



ZC Inflation Swap Clearing

Risk, Margin and Valuation

V1.2.3

15th December 2014

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Document History

Version	Date	Author	Comment
0.1	07-11-13	D Davies	Initial document construction
0.2	08-11-13	V Bouhali	Added Market data, Curve construction, Pricing and Greeks
0.3	11-11-13	P McLornan	Added historic data and stresses
0.4	13-11-13	V Bouhali	Corrected some of the results images
0.5	18-11-13	D Davies	General clean up and grammar corrections
0.8	10-01-14	D Davies	Preparation for Publication & IMMFP Tests
0.9	16-01-14	D Davies	IMMFP test extended
0.9.1	31-01-14	P McLornan	Removed irrelevant section from stress testing
0.9.2	06-02-14	V Bouhali	Modification based on LINKS's remarks
0.9.3	17-02-14	D Davies	Minor edits
0.9.9	29-05-14	D Davies	Significant edits. All survey data updated to 2014 results
1.0.0	30-06-14	D Davies	Finalisation
1.1.0	24-09-14	H Cheah	Revision of stress testing methodology
1.2.0	27-11-14	H Cheah	Added Portfolio Margin document
1.2.3	15-12-14	D Davies	Corrections

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Executive Summary

The LCH.Clearnet Ltd SwapClear service wishes to extend the product offering to incorporate Zero Coupon Inflation Indexed Swaps (hereafter interchangeably referred to as either Zero Coupon Inflation Indexed Swaps or ZCIIS). It is proposed that the service offers clearing for the most liquid indices; EUR (HICPxT and French CPIxT), GBP (RPI) and USD (CPI). These indices correspond to the major underlying bond markets for Inflation.

1 Product Definition

The primary purpose of the inflation-linked derivatives market is the transfer of inflation-risk. This is attractive to companies who typically pay or receive inflation cashflows in their natural line of business. Pension funds and LDI companies typically have inflation linked liabilities that they look to hedge. The most common supply to the market of inflation risk is through inflation linked bonds which are commonly issued by governments and supra-national entities, and from major infrastructure projects where future revenue streams are often inflation indexed.

Zero Coupon Inflation-Indexed Swaps are regarded as the most liquid instruments in the Inflation derivatives market. The term 'zero coupon' comes from the fact that there is no exchange of cashflows before the maturity of the swap, though additional upfront fees on the trade are supported. Other types of swaps exist (such as Year-on-Year or LPI swaps) – but they are priced off the more liquid zero coupon instruments, and require stochastic models with volatility inputs. These instruments are not in scope for the offering at this stage.

1.1 Product Scope

The zero coupon inflation swap has become the corner-stone of the inflation derivatives market. Its appeal lies in its relative simplicity and the fact that it can be tailored in terms of factors such as index, duration and sizing, something which isn't possible in the underlying index-linked bond markets.

A zero coupon inflation indexed swap is a “pure inflation” instrument. Cash flows between the two parties in the transaction occur only at maturity and involve the exchange of a notional adjusted for inflation (that has accrued over the lifetime of the deal) against the notional capitalised with a fixed rate. The fixed rate, agreed at inception, reflects the future expectation of inflation. It can also be regarded as the market price of expected inflation over the period of the zero coupon swap.

An Inflation Swap is composed of:

- Inflation Index + Lag
- Start Date
- End Date
- Nominal
- Fixed rate
- Pay or Receive floating leg

Market convention is to talk about the fixed rate when describing the direction of the trade. A pay position means paying the fixed leg and receiving the inflation linked leg. Likewise, a receive position implies receiving the fixed leg. Long and short are generally used in an inflation rates perspective, where a long position implies paying fixed so money is made as inflation rates rise, i.e. if inflation exceeds the deals break even rate. This is cleaner than in the IRS market where “long” is sometimes used to describe the equivalent bond position, and sometimes the interest rate position.

An example term sheet is outlined below to give a flavour of the terms and conditions of the trade/product type.

Trade Date	02-May-2013
Swap Start	04-May-2013
Swap End	04-May-2023
Notional Amount (EUR)	EUR 100m
Payer of Fixed Rate	Counterparty A
Fixed Rate Zero Coupon	$(1 + 2.2\%)^{10} - 1$
Coupon payment date	04-May-2023
Payer of EU HICPx coupon	Counterparty B
EU HICPx zero coupon	(EU HICPx Month End / EU HICPx Month Start) -1
Month end	Feb 2023
Month start	Feb 2013
Reference rate	European HICP excluding tobacco (non-revised)
Reference source	First publication by Eurostat
First Fixing	115.58
Coupon payment date	04-May-2023

Figure 1 : Sample Term Sheet

The zero-coupon swap, above, begins on the 4th May 2013 and concludes on the 4th May 2023 (with the exchange of cash flows occurring at the end of the swap). The initial level of the HICPxT index is already known as the index is lagged (by three months in this case). Thus the inflation leg pays the performance in the price index (i.e. inflation) between the start date, where the price index level is known, and the end date, for which the price index level is currently unknown. The rate specified in the Fixed Leg above is the rate which is quoted on the market and represents an accumulated (or compounded) rate. This rate is also referred to by the market as the "breakeven inflation swap rate".

1.2 Service overview

This section details the breadth of the service that is proposed in terms of the trade eligibility.

		EUR	EUR	GBP	USD
Inflation Type	Swap	ZCIIS	ZCIIS	ZCIIS	ZCIIS
Index		EU-HICPx	FR-CPI	UK-RPI	US-CPI
Standard Method	Index	MonthlyIndex	Interpolated	MonthlyIndex	Interpolated
Indexation Lag		3m	3m	2m	3m
Max Maturity		30Y	30Y	50Y	30Y
Spot starting		Y	Y	Y	Y
Fwd starting		N	N	N	N
Non-std lags		Y	Y	Y	Y
Max Non-std lag		12m	12m	12m	12m
New / backloads		Y	Y	Y	Y

Figure 2 : Eligibility by Index

The above table details the market standard method for the fixing of trades in each of the indices. It is not unusual for trades to be executed where the fixing method is non-standard for the index. Trades will be supported where the fixing method is from a monthly index, or from an interpolated rate in all indices.

1.3 Cashflows

The ZCIIS is defined as a zero coupon, single flow swap between a known fixed rate and an indexed floating amount.

Party B pays Party A the fixed amount $N[(1 + K)^M - 1]$

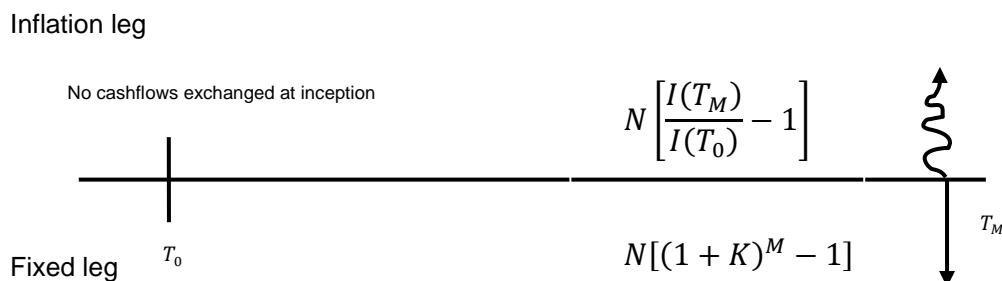
Party A pays Party B the index linked amount $N \left[\frac{I(T_M)}{I(T_0)} - 1 \right]$

In practice, these flows are netted and a single balancing payment is made at the maturity of the swap.

Net flow paid by Party A to Party B $N \left[\frac{I(T_M)}{I(T_0)} - (1 + K)^M \right]$

The cash flows and important time-lines are described diagrammatically below.

Figure 3 : Cashflow Diagram



Summary of notation used

N	The notional amount
K	The fixed rate (observed in the market)
$I(x)$	Observed CPI at time x
M	The length of the swap (measured in years)

1.4 Indices

Any inflation-linked product requires an underlying reference price index on which to calculate the term inflation return. The SwapClear service proposes to reference the most liquid indices in the market. In practice these are the indices which are underpinned by a large and liquid cash bond market. Inflation swaps, like inflation linked bonds, always trade on a non-seasonally adjusted index. This avoids the subjectivity of seasonal adjustment.

1.4.1 The Euro area, HICPxT

The Euro area inflation swap market is currently the most liquid. The benchmark index for the euro area is the Euro HICPxT (Harmonised Index of Consumer Prices excluding Tobacco). The index is published by Eurostat. It is typically published two weeks after the end of the month. For instance the Euro HICPxT index value for May will be published around the 15th June. At Eurostat's discretion, the index may be revised. However in both the cash and the derivatives markets, the unrevised index value is used for the purposes of fixing.

1.4.2 France, CPI

The French market uses the French CPIxT (Consumer Price Index excluding Tobacco). The index for each month is published by INSEE around the 22nd of each month. As with the EUR HICPxT, in the event of revisions, the unrevised index is used. All payments are in EUR.

1.4.3 United Kingdom, RPI

In the UK, inflation-linked securities are linked to the Retail Prices Index (RPI). The index is published around the 15th of each month by the Office for National Statistics (ONS). As per the HICPxT and French CPIxT, in the event of revisions, the unrevised index continues to be used.

1.4.4 United States, CPI-U

In the US, inflation-linked securities are linked to US City Average All Items Consumer Price Index (CPI-U) published by the Bureau of Labor Statistics (BLS). Again, in the event of revisions, the unrevised index continues to be used.

1.5 Lags

The inflation market makes use of indexation lags for the following reasons:

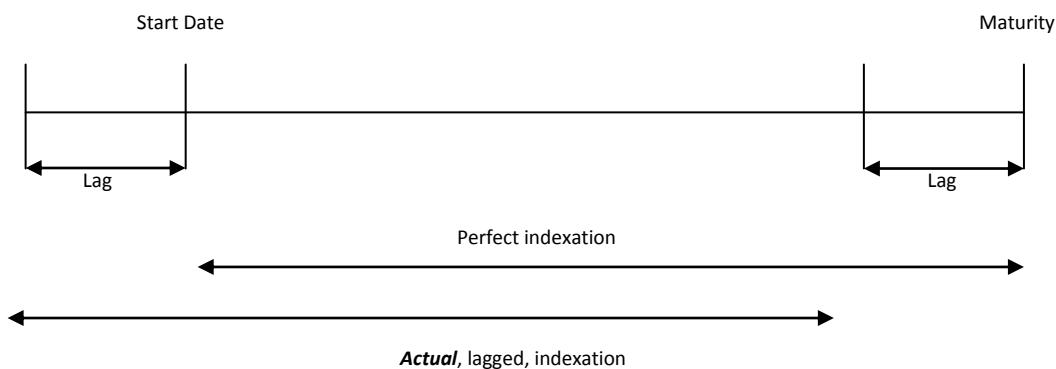
- The process of gathering and computing data required for inflation is time-consuming.
- Bond market participants need a known fixing to compute accrued interest in the event of bond purchase/sale.

Due to these practicalities it is typical that inflation is published around two weeks after the month under consideration. In both the bond and derivative markets the lags are index-dependent; but are usually 2-3 months in duration.

Market	Std Lag
Euro (HICPxT)	3m
French (FR CPI)	3m
UK (UK RPI)	2m
US (CPI-NSA)	3m

A consequence of the indexation lag is that the inflation receiver (buyer) has no “inflation protection” over the final period of the deal. By way of compensation – the inflation buyer will receive the inflation that was observed over the lagged period which precedes the start of the deal. (This is illustrated below by the “Actual, lagged, indexation line”).

Figure 4 : Event timing and lags



1.5.1 Non-standard lags

Whilst the indexation lags mentioned above are market standard, and represent the vast bulk of interbank trades executed; it is also possible to trade non-standard lags and these are typically with end user clients. Non-standard lags introduce small convexity adjustments that, theoretically, should be applied to correct the projected price index. The convexity adjustment in ZCIS with non-standard lags is dependent upon the following:

- The volatility of rates.
- The volatility of the price index.
- The correlation between rates and the price index.
- The length of the deal.
- The length of the lag.

As in the pure interest-rate space – pricing adjustments for unnatural timing requires a more complex model. In the case of the inflation market the most common models are the Jarrow-Yildirim model* and the Inflation Market Model**, which are similar to Hull White and BGM in rates respectively. Models of this nature are difficult to calibrate and can be slow to converge and therefore ultimately unsuitable for a clearing service.

Academic research***, along with feedback from the members, during the Design Authority sessions, indicated that the convexity adjustment is sufficiently small, when the lag is less than one year in total (including the standard lag), and can be ignored. Further it is expected that non-standard lags will not represent a large proportion of the market. Indicative portfolios, provided by members, suggest that non-standard lags make up around 5% of the trade population.

The table below shows the average of member polling for the bid/offer spread in standard inflation swaps with UK RPI index, and the size of adjustment for including non-standard lags. All values are in basis points (e.g. 0.1 represents 1/10th of a basis point).

	Bid/Offer Spread (bp)	2 month standard	6 month	8 month	9 month	12 month	18 month
10y	2.67	0.00	0.07	0.06	0.05	0.09	0.38
20y	2.67	0.00	-0.02	-0.01	-0.01	-0.02	0.24
30y	2.67	0.00	-0.08	-0.11	-0.10	-0.12	0.14
40y	2.67	0.00	-0.10	-0.11	-0.10	-0.11	0.16
50y	2.67	0.00	-0.10	-0.10	-0.10	-0.10	0.18

Figure 5 : Market survey of lag adjustments (UK)

The member survey was taken as positive evidence that the assumption of single curve pricing is sufficiently accurate for the purposes of margining (less than 1 bp off). It is therefore proposed that the service offers non-standard lags up to 1y, for all indices, and prices all these lags using a single inflation curve.

References:

* Pricing Treasury Inflation Protected Securities and Related Derivative securities using an HJM Model (2 June 2003)

**A Market Model for Inflation (Eric Benhamou)

***Convexity adjustments in inflation-linked derivatives (AsiaRisk November 2008 – Brody, Crosby and Li)

2 Market Data

Three market data components are needed to price a ZCIIS:

- An Inflation curve to calculate the index level at maturity of a swap
- A Seasonality adjustment to correct the index level for monthly deviations
- An Interest Rates curve to discount the expected cash flow from payment date to pricing date

2.1 OIS

The formula for Inflation incorporates the PV01 (or BPV) of the forward Inflation level. This is derived from the discount curve (OIS). SwapClear already has this data readily available for the existing product set and the market data will be identical.

ZCIIS have long maturities: 30Y for EUR, FR and US indices and 50Y for UK index. The discounting curve needs to reach these maturities so expected cash flows can be discounted. Before the 19th of June 2007, the SwapClear historic OIS curves do not contain any rates after a 2Y maturity. In such cases where the historic rates are unavailable in OIS space, the LIBOR swaps equivalent will be used.

2.2 Inflation Curve

Building an inflation curve is a key component in the risk management of inflation derivatives. Unlike the interest rate markets, where there are almost “continuous” quotes (i.e. LIBORs are quoted daily) to build the curve upon, the inflation curve is comprised of a set of discrete points (i.e. there are only 12 observations of an index per year). In relation to the selection of instruments used for calibration it is proposed that zero coupon inflation indexed swaps are used (as indicated previously – these are the most liquid instruments in the market and as a corollary implicitly contain the necessary inflation expectations).

Canty et al (2012)* suggest that an inflation curve builder should embody the following characteristics:

- It should re-price accurately all of the input instruments.
- It should have a sensible interpolation scheme to price non-quoted instruments.
- It should contain some description of the seasonal nature of the index.

Reference:

* Inflation Markets: A Comprehensive and Cohesive Guide (7 August 2012)

2.2.1 Input instruments

The underlying instruments used to build the curve will be zero coupon inflation indexed swaps. The instruments will have the following date definitions:

- Swap start date
 - Market data date + currency lag.
 - Business roll date convention: Following.
- Swap end date
 - Swap start date + tenor.
 - Unadjusted.
- Swap fixing date
 - Swap end date – standard lag.
 - Unadjusted.
- Swap payment date
 - Set to swap end date.
 - Business roll date convention: Modified Following.

Further; as the service will only deal with spot starting inflation swaps – the quotes should be for spot starting swaps except for HICPxT index where market convention is to quote from trade date and even if the spot date has moved to the next month, the observed fixing does not change compared to yesterday (e.g. trade date=29 Nov 13, spot=02 Dec 13, fixing= August 13)

2.2.2 Short Maturity instruments

Convention for inflation trading desks is to build the curve at the short end using monthly ZC swaps, specifically between the 1 month and 24 month points. These, monthly, instruments may be constructed in a number of ways:

1. Broken-maturity ZC swaps.

$$\left(\frac{CPI_n}{CPI_0}\right)^{\frac{1}{T}} - 1$$

Where T is computed using a bond (30/360) day count basis and represents the time period between the market data date and the underlying curve instruments end date.

2. 1y ZC swaps up to 1y, followed by 2y ZC swaps up to 2y.

Where the instruments up to the 1y pillar reference the CPI level that was observed 1y ago, relative to the swap end date, versus the projected index values. Instruments beyond 1y reference the CPI level that was observed 2y ago, relative to the swap end date, versus the projected index values.

3. 1y ZC swaps up to 1y, followed by broken-maturity swaps up to 2y.

Partial mixture of options 2 & 1 above. (Up to 1y; the instruments reference the CPI levels that were observed 1y ago relative to the pillar). Beyond 1y broken maturity ZC swaps are used in the same manner as point 1 above.

Based upon system limitations in Murex, and agreement from the design authority members; option 1 above was chosen. Members have agreed to supply quotes conforming to this convention. It should be noted that all three methods will give the same projected forward index value, and the differing rates and conventions are merely algebraic transformations of this index projection.

2.2.3 Tenors

The following tenors will be requested from members.

UK RPI	US CPI	EU HICPxT	FR CPIxT
1M	1M	-	1M
2M	2M	2M	2M
3M	3M	3M	3M
4M	4M	4M	4M
5M	5M	5M	5M
6M	6M	6M	6M
7M	7M	7M	7M
8M	8M	8M	8M
9M	9M	9M	9M
10M	10M	10M	10M
11M	11M	11M	11M
-	12M	-	12M
1Y	1Y	1Y	1Y
13M	13M	13M	13M
14M	14M	14M	14M
15M	15M	15M	15M
16M	16M	16M	16M
17M	17M	17M	17M
18M	18M	18M	18M
19M	19M	19M	19M
20M	20M	20M	20M
21M	21M	21M	21M
22M	22M	22M	22M
23M	23M	23M	23M
-	24M	-	24M
2Y	2Y	2Y	2Y
3Y	3Y	3Y	3Y
4Y	4Y	4Y	4Y
5Y	5Y	5Y	5Y
6Y	6Y	6Y	6Y
7Y	7Y	7Y	7Y
8Y	8Y	8Y	8Y
9Y	9Y	9Y	9Y
10Y	10Y	10Y	10Y
12Y	12Y	12Y	12Y
15Y	15Y	15Y	15Y
20Y	20Y	20Y	20Y
25Y	25Y	25Y	25Y
30Y	30Y	30Y	30Y
35Y	-	-	-
40Y	-	-	-
45Y	-	-	-
50Y	-	-	-

The USD and FRF markets trade on an interpolated index level whereas the GBP and EUR markets use a simple monthly quote. The interpolation mechanism can lead to some instability of the curve construction. To ensure that this instability cannot occur, for the points up to (and inclusive of) 2 year, the USD and FRF quotes sourced will be for simple monthly index fixings. The monthly fixings are more explicitly linked to each of the forward projections.

2.3 Seasonality

Inflation is highly seasonal. Food and energy prices ensure that inflation is also volatile from month to month. It is subsequently an important aspect of modelling, as both inflation bond and derivative markets, use non-seasonally adjusted inflation indices as an underlying (As the seasonality adjustment is very subjective).

Seasonality may be modelled as an additive or a multiplicative adjustment. It is proposed that the seasonality used by the service is multiplicative as in this case seasonal effects move proportionately with the behaviour of the underlying index. Whereas with additive seasonality; the effects are broadly constant irrespective of the directionality of the index. Conversely; additive adjustments require intensive numerical calculation, within a VaR framework, which makes the method unsuitable for VaR calculation. To strengthen the case further – multiplicative seasonality is regarded as market standard and the member banks have agreed to provide the seasonality curves in this format on an on-going basis.

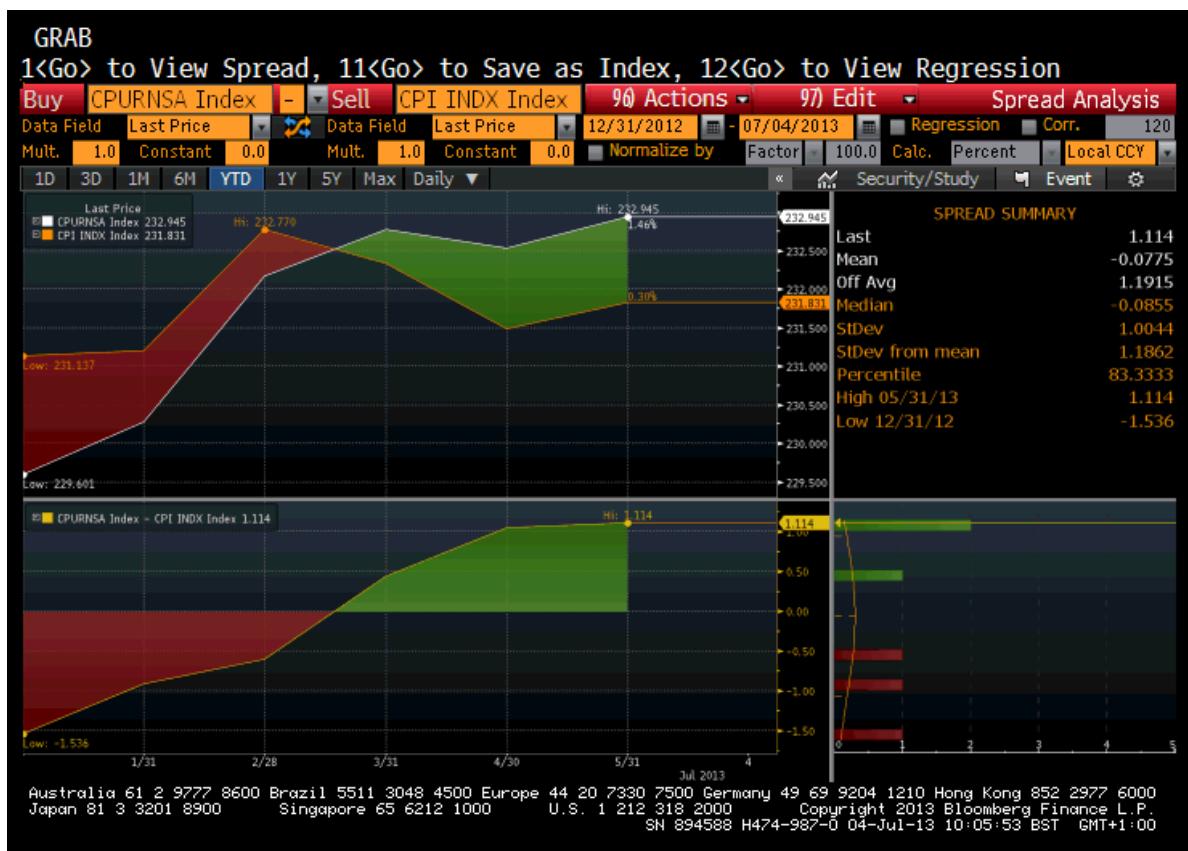


Figure 6: US (Non-)Seasonally adjusted CPI Index values, Source Bloomberg 2013

2.4 Historic Index Values

Inflation indices are published by the relevant governmental authority on a lag basis. Inflation index fixings are directly available from Bloomberg. Fixings per Index and lag need to be imported within the system as ZCIIS start date fixing is based on the available published fixing linked to the lag. Also forward CPI levels of the Inflation index are constructed in relation to the last known fixing of the Inflation index.

Inflation fixings published by the relevant governmental authority are sourced from Bloomberg and already imported in Murex the day they are published. Only the unrevised fixings are used.

2.5 Member Contributions

A minimum of five members need to supply Inflation data and Seasonality five times a day, either through a standard data supplier (Reuters or Bloomberg) or via direct contribution to the CCP. Members that want to clear Inflation Swaps may be required to supply data. A threshold has been established of 250 trades per year per index over which a member will be obliged to supply data for that index. LCH reserves the right to include or exclude member submissions from its official data cleansing and aggregation to ensure that only active market players are incorporated into the benchmark valuation and risk curves.

2.5.1 Inflation CPI levels

Members will provide market swap quotes for all tenors defined in section 2.2.3. These quotes will be for market standard trade definitions, in standard market sizes and under the assumption that the trade is cleared, so as to reduce credit considerations.

2.5.2 Seasonality

Inflation curves will be configured to use Multiplicative seasonality. Seasonality surfaces will be sourced from LCH members and processed to build one unique seasonality surface per index. Members should quote each of the 12 months.

Multiplicative adjustments are naturally in a scale close to 1. Quotes of up to 8 decimal place precision will be supported.

In order for a seasonality surface to be used, it must satisfy the following property: The product of monthly seasonality for a given inflation index equals one.

$$\prod_{i=1}^{12} S_i = 1 \pm \varepsilon$$

Where:

S_i : Seasonality for month i

$\varepsilon = 10^{-8}$: Epsilon precision due to 8 decimal quotes

2.6 Aggregation and Cleansing

2.6.1 Inflation CPI levels

Inflation swap quotes will be aggregated as per the existing process used for IRS quotes. Contributors will be toped and tailed based on how far they are from the average and the remaining contributors will be averaged.

Each swap tenor will be treated separately; therefore a contributor can be included for the first tenor and excluded for all the others.

2.6.2 Seasonality

Seasonality cleansing will be done in three steps: a selection of contributors, an averaging of the selected contributors and a normalisation of the averaged data.

Seasonality surfaces will be treated differently to IRS quotes and Inflation CPI levels; contributors will be even entirely excluded or entirely included for all 12 months. The selection is based on how far from the average a contributor is for each month:

$$Rank = \sum_{i=1}^{12} (S_i - \tilde{S}_i)^2$$

Where:

S_i : Seasonality for month i

\tilde{S}_i : Average seasonality for month i

Contributors are toped and tailed based on the above result and an average of the remaining contributors is done. Finally the seasonality produced is re-scaled so it is correct as a multiplicative seasonality.

$$S'_i = S_i \cdot \left(\prod_{i=1}^{12} S_i \right)^{-\frac{1}{12}}$$

Where:

S'_i : Re-scaled seasonality for month i

S_i : Seasonality for month i

*Seasonality cleansing algorithm available in Appendix7.

