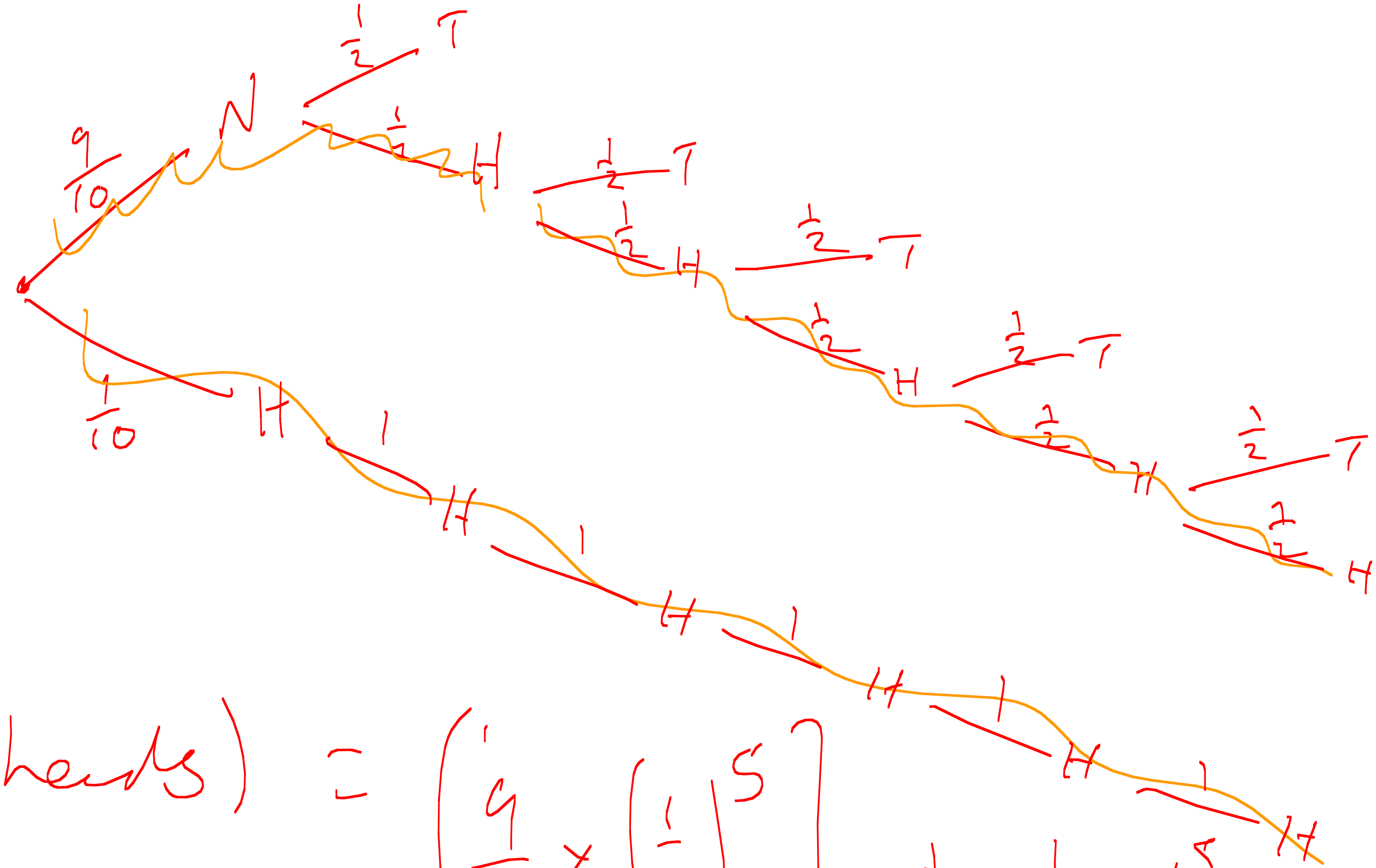
1

N = Normal coin H = 2 headed



$$p(5 \text{ heads}) = \left[\frac{9}{10} \times \left(\frac{1}{2}\right)^5\right] + \frac{1}{10} \times 1^5 = \frac{41}{320} \approx 13\%$$

