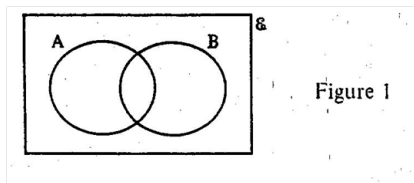


Probability and Statistics Problems

1. Draw six diagrams similar to figure 1 and shade the following sets



- (a) $A \cap B$
 - (b) $A \cup B$
 - (c) A'
 - (d) $A' \cap B$
 - (e) $B' \cap A$
 - (f) $(B \cup A)'$
2. There are 176 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.
 - 112 take systems support
 - 78 take developing software
 - 81 take networking
 - 41 take developing software and systems support
 - 34 take networking and developing software
 - 43 take systems support and networking
 - 8 take all three extra options

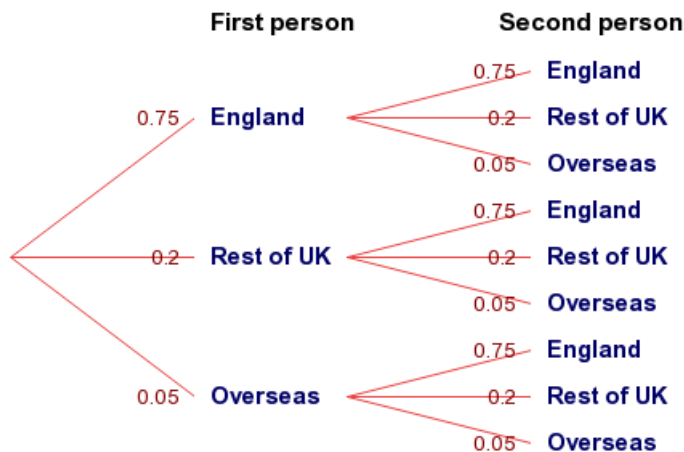
- (a) Draw a Venn Diagram to represent this information.

A student from the course is chosen at random. Find the probability that the student takes

- (b) none of the three extra options
- (c) networking only.

Students who want to become technicians take systems support or networking. Given that a randomly chosen student wants to become a technician,

- (d) find the probability that this student takes all three extra options
3. In a large town 75% of the population were born in England, 20% in the rest of the UK and 5% abroad. Two people are selected at random.



You may use the above tree diagram in answering this question. Find the probability that

- both these people were born in the rest of the UK.
 - at least one of these people was born in England.
 - neither of these people was born overseas.
- (d) Find the probability that both these people were born in the rest of the UK given that neither was born overseas.
- (e) 6 people are selected at random. Find the probability that at least one of them was not born in England.
- (f) An interviewer selects n people at random. The interviewer wishes to ensure that the probability that at least one of them was not born in England is more than 80%. Find the least possible value of n .
4. The events A and B are independent such that $P(A) = 0.14$ and $P(B) = 0.23$. Find
- $P(A \cap B)$
 - $P(A \cup B)$
 - $P(A|B')$
5. The discrete random variable X can take only the values 8, 9 or 10. For these values the cumulative distribution function is defined by

$$F(x) = \frac{(x+k)^2}{144} \quad \text{for } x = 8, 9, 10$$

where k is a positive integer

- (a) Find k .
 - (b) Find the probability distribution of X .
6. The discrete random variable X has the probability function $f(x)$ defined by

$$f(x) = kx^2 \quad x = 2, 3, 4, 5, 6$$

- (a) Construct a table showing the probability distribution of the random variable X .
 - (b) Find the value of k .
 - (c) Find $E(X)$ and $Var(X)$.
 - (d) Find the mean and variance of the random variable Y where $Y = 3X - 7$
7. Bob plays 12 squash games. In each game he either wins or loses.
- (a) State, in this context, two conditions needed for a binomial distribution to arise.
 - (b) Assuming these conditions are satisfied, define a variable in this context which has a binomial distribution.
 - (c) The random variable X has the distribution $B(24, p)$ where $0 < p < 1$. Given that $P(X = 7) = P(X = 6)$ find the value of p .
8. The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx^2(7-x) & 0 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that $k = \frac{12}{2401}$
 - (b) Find $E(X)$
 - (c) Find $P(X < 4)$
9. The length of a telephone call made to a company is denoted by the continuous random variable T . It is modelled by the probability density function

$$f(x) = \begin{cases} kt & 0 \leq t \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the value of k is $\frac{1}{18}$
- (b) Find $P(T > 1)$
- (c) Calculate an exact value for $E(T)$ and for $Var(T)$

10. The continuous random variable X has cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4}x^2(5 - x^2) & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

- (a) Find $P(X > 0.6)$
- (b) Find the probability density function $f(x)$ of X
- (c) Calculate $E(X)$ and show that, to 3 decimal places, $Var(X) = 0.057$

For the following questions use Standard Normal Distribution tables which can be found at the back of any statistics or probability text book

11. The lifetimes of bulbs used in a lamp are normally distributed. A company X sells bulbs with a mean lifetime of 856 hours and a standard deviation of 58 hours.

- (a) Find the probability of a bulb, from company X , having a lifetime of less than 833 hours
- (b) In a box of 400 bulbs, from company X , find the expected number having a lifetime of less than 833 hours.

A rival company Y sells bulbs with a mean lifetime of 882 hours and 19% of these bulbs have a lifetime of less than 830 hours.

- (c) Find the standard deviation of the lifetimes of bulbs from company Y .

Both companies sell bulbs for the same price.

- (d) State which company you would recommend

12. In large-scale tree-felling operations, a machine cuts down trees, strips off the branches and then cuts the trunks into logs of length X metres for transporting to a sawmill. It may be assumed that values of X are normally distributed with mean 4.3 and standard deviation 0.17.

- (a) Determine $P(X < 4.5)$
- (b) Determine $P(X > 4)$
- (c) Determine $P(4 < X < 4.5)$

13. A second hand car dealer has 10 cars for sale. She decides to investigate the link between the age of the cars, x years, and the mileage, y thousand miles. The data collected from the cars are shown in the table below.

Age (years) x	2.5	3.5	5.5	4.5	5.5	5.5	5	4.5	4	4.5
Mileage (1000) y	33	24	39	31	34	32	22	45	21	40

You may assume that

$$\sum x = 45; \quad \sum y = 321; \quad \sum x^2 = 221; \quad \sum xy = 1460;$$

- (a) Find S_{xx} and S_{xy}
 - (b) Find the equation of the regression line of y on x in the form $y = a + bx$.
Give the values of a and b to 2 decimal places
 - (c) Give a practical interpretation of the slope b
 - (d) Using your answer to part (b), find the mileage predicted by the regression line for a 45 year old car
14. A sample of bivariate data was taken and the results were summarised as follows:

$$n = 10; \quad \sum x = 771; \quad \sum x^2 = 60379; \quad \sum y = 723;$$

$$\sum y^2 = 53125; \quad \sum xy = 55905;$$

Show that the value of the the product moment correlation coefficient is 0181, correct to 3 significant figures