3 Inflation Curve

The forward CPI level of the Inflation index is determined from an inflation curve. In general the Inflation curve should change depending on the Inflation Index lag. It has been confirmed that for lags up to one year, using one curve for all lags instead of one curve per lag does not add a large error.

3.1 Curve definition

3.1.1 Variation Margin Curves

For Variation Margin (VM), curves will have a monthly granularity in the short end of the curve. This ensures that the maximum market information is reflected in the pricing without resorting to more generalised seasonality adjustments.

GBP	USD	EUR	FRF
1M	1M	-	1M
2M	2M	2M	2M
3M	3M	3M	3M
4M	4M	4M	4M
5M	5M	5M	5M
6M	6M	6M	6M
7M	7M	7M	7M
8M	8M	8M	8M
9M	9M	9M	9M
10M	10M	10M	10M
11M	11M	11M	11M
1Y	1Y	1Y	1Y
13M	13M	13M	13M
14M	14M	14M	14M
15M	15M	15M	15M
16M	16M	16M	16M
17M	17M	17M	17M
18M	18M	18M	18M
19M	19M	19M	19M
20M	20M	20M	20M
21M	21M	21M	21M
22M	22M	22M	22M
23M	23M	23M	23M
2Y	2Y	2Y	2Y
3Y	3Y	3Y	3Y
4Y	4Y	4Y	4Y
5Y	5Y	5Y	5Y
6Y	6Y	6Y	6Y
7Y	7Y	7Y	7Y
8Y	8Y	8Y	8Y
9Y	9Y	9Y	9Y
10Y	10Y	10Y	10Y
12Y	12Y	12Y	12Y
15Y	15Y	15Y	15Y
20Y	20Y	20Y	20Y
25Y	25Y	25Y	25Y
30Y	30Y	30Y	30Y
35Y	-	-	-
40Y	-	-	-
45Y	-	-	-
50Y	-	-	-

3.1.2 Initial Margin curves

Initial Margin (IM) curves will be a subset of Variation Margin curves. Tenors will be the same as VM curves at the exception of the short end of the curve where only the 1Y and 2Y tenors remain.

GBP	USD	EUR	FRF
1Y	1Y	1Y	1Y
2Y	2Y	2Y	2Y
3Y	3Y	3Y	3Y
4Y	4Y	4Y	4Y
5Y	5Y	5Y	5Y
6Y	6Y	6Y	6Y
7Y	7Y	7Y	7Y
8Y	8Y	8Y	8Y
9Y	9Y	9Y	9Y
10Y	10Y	10Y	10Y
12Y	12Y	12Y	12Y
15Y	15Y	15Y	15Y
20Y	20Y	20Y	20Y
25Y	25Y	25Y	25Y
30Y	30Y	30Y	30Y
40Y	-	-	-
50Y	-	-	-

3.1.3 Initial Margin Multiplier curves

In order to compute inflation sensitivities required for the Initial Margin Multiplier, new curves will be defined with specific key points of risk. Use of this reduced curve set ensures that risk is correctly allocated to each bucket directly rather than using a numerical re-allocation from a more granular curve. IMMFP Curves will be a subset of Initial Margin curves, and match key tenors on which positions would likely be hedged in case of default.

GBP	USD	EUR	FRF
2Y	2Y	2Y	2Y
5Y	5Y	5Y	5Y
10Y	10Y	10Y	10Y
20Y	20Y	20Y	20Y
30Y	30Y	30Y	30Y
50Y	-	-	-

3.2 Instrument definition

The underlying instruments used to build the curve will be zero coupon inflation indexed swaps. The instruments will have the following date definitions:

- Swap start date
 - Market data date + currency lag.
 - Business roll date convention: Following.
- Swap end date
 - Swap start date + tenor.
 - Unadjusted.
- Swap fixing date
 - Swap end date – standard lag.
 - Unadjusted.
- Swap payment date
 - Set to swap end date.
 - Business roll date convention: Modified Following.

Further; as the service will only deal with spot starting inflation swaps – the quotes should be for spot starting swaps except for HICPxT index where market convention is to quote from trade date and even if the spot date has moved to the next month, the observed fixing does not change compared to yesterday (e.g. trade date=29 Nov 13, spot=02 Dec 13, fixing= August 13)

3.3 Compounding

For whole year quotes, the compounding index will be computed as the integer number of years in the swap. Where the period covered is an inexact number of years, the compounding index will be measured as (Number of Months/12). This avoids complexity and potential error in using a basis such as 30/360.

As an example a 3 month rate of 2.6% would result in a fixed rate payment of $(1+2.6\%)^{(0.25)} - 1$.

3.4 Lag adjustment

The forward CPI level of the Inflation index is determined from an inflation curve. In general the Inflation curve should change depending on the Inflation Index lag. It has been confirmed that for lags up to one year, using one curve for all lags instead of one curve per lag does not add a large error.

One curve per index will be configured in the system and will use the standard market lag for each index:

- 2M for GBP (RPI index)
- 3M for EUR (Eurowide HICPxT and French CPIxT) and USD (CPI)

3.5 Seasonality

Inflation is expressed at different fixed maturities but between two maturities the index does not evolve with a constant rate. Inflation is subject to seasonality and it is therefore necessary to adjust it on a month to month basis. Two different types of adjustment commonly used: Additive and Multiplicative.

3.5.1 Additive Adjustment

The additive adjustment is a product of monthly seasonality. With the additive adjustment, it is assumed that between two pillars of an Inflation curve, the price index evolves following this equation:

$$CPI_2 = CPI_1 \prod_{i=1}^n (1 + R + S_i)$$

Where

CPI_1 : CPI of left pillar of the current bucket

CPI_2 : CPI of right pillar of the current bucket

n : Number of months in the bucket

R : Average return or Trend (To determine the average return R , this equation is solved numerically for example using the Newton-Raphson algorithm)

S_i : Seasonality factor for month i

Then the interpolation of the CPI for each month p inside a bucket is calculated using the following formula:

$$CPI_p = CPI_1 \prod_{i=1}^p (1 + R + S_i)$$

Where

CPI_p : CPI for month p in the current bucket

CPI_1 : CPI of left pillar of the current bucket

R : Average return or Trend calculated for the current bucket

S_i : Seasonality factor for month i

The additive adjustment method is an expensive calculation as the average return needs to be recomputed every time an Inflation rate is shifted. This makes the additive method computationally extremely expensive in the context of a VaR calculation. This method is not commonly used in the market.

3.5.2 Multiplicative Adjustment

The multiplicative adjustment adds another layer on top of the interpolation method chosen. Assuming we choose a Log-Linear interpolation; between two pillars of an Inflation curve, the inflation evolves following a log linear interpolation of non-seasonally adjusted CPIs:

$$CPI_2 = \frac{S_2 * CPI_1}{S_1} * (1 + R)^n$$

Where:

CPI_1	Left pillar CPI of the current bucket
CPI_2	Right pillar CPI of the current bucket
S_1	Seasonality factor for start month of the bucket
S_2	Seasonality factor for end month of the bucket
R	Average return or Trend
n	Number of months in the bucket

The average return is implied from the above formula and the interpolation of the CPI for each month p inside a bucket is calculated using the following formula:

$$CPI_p = \frac{S_p * CPI_1}{S_1} * (1 + R)^p$$

Where:

S_p	Seasonality factor for month p of the bucket
S_1	Seasonality factor for start month of the bucket
R	Average return or Trend

This adjustment is much more efficient than the additive case and is seen as the market standard. Therefore this is the proposed method for use in the SwapClear service.

3.6 Base elements

CPI levels will be implied from ZCIIS quotes so that the zero coupon inflation swap NPV equals zero.

3.6.1 CPIs

ZCIIS quotes can be used to construct the forward CPI level using the following formula:

$$CPI_n = CPI_0(1 + R_n)^{t_n}$$

Where:

t_n : Maturity year of pillar n (calculation as defined in paragraph 0)

R_n : Zero Coupon Inflation Rate for pillar n

CPI_0 : Reference CPI for a Swap (vary function of the Index lag: 2M for UK RPI, 3M for the others)

3.6.2 CPI Rates

CPI rates are a derived statistic, that are close to market quotes in the sense that they are rates and can be used to calculate index levels. They don't incorporate seasonality effect in them and are built using the last published index as a reference CPI.

CPI rates are deduced from Index levels; there are three main conventions to convert a CPI into a CPI rate:

- Linear:

$$CPI_n = CPI_{Last}(1 + Z_n * T_n)$$

- Yield:

$$CPI_n = CPI_{Last}(1 + Z_n)^{T_n}$$

- Exponential:

$$CPI_n = CPI_{Last} \exp^{Z_n * T_n}$$

Where:

CPI_n

CPI of inflation curve pillar n

CPI_{Last}

Last published fixing

R_n

CPI rate for inflation curve pillar n

T_n

Ratio of number of days from reference date to interpolate over the number of days in a year.

Assuming that CPIs for all pillars are known, Market practice is to use the Yield formula with 30/360 by month basis convention. The CPI rates are calculated using the following formula:

$$Z_n = \left(\frac{S_{Last} \cdot CPI_n}{S_n \cdot CPI_{Last}} \right)^{\frac{1}{t'_n}} - 1$$

Where

CPI_{Last} : Last published index (generally previous month from system date)

CPI_n : CPI for pillar n

S_{Last} : Seasonality for last published index month

S_n : Seasonality for pillar n

t'_n : Time to maturity from fixing date n to last published index date using 30/360 by month basis convention
(Diff Years * 12 + Diff Months) / 12

3.6.3 Reference CPI

There are two different methods to calculate CPI_0 , it is either directly a fixing for a given month (end-of-month case) or a value interpolated between two consecutive months (linear interpolation case).

For the end-of-month case, the reference price level is the published index value specified by the index lag. For example for index UKRPI with a standard 2 months lag on the 12th of December 2012, the reference price level is the fixing observed in October 2012.

For the interpolated case, a new value of the reference price level is calculated every day. For the first day of the month, the reference value will be the same as the end-of-month case. But for the following days, that reference value is calculated by interpolating between that index and the next one. Market practice is to round the reference CPI level to 5 decimals.

The reference price level is calculated following this formula:

$$CPI_0 = CPI_{M-L} + \frac{d - 1}{D} * (CPI_{M-L+1} - CPI_{M-L})$$

Where

M: Current month of the market data date

L: Index lag in month

d: Day of the current month

D: Number of days in the current month

3.7 Interpolation

When pricing an Inflation Swap, its fixing date will not always fall on a pillar of the Inflation curve; therefore an interpolation is needed to handle those cases. Many different types of interpolation commonly used and available in Murex: Linear on CPI, Linear on CPI Rate, Linear on CPI Rate*Time, Log linear in CPI. Log Linear on CPI is a commonly used market standard and is proposed for use.

3.7.1 Linear on CPI

The following formula is used to linearly interpolate in CPI:

$$CPI_p = \frac{CPI_1 * S_p}{S_1} \left[1 + \frac{p}{n} \left(\frac{\frac{CPI_2 - CPI_1}{S_2 - S_1}}{\frac{CPI_1}{S_1}} \right) \right]$$

Where:

CPI_1 : Left pillar CPI of the bucket

CPI_2 : Right pillar CPI of the bucket

CPI_p : CPI for month p in current bucket

S_p : Seasonality factor for month p in bucket

S : Seasonality factor for left pillar in the current bucket

S_2 : Seasonality factor for right pillar in the current bucket

p : Month p in the current bucket

n : Number of months in the current bucket

3.7.2 Linear on CPI Rate

The following formula is used to linearly interpolate in CPI Rate:

$$Z_p = Z_1 \left[1 + \frac{p}{n} \left(\frac{Z_2 - Z_1}{Z_1} \right) \right]$$

The following formula is used to linearly interpolate in CPI Rate * time:

$$Z_p = Z_1 \cdot \frac{T_1}{T_p} \left[1 + \frac{p}{n} \left(\frac{Z_2 \cdot T_2 - Z_1 \cdot T_1}{Z_1 \cdot T_1} \right) \right]$$

Where:

Z_1 : Left pillar CPI rate of the bucket

Z_2 : Right pillar CPI rate of the bucket

Z_p : CPI rate for month p in current bucket

p : Month p in the current bucket

n : Number of months in the current bucket

A CPI Rate is then converted back into a CPI by using the following formula:

$$CPI_p = CPI_{Last} \cdot \frac{S_p}{S_{Last}} \cdot (1 + Z_p)^{t_p}$$

Where:

CPI_p : Left pillar CPI rate of the bucket

CPI_{Last} : Left pillar CPI rate of the bucket

Z_p : CPI rate for month p in current bucket

S_p : Seasonality factor for month p in bucket

S_{Last} : Seasonality factor for the last published fixing month

t_p : Time to maturity from the last published fixing to Month p

3.7.3 Log linear on CPI

The following formula is used to log-linearly interpolate in CPI:

$$CPI_p = S_p \left[\left(\frac{CPI_1}{S_1} \right)^{1-\frac{p}{n}} \left(\frac{CPI_2}{S_2} \right)^{\frac{p}{n}} \right]$$

Where

CPI_1 : CPI of left pillar of the current bucket

CPI_2 : CPI of right pillar of the current bucket

S_p : Seasonality adjustment for month p

S_1 : Seasonality factor for left pillar month of the bucket

S_2 : Seasonality factor for right pillar month of the current bucket

p: Number of months from the fixing date to the previous pillar (fixing date is a month, convention is 1st of the month)

n: Number of months between left and right curve pillars

The market convention to interpolate CPIs between two dates is to do a Log-Linear interpolation on the CPI levels from adjacent pillars around the date to interpolate. Interpolation on the curve is always done for the 1st of a month. There are no distinctions at that point between piecewise and linear indices.

This methodology is extremely close to linear interpolation on rate times time, and only differs as a result of the seasonality impacts.

3.8 Extrapolation

In some rare cases, Inflation Swaps eligible for clearing will fix on dates beyond the last tenor of the Inflation curve. This will only be the case for nonstandard linear interpolated trades on UK RPI and EU HICPxT indices, and will never be beyond a couple of months extrapolation in the worst cases.

In case of extrapolation, the expected behaviour is to use the last information available and do an extrapolation in time. The extrapolation formula will be the same as the interpolation; the only difference is that $CPI_1 < CPI_2 < CPI_p$ and $p > n$:

$$CPI_p = S_p \left[\left(\frac{CPI_1}{S_1} \right)^{1-\frac{p}{n}} \left(\frac{CPI_2}{S_2} \right)^{\frac{p}{n}} \right]$$

CPI_1 : CPI before the last CPI of the curve

CPI_2 : Last CPI of the curve

S_p : Seasonality adjustment for month p

S_1 : Seasonality factor for left pillar month of the bucket

S_2 : Seasonality factor for right pillar month of the current bucket

p: Number of months from the fixing date p to CPI_1 pillar

n: Number of months between CPI_1 and CPI_2 curve pillars

3.9 Inflation Curve Striping

Dates at which CPIs are expressed are generally the instrument fixing dates. In case of piecewise index such as UK RPI or EU HICPxT this is always the case. For linear indices such as FR or US CPI; the interpolation is done in two steps:

- Log CPI interpolation of the index level from the Inflation curve for adjacent months of the fixing date
- Linear interpolation of the two CPIs to express it on the fixing date

In that case, CPIs of the Inflation curve are expressed on the following month of the fixing date by solving a system of 3 equations:

- linear interpolation between current month and following month of the swap fixing date
- log linear growth between last pillar and current month
- log linear growth between last pillar and following month

As an example a 2Y Swap based on US CPI index that fixes on the 12th of April 2015 will have its CPI calibrated on May 2015.

The following equations will be used to calibrate the 2Y instrument:

- $CPI_{12/04/15} = CPI_{Apr\ 15} * \left(1 - \frac{11}{30}\right) + CPI_{May\ 15} * \frac{11}{30}$
- $CPI_{Apr\ 15} = CPI_{May\ 14} * (1 + s)^{T_1} * \frac{S_{Apr}}{S_{May}}$
- $CPI_{May\ 15} = CPI_{May\ 14} * (1 + s)^{T_2} * \frac{S_{Apr}}{S_{May}}$

Where:

$$CPI_{12/04/15} = CPI_0(1 + R_{2Y})^2$$

$CPI_{May\ 14}$: Last pillar of the curve, 1Y Instrument

$CPI_{Apr\ 15}$: Current month of the swap fixing date, **variable to be solved**

$CPI_{May\ 15}$: Following month of the swap fixing date, **variable to be solved**

s : Trend, **variable to be solved**

S_{Apr} : Seasonality for April

S_{May} : Seasonality for May

T_1 : Time to maturity from April 15 to May 14

T_2 : Time to maturity from May 15 to May 14

When a left pillar is not properly defined, as is the case of the first pillar of the curve; the last known fixing will be used as a replacement.

As an example, if the first pillar of the inflation curve is a 1Y Swap based on US CPI index, fixes on the 12th of April 2014 and the last known fixing is February 14. The system of equations becomes the following:

- $CPI_{12/04/14} = CPI_{Apr\ 14} * \left(1 - \frac{11}{30}\right) + CPI_{Feb\ 14} * \frac{11}{30}$
- $CPI_{Apr\ 14} = CPI_{Feb\ 13} * (1 + s)^{T_1} * \frac{S_{Apr}}{S_{Feb}}$
- $CPI_{Feb\ 14} = CPI_{Feb\ 13} * (1 + s)^{T_2} * \frac{S_{Feb}}{S_{May}}$

In case of monthly quotes used to build an Inflation curve, the stripping is much easier and no system of equation needs to be solved to calibrate the curve. The current month is also the last calibrated point and a single equation can be used.

Assuming a 3M Swap based on US CPI index fixes on the 12th of April 2014 and the last calibrated instrument was a 2M Swap.

The system of equation can be resumed as:

$$CPI_{12/04/14} = CPI_{Apr\ 14} * \left(1 - \frac{11}{30}\right) + CPI_{May\ 14} * \frac{11}{30}$$

$CPI_{Apr\ 14}$: Known CPI from the calibrated 2M instrument

Therefore

$$CPI_{May\ 14} = \frac{30}{11} \cdot \left(CPI_0 (1 + R_{3M})^{3/12} - CPI_{Apr\ 14} * \left(1 - \frac{11}{30}\right) \right)$$

3.10 Murex Testing and Results

Four Inflation curves have been configured in Murex, one per index as mentioned above. The recommended configuration is to use Multiplicative Seasonality and to use a Log linear interpolation scheme on index levels.

3.10.1 Testing approach

The validation of Murex for ZCIIS needs to be done for VM and IM space. It is a combination of multiple effects: the Inflation Curve construction and interpolation, the pricing and the sensitivities calculation. In the case of VM, it consists of Par sensitivities used for hedging and in the case of IM all sensitivities for the Inflation curve will be tested with the addition of Cross Gamma used in the Taylor formula.

Stripping

The first point to validate is the construction of Inflation curves for piecewise and linear indices. This validation consists of ensuring that CPI dates, CPIs and CPI Rates for all four indices configured in Murex follow the defined formulas.

For that purpose a spreadsheet available in Appendix 3 has been built to replicate the theoretical formulas for the curve construction.

Checks:

1. CPIs for piecewise indices are built using the defined formula
2. Basis convention should follow the market convention
3. CPIs for linear indices are expressed on the next month of the fixing swap date
4. CPI Rates are deduced from the CPIs using the defined formula

Tests to cover:

- Strip curve for FR CPIxt index
- Strip curve for EU HICPxt index
- Strip curve for US CPI index
- Strip curve for UK RPI index

Interpolation

After making sure that the index levels are expressed on the correct dates with expected values; the next step is to validate the Inflation Curve interpolation.

Checks:

1. Monthly interpolated CPIs from 12 months in the past to the last tenor of the Inflation curve follow a log linear interpolation with a multiplicative seasonality
2. Extrapolation of the Inflation curve is not flat and uses last CPIs

Tests to cover:

- 31Y1M monthly Inflation Swap on FR CPIxt index with a start date 12M in the past
- 31Y1M monthly Inflation Swap on EU HICPxt index with a start date 12M in the past
- 31Y1M monthly Inflation Swap on US CPI index with a start date 12M in the past
- 51Y1M monthly Inflation Swap on UK RPI index with a start date 12M in the past

Seasonality

Seasonality directly impacts the inflation curve interpolation and therefore its interpretation by Murex need to be tested. The tests defined above need to be run for different profiles of Seasonality.

Tests to cover:

1. No seasonality
2. Real market seasonality
3. Flat seasonality (equal to 1)
4. Parabola seasonality
5. Inverse parabola seasonality

Market Quotes

Market Quotes are the main input of an Inflation curve and therefore impact its construction. To make sure an Inflation curve can be constructed independently of market conditions, different profiles need to be tested.

Tests to cover:

1. Flat curve
2. Real market curve
3. Inverted curve
4. Deflation curve
5. Humped curve
6. Inverse humped curve

Month end role

When a new month comes, piecewise indices have their base changed. Seasonality changes, rates change from one day to another but index levels expectation should not jump from one day to another.

Tests to cover:

1. Have the Database at the last open day of a month, roll the system date to the next business day and make sure reference index used by the Inflation curves changes.
2. Have the Database at the last open day of a month, roll the system date to the next business day and make sure index levels on the curve do not jump when updating the quotes.

3.10.2 Results

All tests passed except when curves were configured to not use Seasonality for Linear indices. In that case the FR and US Inflation curves refused to calibrate. This is not an issue as Murex correctly calibrates inflation curves when using multiplicative seasonality and VM will always use seasonality. The interpolation and extrapolation of index levels can be replicated for all cases on a monthly basis.

FR CPIxT results

FR CPIxT No Seasonality				FR CPIxT Flat Curve			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	-1211346110	0.000000000	Fail
Min Index Error	-12515438988	0.000000000	Fail	Min Index Error	-419354198432	0.000000000	Fail
Max Rate Error	0.8936659527	0.000000000	Pass	Min Rate Error	-3.391538165	0.000000000	Fail

FR CPIxT Real Seasonality				FR CPIxT Inverted Curve			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	0.000000000	0.000000000	Pass
Min Index Error	0.000000000	0.000000000	Pass	Min Index Error	0.000000000	0.000000000	Pass
Max Rate Error	0.000000000	0.000000000	Pass	Min Rate Error	0.000000000	0.000000000	Pass

FR CPIxT Seasonality = 1				FR CPIxT Deflation			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	0.000000000	0.000000000	Pass
Min Index Error	0.000000000	0.000000000	Pass	Min Index Error	0.000000000	0.000000000	Pass
Max Rate Error	0.000000000	0.000000000	Pass	Min Rate Error	0.000000000	0.000000000	Pass

FR CPIxT Seasonality Kink Up				FR CPIxT Humped Curve			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	0.000000000	0.000000000	Pass
Min Index Error	0.000000000	0.000000000	Pass	Min Index Error	0.000000000	0.000000000	Pass
Max Rate Error	0.000000000	0.000000000	Pass	Min Rate Error	0.000000000	0.000000000	Pass

FR CPIxT Seasonality Kink Down				FR CPIxT Inverse Humped Curve			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	0.000000000	0.000000000	Pass
Min Index Error	-0.000000000	0.000000000	Pass	Min Index Error	-0.000000000	0.000000000	Pass
Max Rate Error	-0.000000000	0.000000000	Pass	Min Rate Error	-0.000000000	0.000000000	Pass

US CPI results

FR CPIxT No Seasonality				FR CPIxT Flat Curve			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	0.254680724	0.000000000	Fail
Min Index Error	-0.090940000	0.000000000	Pass	Min Index Error	-0.090940000	0.000000000	Pass
Max Rate Error	-0.000000000	0.000000000	Pass	Min Rate Error	-4.671960363	0.000000000	Fail

FR CPIxT Real Seasonality				FR CPIxT Inverted Curve			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	0.000000000	0.000000000	Pass
Min Index Error	0.000000000	0.000000000	Pass	Min Index Error	0.000000000	0.000000000	Pass
Max Rate Error	0.000000000	0.000000000	Pass	Min Rate Error	0.000000000	0.000000000	Pass

FR CPIxT Seasonality = 1				FR CPIxT Deflation			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	0.000000000	0.000000000	Pass
Min Index Error	0.000000000	0.000000000	Pass	Min Index Error	0.000000000	0.000000000	Pass
Max Rate Error	0.000000000	0.000000000	Pass	Min Rate Error	0.000000000	0.000000000	Pass

FR CPIxT Seasonality Kink Up				FR CPIxT Humped Curve			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000	0.000000000	Pass	Max Index Error	0.000000000	0.000000000	Pass
Min Index Error	0.000000000	0.000000000	Pass	Min Index Error	0.000000000	0.000000000	Pass
Max Rate Error	0.000000000	0.000000000	Pass	Min Rate Error	0.000000000	0.000000000	Pass

FR CPIxT Seasonality Kink Down				FR CPIxT Inverse Humped Curve			
Stripping Comparison		Interpolation Comparison		Stripping Comparison		Interpolation Comparison	
	Value	Threshold	Pass/Fail		Value	Threshold	Pass/Fail
Max Index Error	0.000000000000043	0.000000000	Pass	Max Index Error	0.000000000000043	0.000000000	Pass
Min Index Error	-0.0000000000000564	0.000000000	Pass	Min Index Error	-0.000000000000043	0.000000000	Pass
Max Rate Error	0.000000000000041	0.000000000	Pass	Min Rate Error	-0.00000000000003443	0.000000000	Pass

EUR HICPxT results

UK RPI results

4 Pricing Zero Coupon Inflation Swaps

4.1 Theoretical

The valuation of these products is relatively simple as there are only two market inputs, the predicted future level of the index and the discount rate. Discounting in a cleared world is at standard OIS rates as per all other products cleared within SwapClear.

The future prediction of the price index can be estimated from bootstrapping market instruments. ZCIIS are widely traded for standard tenors and market data is readily available, including historic time series. In addition inflation linked government bonds are widely traded and could also be used as a source of data though the bonds are more complex to use with bootstrapping and in some cases include embedded optionality that lies outside the capabilities of a simple model. Bond based data has been used to extend the historic series used in backtesting of the IM models. Section 0 of this document details the calculation of the forward price index.

One complexity in the forecast price curve is that seasonality must be taken into account. This is required as there are expected cyclical cost shifts in some components of the index. A simple example is home heating costs are expected to rise in winter and fall in summer. Seasonality can be estimated using econometric methods on historic indices, or taken as market data inputs direct from members which is the proposed method. The impact of seasonality on pricing is most noticeable at the short end of the curve, and as the product length rises, it more and more averages itself out.

