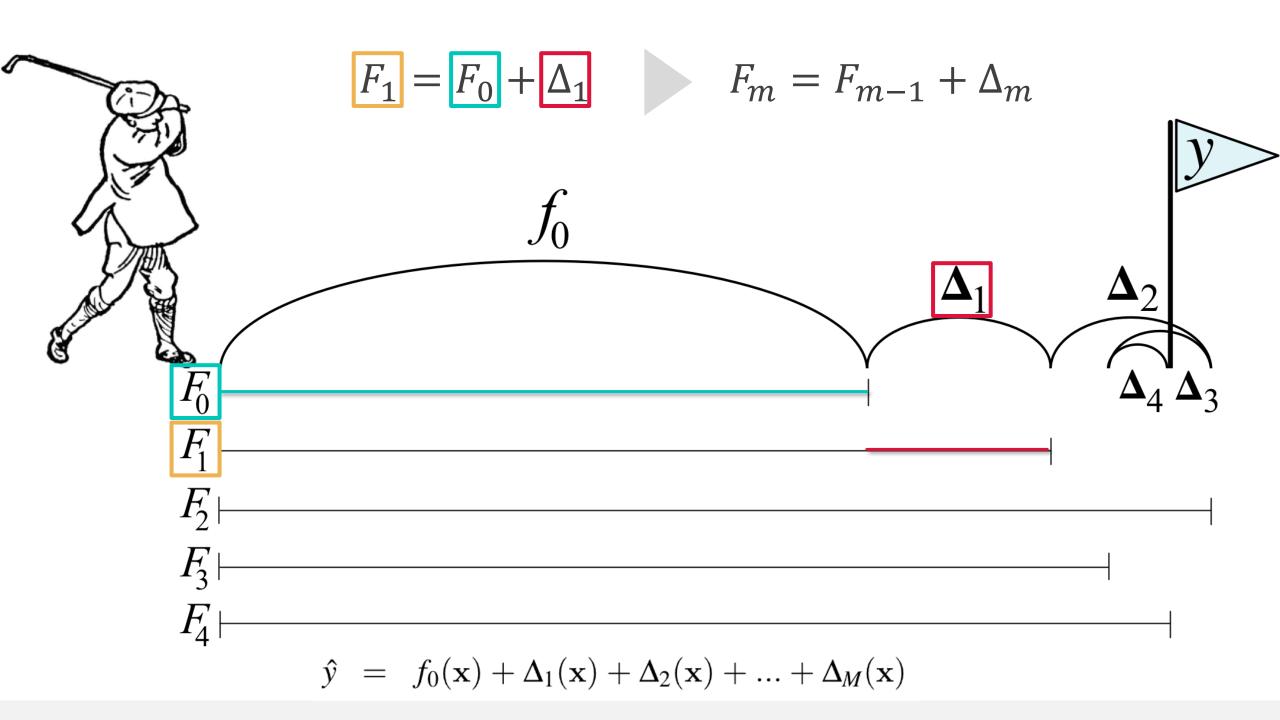


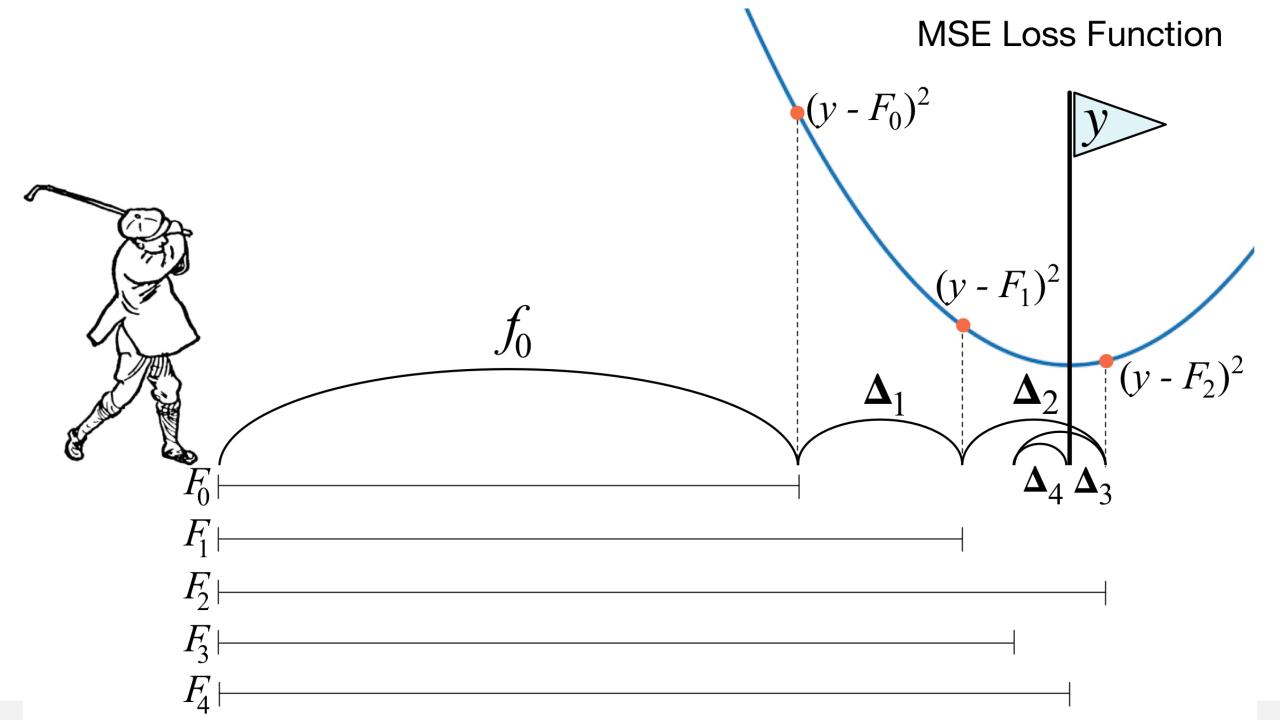


Gradient Boosting = Boosting + Gradient Descent

Greedy Function Approximation: A Gradient Boosting Machine – Jerome Friedman (1999)

A Decision-Theoretic Generalization of On-Line Learning and an Application to Boosting – Y. Freund, and R. Schapire (1997)





Algorithm: $l2boost(X, y, M, \eta)$ returns model F_M

- 1 Let $F_0(X) = \frac{1}{N} \sum_{i=1}^{N} y_i$, mean of target y across all observations
- 2 for m = 1 to M do

Let $r_{m-1} = y - F_{m-1}(X)$ be the residual vector

Train regression tree Δ_m on r_{m-1} , minimizing squared error

$$F_m(X) = F_{m-1}(X) + \eta \Delta_m(X)$$

end

3 return F_M

HYPER-PARAMETERS → **GRID SEARCH**

M= number of steps $\eta=$ learning rate

Gradient Boosting scikit-learn



```
class sklearn.ensemble.GradientBoostingClassifier (
     loss='deviance',
     learning_rate=0.1,
     n_estimators=100,
     subsample=1.0,
     criterion='friedman_mse',
     min_samples_split=2,
     min_samples_leaf=1,
     min_weight_fraction_leaf=0.0,
     max_depth=3,
     min_impurity_decrease=0.0,
     min_impurity_split=None,
     init=None.
     random_state=None,
     max_features=None.
     verbose=0.
     max_leaf_nodes=None,
     warm_start=False,
     presort='auto',
     validation_fraction=0.1,
     n_iter_no_change=None,
     tol=0.0001)
```

The learning rate shrinks the contribution of each tree by learning rate.

There is a trade-off between learning_rate and n_estimators.

 η = learning rate

```
class sklearn.ensemble.GradientBoostingClassifier (
     loss='deviance',
     learning_rate=0.1,
     n_estimators=100,
     subsample=1.0,
     criterion='friedman_mse',
     min_samples_split=2,
     min_samples_leaf=1,
     min_weight_fraction_leaf=0.0,
     max_depth=3,
     min_impurity_decrease=0.0,
     min_impurity_split=None,
     init=None,
     random_state=None,
     max_features=None,
     verbose=0.
     max_leaf_nodes=None,
     warm_start=False,
     presort='auto',
     validation_fraction=0.1,
     n_iter_no_change=None,
     tol=0.0001)
```

The **number of boosting stages** to perform.

