O_p is linear if

① Given a const. λ & a fn. $f(x)$ $\quad O_p(\lambda f(x)) = \lambda O_p(f(x))$

② If $f(x), g(x)$
$O_p(f+g) = O_p(f) + O_p(g)$

I.V.P.	B.V.P.
n^{th} order D.E +	n^{th} order D.E +
n I.C's	n B.C's
$y(x_0) = \alpha_1$	$y(x_1) = \beta_1$
$y'(x_0) = \alpha_2$	$y(x_2) = \beta_2$
⋮	⋮
$y^{(n-1)}(x_0) = \alpha_n$	$y(x_n) = \beta_n$
n Initial Condition	n Boundary Condition,

$\ln y \equiv \log_e y$

$e^{A+B} = e^A \cdot e^B$

$\log y \longrightarrow$ natural log.

In quat fiance.

$\ln y = x(\ln x - 1) + C$ Take exp.

$y = e^{x(\ln x-1)+C} = e^{x(\ln x-1)} \cdot \boxed{e^{C}}_{A}$

$= A\, e^{x(\ln x-1)}$

$$y' + P(x)y = Q(x)$$

I.F. $\quad R(x)$

$$\underbrace{R(x)\left[y' + P(x)y\right]}_{\frac{d}{dx}\left[R(x)y\right]} = R(x)Q(x)$$

$$\frac{d}{dx}\left[R(x)y\right] = R(x)Q(x)$$

$$\int d\left(R(x)y\right) = \int R(x)Q(x)\,dx$$

$$R(x)\,y = \int R(x)\,\varphi(x)\,dx + C$$

$$y = \frac{1}{R(x)} \int R(x)\,\varphi(x)\,dx + \frac{C}{R(x)}$$

$$R(x) = e^{\int P\,dx}$$

$$y' + P\,y = \varphi$$

$$2^{x} y' + 3e^{x} y = x^{2}$$

$$y' + \underbrace{\frac{3}{2^{x}} e^{x}}_{P(x)} y = \frac{1}{2} x^{2}$$

$$\boxed{y''} = f(x, y')$$

Put $y' = p(x)$

$\rightarrow y'' = p'(x)$

$$p' = f(x, p)$$

$$\frac{dp}{dx} = f(x, p)$$

1 $\longrightarrow p$

2 Put p into $y' = p$

$\to y(x)$

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=x^3$$

y is missing.

Put $P = y'$

$P' = y''$

$$xP'+2P=x^3$$

$$\frac{dP}{dx}+\boxed{\frac{2}{x}}P=x^2$$

$$2\int\frac{1}{x}\,dx$$

$R(x) = e$

$Ly = 0$ $\quad L$ – n^{th} order diff op.

$\downarrow$

n sol^{n}s

Set of all sol^{n}s

$y_1 \quad y_2 \quad y_{12} \quad y_3$

$\boxed{\text{Vector space}}$

$y_4 = y_2 + y_3$

$c\,y_1(x)$

$y_{10} = y_8 + y_9$

$\exists$ n solⁿ of $Ly = 0$

$y_1, y_2, \text{------}, y_n$

Take consts. $\alpha_1, \alpha_2, \text{---}, \alpha_n$

linear comb $\alpha_1 y_1 + \alpha_2 y_2 + \text{----} + \alpha_n y_n$

Now let the L.C $= 0$

$\alpha_1 y_1 + \text{----} + \alpha_n y_n = 0$

If $\Rightarrow$ all α_i's $\neq 0$

L.D

If all α_i's $= 0$ Linearly Indep. set

$L_y = 0$ all $\boxed{\text{L.I. set}}$

Now we can't write one sol^n in terms of the other.

$$ay' + by = 0$$

$$a\frac{dy}{dx} = -by$$

$$\int \frac{dy}{y} = -\frac{b}{a}\int dx$$

$$\log y = -\frac{b}{a}x + K$$

$$y = Ce^{\lambda x}$$

$$ay'' + by' + cy = 0$$

1st order

$by' + cy = 0$

2nd order

$ay'' + by' + cy = 0$

$$y = e^{\lambda x} \longrightarrow ay'' + by' + cy = 0$$

$$\underbrace{\left(a\lambda^2 + b\lambda + c\right)}_{\text{A.E.} \quad = 0} e^{\lambda x} = 0$$