Where the fixing lag for the final price index follows the market standard, the above are sufficient to exactly price the product. The PV of a trade is therefore given by discounting the simple payout back to the valuation date:

$$PV = \phi \cdot N \left[\frac{CPI_{T_M}}{CPI_0} - (1 + K)^M \right] \cdot DF(T_{M+L})$$

Where

CPI_{T_M} : Index level observed at the swap fixing date T_M

CPI_0 : Reference Index level observed at the fixing date T_0 = Swap start date – Lag

ϕ : Pay or receive flag related to the Inflation leg: $\phi = \begin{cases} -1, & \text{Pay Inflation} \\ 1, & \text{Receive Inflation} \end{cases}$

K : Fixed rate that nullifies the ZCIIS PV at inception

M : Swap length in integer years

N : Swap notional amount

$DF(T_{M+L})$: Discount factor from swap payment date to pricing date

Non-standard lags introduce small convexity adjustments that theoretically need to be introduced to amend the projected price index. The reasoning is the same as for the more well understood LIBOR in arrears and CMS adjustments in pure rates. The convexity adjustment in ZCIIS with non-standard lags is dependent on the volatility of rates, the volatility of the price index, the correlation between rates and price index, the length of the deal and the length of the lag. In order to correctly evaluate the adjustment a more complex model is required, such as Jarrow-Yildrim, but these are difficult to calibrate and slow to converge. One proposal suggested is to use member supplied curves for different lags that incorporate the adjustments implicitly. Research papers in the public space however show that the size of the adjustment is only significant for lags much longer than 1 year which is the proposed maximum length. Any convexity adjustment would cancel itself out in a VaR calculation. It is therefore proposed that the convexity adjustment can be ignored for LCH scope.

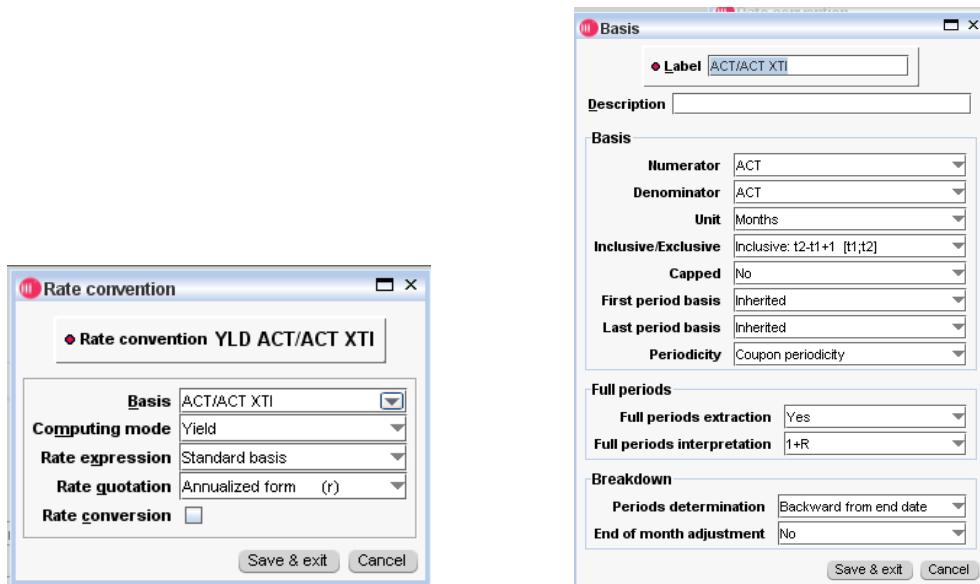
Data was requested from UBS to check the validity of this assumption. The proposed product scope is for lags to 1 year including the standard lag. The table below shows the market convexity adjustment for a variety of tenors and non-standard lag lengths, where the 2M+6M means 6 months beyond the standard 2 month lag. The first column is the standard market quote for the ZCIIS itself. The maximum scale of the adjustment is about 0.5bp which is not significant compared to the bid offer spread associated with this market.

	Standard(2M)	2M+3 M	2M+6 M	2M+9 M	2M+1Y	2M+3Y
10y	3.344%	0.001%	0.003%	0.004%	0.006%	0.017%
20y	3.616%	0.001%	0.003%	0.004%	0.006%	0.017%
30y	3.723%	0.002%	0.003%	0.005%	0.006%	0.018%
40y	3.762%	0.002%	0.003%	0.005%	0.006%	0.019%
50y	3.805%	0.002%	0.003%	0.005%	0.006%	0.019%

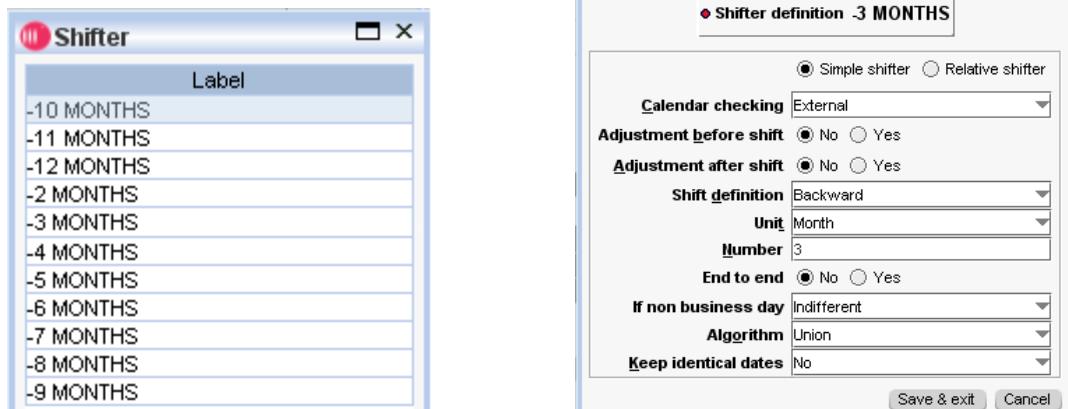
4.2 Murex Setup

4.2.1 Static Data

Basis Convention:



Fixing Lag:



Curve Settings:

Settings

General configuration

Sensitivities	Curves	Pricing	Market parameters	Models	Hedge	Trade settings	
Rate curve	Credit curve	Inflation curve	Correlation curve				
Model type MUREX model Interpolation formula Linear Value to interpolate rate Interpolation before first pillar Flat Interpolation after last pillar Flat Zero rate convention EXP ACT/365 Futures No intermediate interpolation Blocks consistency According to priorities on First date Forwards convention Inherited from currency Zero coupon spread Absolute Ignore fixing All Calibration Standard Business days None Spread curve None Hedge curve No Ext. parameter 0.00 Default entry market rates Maturity step out gap 8 Snapshot generation Never Classification Default Calibration method Global Newton Calibration tolerance 0.00010000 Calibration iterations 30 Accept zero rates Yes Seasonality None Pillar maturity date Last Flow Date FX DV01 Par convention Equivalent deposit							

Settings

General configuration

Sensitivities	Curves	Pricing	Market parameters	Models	Hedge	Trade settings	
Rate curve	Credit curve	Inflation curve	Correlation curve				
Model type MUREX model Interpolation formula Linear Value to interpolate log(CPI) Interpolation before first pillar Flat Interpolation after last pillar Flat Zero rate convention YLD ACT/ACT XTI Futures Standard interpolation Blocks consistency Checking disabled Forwards convention Inherited from currency Zero coupon spread Absolute Ignore fixing No Calibration Curve assignments Business days Shifter Spread curve None Hedge curve No Default entry market rates Maturity step out gap 0 Snapshot generation Never Classification Default CPI spot mode Last known Calibration method Global Newton Calibration tolerance 0.00010000 Calibration iterations 30 Accept zero rates Yes Seasonality Multiplicative							

4.2.2 Market data

Inflation Quotes:

Infl swap FR CPI ZC PW Floating Maturities		Infl swap FR CPI ZC Floating Maturities	
Maturity	Market value	Maturity	Market value
1M	-2.54612	1Y	1.23100 / 33900
2M	1.51896	2Y	1.37600 / 48400
3M	-0.00429	3Y	1.48250 / 57750
4M	1.39732	4Y	1.59625 / 68375
5M	0.63241	5Y	1.69400 / 78400
6M	1.42050	6Y	1.76850 / 86350
7M	0.84846	7Y	1.84750 / 93250
8M	1.43268	8Y	1.92575 / 2.01825
9M	0.96812	9Y	2.00350 / 9850
10M	1.45949	10Y	2.06375 / 15625

OIS Quotes:

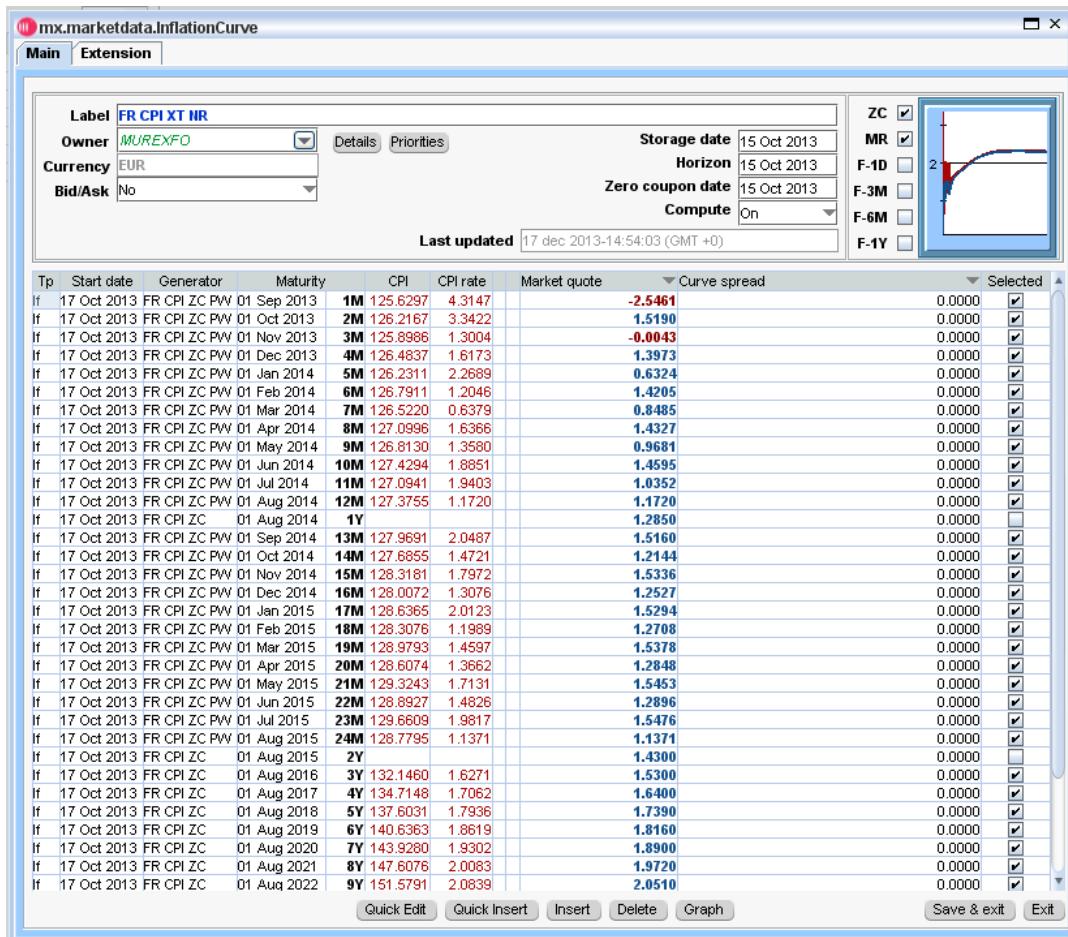
Deposit OIS Floating Maturities	
Maturity	Market value
0/N	0.07800
T/N	0.07800
1W	0.06000 / 11000
2W	0.07600 / 9600
3W	0.08100 / 9600
1M	0.07800 / 9800
2M	0.08500 / 10500
3M	0.09600 / 11600
4M	0.09880 / 13040
5M	0.10520 / 3680
6M	0.11320 / 4480
7M	0.11980 / 5140
8M	0.12780 / 5960
9M	0.13460 / 6640
10M	0.14300 / 7480
11M	0.14900 / 8080
1Y	0.15560 / 8760
15M	0.18000 / 20480
18M	0.20100 / 2900
21M	0.22050 / 5575
2Y	0.25280 / 8500
3Y	-0.05740 / 2240
4Y	-0.03408 / 0.00092
5Y	-0.07460 / 3960
6Y	-0.10480 / 06980
7Y	-0.11555 / 08055
8Y	-0.11773 / 08273
9Y	-0.11045 / 07545

Multiplicative Seasonality:

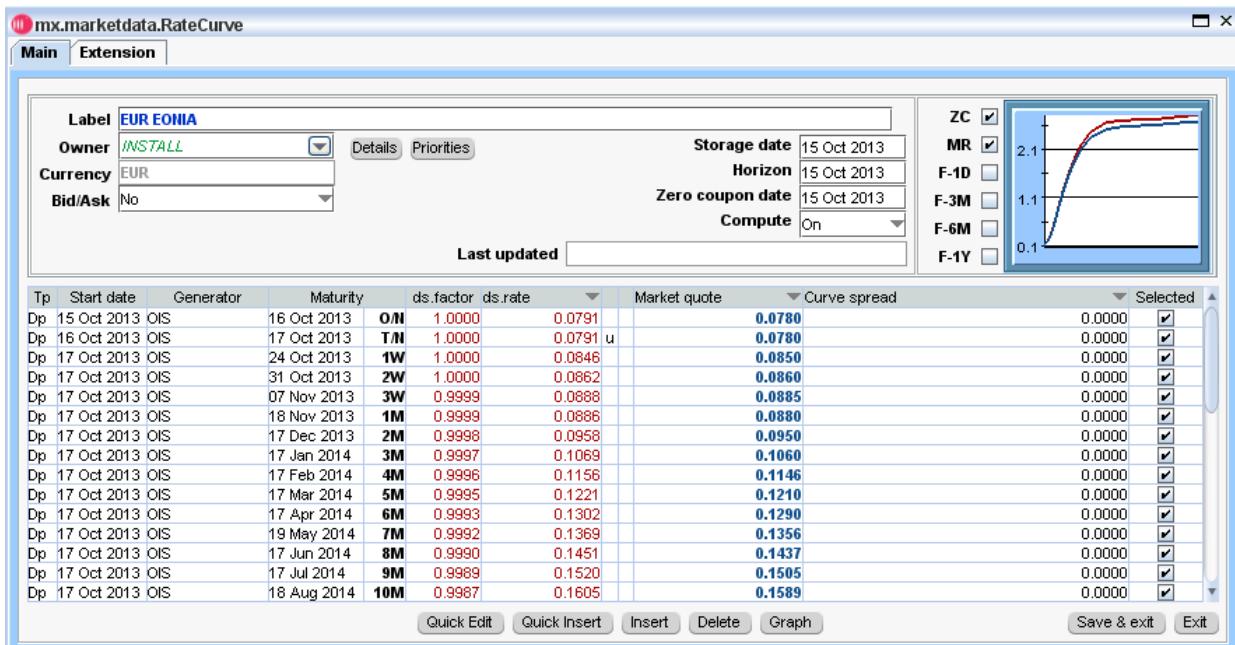
User defined market data			
Input		Output	
Generic group		SEASONALITY	FR CPI SEASONALITY
		Scenario	
		Storage date	15 Oct 2013
		Adj (+)	Adj (x)
Data	0D	0D	0D
January	0.00000000	0.99603083	
February	0.00000000	1.00381706	
March	0.00000000	1.00397102	
April	0.00000000	1.00140635	
May	0.00000000	0.99985376	
June	0.00000000	0.99925567	
July	0.00000000	0.99458301	
August	0.00000000	1.00274792	
September	0.00000000	0.999707902	
October	0.00000000	0.99977746	
November	0.00000000	0.99950348	
December	0.00000000	1.00202370	

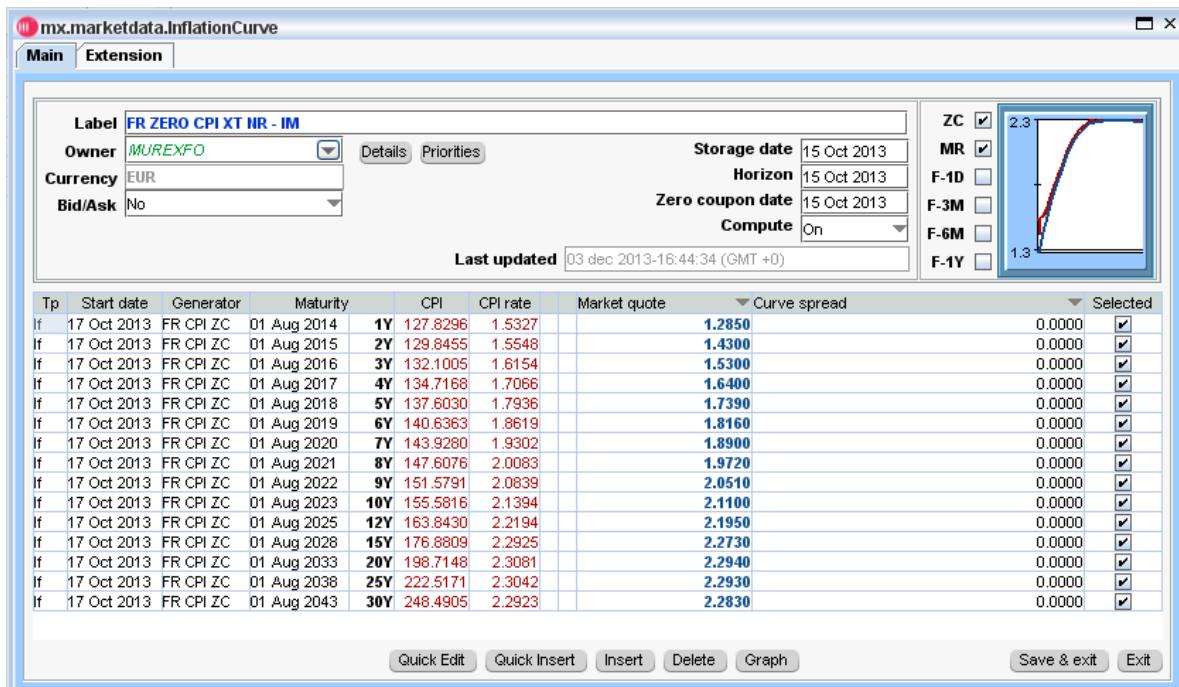
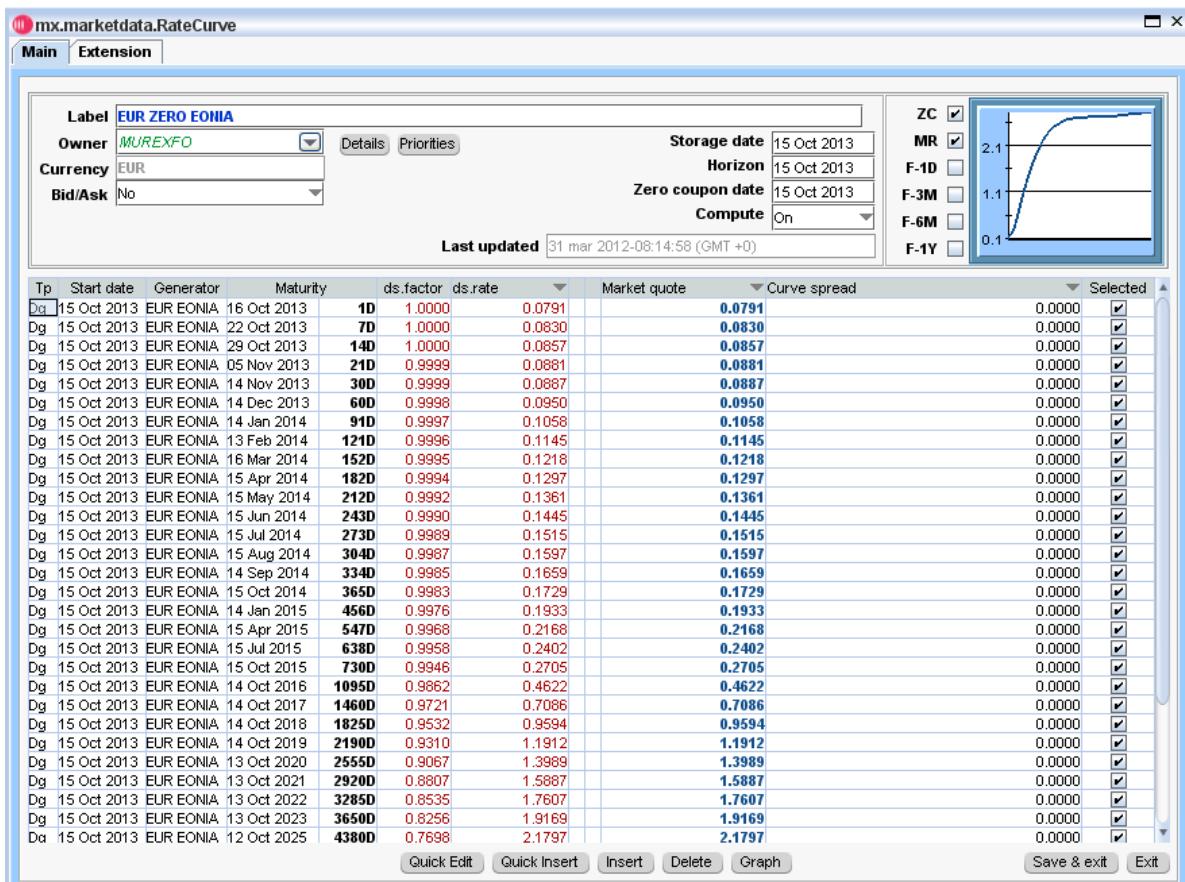
4.2.3 Curves

VM Inflation:



VM OIS:



IM Inflation:

IM OIS:


4.2.4 Curve Generators

Standard Generator

<p style="text-align: center;">Swap generator FR CPI ZC</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%;">Description <input type="text"/></td> <td style="width: 30%; text-align: right;">Phases <input type="button" value="1"/> <input type="button" value="2"/> Legs per phase <input type="button" value="1"/> <input type="button" value="2"/></td> </tr> <tr> <td>Schedules <input type="button" value="Common set across legs"/></td> <td style="text-align: right;">Default amortizing <input type="button" value="None"/></td> </tr> <tr> <td>Stub period <input type="button" value="Up front"/></td> <td style="text-align: right;">Evaluation <input type="button" value="Default"/></td> </tr> <tr> <td>Market quote <input type="button" value="Automatic"/></td> <td style="text-align: right;">Future Cash proceed cut-off <input type="button" value="+01 BD PPS"/></td> </tr> <tr> <td>Settlement delay <input type="button" value="Inherited from currency"/></td> <td></td> </tr> </table> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; border: 1px solid #ccc; padding: 5px;">Main</td> <td style="width: 50%; border: 1px solid #ccc; padding: 5px;">Main</td> </tr> <tr> <td style="padding: 5px;"> <input checked="" type="radio"/> Fixed rate <input type="radio"/> Floating rate Evaluation <input type="button" value="Inherited"/> Currency <input type="button" value="EUR"/> <input type="button" value="Details"/> Start delay <input type="button" value="+2 BUSINESS DAY"/> <input type="button" value="Details"/> Payment calendar <input type="button" value="TGT"/> <input type="button" value="Details"/> </td> <td style="padding: 5px;"> <input checked="" type="radio"/> Fixed rate <input type="radio"/> Floating rate Index <input type="button" value="FR CPI XT"/> <input type="button" value="Details"/> Evaluation <input type="button" value="Inherited"/> Currency <input type="button" value="EUR"/> <input type="button" value="Details"/> </td> </tr> <tr> <td colspan="2" style="text-align: center; padding: 10px;"> Return <input type="button" value="Edit"/> Margin convention <input type="button" value="YLD ACT/ACT XTI"/> <input type="button" value="Details"/> Interest rates rounding <input type="button" value="None"/> <input type="button" value="Details"/> Rate conversion <input type="checkbox"/> </td> </tr> </table>	Description <input type="text"/>	Phases <input type="button" value="1"/> <input type="button" value="2"/> Legs per phase <input type="button" value="1"/> <input type="button" value="2"/>	Schedules <input type="button" value="Common set across legs"/>	Default amortizing <input type="button" value="None"/>	Stub period <input type="button" value="Up front"/>	Evaluation <input type="button" value="Default"/>	Market quote <input type="button" value="Automatic"/>	Future Cash proceed cut-off <input type="button" value="+01 BD PPS"/>	Settlement delay <input type="button" value="Inherited from currency"/>		Main	Main	<input checked="" type="radio"/> Fixed rate <input type="radio"/> Floating rate Evaluation <input type="button" value="Inherited"/> Currency <input type="button" value="EUR"/> <input type="button" value="Details"/> Start delay <input type="button" value="+2 BUSINESS DAY"/> <input type="button" value="Details"/> Payment calendar <input type="button" value="TGT"/> <input type="button" value="Details"/>	<input checked="" type="radio"/> Fixed rate <input type="radio"/> Floating rate Index <input type="button" value="FR CPI XT"/> <input type="button" value="Details"/> Evaluation <input type="button" value="Inherited"/> Currency <input type="button" value="EUR"/> <input type="button" value="Details"/>	Return <input type="button" value="Edit"/> Margin convention <input type="button" value="YLD ACT/ACT XTI"/> <input type="button" value="Details"/> Interest rates rounding <input type="button" value="None"/> <input type="button" value="Details"/> Rate conversion <input type="checkbox"/>		<p style="text-align: center;">Swap generator FR CPI ZC</p> <table style="width: 100%; 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Schedules definition <input type="button" value="Details"/> Payment <input type="button" value="In arrears"/> Rate convention <input type="button" value="YLD ACT/ACT XTI"/> Interest rates rounding <input type="button" value="None"/> Rate conversion <input type="checkbox"/>	Return <input type="button" value="Edit"/> Margin convention <input type="button" value="YLD ACT/ACT XTI"/> <input type="button" value="Details"/> Interest rates rounding <input type="button" value="None"/> Rate conversion <input type="checkbox"/>
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Schedules definition <input type="button" value="Details"/> Calculation start schedule <input type="button" value="Driving schedule"/> Calculation end schedule <input type="button" value="Equal to"/> Payment schedule <input type="button" value="Deduced from"/> Deduction formula <input type="button" value="+0 BUSINESS DAY"/> Fixing schedule <input type="button" value="Equal to"/> Calculations follow fixings <input type="button" value="No (time series)"/> Capital schedules <input type="button" value="No"/> Single period <input type="button" value="Yes"/>	Return <input type="button" value="Edit"/> Index <input type="button" value="FR CPI XT"/> Return type <input type="button" value="Return (P2-P1)/P1"/> Interpolation <input type="button" value="Linear"/> Day count fraction <input type="button" value="Ignore"/> Fixing lag <input type="button" value="-3 MONTHS"/> Leg <input type="button" value="2"/>
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Non Standard Generator