Gradient boosting is fairly robust to over-fitting so a large number usually results in better performance.

M= number of steps

```
class sklearn.ensemble.GradientBoostingClassifier (
     loss='deviance',
     learning_rate=0.1,
     n_estimators=100,
     subsample=1.0,
     criterion='friedman_mse',
     min_samples_split=2,
     min_samples_leaf=1,
     min_weight_fraction_leaf=0.0,
     max_depth=3,
     min_impurity_decrease=0.0,
     min_impurity_split=None,
     init=None.
     random_state=None,
     max_features=None,
     verbose=0.
     max_leaf_nodes=None,
     warm_start=False,
     presort='auto',
     validation_fraction=0.1,
     n_iter_no_change=None,
     tol=0.0001)
```

The **fraction of samples** to be used for fitting the individual base learners.

If smaller than 1.0 this results in Stochastic Gradient Boosting.

Choosing subsample < 1.0 leads to a reduction of variance and an increase in bias.

```
class sklearn.ensemble.GradientBoostingClassifier (
     loss='deviance',
     learning_rate=0.1,
     n_estimators=100,
     subsample=1.0,
     criterion='friedman_mse',
     min_samples_split=2,
     min_samples_leaf=1,
     min_weight_fraction_leaf=0.0,
     max_depth=3,
     min_impurity_decrease=0.0,
     min_impurity_split=None,
     init=None,
     random_state=None,
     max_features=None,
     verbose=0.
     max_leaf_nodes=None,
     warm_start=False,
     presort='auto',
     validation_fraction=0.1,
     n_iter_no_change=None,
     tol=0.0001)
```

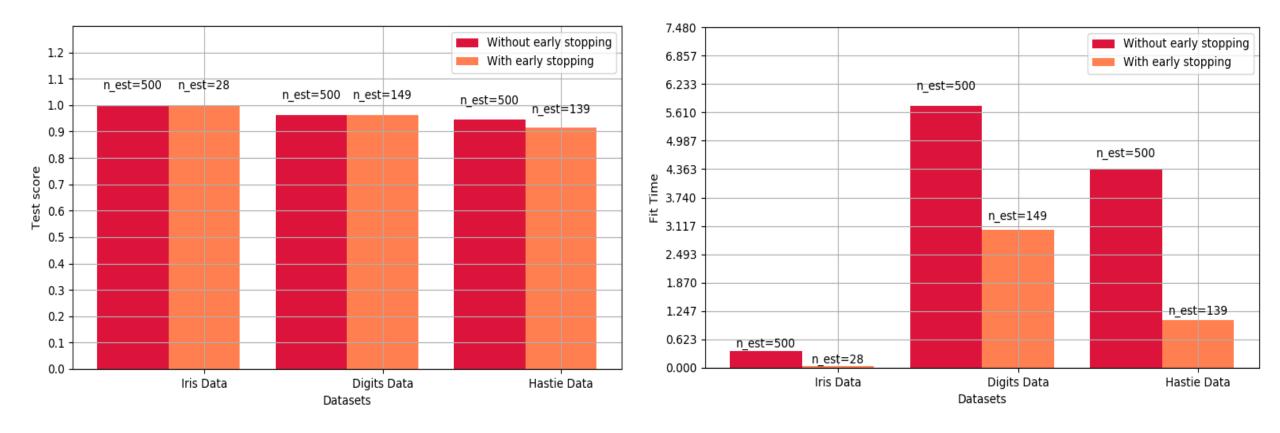
The **proportion of training data** to set aside as validation set for **early stopping**.

Must be between 0 and 1.

