# 1

Consider:

$$p(y,t;y',t') = \frac{1}{2c\sqrt{\pi(t'-t)}} \exp\left(-\frac{(y'-y)^2}{4c^2(t'-t)}\right)$$

We can drop $y, t$ (Substitute $y'-y, t'-t$ etc.) so we only need to work on:

$$p(y',t') = \frac{1}{2c\sqrt{\pi t'}} \exp\left(-\frac{y'^2}{4c^2t'}\right)$$

Calculating the necessary derivatives for the FKE, we get:

$$\begin{aligned}
\frac{\partial p}{\partial t'} &= \frac{1}{2c\sqrt{\pi}} \cdot \left(-\frac{1}{2\sqrt{t'^3}}\right) \exp\left(-\frac{y'^2}{4c^2t'}\right) + \frac{1}{2c\sqrt{\pi t'}} \exp\left(-\frac{y'^2}{4c^2t'}\right) \cdot \left(\frac{y'^2}{4c^2t'^2}\right) \\
&= \frac{1}{2c\sqrt{\pi t'}} \exp\left(-\frac{y'^2}{4c^2t'}\right) \cdot \left(-\frac{1}{2t'} + \frac{y'^2}{4c^2t'^2}\right) \\
&= p \cdot \left(\frac{-2c^2t' + y'^2}{4c^2t'^2}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial p}{\partial y'} &= p \cdot \left(-\frac{y'}{2c^2t'}\right) \\
\frac{\partial^2 p}{\partial y'^2} &= p \cdot \left(-\frac{y'^2}{4c^4t'^2}\right) - \frac{p}{2c^2t'} \\
&= p\left(\frac{y'^2 - 2c^2t'}{4c^4t'^2}\right) \\
&= \frac{1}{c^2}\left[p\left(\frac{y'^2 - 2c^2t'}{4c^2t'^2}\right)\right] \\
&= \frac{1}{c^2}\frac{\partial p}{\partial t'}
\end{aligned}$$

# 2

# 3