<p style="text-align: center;">Swap generator FR CPI ZC PW</p> <p>Description <input type="text"/></p> <p>Schedules Common set across legs <input type="button" value="..."/> Phases <input type="button" value="1"/> Stub period Up front <input type="button" value="..."/> Legs per phase <input type="button" value="2"/></p> <p>Market quote Automatic <input type="button" value="..."/> Settlement delay Inherited from currency <input type="button" value="..."/></p> <p>Main</p> <p style="text-align: center;">(●) Fixed rate (○) Floating rate</p> <p>Evaluation Inherited <input type="button" value="..."/></p> <p>Currency <input type="button" value="EUR"/> <input type="button" value="..."/> Start delay +2 BUSINESS DAY <input type="button" value="..."/> Payment calendar TGT <input type="button" value="..."/></p> <p>Schedules definition <input type="button" value="Details"/></p> <p>Payment In arrears <input type="button" value="..."/></p> <p>Rate convention YLD ACT/ACT XTI <input type="button" value="..."/> Interest rates rounding None <input type="button" value="..."/> Rate conversion <input type="checkbox"/></p> <p>Indexed <input type="checkbox"/> Multi Currency <input type="checkbox"/></p>	<p style="text-align: center;">Default amortizing None <input type="button" value="..."/> Evaluation Default <input type="button" value="..."/></p> <p>Future Cash proceed cut-off <input type="button" value="..."/></p> <p>Main</p> <p style="text-align: center;">(○) Fixed rate (●) Floating rate</p> <p>Index FR CPI XT <input type="button" value="..."/> <input type="button" value="Details"/></p> <p>Evaluation Inherited <input type="button" value="..."/></p> <p>Currency <input type="button" value="EUR"/> <input type="button" value="..."/></p> <p>Return <input type="button" value="Edit"/> Margin convention YLD ACT/ACT XTI <input type="button" value="..."/> Interest rates rounding None <input type="button" value="..."/> Rate conversion <input type="checkbox"/></p> <p>Indexed <input type="checkbox"/> Multi Currency <input type="checkbox"/> Forward start capital <input type="checkbox"/></p>														
<p>Schedules definition <input type="button" value="..."/></p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td>Calculation start schedule Driving schedule <input type="button" value="..."/></td> <td>Calculation end schedule Equal to <input type="button" value="..."/> Calculation start schedule <input type="button" value="..."/></td> </tr> <tr> <td>Payment schedule Deduced from <input type="button" value="..."/></td> <td>Deduction formula +0 DAY MODFOL <input type="button" value="..."/></td> </tr> <tr> <td>Fixing schedule Equal to <input type="button" value="..."/></td> <td>Calculations follow fixings No (time series) <input type="button" value="..."/></td> </tr> <tr> <td>Capital schedules No <input type="button" value="..."/></td> <td>Single period Yes <input type="button" value="..."/> No maturity adjustment <input type="button" value="..."/></td> </tr> </table> <p style="text-align: right;">Close</p> <p>Return <input type="button" value="..."/></p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td>Index FR CPI XT <input type="button" value="..."/></td> </tr> <tr> <td>Return type Initial return (P2-P0)/P0 <input type="button" value="..."/></td> </tr> <tr> <td>Interpolation Piecewise <input type="button" value="..."/></td> </tr> <tr> <td>Day count fraction Ignore <input type="button" value="..."/></td> </tr> <tr> <td>Fixing lag -2 MONTHS <input type="button" value="..."/></td> </tr> <tr> <td>Leg <input type="button" value="2"/></td> </tr> </table> <p style="text-align: right;">Close</p>		Calculation start schedule Driving schedule <input type="button" value="..."/>	Calculation end schedule Equal to <input type="button" value="..."/> Calculation start schedule <input type="button" value="..."/>	Payment schedule Deduced from <input type="button" value="..."/>	Deduction formula +0 DAY MODFOL <input type="button" value="..."/>	Fixing schedule Equal to <input type="button" value="..."/>	Calculations follow fixings No (time series) <input type="button" value="..."/>	Capital schedules No <input type="button" value="..."/>	Single period Yes <input type="button" value="..."/> No maturity adjustment <input type="button" value="..."/>	Index FR CPI XT <input type="button" value="..."/>	Return type Initial return (P2-P0)/P0 <input type="button" value="..."/>	Interpolation Piecewise <input type="button" value="..."/>	Day count fraction Ignore <input type="button" value="..."/>	Fixing lag -2 MONTHS <input type="button" value="..."/>	Leg <input type="button" value="2"/>
Calculation start schedule Driving schedule <input type="button" value="..."/>	Calculation end schedule Equal to <input type="button" value="..."/> Calculation start schedule <input type="button" value="..."/>														
Payment schedule Deduced from <input type="button" value="..."/>	Deduction formula +0 DAY MODFOL <input type="button" value="..."/>														
Fixing schedule Equal to <input type="button" value="..."/>	Calculations follow fixings No (time series) <input type="button" value="..."/>														
Capital schedules No <input type="button" value="..."/>	Single period Yes <input type="button" value="..."/> No maturity adjustment <input type="button" value="..."/>														
Index FR CPI XT <input type="button" value="..."/>															
Return type Initial return (P2-P0)/P0 <input type="button" value="..."/>															
Interpolation Piecewise <input type="button" value="..."/>															
Day count fraction Ignore <input type="button" value="..."/>															
Fixing lag -2 MONTHS <input type="button" value="..."/>															
Leg <input type="button" value="2"/>															

4.2.5 Trade Generators

EUR FR CPI XT ZC 10M LIN	EUR EU HICP XT ZC 10M LIN	GBP GB RPI AI ZC 10M LIN	USD US CPI AI NSA ZC 10M LIN
EUR FR CPI XT ZC 10M PW	EUR EU HICP XT ZC 10M PW	GBP GB RPI AI ZC 11M LIN	USD US CPI AI NSA ZC 10M PW
EUR FR CPI XT ZC 11M LIN	EUR EU HICP XT ZC 11M LIN	GBP GB RPI AI ZC 11M PW	USD US CPI AI NSA ZC 11M LIN
EUR FR CPI XT ZC 11M PW	EUR EU HICP XT ZC 11M PW	GBP GB RPI AI ZC 12M LIN	USD US CPI AI NSA ZC 11M PW
EUR FR CPI XT ZC 12M LIN	EUR EU HICP XT ZC 12M LIN	GBP GB RPI AI ZC 12M PW	USD US CPI AI NSA ZC 12M LIN
EUR FR CPI XT ZC 12M PW	EUR EU HICP XT ZC 12M PW	GBP GB RPI AI ZC 2M LIN	USD US CPI AI NSA ZC 12M PW
EUR FR CPI XT ZC 3M LIN	EUR EU HICP XT ZC 3M LIN	GBP GB RPI AI ZC 2M PW	USD US CPI AI NSA ZC 3M LIN
EUR FR CPI XT ZC 3M PW	EUR EU HICP XT ZC 3M PW	GBP GB RPI AI ZC 3M LIN	USD US CPI AI NSA ZC 3M PW
EUR FR CPI XT ZC 4M LIN	EUR EU HICP XT ZC 4M LIN	GBP GB RPI AI ZC 3M PW	USD US CPI AI NSA ZC 4M LIN
EUR FR CPI XT ZC 4M PW	EUR EU HICP XT ZC 4M PW	GBP GB RPI AI ZC 4M LIN	USD US CPI AI NSA ZC 4M PW
EUR FR CPI XT ZC 5M LIN	EUR EU HICP XT ZC 5M LIN	GBP GB RPI AI ZC 4M PW	USD US CPI AI NSA ZC 5M LIN
EUR FR CPI XT ZC 5M PW	EUR EU HICP XT ZC 5M PW	GBP GB RPI AI ZC 5M LIN	USD US CPI AI NSA ZC 5M PW
EUR FR CPI XT ZC 6M LIN	EUR EU HICP XT ZC 6M LIN	GBP GB RPI AI ZC 5M PW	USD US CPI AI NSA ZC 6M LIN
EUR FR CPI XT ZC 6M PW	EUR EU HICP XT ZC 6M PW	GBP GB RPI AI ZC 6M LIN	USD US CPI AI NSA ZC 6M PW
EUR FR CPI XT ZC 7M LIN	EUR EU HICP XT ZC 7M LIN	GBP GB RPI AI ZC 6M PW	USD US CPI AI NSA ZC 7M LIN
EUR FR CPI XT ZC 7M PW	EUR EU HICP XT ZC 7M PW	GBP GB RPI AI ZC 7M LIN	USD US CPI AI NSA ZC 7M PW
EUR FR CPI XT ZC 8M LIN	EUR EU HICP XT ZC 8M LIN	GBP GB RPI AI ZC 7M PW	USD US CPI AI NSA ZC 8M LIN
EUR FR CPI XT ZC 8M PW	EUR EU HICP XT ZC 8M PW	GBP GB RPI AI ZC 8M LIN	USD US CPI AI NSA ZC 8M PW
EUR FR CPI XT ZC 9M LIN	EUR EU HICP XT ZC 9M LIN	GBP GB RPI AI ZC 8M PW	USD US CPI AI NSA ZC 9M LIN
EUR FR CPI XT ZC 9M PW	EUR EU HICP XT ZC 9M PW	GBP GB RPI AI ZC 9M LIN	USD US CPI AI NSA ZC 9M PW

Schedules definition

Calculation start schedule	Driving schedule
Calculation end schedule	Equal to Calculation start schedule
Payment schedule	Deduced from Calculation start schedule
Deduction formula	+0 DAY MODFOL
Calculations follow fixings	No (time series)
Capital schedules	No
Single period	Yes No maturity adjustment

Return

Index	FR CPI XT
Return type	Return (P2-P1)/P1
Interpolation	Linear
Day count fraction	Ignore
Fixing lag	-3 MONTHS
Leg	2

Swap generator EUR FR CPI XT ZC 3M LIN

Description	Phases 1
Schedules	Independent sets across legs
Stub period	Up front
Market quote	Automatic
Settlement delay	Inherited from currency
Default amortizing	None
Amortizing	Common definition across legs
Evaluation	Default
Future Cash proceed cut-off	+01 BD PPS

Main

Fixed rate	Floating rate
Evaluation	Inherited
Currency	EUR
Start delay	+2 BUSINESS DAY
Payment calendar	TGT
Evaluation	Inherited
Currency	EUR
Start delay	+2 BUSINESS DAY
Payment calendar	TGT
Fixing calendar	TGT
Schedules definition	Details
Return	Edit
Margin convention	YLD ACT/ACT XTI
Interest rates rounding	None
Rate conversion	<input checked="" type="checkbox"/>
Indexed	<input type="checkbox"/>
Multi Currency	<input type="checkbox"/>

4.2.6 Curve Assignment

Seasonality:

Inflation seasonality

Index	Currency	Desk	Seasonality
EU HICP XT			EU HICP SEASONALITY
FR CPI XT			FR CPI SEASONALITY
GB RPI AI			GB RPI SEASONALITY
US CPI AI NSA			US CPI SEASONALITY

Filters

Index	Currency	Desk	Seasonality

Save & exit Cancel

Inflation Curves:

base:EUR																
Currency	Type	Calculation	Curve type	Generator	Index	Archiving Group	Fx Contract	Underlying	Market	Security	Issuer	Category	User	Desk	Rate curve	
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 10M LIN												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 10M PW												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 11M LIN												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 11M PW												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 12M LIN												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 12M PW												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 3M LIN												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 3M PW												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 4M LIN												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 4M PW												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 5M LIN												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 5M PW												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 6M LIN												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 6M PW												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 7M LIN												FR ZERO CPI XT NR - IM
EUR	IR swap	Forwards	Inflation	EUR FR CPI XT ZC 7M PW												FR ZERO CPI XT NR - IM

OIS curves:

Initial Margin/base																	
Currency	Type	Calculation	Index	Generator	Curve type	Archiving Group	Fx Contract	Underlying	Market	Security	Issuer	Category	User	Desk	Group	Typolo	Rate curve
EUR	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	<empty>	
EUR	Infl swap	Discounts															EUR ZERO EONIA
EUR	IR swap	Discounts	EU HICP XT														EUR ZERO EONIA
EUR	IR swap	Discounts	FR CPI XT														EUR ZERO EONIA

4.2.7 Simulation Views

NPV validation:

Global ID	Curren...	Financed NPV
Validation2_163	USD	1,038,842.7014
Validation2_164	EUR	-1,987,678.4887
Validation2_165	USD	7,924,835.1673
Validation2_166	EUR	2,384,608.7437
Validation_1	GBP	4,410,982.2511
Validation_10	EUR	35,980,388.8383
Validation_100	EUR	1,330,573.0885
Validation_101	EUR	1,336,376.4650
Validation_102	EUR	1,338,315.1656
Validation_103	EUR	1,338,315.1656
Validation_104	EUR	1,338,315.1656
Validation_105	EUR	1,340,194.8970
Validation_106	EUR	1,342,153.2779
Validation_107	EUR	1,358,172.1650
Validation_108	EUR	1,365,381.2331
Validation_109	EUR	1,367,698.8227
Validation_11	USD	29,912,746.4764
Validation_110	EUR	1,367,698.8227
Validation_111	GBP	2,392,124.1614
Validation_112	EUR	17,862,3738
Validation_113	USD	46,044.3363
Validation_114	EUR	-605,880.2130

Inflation Delta/Gamma Validation:

Curve name	Date	Inflation (par)	Inflation (Cpi)	Inflation (zero)	Inflation gamma (Par)	Inflation gamma (Zero)	Inflation gamma (Cpi)
FR ZERO CPI XT NR - IM	1Y	-56,934.1845	-2,826,147.6723	-32,665.4205	-1	2	
	2Y	1,014,384.1181	38,483,219.8658	987,376.8329	131	141	
	3Y	620,201.4266	15,404,872.1001	591,285.0373	82	58	
	4Y	-57,074.6611	-1,045,533.7233	-55,824.9546	-19	-20	
	5Y	-35,856.9389	-518,024.1714	-34,398.1373	-12	-10	
	6Y	0.3412			0		
	7Y	-8.8952			-0		
	8Y	227.1908			0		
	9Y	-5,712.4588			-5		
	10Y	141,885.4876	892,838.5432	137,132.5878	132	132	
	12Y	76,275.8845	400,497.4616	74,634.3463	75	68	
	15Y	844.7028			2		
	20Y	-129,669.4766	-305,631.8148	-127,444.2578	-283	-321	
	25Y	160,107.2395	251,146.2539	162,261.1465	493	607	
	30Y	496,766.1641	705,116.5004	492,466.0178	1,332	1,250	
GB ZERO AI 2M - IM	1Y	9,942.2087	1,623,263.2039	9,942.2087	-1	-1	
	2Y	69,520.9967	1,336,012.5963	69,520.9967	8	8	
	3Y	21,539.0146	274,105.9951	21,539.0146	3	2	
	15Y	191,623.0243	288,501.1087	191,623.0243	296	335	
	20Y	199,648.5811	216,917.9504	199,648.5811	330	307	
	25Y	516,437.0556	345,819.3298	516,437.0556	1,180	1,163	
	30Y	282,101.0203	103,616.1753	282,101.0203	953	1,142	
	40Y	564,289.0626	155,459.7237	564,289.0626	1,908	1,713	
	50Y	1,212,641.4302	159,778.2299	1,212,641.4302	5,741	5,728	

OIS Delta/Gamma Validation:

JRS_IM

Outputs DV01 (zero) x DV02 (zero) x

Line breakdowns Curve name Grid Date x

Column breakdowns Outputs x

Curve name	Date	DV01 (zero)	DV02 (zero)
	15 Oct 2013	0.5372	-0.0000
	21 Oct 2013	0.1074	-0.0000
	13 Dec 2013	21.5912	-0.0004
	13 Jan 2014	43.3911	-0.0011
	12 Feb 2014	47.2632	-0.0016
	15 Mar 2014	10.2968	-0.0004
	14 Jun 2014	-123.4225	0.0084
	14 Jul 2014	-18.9881	0.0013
	14 Oct 2014	154.2750	-0.0155
EUR ZERO EONIA	13 Jan 2015	-70.9623	0.0099
	14 Apr 2015	219.5868	-0.0340
	14 Jul 2015	6.5707	-0.0010
	14 Oct 2015	-13,250.6115	3.1200
	13 Oct 2016	-7,278.2298	1.7136
	13 Oct 2017	-6,907.9878	3.0244
	13 Oct 2018	-3,270.8230	1.3391
	13 Oct 2019	564.6097	-0.3037
	12 Oct 2020	-0.0000	
	12 Oct 2021	-0.0000	
	12 Oct 2022	-0.0000	
	12 Oct 2023	-3,646.7967	3.8579
	11 Oct 2025	-2,312.2702	2.5637
	10 Oct 2028	-107,704.1951	161.7333
	09 Oct 2033	-44,797.0540	105.0713
	08 Oct 2038	-229,300.6210	588.0624
	07 Oct 2043	-210,719.0432	601.6922
	05 Oct 2048	-0.0000	0.0000
GBP ZERO SONIA	15 Oct 2013	-2.1846	0.0000
	21 Oct 2013	-0.4369	0.0000
	13 Dec 2013	-47.8941	0.0008
	13 Jan 2014	-5.1315	0.0001

Inflation Cross Gamma Validation:

_Inflation_Cross_IM

Global ID: ALL

Outputs: PAR PAR Zero ZC CPI ZC

Line breakdowns: Curve name Curve name Inflation date Rate date

Column breakdowns: Outputs

Curve name	Curve name	Inflation date	Rate date	PAR PAR	Zero ZC	CPI ZC
FR ZERO CPI XT NR - IM	EUR ZERO EONIA	01 Aug 2014	13 Dec 2013	-0.0400	-0.0382	-24
			13 Jan 2014	-0.0077	-0.0074	-5
			14 Oct 2014	6.1321	5.8542	625
			13 Jan 2015	-1.2212	-1.1709	-90
			14 Apr 2015	-2.1499	-2.0883	-165
			14 Jul 2015	-0.0184	-0.0183	-1
			14 Oct 2015	3.4406		
			13 Oct 2016	1.8181		
			13 Oct 2017	-0.0003		
			13 Oct 2018	-0.0002		
			13 Jan 2015	-0.2380	-0.2276	-9
			14 Apr 2015	-1.6750	-1.6012	-64
			14 Jul 2015	-0.0324	-0.0310	-1
01 Aug 2015	01 Aug 2015	01 Aug 2015	14 Oct 2015	-153.6255	-149.5499	-5,827
			13 Oct 2016	-81.1791	-79.0519	-3,080
			13 Oct 2017	0.0156		
			13 Oct 2018	0.0095		
			12 Oct 2023	-0.0000		
			14 Oct 2015	-94.4383	-90.3142	-2,353
			13 Oct 2016	-50.8296	-48.6099	-1,266
			13 Oct 2017	-0.5218		
			13 Oct 2018	-0.3172		
			12 Oct 2023	0.0000		
			11 Oct 2025	0.0000		
			13 Oct 2017	15.5403	15.2001	285
01 Aug 2017	01 Aug 2017	01 Aug 2017	13 Oct 2018	9.4474	9.2406	173
			12 Oct 2023	-0.0000		
			11 Oct 2025	-0.0000		
			13 Oct 2017	9.7904	9.3659	141
			13 Oct 2018	5.9519	5.6938	86

4.3 Murex Testing and Results

4.3.1 Testing approach

The pricing formula can be split in two different aspects: deal parameters and market data. A portfolio of ZCIIS was constructed to test all deal parameters and pricing repeated under a number of different market conditions.

Deal parameters

The pricing of a ZCIIS can be computed as a difference of two cash flows (fixed and floating leg) discounted from the payment date to the pricing date. Four different parameters can be listed to cover the tests.

A. Fixed leg Cash flow

The fixed leg cash flow is calculated as follows:

$$CF = N[(1 + K)^M - 1]$$

Tests to cover:

1. Time to maturity calculation follows 30/360 by month basis convention
2. Time to maturity is calculated from Swap Start Date to Swap End Date
3. Cash flow is signed based on Buy/Sell flag

B. Reference Index level

The reference index level is calculated following this formula:

$$CPI_0 = CPI_{M-L} + \frac{d-1}{D} * (CPI_{M-L+1} - CPI_{M-L})$$

Tests to cover:

1. For two identical ZCIIS except the fixing lag, the reference index level changes
2. D is calculated using the number of days in the swap start date month (not the fixing date month)
3. Reference CPI is rounded to 5 decimals for linear interpolated indices

Test cases for all 4 indices:

- 3M and 4M lag ZCIIS with same swap start date should not point to the same reference CPI

Test cases for linear FR/US indices and non standard UK/EU indices:

- D=30 for a 3M lag ZCIIS with a Swap End date in April
- D=31 for a 3M lag ZCIIS with a Swap End date in December
- D=28 for a 5M lag ZCIIS with a Swap End date in February 2015
- D=29 for a 5M lag ZCIIS with a Swap End date in February 2016
- Spot 1Y ZCIIS with 3M/4M/5M/6M/7M/8M/9M/10M/11M/12M lags all have a reference CPI rounded to 5 decimals

C. Forward Index level

Forward index levels are interpolated from the Inflation Curve and the interpolation/extrapolation part has been tested previously. The remaining test to perform is to make sure that a linear interpolation occurs for standard FR and US indices and non standard UK and EU indices.

Tests to cover:

1. CPI before the first pillar of the Inflation Curve is correctly interpolated
2. CPI on a pillar of the Inflation Curve is correctly read
3. CPI in between two pillars of the curve is correctly interpolated
4. CPI interpolated for linear indices is correctly interpolated from current and next month
5. CPI extrapolated for linear indices is correctly calculated

Test cases for all 4 indices:

- 1Y backdated ZCIIS with 1 day left before swap end date
- Spot ZCIIS with maturity available on the inflation curve and fixed rate equal to par rate
- 6M backdated ZCIIS with 8M/9M maturity and 2M/3M lag
- Spot ZCIIS with maturity date not available on the inflation curve

D. Cash flow Discounting

Discount of a cash flow is done the same way for a zero coupon IRS or Inflation Swap. It is calculated from the payment date to the pricing date. This functionality is used in all existing products but was tested for inflation swaps for completeness.

Tests to cover:

1. Discount factor is calculated from the payment date to the pricing date
2. With zero coupon rates at 0, discounted and undiscounted cash flows are the same

Test cases for all 4 indices:

- Use previous trades and make sure the discounted PV is correct when compared to the spreadsheet in Appendix
- Hard code the zero coupon rates at 0 and check that discounted and undiscounted flows are identical

Market data

On top of deal parameters that need to be tested, market data have an import role in the pricing. To make sure the pricing is correct; tests must be conducted for different profile of curves to ensure that a wide variety of cases are covered.

There are two market data involved in the pricing of a ZCIIS: an Inflation curve and an Interest Rates curve. A combination of different curve profiles will be applied so that Murex can be validated for any market condition. OIS curves are already well known and used by LCH to discount OIS Swaps and therefore the number of combination applied to them will be less important than on the Inflation curves which are entirely new.

The following 15 combinations of profile were tested:

Inflation Curve:

- Standard market curve
- Negative rates in the short end (up to 3 years)
- Flat positive curve
- Kink up centred on the 10y tenor
- Kink down centred on the 10y tenor

Interest Rate Curve:

- Standard market curve
- Negative rates in the short end (up to 3 years)
- Steep rising rates in the short end (up to 3 years)

4.3.2 Test portfolio

The same test portfolio containing 268 deals to cover the four main parameters defined previously has been used to test the PV and sensitivities calculation. Deals are split in two: one half to cover the standard interpolation case (linear for GBP and EUR and piecewise for USD and FRF) and the second half for non standard interpolation where the previous mapping is reversed.

The following trades have been used to test the fixed leg Cash flow calculation.