$\llcorner$ o?

$$ay'' + by' + cy = 0 \longrightarrow a\lambda^2 + b\lambda + c \dots$$

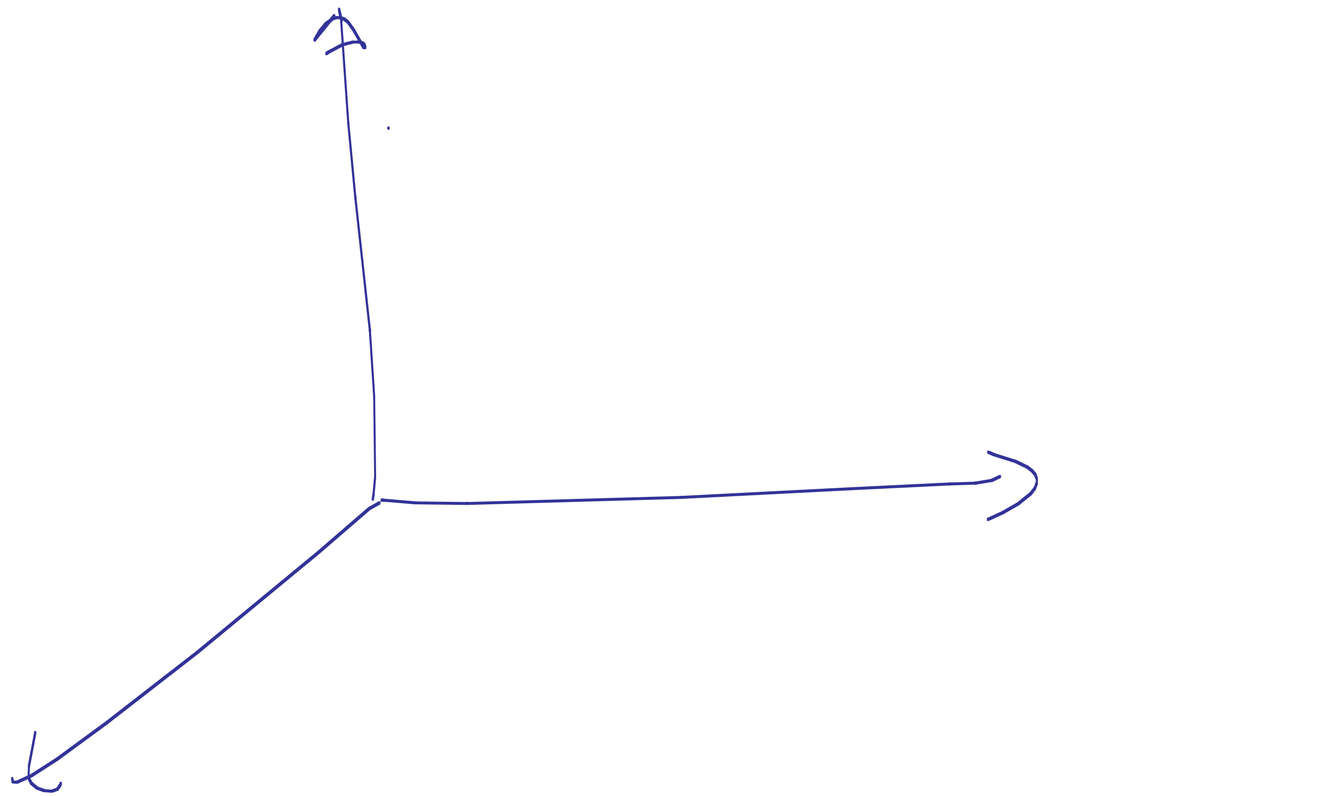
$$e^{-\frac{b}{2a}x} \quad y = x e^{2x} \longrightarrow \underbrace{ay'' + by' + cy = 0}$$