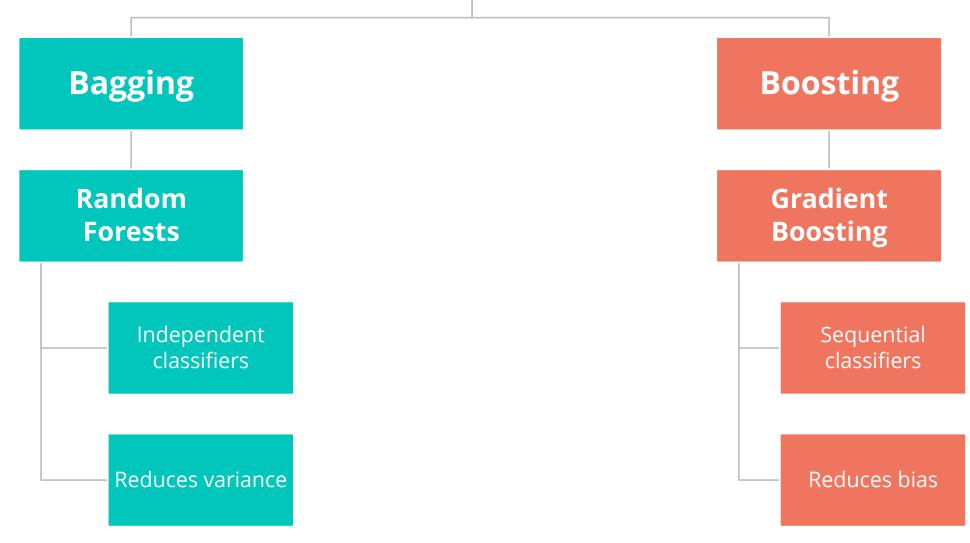
Only used if n_iter_no_change is set to an integer.

```
class sklearn.ensemble.GradientBoostingClassifier (
     loss='deviance',
     learning_rate=0.1,
     n_estimators=100,
     subsample=1.0,
     criterion='friedman_mse',
     min_samples_split=2,
     min_samples_leaf=1,
     min_weight_fraction_leaf=0.0,
     max_depth=3,
     min_impurity_decrease=0.0,
     min_impurity_split=None,
     init=None,
     random_state=None,
     max_features=None,
     verbose=0.
     max_leaf_nodes=None,
     warm_start=False,
     presort='auto',
     validation_fraction=0.1,
     n_iter_no_change=None,
     tol=0.0001)
```

n_iter_no_change is used to decide
 if early stopping will be used
 to terminate training
when validation score is not improving.



Ensembling



Further Reading: Gradient Boosting

Gradient boosting: Distance to target

Terence Parr and Jeremy Howard

https://explained.ai/gradient-boosting/L2-loss.html

3.2.4.3.5. sklearn.ensemble.GradientBoostingClassifier

https://scikit-learn.org/stable/modules/generated/sklearn.ensemble.GradientBoostingClassifier.html

Boosting algorithm: AdaBoost



https://towardsdatascience.com/boosting-algorithm-adaboost-b6737a9ee60c

XGBoost Documentation

https://xgboost.readthedocs.io/en/latest/



Summary



Complexity

Neural Network

Support Vector Machine

Gradient Boosting

Random Forest

Classification/Regression Tree

Linear/Logistic Regression

Interpretability

Method	Advantages	Disadvantages
Linear / Logistic Regression	 Model fit diagnostics Interpretable coefficients, even for categorical predictors[†] Tests for predictors 	Simple/linear relationshipsUnable to handle missing valuesBinarize categorical predictors
Classification and Regression Trees	 Categorical predictors[†] Handles missing values and non-linear relationships Visualization/interpretation Variable importance 	Prone to overfittingSlow for categorical data with many levels
Random Forest & Gradient Boosting	 Categorical predictors[†] & missing values Variable importance Controls overfitting (bias and/or variance) 	No visualizationSlow for large datasetsHyper-parameters is required
Support Vector Machine	 Vary complexity by changing kernel/tuning parameters 	O Hard to visualize/interpretO Hyper-parameters is requiredO Binarize categorical predictors
Neural Network	Vary complexity by changing number of layers/tuning parametersHigh accuracy	 Hard to visualize/interpret, "black box" Tuning parameters, stopping criteria Binarize categorical predictors