Standard interpolation:

Nominal	Interpolation	Index	Index Lag	Pay Inflation	Swap	Fixed Rate	Currency	Currency Lag	Start Date	End Date	Payment Date
100,000,000	Piecewise	GB	2 month	No	2Y	1.00%	GBP	0 days	14/10/2013	14/10/2015	14/10/2015
100,000,000	Piecewise	GB	2 month	Yes	2Y	1.00%	GBP	0 days	14/10/2013	14/10/2015	14/10/2015
100,000,000	Piecewise	EUR	3 month	No	5Y	1.50%	EUR	2 days	16/10/2013	16/10/2018	16/10/2018
100,000,000	Piecewise	EUR	3 month	Yes	5Y	1.50%	EUR	2 days	16/10/2013	16/10/2018	16/10/2018
100,000,000	Linear	US	3 month	No	126M	2.77%	USD	2 days	17/04/2013	17/04/2024	17/04/2024
100,000,000	Linear	US	3 month	Yes	126M	2.77%	USD	2 days	17/04/2013	17/04/2024	17/04/2024
100,000,000	Linear	FR	3 month	No	30Y	3.00%	EUR	2 days	16/10/2013	16/10/2043	16/10/2043
100,000,000	Linear	FR	3 month	Yes	30Y	3.00%	EUR	2 days	16/10/2013	16/10/2043	16/10/2043
100,000,000	Piecewise	GB	6 month	No	26Y	-2.50%	GBP	0 days	15/10/2012	15/10/2038	15/10/2038
100,000,000	Piecewise	EUR	7 month	Yes	16Y	4.10%	EUR	2 days	16/10/2012	16/10/2028	16/10/2028
100,000,000	Linear	US	8 month	No	10Y	-1.11%	USD	2 days	16/10/2012	16/10/2022	17/10/2022
100,000,000	Linear	FR	12 month	Yes	25Y	0.50%	EUR	2 days	16/10/2012	16/10/2037	16/10/2037
100,000,000	Piecewise	GB	2 month	No	37Y	1.00%	GBP	0 days	15/10/2012	15/10/2049	15/10/2049
100,000,000	Piecewise	EUR	3 month	No	24Y	1.50%	EUR	2 days	16/10/2012	16/10/2036	16/10/2036
100,000,000	Linear	US	3 month	No	10Y	2.77%	USD	2 days	16/10/2012	16/10/2022	17/10/2022
100,000,000	Linear	FR	3 month	Yes	30Y	3.00%	EUR	2 days	16/10/2012	16/10/2042	16/10/2042
100,000,000	Piecewise	GB	6 month	Yes	6M	1.11%	GBP	0 days	15/07/2013	15/07/2014	15/07/2014
100,000,000	Piecewise	EUR	5 month	No	1M	1.50%	EUR	2 days	18/02/2013	18/02/2014	18/02/2014
100,000,000	Linear	US	6 month	No	10Y	2.77%	USD	2 days	16/01/2014	16/01/2024	16/01/2024
100,000,000	Linear	FR	6 month	No	29Y	3.00%	EUR	2 days	16/01/2014	16/01/2043	16/01/2043

Non standard interpolation:

Nominal	Interpolation	Index	Index Lag	Pay Inflation	Swap	Fixed Rate	Currency	Currency Lag	Start Date	End Date	Payment Date
100,000,000	Piecewise	GB	2 month	No	2Y	1.00%	GBP	0 days	14/10/2013	14/10/2015	14/10/2015
100,000,000	Piecewise	GB	2 month	Yes	2Y	1.00%	GBP	0 days	14/10/2013	14/10/2015	14/10/2015
100,000,000	Piecewise	EUR	3 month	No	5Y	1.50%	EUR	2 days	16/10/2013	16/10/2018	16/10/2018
100,000,000	Piecewise	EUR	3 month	Yes	5Y	1.50%	EUR	2 days	16/10/2013	16/10/2018	16/10/2018
100,000,000	Linear	US	3 month	No	126M	2.77%	USD	2 days	17/04/2013	17/04/2024	17/04/2024
100,000,000	Linear	US	3 month	Yes	126M	2.77%	USD	2 days	17/04/2013	17/04/2024	17/04/2024
100,000,000	Linear	FR	3 month	No	30Y	3.00%	EUR	2 days	16/10/2013	16/10/2043	16/10/2043
100,000,000	Linear	FR	3 month	Yes	30Y	3.00%	EUR	2 days	16/10/2013	16/10/2043	16/10/2043
100,000,000	Piecewise	GB	6 month	No	26Y	-2.50%	GBP	0 days	15/10/2012	15/10/2038	15/10/2038
100,000,000	Piecewise	EUR	7 month	Yes	16Y	4.10%	EUR	2 days	16/10/2012	16/10/2028	16/10/2028
100,000,000	Linear	US	8 month	No	10Y	-1.11%	USD	2 days	16/10/2012	16/10/2022	17/10/2022
100,000,000	Linear	FR	12 month	Yes	25Y	0.50%	EUR	2 days	16/10/2012	16/10/2037	16/10/2037
100,000,000	Piecewise	GB	2 month	No	37Y	1.00%	GBP	0 days	15/10/2012	15/10/2049	15/10/2049
100,000,000	Piecewise	EUR	3 month	No	24Y	1.50%	EUR	2 days	16/10/2012	16/10/2036	16/10/2036
100,000,000	Linear	US	3 month	No	10Y	2.77%	USD	2 days	16/10/2012	16/10/2022	17/10/2022
100,000,000	Linear	FR	3 month	Yes	30Y	3.00%	EUR	2 days	16/10/2012	16/10/2042	16/10/2042
100,000,000	Piecewise	GB	6 month	Yes	6M	1.11%	GBP	0 days	15/07/2013	15/07/2014	15/07/2014
100,000,000	Piecewise	EUR	5 month	No	1M	1.50%	EUR	2 days	18/02/2013	18/02/2014	18/02/2014
100,000,000	Linear	US	6 month	No	10Y	2.77%	USD	2 days	16/01/2014	16/01/2024	16/01/2024
100,000,000	Linear	FR	6 month	No	29Y	3.00%	EUR	2 days	16/01/2014	16/01/2043	16/01/2043

The following deals have been used to test the reference index calculation.
Standard interpolation:

Nominal	Interpolation	Index	Index Lag	Pay Inflation	Swap	Fixed Rate	Currency	Currency Lag	Start Date	End Date	Payment Date
100,000,000	Piecewise	GB	3 month	No	6M	1.00%	GBP	0 days	15/04/2013	15/04/2014	15/04/2014
100,000,000	Piecewise	GB	4 month	No	6M	1.20%	GBP	0 days	15/04/2013	15/04/2014	15/04/2014
100,000,000	Piecewise	EUR	3 month	No	8M	1.50%	EUR	2 days	18/06/2013	18/06/2014	18/06/2014
100,000,000	Piecewise	EUR	4 month	No	8M	1.00%	EUR	2 days	18/06/2013	18/06/2014	18/06/2014
100,000,000	Linear	US	3 month	No	26M	2.50%	USD	2 days	18/12/2012	18/12/2015	18/12/2015
100,000,000	Linear	US	4 month	No	26M	1.60%	USD	2 days	18/12/2012	18/12/2015	18/12/2015
100,000,000	Linear	FR	3 month	No	27Y	1.80%	EUR	2 days	16/10/2013	16/10/2040	16/10/2040
100,000,000	Linear	FR	4 month	No	27Y	2.00%	EUR	2 days	16/10/2013	16/10/2040	16/10/2040
100,000,000	Linear	US	3 month	No	3Y April	1.60%	USD	2 days	17/12/2012	17/12/2015	17/12/2015
100,000,000	Linear	FR	3 month	No	3Y April	2.00%	EUR	2 days	17/12/2012	17/12/2015	17/12/2015
100,000,000	Linear	US	3 month	No	11Y Dec	1.70%	USD	2 days	17/12/2012	17/12/2024	17/12/2024
100,000,000	Linear	FR	3 month	No	11Y Dec	1.80%	EUR	2 days	17/12/2012	17/12/2024	17/12/2024
100,000,000	Linear	US	5 month	No	Feb-15	1.15%	USD	2 days	28/02/2011	28/02/2015	27/02/2015
100,000,000	Linear	FR	6 month	No	Feb-15	1.12%	EUR	2 days	29/02/2012	28/02/2015	27/02/2015
100,000,000	Linear	US	5 month	No	Feb-16	1.00%	USD	2 days	29/02/2008	29/02/2016	29/02/2016
100,000,000	Linear	FR	6 month	No	Feb-16	1.20%	EUR	2 days	29/02/2008	29/02/2016	29/02/2016
100,000,000	Linear	FR	3 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	FR	4 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	FR	5 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	FR	6 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	FR	7 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	3 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	4 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	5 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	6 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	7 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014

Non standard interpolation:

Nominal	Interpolation	Index	Index Lag	Pay Inflation	Swap	Fixed Rate	Currency	Currency Lag	Start Date	End Date	Payment Date
100,000,000	Piecewise	GB	3 month	No	6M	1.00%	GBP	0 days	15/04/2013	15/04/2014	15/04/2014
100,000,000	Piecewise	GB	4 month	No	6M	1.20%	GBP	0 days	15/04/2013	15/04/2014	15/04/2014
100,000,000	Piecewise	EUR	3 month	No	8M	1.50%	EUR	2 days	18/06/2013	18/06/2014	18/06/2014
100,000,000	Piecewise	EUR	4 month	No	8M	1.00%	EUR	2 days	18/06/2013	18/06/2014	18/06/2014
100,000,000	Linear	US	3 month	No	26M	2.50%	USD	2 days	18/12/2012	18/12/2015	18/12/2015
100,000,000	Linear	US	4 month	No	26M	1.60%	USD	2 days	18/12/2012	18/12/2015	18/12/2015
100,000,000	Linear	FR	3 month	No	27Y	1.80%	EUR	2 days	16/10/2013	16/10/2040	16/10/2040
100,000,000	Linear	FR	4 month	No	27Y	2.00%	EUR	2 days	16/10/2013	16/10/2040	16/10/2040
100,000,000	Linear	US	3 month	No	3Y April	1.60%	USD	2 days	16/04/2012	16/04/2015	16/04/2015
100,000,000	Linear	FR	3 month	No	3Y April	2.00%	EUR	2 days	16/04/2012	16/04/2015	16/04/2015
100,000,000	Linear	US	3 month	No	11Y Dec	1.70%	USD	2 days	17/12/2012	17/12/2023	18/12/2023
100,000,000	Linear	FR	3 month	No	11Y Dec	1.80%	EUR	2 days	17/12/2012	17/12/2023	18/12/2023
100,000,000	Linear	US	5 month	No	Feb-15	1.15%	USD	2 days	28/02/2011	28/02/2015	27/02/2015
100,000,000	Linear	FR	6 month	No	Feb-15	1.12%	EUR	2 days	29/02/2012	28/02/2015	27/02/2015
100,000,000	Linear	US	5 month	No	Feb-16	1.00%	USD	2 days	29/02/2008	29/02/2016	29/02/2016
100,000,000	Linear	FR	6 month	No	Feb-16	1.20%	EUR	2 days	29/02/2008	29/02/2016	29/02/2016
100,000,000	Linear	FR	3 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	FR	4 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	FR	5 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	FR	6 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	FR	7 month	Yes	2Y	1.00%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	3 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	4 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	5 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	6 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Linear	US	7 month	Yes	2Y	1.20%	USD	2 days	16/10/2013	16/10/2014	16/10/2014

The following deals have been used to test the forward index calculation.
Standard interpolation:

Nominal	Interpolation	Index	Index Lag	Pay Inflation	Swap	Fixed Rate	Currency	Currency Lag	Start Date	End Date	Payment Date
100,000,000	Piecewise	GB	2 month	No	1Y	0.90%	GBP	0 days	16/10/2012	16/10/2013	16/10/2013
100,000,000	Piecewise	EUR	3 month	No	1Y	1.50%	EUR	2 days	16/10/2012	16/10/2013	16/10/2013
100,000,000	Linear	US	3 month	No	1Y	1.70%	USD	2 days	16/10/2012	16/10/2013	16/10/2013
100,000,000	Linear	FR	3 month	No	1Y	1.40%	EUR	2 days	16/10/2012	16/10/2013	16/10/2013
100,000,000	Linear	FR	3 month	No	30Y	2.28%	EUR	2 days	16/10/2013	16/10/2043	16/10/2043
100,000,000	Linear	FR	3 month	No	1Y	1.28%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Piecewise	GB	2 month	No	6M	3.18%	GBP	0 days	14/10/2013	14/10/2015	14/10/2015
100,000,000	Piecewise	GB	2 month	No	50Y	3.74%	GBP	0 days	14/10/2013	14/10/2063	15/10/2063
100,000,000	Linear	US	3 month	No	3Y	1.91%	USD	2 days	16/10/2013	16/10/2016	17/10/2016
100,000,000	Linear	US	3 month	No	15Y	2.73%	USD	2 days	16/10/2013	16/10/2028	16/10/2028
100,000,000	Piecewise	EUR	3 month	No	25Y	2.16%	EUR	2 days	16/10/2013	16/10/2038	18/10/2038
100,000,000	Piecewise	EUR	3 month	No	5Y	1.47%	EUR	2 days	16/10/2013	16/10/2018	16/10/2018
100,000,000	Piecewise	GB	2 month	No	8M	0.90%	GBP	0 days	14/12/2012	14/12/2013	16/12/2013
100,000,000	Piecewise	EUR	3 month	No	9M	1.50%	EUR	2 days	16/01/2013	16/01/2014	16/01/2014
100,000,000	Linear	US	3 month	No	8M	1.70%	USD	2 days	18/12/2012	18/12/2013	18/12/2013
100,000,000	Linear	FR	3 month	No	8M	1.40%	EUR	2 days	18/12/2012	18/12/2013	18/12/2013
100,000,000	Piecewise	GB	2 month	No	17Y	2.01%	GBP	0 days	14/10/2013	14/10/2030	14/10/2030
100,000,000	Piecewise	EUR	3 month	No	24Y	-2.00%	EUR	2 days	16/10/2013	16/10/2037	16/10/2037
100,000,000	Linear	US	3 month	No	11Y	0.00%	USD	2 days	16/10/2013	16/10/2024	16/10/2024
100,000,000	Linear	FR	3 month	Yes	29Y	3.50%	EUR	2 days	16/10/2013	16/10/2042	16/10/2042
100,000,000	Piecewise	GB	2 month	No	4Y	2.01%	GBP	0 days	29/02/2012	29/02/2016	29/02/2016
100,000,000	Piecewise	EUR	3 month	No	7Y	1.78%	EUR	2 days	29/02/2012	28/02/2019	28/02/2019
100,000,000	Linear	US	3 month	No	27Y	0.00%	USD	2 days	29/02/2012	28/02/2039	28/02/2039
100,000,000	Linear	FR	3 month	Yes	6y	3.50%	EUR	2 days	29/02/2012	28/02/2018	28/02/2018

Non standard interpolation:

Nominal	Interpolation	Index	Index Lag	Pay Inflation	Swap	Fixed Rate	Currency	Currency Lag	Start Date	End Date	Payment Date
100,000,000	Piecewise	GB	2 month	No	1Y	0.90%	GBP	0 days	16/10/2012	16/10/2013	16/10/2013
100,000,000	Piecewise	EUR	3 month	No	1Y	1.50%	EUR	2 days	16/10/2012	16/10/2013	16/10/2013
100,000,000	Linear	US	3 month	No	1Y	1.70%	USD	2 days	16/10/2012	16/10/2013	16/10/2013
100,000,000	Linear	FR	3 month	No	1Y	1.40%	EUR	2 days	16/10/2012	16/10/2013	16/10/2013
100,000,000	Linear	FR	3 month	No	30Y	2.28%	EUR	2 days	16/10/2013	16/10/2043	16/10/2043
100,000,000	Linear	FR	3 month	No	1Y	1.28%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Piecewise	GB	2 month	No	6M	3.18%	GBP	0 days	14/10/2013	14/10/2015	14/10/2015
100,000,000	Piecewise	GB	2 month	No	50Y	3.74%	GBP	0 days	14/10/2013	14/10/2063	15/10/2063
100,000,000	Linear	US	3 month	No	3Y	1.91%	USD	2 days	16/10/2013	16/10/2016	17/10/2016
100,000,000	Linear	US	3 month	No	15Y	2.73%	USD	2 days	16/10/2013	16/10/2028	16/10/2028
100,000,000	Piecewise	EUR	3 month	No	25Y	2.16%	EUR	2 days	16/10/2013	16/10/2038	18/10/2038
100,000,000	Piecewise	EUR	3 month	No	5Y	1.47%	EUR	2 days	16/10/2013	16/10/2018	16/10/2018
100,000,000	Piecewise	GB	2 month	No	8M	0.90%	GBP	0 days	14/12/2012	14/12/2013	16/12/2013
100,000,000	Piecewise	EUR	3 month	No	9M	1.50%	EUR	2 days	16/01/2013	16/01/2014	16/01/2014
100,000,000	Linear	US	3 month	No	8M	1.70%	USD	2 days	18/12/2012	18/12/2013	18/12/2013
100,000,000	Linear	FR	3 month	No	8M	1.40%	EUR	2 days	18/12/2012	18/12/2013	18/12/2013
100,000,000	Piecewise	FR	3 month	No	1Y	1.28%	EUR	2 days	16/10/2013	16/10/2014	16/10/2014
100,000,000	Piecewise	FR	3 month	No	24Y	-2.00%	EUR	2 days	16/10/2013	16/10/2037	16/10/2037
100,000,000	Linear	US	3 month	No	11Y	0.00%	USD	2 days	16/10/2013	16/10/2024	16/10/2024
100,000,000	Linear	FR	3 month	Yes	29Y	3.50%	EUR	2 days	16/10/2013	16/10/2042	16/10/2042
100,000,000	Piecewise	GB	2 month	No	4Y	2.01%	GBP	0 days	29/02/2012	29/02/2016	29/02/2016
100,000,000	Piecewise	EUR	3 month	No	7Y	1.78%	EUR	2 days	29/02/2012	28/02/2019	28/02/2019
100,000,000	Linear	US	3 month	No	27Y	0.00%	USD	2 days	29/02/2012	28/02/2039	28/02/2039
100,000,000	Linear	FR	3 month	Yes	6y	3.50%	EUR	2 days	29/02/2012	28/02/2018	28/02/2018

The following deals have been used to test the linear interpolation.
Standard interpolation:

Non standard interpolation:

4.3.3 Results

Tests have shown one bug in Murex:

- Fixed index levels can't be rounded to 5 decimals without the forward index to be rounded in case of linear interpolation. The error is less than £10 on a 50Y trade with 100 million notional. The workaround is to always round CPIs (past and fwd) at 5 decimals to have the correct cash flow generated at the end of a ZCIIS.

Tests have been performed for the 15 combinations of Inflation and Interest rate curves profile defined above.

1) Standard Inflation curve / Standard IRS curve

Threshhold	VM - PV			Threshhold	IM - PV		
	Min Error	Max Error	Pass/Fail		Min Error	Max Error	Pass/Fail
0.001	-	0.00000	0.00000	GB	-	0.00000	0.00000
GB	-	0.00000	0.00000	EUR	-	0.00000	0.00000
EUR	-	0.00000	0.00000	FR	-	0.00000	0.00000
FR	-	0.00000	0.00000	US	-	0.00000	0.00000
US	-	0.00000	0.00000				

2) Negative Inflation curve / Standard IRS curve

Threshhold	VM - PV			Threshhold	IM - PV		
	Min Error	Max Error	Pass/Fail		Min Error	Max Error	Pass/Fail
0.001	-	0.00000	0.00000	GB	-	0.00000	0.00000
GB	-	0.00000	0.00000	EUR	-	0.00000	0.00000
EUR	-	0.00000	0.00000	FR	-	0.00000	0.00000
FR	-	0.00000	0.00000	US	-	0.00000	0.00000
US	-	0.00000	0.00000				

3) Flat Inflation curve / Standard IRS curve

Threshhold	VM - PV			Threshhold	IM - PV		
	Min Error	Max Error	Pass/Fail		Min Error	Max Error	Pass/Fail
0.001	-	0.00000	0.00000	GB	-	0.00000	0.00000
GB	-	0.00000	0.00000	EUR	-	0.00000	0.00000
EUR	-	0.00000	0.00000	FR	-	0.00000	0.00000
FR	-	0.00000	0.00000	US	-	0.00000	0.00000
US	-	0.00000	0.00000				

4) Kink up Inflation curve / Standard IRS curve

Threshhold	VM - PV			Threshhold	IM - PV		
	Min Error	Max Error	Pass/Fail		Min Error	Max Error	Pass/Fail
0.001	-	0.00000	0.00000	GB	-	0.00000	0.00000
GB	-	0.00000	0.00000	EUR	-	0.00000	0.00000
EUR	-	0.00000	0.00000	FR	-	0.00000	0.00000
FR	-	0.00000	0.00000	US	-	0.00000	0.00000
US	-	0.00000	0.00000				

5) Kink down Inflation curve / Standard IRS curve

Threshhold	VM - PV			Threshhold	IM - PV		
	Min Error	Max Error	Pass/Fail		Min Error	Max Error	Pass/Fail
0.001	-	0.00000	0.00000	GB	-	0.00000	0.00000
GB	-	0.00000	0.00000	EUR	-	0.00000	0.00000
EUR	-	0.00000	0.00000	FR	-	0.00000	0.00000
FR	-	0.00000	0.00000	US	-	0.00000	0.00000
US	-	0.00000	0.00000				

6) Standard Inflation curve / Negative IRS curve

Threshhold	VM - PV			Threshhold	IM - PV		
	Min Error	Max Error	Pass/Fail		Min Error	Max Error	Pass/Fail
0.001	-	0.00000	0.00000	GB	-	0.00000	0.00000
GB	-	0.00000	0.00000	EUR	-	0.00000	0.00000
EUR	-	0.00000	0.00000	FR	-	0.00000	0.00000
FR	-	0.00000	0.00000	US	-	0.00000	0.00000
US	-	0.00000	0.00000				

15) Kink down Inflation curve / Steep rising IRS curve

Threshhold	VM - PV		
	Min Error	Max Error	Pass/Fail
0.001	-	0.000000	0.000000
GB	-	0.000000	Pass
EUR	-	0.000000	Pass
FR	-	0.000000	Pass
US	-	0.000000	Pass

Threshhold	IM - PV		
	Min Error	Max Error	Pass/Fail
0.001	-	0.000000	0.000000
GB	-	0.000000	Pass
EUR	-	0.000000	Pass
FR	-	0.000000	Pass
US	-	0.000000	Pass

5 Greeks

Sensitivities for Piecewise and Linear indices follow the same equations. In the case of Linear indices, CPI_p is linearly interpolated from the two surrounding month CPIs and the chain rules need to be applied for each index. One other major difference between the two types of index is that curve stripping of linear indices creates a dependency to previous pillars; therefore the translation of a CPI rate sensitivity into a Par sensitivity is done using Jacobian/Hessian matrix. There is not a one to one relation between a CPI rate sensitivity and a Par sensitivity on a tenor i.e. a CPI rate sensitivity for Linear indices on the 3Y tenor will create Par sensitivity on 1Y, 2Y and 3Y tenors, though the bulk of this risk will lie at the 3y point.

5.1 Inflation Sensitivities

5.1.1 CPI Delta

CPI Delta is the derivative of the present value of a trade regarding the CPIs on the curve. Following the chain rule, it is a composition of two partial derivatives: the derivative of the PV against the CPI used by a trade and the projection on the two surrounding tenors of the curve.

$$\frac{\partial PV}{\partial CPI_n} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial CPI_n}$$

Where

CPI_p : CPI of the deal, calculated from swap start date and using the deal lag

CPI_n : CPI for pillar n of the Inflation Curve

Calculation of CPI Delta is the same for piecewise or linear indices; the only difference is that in case of linear indices, the PV is sensitive to two CPIs.

5.1.2 CPI Rate Delta

CPI Rate Delta is the derivative of the present value of a trade regarding the CPI rates on the curve. Following the chain rule; it is a composition of three partial derivatives: the derivative of the PV against the CPI used by a trade, the translation of that CPI sensitivity into a CPI Rate sensitivity and its projection on the two surrounding tenors of the curve.

$$\frac{\partial PV}{\partial Z_n} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n}$$

Where

CPI_p : CPI of the deal, calculated from swap start date and using the deal lag

Z_p : CPI Rate of the deal

Z_n : CPI Rate for pillar n of the Inflation Curve

5.1.3 Par Delta

Par Delta is the first derivative with respect to Par rates. It can be calculated as a composition of four derivatives: the derivative of the PV against the CPI observed, then composed with the derivative of that CPI against its CPI rate then the projection on the CPI rates of the curve and subsequently taking the derivative of the CPI rate with respect to the par rate.

$$\frac{\partial PV}{\partial R_n} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial Z_n}{\partial R_n}$$

Where

CPI_p : CPI of the deal, calculated from swap start date and using the deal lag

Z_p : CPI Rate of the deal

Z_n : CPI Rate for pillar n of the Inflation Curve
 R_n : Par Rate for pillar n of the Inflation Curve

In the case of linear indices, there is not a one to one relationship between CPI rate and Par rate. Multiple CPI rate sensitivities contribute to a Par rate sensitivity; therefore Par Delta can be written as a sum of contributions:

$$\frac{\partial PV}{\partial R_n} = \sum_{k=1,2} \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_k} \cdot \frac{\partial Z_k}{\partial R_n}$$

Assuming fixing(s) for a ZCIS falls in between date 3 and 4, Par Deltas will look as follow:

	Piecewise index	Linear index
Date_{1_{INF}}	0	$\frac{\partial PV}{\partial R_{1INF}}$
Date_{2_{INF}}	0	$\frac{\partial PV}{\partial R_{2INF}}$
Date_{3_{INF}}	$\frac{\partial PV}{\partial R_{3INF}}$	$\frac{\partial PV}{\partial R_{3INF}}$
Date_{4_{INF}}	$\frac{\partial PV}{\partial R_{4INF}}$	$\frac{\partial PV}{\partial R_{4INF}}$

In the linear index case, sensitivity on Par rates 1 and 2 is created from adjacent CPI rates 3 and 4 (adjacent tenors to the trade fixing).

5.1.4 CPI Gamma

CPI Gamma is the second derivative of the PV against the CPI of the Inflation curve. Gamma can be represented as a matrix:

$$\begin{matrix} & \textbf{Date}_1 & \textbf{Date}_2 & \textbf{Date}_3 \\ \textbf{Date}_1 & \frac{\partial^2 PV}{\partial CPI_1^2} & & \\ \textbf{Date}_2 & & \frac{\partial^2 PV}{\partial CPI_2 \partial CPI_1} & 0 \\ \textbf{Date}_3 & \frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} & & 0 \\ & 0 & 0 & 0 \end{matrix}$$

A trade can only be at maximum sensitive to two CPIs of the Inflation curve. The matrix is symmetric; the cross sensitivities are identical: $\frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} = \frac{\partial^2 PV}{\partial CPI_2 \partial CPI_1}$.

In Murex: CPI index Delta and Gamma are not projected on the Inflation curve, CPI index is a constant therefore Murex Gamma is always equal to zero.

Diagonal term:

$$\begin{aligned} \frac{\partial^2 PV}{\partial CPI_n^2} &= \frac{\partial}{\partial CPI_n} \left(\frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial CPI_n} \right) \\ &\Leftrightarrow \frac{\partial^2 PV}{\partial CPI_n \partial CPI_p} \cdot \frac{\partial CPI_p}{\partial CPI_n} + \frac{\partial^2 CPI_p}{\partial CPI_n^2} \cdot \frac{\partial PV}{\partial CPI_p} \end{aligned}$$

$$\Rightarrow \frac{\partial^2 PV}{\partial CPI_n^2} = \frac{\partial^2 CPI_p}{\partial CPI_n^2} \cdot \frac{\partial PV}{\partial CPI_p}$$

In the case of a trade fixing on a curve pillar ($CPI_p = CPI_n$) the projection on the curve doesn't exist anymore ($\frac{\partial^2 CPI_p}{\partial CPI_n^2} = 0$) and Gamma equals zero.

