Gautam Buddha University

Engineering Mathematics-III (MA-201)

Second semester (2016-2017)

Tutorial Sheet-3

Exact Differential Equations, Integrating Factors (I.F) for $M(x,y)dx + N(x,y)dy = 0$.

Q.1 For the differential equation $M(x,y)dx + N(x,y)dy = 0$, show that

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = g(x), \text{ a function of } x \text{ only} \quad \Longleftrightarrow \quad \mu = e^{\int g(x)dx} \text{ is an IF.}$$

Q.2 For the differential equation $M(x,y)dx + N(x,y)dy = 0$, show that

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = g(y), \text{ a function of } y \text{ only} \quad \Longleftrightarrow \quad \mu = e^{\int g(y)dy} \text{ is an IF.}$$

Q.3 Show that if

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{Ny - Mx} = g(z), \quad \text{a function of product } z = xy$$

then $\mu = e^{\int g(z)dz}$ is an IF for the differential equation $M(x,y)dx + N(x,y)dy = 0$.

Q.4 Show that if

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N - M} = g(z), \quad \text{a function of sum } z = x + y$$

then $\mu = e^{\int g(z)dz}$ is an IF for the differential equation $M(x,y)dx + N(x,y)dy = 0$.

Q. 5 If $Mx + Ny = 0$ for the differential equation $M(x,y)dx + N(x,y)dy = 0$ then show that

$$\frac{1}{xy}, \frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{x^2 + y^2}, \cdots, \text{etc are various IFs.}$$

Q. 6 Solve the following DEs by first finding an I.F. μ

(a) $(xy - 1)dx + (x^2 - xy)dx = 0$

(b) $(x + 2)\sin y\, dx + x\cos y\, dy = 0$

(c) $(x + 3y^2)dx + 2xy\, dy = 0$

(d) $(x^3 + xy^3)dx + 3y^2\, dy = 0$

(e) $(y\ln y - 2xy)dx + (x + y)dy = 0$

(f) $y\, dx + (2x - ye^y)dy = 0$

(g) $y^2\, dx + x\, dy - y\, dx = 0$

(h) $y(1 + 6xy)dx + (4y - x)dy = 0$

Q. 7 Solve the following DEs by first finding an I.F.

(a) $ydx + x(1 + y)dx = 0$

(b) $x\, dy + 2y\, dx = xy\, dy$

(c) $(y^2 + xy + 1)dx + (x^2 + xy + 1)dy = 0$

(d) $y\, dx + (x + 3x^3y^4)dy = 0$

(e) $e^y\, dx + e^x\, dy = 0$

Q. 8 Solve the following DEs by inspection:

(a) $x\, dy - y\, dx = (1 + y^2)dy$ [Hint: $d(-\frac{x}{y}) = \frac{xdy - ydx}{y^2}$]

(b) $x\, dy + y\, dx = \sqrt{xy}\, dy$ [Hint: $d(2\sqrt{xy}) = \sqrt{\frac{y}{x}}\, dx + \sqrt{\frac{x}{y}}\, dy$]

(c) $dy + (y/x)dx = \sin x\, dx$

(d) $y\, dx - x\, dy = xy^3\, dy$