$$c_1 e^{iqx} + c_2 e^{-iqx}$$

$$c_1\left[\cos qx + i\sin qx\right] + c_2\left[\cos qx - i\sin qx\right]$$

$$\cos qx\,\underbrace{\left[c_1 + c_2\right]}_{A} + \sin qx\,\underbrace{\left[c_1 - c_2\right]i}_{B}$$

$$y_1 = e^{px}\cos qx \qquad \boxed{y_2 = i\,e^{px}\sin qx}$$



$\lambda = 0 \quad 0 \quad 0 \quad 0$

$e^{\lambda x} \qquad x e^{\lambda x} \qquad x^2 e^{\lambda x} \qquad x^3 e^{\lambda}$

$y_3 = 1 \qquad y_4 = x \qquad y_5 = x^2 \qquad y_6 = x^3$

$$\boxed{\cos 3x}$$

$$y_p = A\cos 3x + B\sin 3x$$

2 coeff.

$$y_p = K\,\mathrm{Re}\left[e^{i3x}\right]$$

$$ay'' + by' + cy = \alpha \cos\beta x$$

$$y_p = \boxed{A\cos\beta x + B\sin\beta x}$$

$$\boxed{e^{\pm i\theta}}$$

$$I = \int e^{x} \cos x \, dx$$

$$I = \int e^{x} \mathrm{Re}\, e^{+ix} \, dx = \mathrm{Re} \int e^{(1+i)x} \, dx$$

$$\sin x = \mathrm{Im}\, e^{ix}$$

$$ay'' + by' + cy = \alpha e^{\boxed{\beta} x}$$

$$y_c = e^{\beta x}\left[A + Bx\right] \qquad y_p = Ce^{\beta x}_{x^2}$$

Ex:

$$ax\frac{dy}{dx} + by = 0$$

$$ax^2 \frac{d^2y}{dx^2} + \beta x \frac{dy}{dx} + cy = 0$$

Let $y = x^\lambda \longrightarrow$

$$\underbrace{\left[a\lambda^2 + (\beta - a)\lambda + c\right]}_{=0} x^\lambda = 0$$

Case : 3 $\qquad \lambda = \alpha \pm i\beta$

$$y = Ax^{\alpha+i\beta} + Bx^{\alpha-i\beta} = x^{\alpha}\underbrace{\left(Ax^{i\beta} + Bx^{-i\beta}\right)}$$

$$x^{\pm i\beta} = e^{\log x^{\pm i\beta}} = e^{\pm i\boxed{\beta\log x}}$$

$$= e^{\pm i\boxed{\theta}}$$

$\cos(\beta\log x) + i\sin(\beta\log x)$

$$ay'' + by' + cy = 0$$

$$y = e^{\lambda x}$$

$$a\lambda^2 + b\lambda + c = 0$$

Euler

$$ax^2 y'' + bxy' + cy = 0$$

$$y = x^\lambda$$

$$a\lambda^2 + (b-a)\lambda + c = 0$$

Euler D.E. $\xrightarrow{\quad t=\log x \quad}$ Const Coeff Linear

$y(x)$ $\qquad\qquad\qquad\qquad$ $y(t)$

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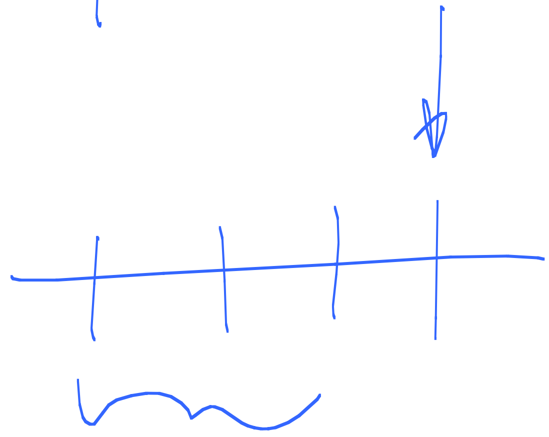

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