Cross term:

$$\begin{aligned} \frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} &= \frac{\partial}{\partial CPI_2} \left(\frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial CPI_1} \right) \\ \Leftrightarrow \frac{\partial^2 PV}{\partial CPI_p \partial CPI_2} \cdot \frac{\partial CPI_p}{\partial CPI_1} + \frac{\partial^2 CPI_p}{\partial CPI_1 \partial CPI_2} \cdot \frac{\partial PV}{\partial CPI_p} \\ \Rightarrow \frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} &= \frac{\partial^2 CPI_p}{\partial CPI_1 \partial CPI_2} \cdot \frac{\partial PV}{\partial CPI_p} \end{aligned}$$

Details:

$$\begin{aligned} \frac{\partial^2 PV}{\partial CPI_p \partial CPI_i} &= \frac{\partial}{\partial CPI_p} \left(\frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial CPI_i} \right) = \frac{\partial^2 PV}{\partial CPI_p^2} \cdot \frac{\partial CPI_p}{\partial CPI_i} + \frac{\partial^2 CPI_p}{\partial CPI_p \partial CPI_i} \cdot \frac{\partial PV}{\partial CPI_p} = 0 \\ \frac{\partial^2 CPI_p}{\partial CPI_n \partial CPI_p} &= \frac{\partial}{\partial CPI_n} \left(\frac{\partial CPI_p}{\partial CPI_p} \right) = 0 \\ \frac{\partial^2 PV}{\partial CPI_p^2} &= \frac{\partial}{\partial CPI_p} \left(\frac{\partial PV}{\partial CPI_p} \right) = \frac{\partial}{\partial CPI_p} (cst) = 0 \end{aligned}$$

5.1.5 CPI Rate Gamma

CPI Gamma is null in some cases, therefore a way to calculate the second derivative is to skip the first part of the chain rule and directly express the PV function of CPI rates. Gamma can be represented as a matrix:

$$\begin{matrix} Date_1 & Date_2 & Date_3 \\ \begin{pmatrix} \frac{\partial^2 PV}{\partial Z_1^2} & \frac{\partial^2 PV}{\partial Z_2 \partial Z_1} & 0 \\ \frac{\partial^2 PV}{\partial Z_1 \partial Z_2} & \frac{\partial^2 PV}{\partial Z_2^2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ Date_2 \\ Date_3 \end{matrix}$$

A trade can only be at maximum sensitive to two CPIs of the Inflation curve. The matrix is symmetric as the cross sensitivities are identical: $\frac{\partial^2 PV}{\partial Z_2 \partial Z_1} = \frac{\partial^2 PV}{\partial Z_1 \partial Z_2}$.

Diagonal term:

$$\begin{aligned} \frac{\partial^2 PV}{\partial Z_n^2} &= \frac{\partial}{\partial Z_n} \left(\frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \right) \\ \Leftrightarrow \frac{\partial^2 PV}{\partial Z_n \partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} + \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial PV}{\partial Z_p} \\ \Rightarrow \frac{\partial^2 PV}{\partial Z_n^2} &= \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial^2 CPI_p}{\partial Z_p^2} \cdot \left(\frac{\partial Z_p}{\partial Z_n} \right)^2 + \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial PV}{\partial Z_p} \end{aligned}$$

Or

$$\frac{\partial^2 PV}{\partial Z_n^2} = \frac{\partial^2 PV}{\partial Z_p^2} \cdot \left(\frac{\partial Z_p}{\partial Z_n} \right)^2 + \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial PV}{\partial Z_p}$$

Cross term:

$$\begin{aligned} \frac{\partial^2 PV}{\partial Z_1 \partial Z_2} &= \frac{\partial}{\partial Z_1} \left(\frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_2} \right) \\ &\Leftrightarrow \frac{\partial^2 PV}{\partial Z_p \partial Z_1} \cdot \frac{\partial Z_p}{\partial Z_2} + \frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2} \cdot \frac{\partial PV}{\partial Z_p} \end{aligned}$$

$$\Rightarrow \frac{\partial^2 PV}{\partial Z_1 \partial Z_2} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial^2 CPI_p}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_1} \cdot \frac{\partial Z_p}{\partial Z_2} + \frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2} \cdot \frac{\partial PV}{\partial Z_p}$$

or

$$\frac{\partial^2 PV}{\partial Z_1 \partial Z_2} = \frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_1} \cdot \frac{\partial Z_p}{\partial Z_2} + \frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2} \cdot \frac{\partial PV}{\partial Z_p}$$

Details:

$$\begin{aligned} \frac{\partial PV}{\partial Z_n} &= \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \\ \frac{\partial^2 Z_p}{\partial Z_n \partial Z_p} &= \frac{\partial}{\partial Z_n} \left(\frac{\partial Z_p}{\partial Z_p} \right) = 0 \\ \frac{\partial^2 PV}{\partial Z_n \partial Z_p} &= \frac{\partial}{\partial Z_p} \left(\frac{\partial PV}{\partial Z_n} \right) = \frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_n} + \frac{\partial^2 Z_p}{\partial Z_n \partial Z_p} \cdot \frac{\partial PV}{\partial Z_p} = \frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_n} \\ \frac{\partial^2 PV}{\partial Z_p \partial Z_1} &= \frac{\partial}{\partial Z_p} \left(\frac{\partial PV}{\partial Z_1} \right) = \frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_1} + \frac{\partial^2 Z_p}{\partial Z_p \partial Z_1} \cdot \frac{\partial PV}{\partial Z_p} = \frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_1} \\ \frac{\partial^2 PV}{\partial Z_p \partial CPI_p} &= \frac{\partial}{\partial CPI_p} \left(\frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} \right) = \frac{\partial^2 PV}{\partial CPI_p^2} \cdot \frac{\partial CPI_p}{\partial Z_p} + \frac{\partial^2 CPI_p}{\partial Z_p \partial CPI_p} \cdot \frac{\partial PV}{\partial CPI_p} = 0 \\ \frac{\partial^2 PV}{\partial Z_p^2} &= \frac{\partial}{\partial Z_p} \left(\frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} \right) = \frac{\partial^2 PV}{\partial Z_p \partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} + \frac{\partial^2 CPI_p}{\partial Z_p^2} \cdot \frac{\partial PV}{\partial CPI_p} = \frac{\partial^2 CPI_p}{\partial Z_p^2} \cdot \frac{\partial PV}{\partial CPI_p} \end{aligned}$$

5.1.6 Par Gamma

Par Gamma can be calculated as an extension of CPI Rate gamma with the chain rule. There is a major difference in Piecewise and Linear indices in the way the matrix is filled:

Piecewise Gamma:

$$\begin{matrix} & \boldsymbol{Date_1} & \boldsymbol{Date_2} & \boldsymbol{Date_3} \\ \boldsymbol{Date_1} & 0 & 0 & 0 \\ \boldsymbol{Date_2} & 0 & \frac{\partial^2 PV}{\partial R_2^2} & \frac{\partial^2 PV}{\partial R_3 \partial R_2} \\ \boldsymbol{Date_3} & 0 & \frac{\partial^2 PV}{\partial R_2 \partial R_3} & \frac{\partial^2 PV}{\partial R_3^2} \end{matrix}$$

A trade can only be sensitive to two Par rates of the Inflation curve at maximum. The matrix is symmetric; cross sensitivities are identical: $\frac{\partial^2 PV}{\partial R_2 \partial R_3} = \frac{\partial^2 PV}{\partial R_3 \partial R_2}$.

Diagonal term:

$$\begin{aligned}
 \frac{\partial^2 PV}{\partial R_n^2} &= \frac{\partial}{\partial R_n} \left(\frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial R_n} \cdot \frac{\partial Z_n}{\partial R_n} \right) \\
 &\Leftrightarrow \frac{\partial}{\partial R_n} \left(\frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \right) \cdot \frac{\partial Z_n}{\partial R_n} + \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial^2 Z_n}{\partial R_n^2} \\
 &\Leftrightarrow \left(\frac{\partial^2 PV}{\partial R_n \partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} + \frac{\partial^2 Z_p}{\partial Z_n \partial R_n} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \frac{\partial Z_n}{\partial R_n} + \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial^2 Z_n}{\partial R_n^2} \\
 &\Leftrightarrow \left(\frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial Z_n}{\partial R_n} \cdot \frac{\partial Z_p}{\partial Z_n} + \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial Z_n}{\partial R_n} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \frac{\partial Z_n}{\partial R_n} + \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial^2 Z_n}{\partial R_n^2} \\
 \Rightarrow \frac{\partial^2 PV}{\partial R_n^2} &= \left(\frac{\partial^2 PV}{\partial Z_p^2} \cdot \left(\frac{\partial Z_p}{\partial Z_n} \right)^2 + \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \left(\frac{\partial Z_n}{\partial R_n} \right)^2 + \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial^2 Z_n}{\partial R_n^2} \\
 &\Leftrightarrow \frac{\partial^2 PV}{\partial R_n^2} = \frac{\partial^2 PV}{\partial Z_n^2} \cdot \left(\frac{\partial Z_n}{\partial R_n} \right)^2 + \frac{\partial PV}{\partial Z_n} \cdot \frac{\partial^2 Z_n}{\partial R_n^2}
 \end{aligned}$$

Cross term:

$$\begin{aligned}
 \frac{\partial^2 PV}{\partial R_i \partial R_j} &= \frac{\partial}{\partial R_i} \left(\frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_j} \cdot \frac{\partial Z_j}{\partial R_j} \right) \\
 &\Leftrightarrow \frac{\partial}{\partial R_i} \left(\frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_j} \right) \cdot \frac{\partial Z_j}{\partial R_j} + \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_j} \cdot \frac{\partial^2 Z_j}{\partial R_j \partial R_i} \\
 &\Leftrightarrow \left(\frac{\partial^2 PV}{\partial Z_p \partial R_i} \cdot \frac{\partial Z_p}{\partial Z_j} + \frac{\partial^2 Z_p}{\partial Z_j \partial R_i} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \frac{\partial Z_j}{\partial R_j} + \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_j} \cdot \frac{\partial^2 Z_j}{\partial R_j \partial R_i} \\
 \Leftrightarrow \left(\left(\frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_i} \cdot \frac{\partial Z_i}{\partial R_i} + \frac{\partial^2 Z_i}{\partial R_i \partial Z_p} \cdot \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_i} \right) \cdot \frac{\partial Z_p}{\partial Z_j} + \frac{\partial^2 Z_p}{\partial Z_i \partial Z_j} \cdot \frac{\partial Z_i}{\partial R_i} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \frac{\partial Z_j}{\partial R_j} + \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_j} \cdot \frac{\partial^2 Z_j}{\partial R_j \partial R_i} \\
 \Rightarrow \frac{\partial^2 PV}{\partial R_i \partial R_j} &= \left(\frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_i} \cdot \frac{\partial Z_p}{\partial Z_j} + \frac{\partial^2 Z_p}{\partial Z_i \partial Z_j} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \frac{\partial Z_j}{\partial R_j} \cdot \frac{\partial Z_i}{\partial R_i} \\
 &\Leftrightarrow \frac{\partial^2 PV}{\partial R_i \partial R_j} = \frac{\partial^2 PV}{\partial Z_1 \partial Z_2} \cdot \frac{\partial Z_j}{\partial R_j} \cdot \frac{\partial Z_i}{\partial R_i}
 \end{aligned}$$

Details:

$$\begin{aligned}
 \frac{\partial^2 Z_n}{\partial R_n \partial Z_p} &= 0 \\
 \frac{\partial^2 Z_p}{\partial Z_n \partial R_n} &= \frac{\partial}{\partial Z_n} \left(\frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial Z_n}{\partial R_n} \right) = \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial Z_n}{\partial R_n} + \frac{\partial^2 Z_n}{\partial R_n \partial Z_n} \cdot \frac{\partial Z_p}{\partial Z_n} = \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial Z_n}{\partial R_n} \\
 \frac{\partial^2 PV}{\partial R_n \partial Z_p} &= \frac{\partial}{\partial Z_p} \left(\frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial Z_n}{\partial R_n} \right) = \frac{\partial}{\partial Z_p} \left(\frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \right) \cdot \frac{\partial Z_n}{\partial R_n} + \frac{\partial^2 Z_n}{\partial R_n \partial Z_p} \cdot \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \\
 &\Leftrightarrow \left(\frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_n} + \frac{\partial^2 Z_p}{\partial Z_n \partial Z_p} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \frac{\partial Z_n}{\partial R_n} + \frac{\partial^2 Z_n}{\partial R_n \partial Z_p} \cdot \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \\
 &\Leftrightarrow \frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial Z_n}{\partial R_n} \\
 \frac{\partial^2 Z_j}{\partial R_j \partial R_i} &= 0 \text{ in case of Piecewise index}
 \end{aligned}$$

Linear Gamma:

$$\begin{array}{c}
 \textbf{Date}_1 \left(\begin{array}{ccc} \textbf{Date}_1 & \textbf{Date}_2 & \textbf{Date}_3 \\ \frac{\partial^2 PV}{\partial R_1^2} & \frac{\partial^2 PV}{\partial R_2 \partial R_1} & \frac{\partial^2 PV}{\partial R_3 \partial R_1} \\ \hline \end{array} \right) \\
 \textbf{Date}_2 \left(\begin{array}{ccc} \textbf{Date}_2 & \textbf{Date}_2 & \textbf{Date}_2 \\ \frac{\partial^2 PV}{\partial R_1 \partial R_2} & \frac{\partial^2 PV}{\partial R_2^2} & \frac{\partial^2 PV}{\partial R_3 \partial R_2} \\ \hline \end{array} \right) \\
 \textbf{Date}_3 \left(\begin{array}{ccc} \textbf{Date}_3 & \textbf{Date}_3 & \textbf{Date}_3 \\ \frac{\partial^2 PV}{\partial R_1 \partial R_3} & \frac{\partial^2 PV}{\partial R_2 \partial R_3} & \frac{\partial^2 PV}{\partial R_3^2} \\ \hline \end{array} \right)
 \end{array}$$

A trade can only be sensitive to two CPIs of the Inflation curve but those will create Par sensitivity on all previous tenors. The matrix is symmetric as cross sensitivities are identical:

$$\frac{\partial^2 PV}{\partial R_2 \partial R_3} = \frac{\partial^2 PV}{\partial R_3 \partial R_2}, \quad \frac{\partial^2 PV}{\partial R_1 \partial R_2} = \frac{\partial^2 PV}{\partial R_2 \partial R_1} \text{ and } \frac{\partial^2 PV}{\partial R_1 \partial R_3} = \frac{\partial^2 PV}{\partial R_3 \partial R_1}.$$

The chain rule for Gamma can be written as a sum of contributions; by using Francesco Faà Di Bruno's formula who generalized the chain rule formula to higher derivatives, we can write:

$$\begin{aligned}
 \frac{d^k}{dx^k} f(y) &= \sum \frac{k!}{1^{m_1} \cdots k^{m_k} m_1! \cdots m_k!} f^{(m_1 + \cdots + m_k)}(y) \prod_{j: m_j \neq 0} \frac{d^{m_j} y}{dx^{m_j}} \\
 \frac{\partial^n}{\partial x_1 \cdots \partial x_n} f(y) &= \sum_{\pi} f^{(|\pi|)}(y) \prod_{B \in \pi} \frac{\partial^{|B|} y}{\prod_{j \in B} \partial x_j}
 \end{aligned}$$

And deduce:

$$\begin{aligned}
 \frac{\partial^2 (f \circ g)}{\partial x^2} &= \frac{\partial^2 f}{\partial g^2} \cdot \left(\frac{\partial g}{\partial x} \right)^2 + \frac{\partial f}{\partial g} \cdot \frac{\partial^2 g}{\partial x^2} \\
 \frac{\partial^2 (f \circ g)}{\partial x_i \partial x_j} &= \sum_k \frac{\partial f}{\partial u_k} \cdot \frac{\partial^2 g_k}{\partial x_i \partial x_j} + \sum_{k,l} \frac{\partial^2 f}{\partial u_k \partial u_l} \cdot \frac{\partial g_k}{\partial x_i} \cdot \frac{\partial g_l}{\partial x_j}
 \end{aligned}$$

Where: $u = g(x)$

Diagonal terms:

$$\frac{\partial^2 PV}{\partial R_n^2} = \frac{\partial}{\partial R_n} \left(\sum_{k=1,2} \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_k} \cdot \frac{\partial Z_k}{\partial R_n} \right)$$

$$\Rightarrow \frac{\partial^2 PV(p)}{\partial R_n^2} = \sum_{k=1,2} \left(\frac{\partial PV}{\partial Z_k} \cdot \frac{\partial^2 Z_k}{\partial R_n^2} + \frac{\partial^2 PV}{\partial Z_k^2} \cdot \left(\frac{\partial Z_k}{\partial R_n} \right)^2 \right)$$

Cross terms:

$$\begin{aligned}
 \frac{\partial^2 PV}{\partial R_i \partial R_j} &= \frac{\partial}{\partial R_i} \left(\sum_{k=1,2} \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_k} \cdot \frac{\partial Z_k}{\partial R_j} \right) \\
 \Rightarrow \frac{\partial^2 PV(p)}{\partial R_i \partial R_j} &= \sum_{k=1,2} \frac{\partial PV}{\partial Z_k} \cdot \frac{\partial^2 Z_k}{\partial R_i \partial R_j} + \sum_{k,l=1,2} \frac{\partial^2 PV}{\partial Z_k \partial Z_l} \cdot \frac{\partial Z_k}{\partial R_i} \cdot \frac{\partial Z_l}{\partial R_j}
 \end{aligned}$$

or

$$\frac{\partial^2 PV(p)}{\partial R_i \partial R_j} = \frac{\partial^2 PV}{\partial Z_1^2} \cdot \frac{\partial Z_1}{\partial R_i} \cdot \frac{\partial Z_1}{\partial R_j} + \frac{\partial^2 PV}{\partial Z_1 \partial Z_2} \cdot \frac{\partial Z_1}{\partial R_i} \cdot \frac{\partial Z_2}{\partial R_j} + \frac{\partial^2 PV}{\partial Z_2^2} \cdot \frac{\partial Z_2}{\partial R_i} \cdot \frac{\partial Z_2}{\partial R_j} + \frac{\partial^2 PV}{\partial Z_2 \partial Z_1} \cdot \frac{\partial Z_2}{\partial R_i} \cdot \frac{\partial Z_1}{\partial R_j} + \frac{\partial PV}{\partial Z_2} \cdot \frac{\partial^2 Z_2}{\partial R_j \partial R_i}$$
$$+ \frac{\partial PV}{\partial Z_1} \cdot \frac{\partial^2 Z_1}{\partial R_j \partial R_i}$$

Cross Gamma details:



CrossGammaDetails.
docx

5.2 Interest Rate Sensitivities

Piecewise or linear interpolation types do not change the interest rate risk; the interest rate risk only comes from the discounting risk of a unique cash flow. It is therefore zero when the trade is at market (NPV =0) and will change based on the cash flow size. Where a trade has a positive PV, it will have a negative IR delta (value drops as rates rise) and the reverse for a negative PV.

5.2.1 Zero Delta

In the general case where the payment date of a ZCIIS falls in between two tenors of the IR curve, the sensitivities related to the two tenors can be written as follow:

$$\frac{\partial PV}{\partial ZC_1} = -t \cdot (1 - \alpha) \cdot PV$$

$$\frac{\partial PV}{\partial ZC_2} = -t \cdot \alpha \cdot PV$$

Where:

$$\alpha = \frac{Date_p - Date_1}{Date_2 - Date_1}$$

Date₂: Date of right tenor

Date₁: Date of left tenor

Date_p: Swap payment date

t: Time to maturity from payment date to pricing date using ACT/365 basis convention

PV: Present value of the deal

In the case where the payment date falls on a tenor of the IR curve, the sensitivity can be resumed to one unique equation:

$$\frac{\partial PV}{\partial ZC} = -t \cdot PV$$

5.2.2 Zero Gamma

Gamma is a diagonal matrix where for a single trade only four terms are not null. In case the payment date falls on an IR tenor, the matrix is filled up with only one term ($\frac{\partial^2 PV}{\partial ZC^2}$).

$$\begin{matrix} & \boldsymbol{Date_1} & \boldsymbol{Date_2} & \boldsymbol{Date_3} \\ \boldsymbol{Date_1} & 0 & 0 & 0 \\ \boldsymbol{Date_2} & 0 & \frac{\partial^2 PV}{\partial ZC_2^2} & \frac{\partial^2 PV}{\partial ZC_3 \partial ZC_2} \\ \boldsymbol{Date_3} & 0 & \frac{\partial^2 PV}{\partial ZC_2 \partial ZC_3} & \frac{\partial^2 PV}{\partial ZC_3^2} \end{matrix}$$

Diagonal term:

$$\frac{\partial^2 PV}{\partial ZC_1^2} = t^2 \cdot (1 - \alpha)^2 \cdot PV$$

$$\frac{\partial^2 PV}{\partial ZC_2^2} = t^2 \cdot \alpha^2 \cdot PV$$

Cross term:

$$\frac{\partial^2 PV}{\partial ZC_1 \partial ZC_2} = t^2 \cdot (1 - \alpha) \alpha \cdot PV$$

5.3 Cross Gamma Sensitivities

Inflation and Interest rate risks are independent, therefore the order of derivation is not important and Cross Gamma can be treated as the Inflation Delta projected composed with the Interest Rate Delta.

Current assumptions are that LCH will use Cross Gamma between Par Inflation rates and Zero Coupon IRS rates.

$$\text{Cross Gamma} = \frac{\partial^2 PV}{\partial ZC_{IRS} \partial R_{INF}}$$

LCH IRS curves always have an overnight point (O/N for VM or 1D for IM curves); therefore a trade payment date can't be before the first pillar of the IRS curve. Based on eligibility rules, Inflation trades can't go beyond 50Y and there is a tenor above any payment date of trade (no extrapolation is needed). There are therefore six combinations of date on which to calculate Cross Gamma.

5.3.1 Par-ZC Cross Gamma

The Cross Gamma matrix is not symmetric but deriving on the zero coupons IRS space and then Par Inflation rate is identical as doing it the other way around.

	Date_{1_{IRS}}	Date_{2_{IRS}}	Date_{3_{IRS}}	Date_{4_{IRS}}	Date_{5_{IRS}}
Date_{1_{INF}}	$\frac{\partial PV}{\partial R_{1INF} \partial ZC_{1IRS}}$	$\frac{\partial PV}{\partial R_{1INF} \partial ZC_{2IRS}}$	$\frac{\partial PV}{\partial R_{1INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{1INF} \partial ZC_{4IRS}}$	$\frac{\partial PV}{\partial R_{1INF} \partial ZC_{5IRS}}$
Date_{2_{INF}}	$\frac{\partial PV}{\partial R_{2INF} \partial ZC_{1IRS}}$	$\frac{\partial PV}{\partial R_{2INF} \partial ZC_{2IRS}}$	$\frac{\partial PV}{\partial R_{2INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{2INF} \partial ZC_{4IRS}}$	$\frac{\partial PV}{\partial R_{2INF} \partial ZC_{5IRS}}$
Date_{3_{INF}}	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{1IRS}}$	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{2IRS}}$	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{4IRS}}$	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{5IRS}}$
Date_{4_{INF}}	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{1IRS}}$	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{2IRS}}$	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{4IRS}}$	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{5IRS}}$

The same formulas defined for Par Inflation Delta and Zero IRS Delta can be applied. The same differences are found between linear and piecewise indices: cross gamma is composed of four terms for piecewise vs two columns for linear indices.

	Piecewise index				Linear index		
	Date_{1_{IRS}}	Date_{2_{IRS}}	Date_{3_{IRS}}	Date_{4_{IRS}}	Date_{1_{IRS}}	Date_{2_{IRS}}	Date_{3_{IRS}}
Date_{1_{INF}}	0	0	0	0	0	$\frac{\partial PV}{\partial R_{1INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{1INF} \partial ZC_{4IRS}}$
Date_{2_{INF}}	0	0	0	0	0	$\frac{\partial PV}{\partial R_{2INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{2INF} \partial ZC_{4IRS}}$
Date_{3_{INF}}	0	0	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{4IRS}}$	0	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{3INF} \partial ZC_{4IRS}}$
Date_{4_{INF}}	0	0	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{4IRS}}$	0	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{3IRS}}$	$\frac{\partial PV}{\partial R_{4INF} \partial ZC_{4IRS}}$

The general formula when a payment date falls in between two IRS curve pillars is the following:

$$\frac{\partial PV}{\partial R_i \partial ZC_1} = -t_p \cdot (1 - \alpha) \cdot \frac{\partial PV}{\partial R_i} / 10000$$

$$\frac{\partial PV}{\partial R_i \partial ZC_2} = -t_p \cdot \alpha \cdot \frac{\partial PV}{\partial R_i} / 10000$$

Where:

t_p : Time to maturity from payment date to pricing date using ACT/365 convention

$\frac{\partial PV}{\partial R_i}$: Inflation Par delta projected on pillar i

$$\alpha = \frac{Date_p - Date_1}{Date_2 - Date_1}$$

$Date_2$: Date of right tenor

$Date_1$: Date of left tenor

$Date_p$: Swap payment date

5.3.2 CPI Rate-ZC Cross Gamma

CPI Rate – ZC Cross Gamma is internally used by Murex during TDG calculation. There is currently a bug in Murex due to the wrong projection of CPI Rate delta on curve pillars. They correctly calculate the CPI Rate Delta at trade level but do a linear projection on the curve instead of following the Log CPI interpolation. The scale of this error is not significant.

The general formula when a payment date falls in between two IRS curve pillars is the following:

$$\frac{\partial PV}{\partial Z_i \partial Z C_1} = -t_p \cdot (1 - \alpha) \cdot \frac{\partial PV}{\partial Z_i} / 10000$$

$$\frac{\partial PV}{\partial Z_i \partial Z C_2} = -t_p \cdot \alpha \cdot \frac{\partial PV}{\partial Z_i} / 10000$$

Where:

t_p : Time to maturity from payment date to pricing date using ACT/365 convention

$\frac{\partial PV}{\partial Z_i}$: Inflation CPI Rte delta projected on pillar i

$$\alpha = \frac{Date_p - Date_1}{Date_2 - Date_1}$$

$Date_2$: Date of right tenor

$Date_1$: Date of left tenor

$Date_p$: Swap payment date

6 Inflation Sensitivity Formulas

Piecewise or Linear interpolation does not change the sensitivity formulas up to the Par sensitivities. In case of a linear interpolation, sensitivity formulas need to be applied twice with the correct linear coefficient for each one. Formulas that define interpolation and the link between CPI and CPI rates (Z) are identical for linear and piecewise interpolation.