② $$P(H \mid 5\text{heads}) = \frac{P(5\text{heads} \mid H) \times P(H)}{P(5\text{heads})}$$

$$= \frac{1 \times \frac{1}{10}}{41/320} \approx 78\%$$

(2) Consider a test for a disease. Let

$D+$: person tested has the disease

$D-$: " " does not " "

Y : Test is positive

N : Test is negative

95% accurate

1% of population has the disease.

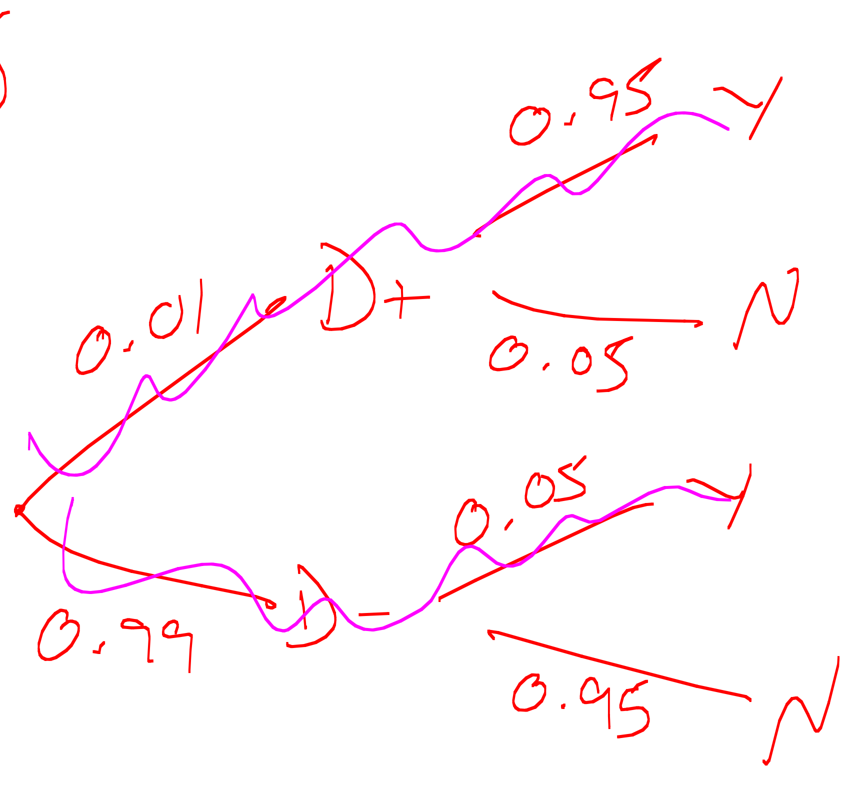
$P(D- \mid Y)$

$$P(D-|Y) = \frac{P(Y|D-) \times P(D-)}{P(Y)}$$

$$P(Y|D-) = 0.05$$

$$P(D-) = 0.99$$

$$P(Y) = 0.059$$



$$P(D-|Y) = \frac{0.05 \times 0.99}{0.059}$$

$$= 0.839$$

$$\approx 84\%$$

very high!

5% ✗

50% ?

21% ✗

80% ?

Birthday problem

60 people

What is the prob that at least 2 people share the same birthday?

$$P(S) = P(\text{someone shares with at least someone else})$$

$$P(d) = P(\text{all 60 have different birthdays})$$

$$P(S) + P(d) = 1$$

$p(d)$

Person 1 $\dfrac{365}{365} \times$ $= 0.5\%$

2 $\dfrac{364}{365} \times$

3 $\dfrac{363}{365} \times$

$\vdots$

60 $\dfrac{306}{365}$

$p(s) = 1 - 0.5\%$

$= 99.5\%$

Monty Hall