Gautam Buddha University

Engineering Mathematics-III (MA-201) Second semester (2016-2017)

Tutorial Sheet-1

Formulation of differential equation, Types of differential equations, Order and degree of differential equation, Method of Separable variables.

Question.1 Check whether the following differential equations are linear/non-linear, homogeneous/non-homogeneous. Also determine the order and degree (if possible) the differential equations.

(a)
$$[1 + (\frac{dy}{dx})^2]^{1/2} = 5y$$
,

(b)
$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 6y = \log x$$
,

(c)
$$\frac{d^2y}{dx^2} + 2(\frac{dy}{dx})^2 + y = x$$
,

(d)
$$\left(\frac{d^3y}{dx^3}\right)^2 + 3\left(\frac{dy}{dx}\right) + 4y = 0$$

(e)
$$\left[1 + (\frac{dy}{dx})^2\right]^{1/2} = \frac{d^2y}{dx^2}$$
,

(f)
$$x^3 \frac{d^3y}{dx^3} + x^2 (\frac{d^2y}{dx^2})^2 + y^4 = 0$$
,

$$(g) \sin^{-1}(\frac{dy}{dx}) = x + y.$$

Question.2 Obtain various differential equations whose general solutions are given by:

1.
$$y^2 = 4a(x+a)$$
,

$$2. \ y = a\cos(x+3),$$

3.
$$xy = Ae^x + Be^{-x} + x^2$$
,

4.
$$x^2 + y^2 = a^2$$
.

5.
$$y = 2cx - c^2$$
,

6.
$$y = c \sin x$$
,

7.
$$y = ae^{-x} + be^{-2x} + ce^{-3x}$$
.

Question.3 Define homogeneous differential equation. Show that

- (a) it can be transformed to a separable variable form and hence always solvable.
- (a) a straight line through origin intersects all integral curves of a homogeneous differential equation at the same angle.

Question.4 Verify that the given function satisfies the differential equation

(a)
$$y = ce^{-x^2}$$
, $y' + 2xy = 0$ (b) $y = x \log x - x$, $y' = \log x$,

(b)
$$y = \sin^{-1} x$$
, $y'' = \frac{x}{(1 - x^2)^{3/2}}$ (b) $y = \sec x + \tan x$, $(1 - \sin x)^2 y'' = \cos x$.

Question.5 Define all types of solutions.

Q.6 Solve the following differential equations by variables separable method:

(a)
$$(1+x^2)dy - (1+y^2)dx = 0$$

(b)
$$(1-x^2)(1-y)dx = xy(1+y)dy$$

(c)
$$x^2(1+y)dx + y^2(x-1)dy = 0$$

(d)
$$\sqrt{(1-x^2)}dy + \sqrt{(1-y^2)}dx = 0$$

(e)
$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

(f)
$$\frac{dy}{dx} = (x+y)^2$$