6.1 Piecewise Interpolation

In case of a piecewise interpolation, a trade only requests one index level. Choosing a log linear index level interpolation, in the general the PV formula can be written case as follows:

$$PV = \phi \cdot N \cdot \left[\frac{S_p}{CPI_0} \left(\frac{CPI_1}{S_1} \right)^{1-\frac{p}{n}} \left(\frac{CPI_2}{S_2} \right)^{\frac{p}{n}} - (1+K)^{t_3} \right] \cdot Df$$

Or

$$PV = \phi \cdot N \cdot \left[\frac{S_p}{CPI_0} \left[\frac{CPI_{S0}(1+R_1)^{t_1}}{S_1} \right]^{1-\frac{p}{n}} \cdot \left[\frac{CPI_{S0}(1+R_2)^{t_2}}{S_2} \right]^{\frac{p}{n}} - (1+K)^{t_3} \right] \cdot Df$$

Or

$$PV = \phi \cdot N \cdot \left[\frac{CPI_{Last} \cdot S_p}{CPI_0 \cdot S_{Last}} (1+Z_1)^{t'_1(1-\frac{p}{n})} \cdot (1+Z_2)^{t'_2\frac{p}{n}} - (1+K)^{t_3} \right] \cdot Df$$

Where

CPI_{S0} : Reference CPI, calculated from spot date and using the standard lag

CPI_0 : Reference CPI of the deal, calculated from swap start date and using the deal lag

R_1 : Par Inflation rate for the left pillar

R_2 : Par Inflation rate for the right pillar

S_1 : Seasonality factor for left pillar month

S_2 : Seasonality factor for right pillar month

S_p : Seasonality adjustment for month p

p : Number of months from the fixing date to the left pillar

n : Number of months between left and right pillars

K : Fixed rate

N : Nominal

t_1 : Time to maturity for the left pillar using 30/360 by month basis convention

t_2 : Time to maturity for the right pillar using 30/360 by month basis convention

t_3 : Time to maturity for the swap following using 30/360 by month basis convention

$Df = \exp^{-Zc*t_3}$: Discount factor from the payment date to the pricing date using ACT/365 basis convention

ϕ : Pay or receive flag related to the Inflation leg: $\phi = \begin{cases} -1, & \text{Pay Inflation} \\ 1, & \text{Receive Inflation} \end{cases}$

6.1.1 CPI Delta

CPI Delta for piecewise indices is the composition of two derivatives: the derivative of the PV against the CPI observed and the projection on the CPIs of the curve.

$$\frac{\partial PV}{\partial CPI_1} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial CPI_1}$$

$$\frac{\partial PV}{\partial CPI_2} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial CPI_2}$$

-
- a) Sensitivity on the Index level

$$\frac{\partial PV}{\partial CPI_p} = \frac{\rho \cdot N}{CPI_0} \cdot Df$$

- b) Projection on Index levels of the curve

In between two pillars

$$\begin{aligned}\frac{\partial CPI_p}{\partial CPI_1} &= \frac{S_p \cdot \left(1 - \frac{p}{n}\right)}{S_1^{1-\frac{p}{n}}} CPI_1^{-\frac{p}{n}} \left(\frac{CPI_2}{S_2}\right)^{\frac{p}{n}} \\ \frac{\partial CPI_p}{\partial CPI_2} &= \frac{S_p \cdot \frac{p}{n}}{S_2^{\frac{p}{n}}} CPI_2^{\frac{p}{n}-1} \left(\frac{CPI_1}{S_1}\right)^{1-\frac{p}{n}}\end{aligned}$$

Before first pillars

$$\frac{\partial CPI_p}{\partial CPI_2} = \frac{S_p \cdot \frac{p}{n}}{S_2^{\frac{p}{n}}} CPI_2^{\frac{p}{n}-1} \left(\frac{CPI_{Last}}{S_{Last}}\right)^{1-\frac{p}{n}}$$

On a pillar

$$\frac{\partial CPI_p}{\partial CPI} = 1$$

6.1.2 CPI Rate Delta

CPI Rate Delta can be calculated as the composition of three derivatives: the derivative of the PV against the CPI observed, then composed with the derivative of that CPI against its CPI rate and finally the projection on the CPI rates of the curve tenors.

$$\frac{\partial PV}{\partial Z_1} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_1}$$

$$\frac{\partial PV}{\partial Z_2} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_2}$$

- a) CPI against its CPI rate

$$\frac{\partial CPI_p}{\partial Z_p} = CPI_{Last} \cdot \frac{S_p}{S_{Last}} \cdot t_p \cdot (1 + Z_p)^{t_p-1}$$

- b) CPI rate against curve pillars

In between two pillars:

$$\frac{\partial Z_p}{\partial Z_1} = \frac{t_1 \cdot \left(1 - \frac{p}{n}\right)}{t_p} \cdot (1 + Z_1)^{\frac{t_1 \cdot \left(1 - \frac{p}{n}\right)}{t_p} - 1} (1 + Z_2)^{\frac{t_2 \cdot \frac{p}{n}}{t_p}}$$

$$\frac{\partial Z_p}{\partial Z_2} = \frac{t_2 \cdot \frac{p}{n}}{t_p} \cdot (1 + Z_1)^{\frac{t_1(1-\frac{p}{n})}{t_p}} \cdot (1 + Z_2)^{\frac{t_2 \frac{p}{n}-1}{t_p}}$$

On / before first pillar:

$$\frac{\partial Z_p}{\partial Z_2} = 1$$

6.1.3 Par Delta

Par Delta can be calculated as a composition of four derivatives: the derivative of the PV against the CPI observed, composed with the derivative of that CPI against its CPI rate, projected on the CPI rates of the curve and subsequently taking the derivative of the CPI rate with respect to the par rate.

$$\frac{\partial PV}{\partial R_1} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_1} \cdot \frac{\partial Z_1}{\partial R_1}$$

$$\frac{\partial PV}{\partial R_2} = \frac{\partial PV}{\partial CPI_p} \cdot \frac{\partial CPI_p}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_2} \cdot \frac{\partial Z_2}{\partial R_2}$$

The first three partial derivatives have been calculated in the previous steps. The forth derivative in the chain is calculated below.

$$Z_n = \left(\frac{CPI_{S0}}{CPI_{Last}} \cdot \frac{S_{Last}}{S_n} \right)^{\frac{1}{t'_n}} (1 + R_n)^{\frac{t_n}{t'_n}} - 1$$

$$\frac{\partial Z_n}{\partial R_n} = \left(\frac{CPI_{S0}}{CPI_{Last}} \cdot \frac{S_{Last}}{S_n} \right)^{\frac{1}{t'_n}} \cdot \frac{t_n}{t'_n} \cdot (1 + R_n)^{\frac{t_n}{t'_n}-1}$$

6.1.4 CPI Rate Delta (direct derivation)

In between two pillars:

The general formulas when an index level falls in between two curve pillars are the following:

$$\frac{\partial PV}{\partial Z_1} = \Phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t_1 \left(1 - \frac{p}{n} \right) (1 + Z_1)^{t_1(1-\frac{p}{n})-1} \cdot (1 + Z_2)^{t_2 \frac{p}{n}}$$

$$\frac{\partial PV}{\partial Z_2} = \Phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t_2 \frac{p}{n} \cdot (1 + Z_1)^{t_1(1-\frac{p}{n})} \cdot (1 + Z_2)^{t_2 \frac{p}{n}-1}$$

Before first pillar:

When an index level is observed before the first pillar of the curve, the trade is only sensitive to the right pillar as the left is the last published index and is fixed.

$$\frac{\partial PV}{\partial Z_2} = \Phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t_p \cdot (1 + Z_2)^{t_p-1}$$

On a pillar:

When an index level falls on a pillar date, the trade is only sensitive on that unique pillar and the delta formula is much simpler.

$$\frac{\partial PV}{\partial Z} = \phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t(1+Z)^{t-1}$$

6.1.5 Par Delta (direct derivation)

In between two pillars:

The general formulas when an index level falls in between two curve pillars are the following:

$$\begin{aligned}\frac{\partial PV}{\partial R_1} &= \phi \cdot \frac{CPI_{S0}}{CPI_0} \cdot N \cdot Df \cdot S_p \cdot \left(\frac{(1+R_2)^{t_2}}{S_2} \right)^{\frac{p}{n}} \cdot \frac{t_1 (1-\frac{p}{n})}{S_1^{1-\frac{p}{n}}} \cdot (1+R_1)^{t_1(1-\frac{p}{n})-1} \\ \frac{\partial PV}{\partial R_2} &= \phi \cdot \frac{CPI_{S0}}{CPI_0} \cdot N \cdot Df \cdot S_p \cdot \left(\frac{(1+R_1)^{t_1}}{S_1} \right)^{1-\frac{p}{n}} \cdot \frac{t_2 \frac{p}{n}}{S_2^{\frac{p}{n}}} \cdot (1+R_2)^{\frac{p}{n}t_2-1}\end{aligned}$$

Before first pillar:

When an index level is observed before the first pillar of the curve, the trade is only sensitive to the right pillar as the left is the last published index and is fixed.

$$\frac{\partial PV}{\partial R_2} = \phi \cdot \frac{N \cdot Df \cdot S_p}{CPI_0} \cdot \left(\frac{CPI_{Last}}{S_1} \right)^{1-\frac{p}{n}} \cdot \left(\frac{CPI_{S0}}{S_2} \right)^{\frac{p}{n}} \cdot t_2 \cdot \frac{p}{n} \cdot (1+R_2)^{\frac{p}{n}t_2-1}$$

On a pillar:

When an index level falls on a pillar date, the trade is only sensitive on that unique pillar and the delta formula is much simpler.

$$\frac{\partial PV}{\partial R} = \phi \cdot \frac{N \cdot CPI_{S0} \cdot Df}{CPI_0} \cdot t \cdot (1+R)^{t-1}$$

6.1.6 CPI Gamma

CPI Gamma is the second order derivative with respect to CPI levels. It is a symmetric matrix; where only four terms are not null.

$$\begin{matrix} Date_1 & Date_2 & Date_3 \\ \frac{\partial^2 PV}{\partial CPI_1^2} & \frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} & 0 \\ \frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} & \frac{\partial^2 PV}{\partial CPI_2^2} & 0 \\ 0 & 0 & 0 \end{matrix}$$

Based on the chain rules define above, only two new terms have not been defined: $\frac{\partial^2 CPI_p}{\partial CPI_1 \partial CPI_2}$ and $\frac{\partial^2 CPI_p}{\partial CPI_n^2}$.

a) Diagonal terms:

$$\frac{\partial^2 PV}{\partial CPI_n^2} = \frac{\partial^2 CPI_p}{\partial CPI_n^2} \cdot \frac{\partial PV}{\partial CPI_p}$$

In between two pillars

$$\frac{\partial^2 PV}{\partial CPI_1^2} = -\frac{\phi \cdot N}{CPI_0} \cdot Df \cdot \frac{p}{n} \cdot \left(1 - \frac{p}{n}\right) \cdot \frac{S_p}{S_1^{1-\frac{p}{n}}} CPI_1^{-\frac{p}{n}-1} \left(\frac{CPI_2}{S_2}\right)^{\frac{p}{n}}$$

$$\frac{\partial^2 PV}{\partial CPI_2^2} = \frac{\phi \cdot N}{CPI_0} \cdot Df \cdot \frac{p}{n} \cdot \left(\frac{p}{n} - 1\right) \cdot \frac{S_p}{S_2^{\frac{p}{n}}} CPI_2^{\frac{p}{n}-2} \left(\frac{CPI_1}{S_1}\right)^{1-\frac{p}{n}}$$

Before first pillars

$$\frac{\partial CPI_p}{\partial CPI_2} = \frac{\phi \cdot N}{CPI_0} \cdot Df \cdot \frac{p}{n} \cdot \left(\frac{p}{n} - 1\right) \cdot \frac{S_p}{S_2^{\frac{p}{n}}} CPI_2^{\frac{p}{n}-2} \left(\frac{CPI_{Last}}{S_{Last}}\right)^{1-\frac{p}{n}}$$

On a pillar

$$\frac{\partial^2 PV}{\partial CPI^2} = 0$$

b) Cross term:

$$\frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} = \frac{\partial^2 CPI_p}{\partial CPI_1 \cdot \partial CPI_2} \cdot \frac{\partial PV}{\partial CPI_p}$$

In between two pillars

$$\frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} = \frac{p}{n} \cdot \left(1 - \frac{p}{n}\right) \frac{S_p}{S_1^{1-\frac{p}{n}} \cdot S_2^{\frac{p}{n}}} CPI_1^{-\frac{p}{n}} CPI_2^{\frac{p}{n}-1}$$

Before first pillar - on a pillar

$$\frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} = 0$$

6.1.7 CPI Rate Gamma

CPI Rate Gamma is the second order derivative with respect to CPI Rates. It is a symmetric matrix; where only four terms are not null.

$$\begin{matrix} Date_1 & Date_2 & Date_3 \\ \frac{\partial^2 PV}{\partial Z_1^2} & \frac{\partial^2 PV}{\partial Z_1 \partial Z_2} & 0 \\ \frac{\partial^2 PV}{\partial Z_1 \partial Z_2} & \frac{\partial^2 PV}{\partial Z_2^2} & 0 \\ 0 & 0 & 0 \end{matrix}$$

a) Diagonal terms:

$$\frac{\partial^2 PV}{\partial Z_n^2} = \frac{\partial^2 PV}{\partial Z_p^2} \cdot \left(\frac{\partial Z_p}{\partial Z_n}\right)^2 + \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial PV}{\partial Z_p}$$

b) Cross term:

$$\frac{\partial^2 PV}{\partial Z_1 \partial Z_2} = \frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_1} \cdot \frac{\partial Z_p}{\partial Z_2} + \frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2} \cdot \frac{\partial PV}{\partial Z_p}$$

From the Gamma CPI rate chain rule, we need the following terms to be able to calculate it:

- $\frac{\partial PV}{\partial Z_p}$: First part of the chain rule in CPI rate Delta calculation
- $\frac{\partial Z_p}{\partial Z_n}$: Second part of the CPI rate Delta calculation; projection of CPI rate delta on the curve
- $\frac{\partial^2 PV}{\partial Z_p^2}$: New term; second order derivative of the PV to its CPI rate
- $\frac{\partial^2 Z_p}{\partial Z_n^2}$: New term; second order derivative of CPI rate on the curve
- $\frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2}$: New term; first cross order derivative of CPI rate on the curve

a) Sensitivity on trade CPI rate

$$\frac{\partial^2 PV}{\partial Z_p^2} = \rho \cdot N \cdot Df \frac{CPI_{Last}}{CPI_0 \cdot S_{Last}} \cdot S_p \cdot t_p \cdot (t_p - 1) \cdot (1 + Z_p)^{t_p - 2}$$

b) Projection on CPI rates of the Inflation curve

In between two pillars

$$\frac{\partial^2 Z_p}{\partial Z_1^2} = \frac{t_1 \cdot (1 - \frac{p}{n})}{t_p} \cdot \left(\frac{t_1 \cdot (1 - \frac{p}{n})}{t_p} - 1 \right) (1 + Z_1)^{\frac{t_1 \cdot (1 - \frac{p}{n})}{t_p} - 2} (1 + Z_2)^{\frac{t_2 \cdot p}{t_p}}$$

$$\frac{\partial^2 Z_p}{\partial Z_2^2} = \frac{t_2 \cdot \frac{p}{n}}{t_p} \cdot \left(\frac{t_2 \cdot \frac{p}{n}}{t_p} - 1 \right) \cdot (1 + Z_1)^{\frac{t_1 \cdot (1 - \frac{p}{n})}{t_p}} (1 + Z_2)^{\frac{t_2 \cdot \frac{p}{n}}{t_p} - 2}$$

$$\frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2} = \frac{t_1 \cdot (1 - \frac{p}{n})}{t_p} \cdot \frac{t_2 \cdot \frac{p}{n}}{t_p} (1 + Z_1)^{\frac{t_1 \cdot (1 - \frac{p}{n})}{t_p} - 1} (1 + Z_2)^{\frac{t_2 \cdot \frac{p}{n}}{t_p} - 1}$$

Before first pillar/ On a pillar

$$Z_p = Z_n \Rightarrow \frac{\partial^2 Z_p}{\partial Z_n^2} = \frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2} = 0$$

The chain rule can be simplified and the CPI rate Gamma can be directly written as:

$$\frac{\partial^2 PV}{\partial Z_n^2} = \frac{\partial^2 PV}{\partial Z_p^2} = \rho \cdot N \cdot Df \frac{CPI_{Last}}{CPI_0 \cdot S_{Last}} \cdot S_p \cdot t_p \cdot (t_p - 1) \cdot (1 + Z_p)^{t_p - 2}$$

6.1.8 Par Gamma

Par Gamma is the second order derivative with respect to Par rates. It is a symmetric matrix; where only four terms are not null.

$$\begin{aligned} & Date_1 \begin{pmatrix} Date_1 & Date_2 & Date_3 \\ \frac{\partial^2 PV}{\partial R_1^2} & \frac{\partial^2 PV}{\partial R_1 \partial R_2} & 0 \\ \frac{\partial^2 PV}{\partial R_1 \partial R_2} & \frac{\partial^2 PV}{\partial R_2^2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ & Date_2 \begin{pmatrix} Date_1 & Date_2 & Date_3 \\ \frac{\partial^2 PV}{\partial R_1^2} & \frac{\partial^2 PV}{\partial R_1 \partial R_2} & 0 \\ \frac{\partial^2 PV}{\partial R_1 \partial R_2} & \frac{\partial^2 PV}{\partial R_2^2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ & Date_3 \begin{pmatrix} Date_1 & Date_2 & Date_3 \\ \frac{\partial^2 PV}{\partial R_1^2} & \frac{\partial^2 PV}{\partial R_1 \partial R_2} & 0 \\ \frac{\partial^2 PV}{\partial R_1 \partial R_2} & \frac{\partial^2 PV}{\partial R_2^2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

There is a one to one relationship in between index levels and par rates; therefore derivatives to Par rates can be calculated without using the chain rule.

a) Diagonal terms:

$$\frac{\partial^2 PV}{\partial R_n^2} = \left(\frac{\partial^2 PV}{\partial Z_p^2} \cdot \left(\frac{\partial Z_p}{\partial Z_n} \right)^2 + \frac{\partial^2 Z_p}{\partial Z_n^2} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \left(\frac{\partial Z_n}{\partial R_n} \right)^2 + \frac{\partial PV}{\partial Z_p} \cdot \frac{\partial Z_p}{\partial Z_n} \cdot \frac{\partial^2 Z_n}{\partial R_n^2}$$

b) Cross term:

$$\frac{\partial^2 PV}{\partial R_i \partial R_j} = \left(\frac{\partial^2 PV}{\partial Z_p^2} \cdot \frac{\partial Z_p}{\partial Z_i} \cdot \frac{\partial Z_p}{\partial Z_j} + \frac{\partial^2 Z_p}{\partial Z_i \partial Z_j} \cdot \frac{\partial PV}{\partial Z_p} \right) \cdot \frac{\partial Z_j}{\partial R_j} \cdot \frac{\partial Z_i}{\partial R_i}$$

From the Par Gamma chain rule, we need the following terms to be able to calculate it:

- $\frac{\partial PV}{\partial Z_p}$: First part of the chain rule in CPI rate Delta calculation
- $\frac{\partial Z_p}{\partial Z_n}$: Second part of the CPI rate Delta calculation; projection of CPI rate delta on the curve
- $\frac{\partial^2 PV}{\partial Z_p^2}$: Second order derivative of the PV to its CPI rate
- $\frac{\partial^2 Z_p}{\partial Z_n^2}$: Second order derivative of CPI rate on the curve
- $\frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2}$: First cross order derivative of CPI rate on the curve
- $\frac{\partial Z_n}{\partial R_n}$: Last part of the chain rule in Par Delta calculation
- $\frac{\partial^2 Z_n}{\partial R_n^2}$: New term; second order derivative of the CPI rate with respect to the par rate

$$\frac{\partial^2 Z_n}{\partial R_n^2} = \left(\frac{CPI_{S0}}{CPI_{Last}} \cdot \frac{S_{Last}}{S_n} \right)^{\frac{1}{t_n}} \cdot \frac{t_n}{t'_n} \cdot \left(\frac{t_n}{t'_n} - 1 \right) \cdot (1 + R_n)^{\frac{t_n}{t'_n} - 2}$$

6.1.9 CPI Rate Gamma (direct derivation)

a) Diagonal term:

In between two pillars:

$$\frac{\partial^2 PV}{\partial Z_1^2} = \phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t_1 \left(1 - \frac{p}{n} \right) \cdot \left(t_1 \left(1 - \frac{p}{n} \right) - 1 \right) \cdot (1 + Z_1)^{t_1(1-\frac{p}{n})-2} \cdot (1 + Z_2)^{t_2 \frac{p}{n} - 2}$$

$$\frac{\partial^2 PV}{\partial Z_2^2} = \phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t_2 \frac{p}{n} \cdot \left(t_2 \frac{p}{n} - 1 \right) \cdot (1 + Z_1)^{t_1(1-\frac{p}{n})} \cdot (1 + Z_2)^{t_2 \frac{p}{n} - 2}$$

Before first pillar:

$$\frac{\partial^2 PV}{\partial Z_2^2} = \phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t_p \cdot (t_p - 1) \cdot (1 + Z_2)^{t_p - 2}$$

On a pillar:

$$\frac{\partial^2 PV}{\partial Z^2} = \phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t \cdot (t - 1) \cdot (1 + Z)^{t - 2}$$

b) Cross term:

In between two pillars:

$$\frac{\partial^2 PV}{\partial Z_1 \partial Z_2} = \phi \cdot N \cdot Df \cdot \frac{S_p \cdot CPI_{Last}}{S_{Last} \cdot CPI_0} \cdot t_2 \cdot \frac{p}{n} \cdot t_1 \left(1 - \frac{p}{n} \right) (1 + Z_1)^{t_1(1-\frac{p}{n})-1} \cdot (1 + Z_2)^{t_2 \frac{p}{n} - 1}$$

Before first pillar/ On a pillar

$$\frac{\partial^2 PV}{\partial Z_1 \partial Z_2} = 0$$

6.1.10 Par Gamma (direct derivation)

a) Diagonal term:

In between two pillars

$$\frac{\partial^2 PV}{\partial R_1^2} = \phi \cdot N \cdot Df \cdot \frac{CPI_{S0}}{CPI_0} \cdot S_p \cdot \left(\frac{(1+R_2)^{t_2}}{S_2} \right)^{\frac{p}{n}} \cdot \frac{t_1 \left(1 - \frac{p}{n} \right) \cdot \left(t_1 \left(1 - \frac{p}{n} \right) - 1 \right)}{S_1^{1-\frac{p}{n}}} \cdot (1+R_1)^{t_1(1-\frac{p}{n})-2}$$

$$\frac{\partial^2 PV}{\partial R_2^2} = \phi \cdot N \cdot Df \cdot \frac{CPI_{S0}}{CPI_0} \cdot S_p \cdot \left(\frac{(1+R_1)^{t_1}}{S_1} \right)^{1-\frac{p}{n}} \cdot \frac{t_2 \frac{p}{n} \cdot \left(t_2 \frac{p}{n} - 1 \right)}{S_2^{\frac{p}{n}}} \cdot (1+R_2)^{\frac{p}{n}t_2-2}$$

Before first pillar:

$$\frac{\partial^2 PV}{\partial R_2^2} = \phi \cdot \frac{N \cdot Df \cdot S_p}{CPI_0} \cdot \left(\frac{CPI_{Last}}{S_1} \right)^{1-\frac{p}{n}} \cdot \left(\frac{CPI_{S0}}{S_2} \right)^{\frac{p}{n}} \cdot t_2 \cdot \frac{p}{n} \cdot \left(\frac{p}{n} \cdot t_2 - 1 \right) \cdot (1+R_2)^{\frac{p}{n}t_2-2}$$

On a pillar:

$$\frac{\partial^2 PV}{\partial R^2} = \phi \cdot \frac{N \cdot CPI_{S0} \cdot Df}{CPI_0} \cdot t \cdot (t-1) \cdot (1+R)^{t-2}$$

b) Cross term:

In between two pillars

$$\frac{\partial^2 PV}{\partial R_1 \partial R_2} = \phi \cdot N \cdot Df \cdot \frac{CPI_{S0}}{CPI_0} \cdot S_p \cdot (1+R_2)^{\frac{p}{n}t_2-1} \cdot \frac{t_1 \left(1 - \frac{p}{n} \right) t_2 \frac{p}{n}}{S_1^{1-\frac{p}{n}} * S_2^{\frac{p}{n}}} \cdot (1+R_1)^{t_1(1-\frac{p}{n})-1}$$

Before first pillar/ On a pillar

$$\frac{\partial^2 PV}{\partial R_1 \partial R_2} = 0$$

6.1.11 Cross Gamma: Par Inflation / ZC IRS

Cross Gamma is a matrix that mixes interest rate and inflation Deltas. In case of piecewise interpolation, the matrix has only four non null terms at maximum.

$$\begin{matrix} Date_{INF_1} & Date_{IRS_1} & Date_{IRS_2} & Date_{IRS_3} \\ Date_{INF_2} & \frac{\partial^2 PV}{\partial R_1 \partial ZC_1} & \frac{\partial^2 PV}{\partial R_1 \partial ZC_2} & 0 \\ Date_{INF_3} & \frac{\partial^2 PV}{\partial R_2 \partial ZC_1} & \frac{\partial^2 PV}{\partial R_2 \partial ZC_2} & 0 \\ & 0 & 0 & 0 \end{matrix}$$

As defined in the first part, there are therefore 6 different cases to calculate Cross Gamma and project it on both curves.

