Gautam Buddha University

Engineering Mathematics-III (MA-201)

Second semester (2016-2017)

Tutorial Sheet-5.Part-1

General Linear Differential equations, Homogeneous Linear DEs with Real Constants Coefficients, Cauchy-Euler Equations.

$$\text{Homegeneous LDE of order-2 :} \quad P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0 \qquad (*)$$

$$\text{Cauchy-Euler Equation of order n :} \quad x^n\frac{d^ny}{dx^n} + p_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \cdots + p_{n-1}x\frac{dy}{dx} + p_ny = 0 \qquad \text{(CE)}$$

Q. 1 Let y_1, y_2 be any two solutions of equation $(*)$ on interval I. Suppose that $W(x)$ is the Wronskian of y_1, y_2. Then either $W(x) = 0$ for all $x \in I$ or it is nowhere zero over I. Hence for two independent solutions y_1, y_2 of $(*)$, $W(x) \neq 0$ for all $x \in I$

Q. 2 Consider functions $y_1(x) = x^3$ and $y_2(x) = x^2|x|$ on interval $I = [-1, 1]$. Let $W(x)$ be the Wronskian of y_1, y_2. Show that

(a) $W(x) = 0$ for all $x \in I$.

(b) y_1, y_2 are LI on I

Do you think this contradicts $Q.1$? if not explain?

Q. 3 Find the general solutions of following homogeneous linear differential equations with real constant coefficients:

(a) $(D^3 - 7D - 6)y = 0$

(b) $(D^3 - D^2 - D + 1)y = 0$

(c) $(D^4 - 4)y = 0$

(d) $(D^4 + 4)y = 0$

(d) $(D^2 + 1)^3(D - 5)^2(D + \sqrt{3})y = 0$

(e) $(D^2 + D + 1)^2(D^2 + 6D + 5)^2y = 0$

Here D represents the differential operator, i.e., $D \equiv \dfrac{d}{dx}$.

Q.4 (Cauchy-Euler Equation)

(a) Consider $x^2y'' + p_1\,xy' + p_2y = 0$, where p_1, p_2 are constants. Show that the change of independent variable x to variable z via function $z = \ln x$ transforms CE equation to a linear DE with constants coefficients. (Answer: $\dfrac{d^2y}{dz^2} + (p_1 - 1)\dfrac{dy}{dz} + p_2y = 0$)

(b) Transform the equation $x^3y''' + p_1\,x^2y'' + \ p_2\,xy' + p_3y = 0$, where p_1, p_2, p_3 are constants to a linear DE with constant coefficients by change of independent variable via, $z = \ln x$.
(Answer: $\dfrac{d^3y}{dz^3} + (p_1 - 3)\dfrac{d^2y}{dz^2} + (p_2 - p_1 + 2)\dfrac{dy}{dz} + p_3y = 0$)

(c) Could you generalize this for higher orderdifferential equation (CE)?

Q.5 Find the general solutions of following Cauchy-Euler equations :

(a) $(x^2D^2 + 3xD + 1)y = 0$

(b) $(9x^2D^2 + 3xD + 10)y = 0$

(c) $(x^3D^3 + 9x^2D^2 + 18xD + 6)y = 0$

(d) $(x^3D^3 + 5x^2D^2 + 5xD + 1)y = 0$. Where D represents the differential operator, i.e., $D \equiv \dfrac{d}{dx}$.