Gautam Buddha University

Engineering Mathematics-III (MA-201)
Second semester (2016-2017)

Tutorial Sheet-4

Linear Differential equations: Homogeneous Linear DEs with Real Constants Coefficients.

$$\text{LDE}: \quad P_0(x)\frac{d^n y}{dx^n} + P_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_{n-1}(x)\frac{dy}{dx} + P_n(x)y = R(x) \tag{NH}$$

$$\text{corresponding homogeneous LDE}: \quad P_0(x)\frac{d^n y}{dx^n} + P_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_{n-1}(x)\frac{dy}{dx} + P_n(x)y = 0 \tag{H}$$

$$\text{Homogeneous LDE with constants coefficients}: \quad \frac{d^n y}{dx^n} + p_1\frac{d^{n-1}y}{dx^{n-1}} + \cdots + p_{n-1}\frac{dy}{dx} + p_n y = 0 \tag{HWC}$$

Q. 1 Write a note on the following:

(a) Complementary function and Particular integral.

(b) Wronskian of functions, a relation between wronskian and linearly independent solutions.

(c) Form of the general solution of a non-homogeneous linear differential equation (see Q.4).

Q. 2 Let $y_1, y_2, \cdots, y_m$ be any m-solutions of equation (H). Then show that their linear combination:

$$y = c_1y_1 + c_2y_2 + \cdots + c_my_m.$$

is also a solution of (H).

Q. 3 Let y_h be a solution of equation (H) and y_p a solution of equation (NH). Then show that $y_h + y_p$ is a solution of equation (NH).

Q. 4 Let $y_h = f(x, c_1, c_2, \cdots, c_n)$ be the general solution of equation (H) and y_p be a particular solution of equation (NH). Then show that every solution of (NH) is given by $f(x, c_1, c_2, \cdots, c_n) + y_p$ for certain values of $c_1, c_2, \cdots, c_n$. Hence conclude that the general solution of (NH) is $y_h + y_p$, that is sum of CF and PI.

Q. 5 Show that if a complex valued function $y = u(x) + iv(x)$ is a solution of equation (HWC) then so are the real valued functions $u(x)$ and $v(x)$. Hence prove that the conjugation of y, viz, $\overline{y} = u(x) - iv(x)$ is also a solution of equation (HWC).

Q. 6 Suppose $y_1, y_2, \cdots, y_n$ are n-solutions of equation (H) over I and $W(x)$ is the Wronskian of $y_1, y_2, \cdots, y_n$. Then show that

(a) the set $\{y_1, y_2, \cdots, y_n\}$ is LI on I if and only if Wronskian $W(x) \neq 0$ for all $x \in I$

(b) either $W(x) \equiv 0$ on I or it is nowhere zero over I

Q. 7 Suppose $y_1, y_2, \cdots, y_n$ are n LI solutions of equation (H). Then any solution $y(x)$ of (H) can be written as linear combination $y = c_1y_1 + c_2y_2 + \cdots + c_ny_n$ for some suitable values of $c_1, \cdots, c_n$.