In between two IRS and INF tenors:

The general formulas when an index level falls in between two curve pillars are the following:

$$\frac{\partial PV}{\partial R_1 \partial ZC_1} = -t_p \cdot (1-\alpha) \cdot \phi \cdot \frac{CPI_{S0}}{CPI_0} \cdot N \cdot Df \cdot S_p \cdot \left[\frac{(1+R_2)^{t_2}}{S_2} \right]^{\frac{p}{n}} \cdot \frac{t_1 \left(1 - \frac{p}{n} \right)}{S_1^{1-\frac{p}{n}}} \cdot (1+R_1)^{t_1(1-\frac{p}{n})-1}$$

$$\frac{\partial PV}{\partial R_1 \partial ZC_2} = -t_p \cdot \alpha \cdot \phi \cdot \frac{CPI_{S0}}{CPI_0} \cdot N \cdot Df \cdot S_p \cdot \left[\frac{(1+R_2)^{t_2}}{S_2} \right]^{\frac{p}{n}} \cdot \frac{t_1 \left(1 - \frac{p}{n}\right)}{S_1^{1-\frac{p}{n}}} \cdot (1+R_1)^{t_1(1-\frac{p}{n})-1}$$

$$\frac{\partial PV}{\partial R_2 \partial ZC_1} = -t_p \cdot (1-\alpha) \cdot \phi \cdot \frac{CPI_{S0}}{CPI_0} \cdot N \cdot Df \cdot S_p \cdot \left[\frac{(1+R_1)^{t_1}}{S_1} \right]^{1-\frac{p}{n}} \cdot \frac{t_2 \frac{p}{n}}{S_2^{\frac{p}{n}}} \cdot (1+R_2)^{\frac{p}{n}t_2-1}$$

$$\frac{\partial PV}{\partial R_2 \partial ZC_2} = -t_p \cdot \alpha \cdot \phi \cdot \frac{CPI_{S0}}{CPI_0} \cdot N \cdot Df \cdot S_p \cdot \left[\frac{(1+R_1)^{t_1}}{S_1} \right]^{1-\frac{p}{n}} \cdot \frac{t_2 \frac{p}{n}}{S_2^{\frac{p}{n}}} \cdot (1+R_2)^{\frac{p}{n}t_2-1}$$

Where:

$$\alpha = \frac{Date_p - Date_1}{Date_2 - Date_1}$$

Date₂: Date of right tenor

Date₁: Date of left tenor

Date_p: Swap payment date

Before first tenor of Inflation curve and on an IRS tenor:

When an index level is observed before the first pillar of the curve, the trade is only sensitive to the right pillar as the left is the last published index and is fixed.

$$\frac{\partial PV}{\partial R_2 \partial ZC} = -t_p \cdot \phi \cdot \frac{N \cdot Df \cdot S_p}{CPI_0} \cdot \left(\frac{CPI_{Last}}{S_1} \right)^{1-\frac{p}{n}} \cdot \left(\frac{CPI_{S0}}{S_2} \right)^{\frac{p}{n}} \cdot t_2 \cdot \frac{p}{n} \cdot (1+R_2)^{\frac{p}{n}t_2-1}$$

Before first tenor of Inflation curve and in between two IRS tenors:

$$\frac{\partial PV}{\partial R_2 \partial ZC_1} = -t_p \cdot (1-\alpha) \cdot \phi \cdot \frac{N \cdot Df \cdot S_p}{CPI_0} \cdot \left(\frac{CPI_{Last}}{S_1} \right)^{1-\frac{p}{n}} \cdot \left(\frac{CPI_{S0}}{S_2} \right)^{\frac{p}{n}} \cdot t_2 \cdot \frac{p}{n} \cdot (1+R_2)^{\frac{p}{n}t_2-1}$$

$$\frac{\partial PV}{\partial R_2 \partial ZC_2} = -t_p \cdot \alpha \cdot \phi \cdot \frac{N \cdot Df \cdot S_p}{CPI_0} \cdot \left(\frac{CPI_{Last}}{S_1} \right)^{1-\frac{p}{n}} \cdot \left(\frac{CPI_{S0}}{S_2} \right)^{\frac{p}{n}} \cdot t_2 \cdot \frac{p}{n} \cdot (1+R_2)^{\frac{p}{n}t_2-1}$$

On a tenor of Inflation curve and IRS curve:

When an index level falls on a pillar date, the trade is only sensitive to that unique pillar and the delta formula is much simpler.

$$\frac{\partial PV}{\partial R \partial ZC} = -t_p \cdot \phi \cdot \frac{N \cdot CPI_{S0} \cdot Df}{CPI_0} \cdot t_e \cdot (1+R)^{t_e-1}$$

Where:

t_p: Time to maturity from payment date to pricing date using ACT/365 basis convention

t_e: Time to maturity from swap end date using 30/360 by month basis convention

On a tenor of Inflation curve and in between IRS curve tenors:

$$\frac{\partial PV}{\partial R \partial ZC_1} = -t_p \cdot (1-\alpha) \cdot \phi \cdot \frac{N \cdot CPI_{S0} \cdot Df}{CPI_0} \cdot t_e \cdot (1+R)^{t_e-1}$$

$$\frac{\partial PV}{\partial R \partial ZC_2} = -t_p \cdot \alpha \cdot \phi \cdot \frac{N \cdot CPI_{S0} \cdot Df}{CPI_0} \cdot t_e \cdot (1+R)^{t_e-1}$$

In between two Inflation tenors and on IRS tenor:

$$\frac{\partial PV}{\partial R_1 \partial ZC} = -t_p \cdot \phi \cdot \frac{CPI_{S0}}{CPI_0} \cdot N \cdot Df \cdot S_p \cdot \left[\frac{(1+R_2)^{t_2}}{S_2} \right]^{\frac{p}{n}} \cdot \frac{t_1 \left(1 - \frac{p}{n}\right)}{S_1^{1-\frac{p}{n}}} \cdot (1+R_1)^{t_1(1-\frac{p}{n})-1}$$

$$\frac{\partial PV}{\partial R_2 \partial ZC} = -t_p \cdot \phi \cdot \frac{CPI_{S0}}{CPI_0} \cdot N \cdot Df \cdot S_p \cdot \left[\frac{(1+R_1)^{t_1}}{S_1} \right]^{1-\frac{p}{n}} \cdot \frac{t_2 \frac{p}{n}}{S_2^{\frac{p}{n}}} \cdot (1+R_2)^{\frac{p}{n}t_2-1}$$

6.2 Linear Interpolation

$$PV = \phi \cdot N \cdot \left[\frac{\alpha \cdot CPI_{P-L} + (1 - \alpha) \cdot CPI_{P-L+1}}{CPI_0} - (1 + K)^{t_3} \right] \cdot Df$$

Where

CPI_0 : Reference CPI of the deal, calculated from swap start date and using the deal lag

$\alpha = 1 - \frac{d-1}{D}$: Coefficient of the linear interpolation

$CPI_0 = \alpha \cdot CPI_{M-L} + (1 - \alpha) \cdot CPI_{M-L+1}$

CPI_{P-L} : CPI of the current month where the index level fixes

CPI_{P-L+1} : CPI of the month following CPI_{P-L}

M: Current month of the market data date

L: Index lag in month

d: Day of the Swap End date

D: Number of days in the Swap End date month

P: Swap End date

K: Fixed rate

N: Nominal

t_3 : Time to maturity for the swap following using 30/360 by month basis convention

$Df = \exp^{-Zc * t_3}$: Discount factor from the payment date to the pricing date using ACT/365 basis convention

ϕ : Pay or receive flag related to the Inflation leg: $\phi = \begin{cases} -1, & \text{Pay Inflation} \\ 1, & \text{Receive Inflation} \end{cases}$

6.2.1 CPI Delta

CPI Delta for linear indices is the composition of two derivatives: the derivative of the PV against the two CPIs used in the linear interpolation and the projection on the CPIs of the curve.

$$\frac{\partial PV}{\partial CPI_1} = \frac{\partial PV}{\partial CPI_{P-L}} \cdot \frac{\partial CPI_{P-L}}{\partial CPI_1} + \frac{\partial PV}{\partial CPI_{P-L+1}} \cdot \frac{\partial CPI_{P-L+1}}{\partial CPI_1}$$

$$\frac{\partial PV}{\partial CPI_2} = \frac{\partial PV}{\partial CPI_{P-L}} \cdot \frac{\partial CPI_{P-L}}{\partial CPI_2} + \frac{\partial PV}{\partial CPI_{P-L+1}} \cdot \frac{\partial CPI_{P-L+1}}{\partial CPI_2}$$

a) Sensitivity on Index levels

$$\frac{\partial PV}{\partial CPI_{P-L}} = \frac{\phi \cdot N}{CPI_0} \cdot \left(1 - \frac{d-1}{D} \right) \cdot Df$$

$$\frac{\partial PV}{\partial CPI_{P-L+1}} = \frac{\phi \cdot N}{CPI_0} \cdot \frac{d-1}{D} \cdot Df$$

b) Projection on Index levels of the curve

In between two pillars

$$\frac{\partial CPI_p}{\partial CPI_1} = \frac{S_p \cdot \left(1 - \frac{p}{n} \right)}{S_1^{1-\frac{p}{n}}} CPI_1^{-\frac{p}{n}} \left(\frac{CPI_2}{S_2} \right)^{\frac{p}{n}}$$

$$\frac{\partial CPI_p}{\partial CPI_2} = \frac{S_p \cdot \frac{p}{n}}{S_2^{\frac{p}{n}}} \cdot CPI_2^{\frac{p}{n}-1} \left(\frac{CPI_1}{S_1} \right)^{1-\frac{p}{n}}$$

Before first pillars

$$\frac{\partial CPI_p}{\partial CPI_2} = \frac{S_p \cdot \frac{p}{n}}{S_2^{\frac{p}{n}}} CPI_2^{\frac{p}{n}-1} \left(\frac{CPI_{Last}}{S_{Last}} \right)^{1-\frac{p}{n}}$$

On a pillar

$$\frac{\partial CPI_p}{\partial CPI} = 1$$

6.2.2 CPI Rate Delta

CPI Rate Delta for linear indices is the composition of three derivatives: the derivative of the PV against the two CPIs used in the linear interpolation, the derivative of a CPI against its CPI rate and of the CPI rate sensitivity on the CPI rates of the curve. A deal is only sensitive to a maximum of two CPI rates of the curve.

$$\frac{\partial PV}{\partial Z_n} = \frac{\partial PV}{\partial CPI_{P-L}} \cdot \frac{\partial CPI_{P-L}}{\partial Z_{P-L}} \cdot \frac{\partial Z_{P-L}}{\partial Z_n} + \frac{\partial PV}{\partial CPI_{P-L+1}} \cdot \frac{\partial CPI_{P-L+1}}{\partial Z_{P-L+1}} \cdot \frac{\partial Z_{P-L+1}}{\partial Z_n}$$

a) CPI against its CPI rate

$$\frac{\partial CPI_p}{\partial Z_p} = CPI_{Last} \cdot \frac{SM_p}{S_{Last}} \cdot t_p \cdot (1 + Z_p)^{t_p-1}$$

b) CPI rate against curve pillars

In between two pillars

$$\frac{\partial Z_p}{\partial Z_1} = \frac{t_1 \cdot (1 - \frac{p}{n})}{t_p} \cdot (1 + Z_1)^{\frac{t_1 \cdot (1 - \frac{p}{n})}{t_p} - 1} (1 + Z_2)^{\frac{t_2 \cdot \frac{p}{n}}{t_p}}$$

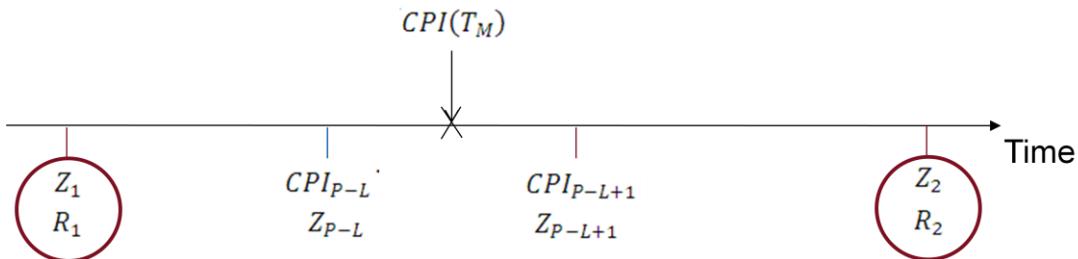
$$\frac{\partial Z_p}{\partial Z_2} = \frac{t_2 \cdot \frac{p}{n}}{t_p} \cdot (1 + Z_1)^{\frac{t_1 \cdot (1 - \frac{p}{n})}{t_p}} (1 + Z_2)^{\frac{t_2 \cdot \frac{p}{n}}{t_p} - 1}$$

Before first pillar- on a pillar

$$\frac{\partial Z_p}{\partial Z} = 1$$

6.2.3 Par Delta

Par Delta for linear indices is more complex because of the projection on different dates and link between index levels and par rates. A way to calculate delta is to use the chain rule: first derive regarding the two CPIs involved in the linear interpolation, then derive those against the zero coupon rate, then project the zero coupon rate sensibilities on adjacent pillars of the curve and finally derive the results against the market quotes.



Where

$CPI(T_M)$: CPI of the trade used for pricing and interpolated from CPI_{P-L} and CPI_{P-L+1}

CPI_{P-L} : CPI of the current month of T_M

CPI_{P-L+1} : CPI of month following CPI_{P-L}

Z_{P-L} : CPI rate of the current month of T_M

Z_{P-L+1} : CPI rate of month following CPI_{P-L}

Z_1 : CPI rate of left pillar of the current bucket

R_1 : Market quote of left pillar of the current bucket (par rate)

Z_2 : CPI rate of right pillar of the current bucket

R_2 : Market quote of right pillar of the current bucket (par rate)

$$\frac{\partial PV}{\partial R_n} = \left(\frac{\partial PV}{\partial CPI_{P-L}} \cdot \frac{\partial CPI_{P-L}}{\partial Z_{P-L}} \cdot \frac{\partial Z_{P-L}}{\partial Z_1} + \frac{\partial PV}{\partial CPI_{P-L+1}} \cdot \frac{\partial CPI_{P-L+1}}{\partial Z_{P-L+1}} \cdot \frac{\partial Z_{P-L+1}}{\partial Z_1} \right) \cdot \frac{\partial Z_1}{\partial R_n} \\ + \left(\frac{\partial PV}{\partial CPI_{P-L+1}} \cdot \frac{\partial CPI_{P-L+1}}{\partial Z_{P-L+1}} \cdot \frac{\partial Z_{P-L+1}}{\partial Z_2} + \frac{\partial PV}{\partial CPI_{P-L}} \cdot \frac{\partial CPI_{P-L}}{\partial Z_{P-L}} \cdot \frac{\partial Z_{P-L}}{\partial Z_2} \right) \cdot \frac{\partial Z_2}{\partial R_n}$$

All the terms involved in the Delta calculation are known except $\frac{\partial Z_i}{\partial R_n}$. This term is a part of the Jacobian matrix which helps convert Zero sensitivity into Par sensitivity.

Jacobian Matrix construction steps:

- Derive Par rates with respect to all CPI rates used in the curve stripping ($\frac{\partial R_i}{\partial Z_{i,j}}$ matrix)
- Derive CPI rates used in the curve stripping against CPI rates at which CPIs are expressed ($\frac{\partial Z_{i,j,P-L}}{\partial Z_{i,p-L+1}}$ matrix)
- Sum both matrices ($\frac{\partial R_i}{\partial Z_i}$ matrix)
- Inverse matrix to get to $\frac{\partial Z_i}{\partial R_i}$ matrix

a) Par rates function of CPI rates

$$R_n = \left[\frac{CPI_{Last}}{CPI_0 \cdot S_{Last}} \cdot [a \cdot S_{n_{P-L}} \cdot (1 + Z_{n_{P-L}})^{t_{n_{P-L}}} + (1 - a) \cdot S_{n_{P-L+1}} \cdot (1 + Z_{n_{P-L+1}})^{t_{n_{P-L+1}}}] \right]^{1/t_n} - 1$$

Where

R_n : Par rate for pillar n

Z_n : CPI rate for pillar n

S_{Last} : Seasonality of last published index month

S_n : Seasonality for pillar n month

t_n : Time to maturity in between the fixing date n and the reference index level date using 30/360 by month basis convention

t_l : Time to maturity in between the fixing date n and the last published index date using 30/360 by month basis convention

α : Coefficient of the linear interpolation $\alpha = 1 - \frac{d-1}{D}$

b) Pre-compute $\frac{\partial R_i}{\partial Z_{i,j}}$ matrix

$$\begin{array}{c} \textbf{Date}_{1_{P-L}} \\ \textbf{Date}_{1_{P-L+1}} \\ \textbf{Date}_{2_{P-L}} \\ \textbf{Date}_{2_{P-L+1}} \\ \textbf{Date}_{3_{P-L}} \\ \textbf{Date}_{3_{P-L+1}} \end{array} \left(\begin{array}{ccc} \textbf{Date}_1 & \textbf{Date}_2 & \textbf{Date}_3 \\ \frac{\partial R_1}{\partial Z_{1_{P-L}}} & 0 & 0 \\ \frac{\partial R_1}{\partial Z_{1_{P-L+1}}} & 0 & 0 \\ 0 & \frac{\partial R_2}{\partial Z_{2_{P-L}}} & 0 \\ 0 & \frac{\partial R_2}{\partial Z_{2_{P-L+1}}} & 0 \\ 0 & 0 & \frac{\partial R_3}{\partial Z_{3_{P-L}}} \\ 0 & 0 & \frac{\partial R_3}{\partial Z_{3_{P-L+1}}} \end{array} \right)$$

In between two pillars

$$\frac{\partial R_n}{\partial Z_{n_{P-L}}} = \left(\frac{CPI_{Last}}{CPI_0 \cdot S_{Last}} \right)^{1/t_n} \cdot \frac{t_{n_{P-L}}}{t_n} \cdot \left(1 - \frac{d-1}{D} \right) \cdot S_{n_{P-L}} \cdot (1 + Z_{n_{P-L}})^{t_{n_{P-L}}} \cdot \left[\left(1 - \frac{d-1}{D} \right) \cdot S_{n_{P-L}} \cdot (1 + Z_{n_{P-L}})^{t_{n_{P-L}}} + \frac{d-1}{D} \cdot S_{n_{P-L+1}} \cdot (1 + Z_{n_{P-L+1}})^{t_{n_{P-L+1}}} \right]^{1/t_{n-1}}$$

$$\frac{\partial R_n}{\partial Z_{n_{P-L+1}}} = \left(\frac{CPI_{Last}}{CPI_0 \cdot S_{Last}} \right)^{1/t_n} \cdot \frac{t_{n_{P-L+1}}}{t_n} \cdot \frac{d-1}{D} \cdot S_{n_{P-L+1}} \cdot (1 + Z_{n_{P-L+1}})^{t_{n_{P-L+1}}} \cdot \left[\left(1 - \frac{d-1}{D} \right) \cdot S_{n_{P-L}} \cdot (1 + Z_{n_{P-L}})^{t_{n_{P-L}}} + \frac{d-1}{D} \cdot S_{n_{P-L+1}} \cdot (1 + Z_{n_{P-L+1}})^{t_{n_{P-L+1}}} \right]^{1/t_{n-1}}$$

Left pillar is fixed

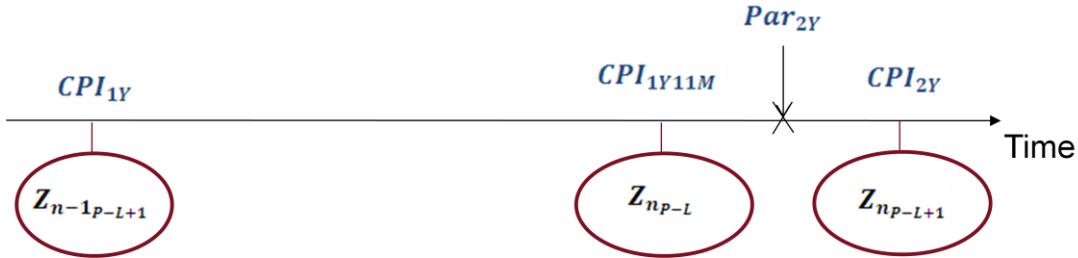
$$CPI_0 (1 + R_n)^{t_n} = \left(1 - \frac{d-1}{D} \right) \cdot CPI_{Last} + \frac{d-1}{D} \cdot CPI_{P-L+1}$$

$$R_n = \left[\frac{CPI_{Last}}{CPI_0} \cdot \left[\left(1 - \frac{d-1}{D} \right) + \frac{d-1}{D} \cdot \frac{S_{n_{P-L+1}}}{S_{Last}} \cdot (1 + Z_{n_{P-L+1}})^{t_{n_{P-L+1}}} \right] \right]^{1/t_n} - 1$$

$$\frac{\partial R_n}{\partial Z_{n_{P-L}}} = 0$$

$$\frac{\partial R_n}{\partial Z_{n_{P-L+1}}} = \left(\frac{CPI_{Last}}{CPI_0} \right)^{1/t_n} \cdot \frac{t_{n_{P-L+1}}}{t_n} \cdot \frac{d-1}{D} \cdot \frac{S_{n_{P-L+1}}}{S_{Last}} \cdot (1 + Z_{n_{P-L+1}})^{t_{n_{P-L+1}}} \cdot \left[\left(1 - \frac{d-1}{D} \right) + \frac{d-1}{D} \cdot \frac{S_{n_{P-L+1}}}{S_{Last}} \cdot (1 + Z_{n_{P-L+1}})^{t_{n_{P-L+1}}} \right]^{1/t_{n-1}}$$

c) Compute $\frac{\partial Z_{i,j_{P-L}}}{\partial Z_{i_{P-L+1}}}$ matrix



$$Z_{np-L} = (1 + Z_{n-1p-L+1})^{\frac{(1-p)}{t_{np-L}} \cdot t_{n-1p-L+1}} \cdot (1 + Z_{np-L+1})^{\frac{(p)}{t_{np-L}} \cdot t_{np-L+1}} - 1$$

Based on the formula that links CPI rates used in the curve stripping; the following $\frac{\partial Z_{i,j,p-L}}{\partial Z_{i,p-L+1}}$ matrix can be built:

$$\begin{matrix} Date_{1p-L+1} & \begin{pmatrix} Date_{1p-L} & Date_{2p-L} & Date_{3p-L} \\ 1 & \frac{\partial Z_{2p-L}}{\partial Z_{1p-L+1}} & 0 \\ 0 & \frac{\partial Z_{2p-L}}{\partial Z_{2p-L+1}} & \frac{\partial Z_{3p-L}}{\partial Z_{2p-L+1}} \\ 0 & 0 & \frac{\partial Z_{3p-L}}{\partial Z_{3p-L+1}} \end{pmatrix} \\ Date_{2p-L+1} \\ Date_{3p-L+1} \end{matrix}$$

In between two pillars

$$\frac{\partial Z_{np-L}}{\partial Z_{n-1p-L+1}} = \frac{(1 - \frac{p}{n}) \cdot t_{n-1p-L+1}}{t_{np-L}} \cdot (1 + Z_{n-1p-L+1})^{\frac{(1-p)}{t_{np-L}} \cdot t_{n-1p-L+1}-1} \cdot (1 + Z_{np-L+1})^{\frac{(p)}{t_{np-L}} \cdot t_{np-L+1}}$$

$$\frac{\partial Z_{np-L}}{\partial Z_{np-L+1}} = \frac{(p)}{t_{np-L}} \cdot t_{np-L+1} \cdot (1 + Z_{n-1p-L+1})^{\frac{(1-p)}{t_{np-L}} \cdot t_{n-1p-L+1}} \cdot (1 + Z_{np-L+1})^{\frac{(p)}{t_{np-L}} \cdot t_{np-L+1}-1}$$

First pillar

$$\frac{\partial Z_{1p-L}}{\partial Z_{1p-L+1}} = 1$$

d) Compute $\frac{\partial R}{\partial Z_{p-L+1}}$ matrix

After calculating $\frac{\partial R}{\partial Z}$ matrix and $\frac{\partial Z_{p-L}}{\partial Z_{p-L+1}}$ matrix, we can deduce $\frac{\partial R}{\partial Z_{p-L+1}}$ matrix by composing both matrices. This can be done using the following formula:

$$\frac{\partial R}{\partial Z} = \frac{\partial R_i}{\partial Z_{i,p-L+1}} + \frac{\partial R_i}{\partial Z_{i-1,p-L}} \cdot \frac{\partial Z_{i-1,p-L}}{\partial Z_{i,p-L+1}}$$

e) Jacobian Matrix

The final matrix to derive a CPI rate regarding Par rates and project the sensitivity on the pillar dates can be calculated by inverting $\frac{\partial R}{\partial Z_{p-L+1}}$.

$$A \cdot B = I_n$$

$$B = A^{-1} = \frac{\partial R}{\partial Z_{p-L+1}} \text{ matrix}$$

6.2.4 CPI Gamma

CPI Gamma can be calculated by applying the chain rule to the PV formula:

$$PV = \phi \cdot N \cdot \left[\frac{a \cdot CPI_{P-L} + (1-a) \cdot CPI_{P-L+1}}{CPI_0} \cdot -(1+K)^{t_3} \right] \cdot Df$$

Terms of the CPI Gamma matrix follow exactly the same formulae as for the piecewise interpolation case except it's applied twice; once for CPI_{P-L} and once for CPI_{P-L+1} . Contrary to Piecewise CPI Gamma, Linear CPI Gamma can't be null; only one of the term can be null at a time (CPI_{P-L} or CPI_{P-L+1}).

a) Diagonal terms:

$$\frac{\partial^2 PV}{\partial CPI_n^2} = a \cdot \frac{\partial^2 PV(P-L)}{\partial CPI_n^2} + (1-a) \cdot \frac{\partial^2 PV(P-L+1)}{\partial CPI_n^2}$$

b) Cross term:

$$\frac{\partial^2 PV}{\partial CPI_1 \partial CPI_2} = a \cdot \frac{\partial^2 PV(P-L)}{\partial CPI_1 \partial CPI_2} + (1-a) \cdot \frac{\partial^2 PV(P-L+1)}{\partial CPI_1 \partial CPI_2}$$

6.2.5 CPI Rate Gamma

CPI Rate Gamma can be calculated by applying the chain rule to the PV formula:

$$PV = \rho \cdot N \cdot \left[\frac{CPI_{Last}}{CPI_0 \cdot S_{Last}} \cdot [a \cdot S_{P-L} \cdot (1+Z_{P-L})^{t_{P-L}} + (1-a) \cdot S_{P-L+1} \cdot (1+Z_{P-L+1})^{t_{P-L+1}}] - (1+K)^{t_3} \right] \cdot Df$$

Terms of the CPI Rate Gamma matrix follows exactly the same formulas as for the piecewise interpolation case except it's applied twice; once for CPI_{P-L} and once for CPI_{P-L+1} .

a) Diagonal terms:

$$\frac{\partial^2 PV}{\partial Z_n^2} = a \cdot \frac{\partial^2 PV(P-L)}{\partial Z_n^2} + (1-a) \cdot \frac{\partial^2 PV(P-L+1)}{\partial Z_n^2}$$

b) Cross term:

$$\frac{\partial^2 PV}{\partial Z_1 \partial Z_2} = a \cdot \frac{\partial^2 PV(P-L)}{\partial Z_1 \partial Z_2} + (1-a) \cdot \frac{\partial^2 PV(P-L+1)}{\partial Z_1 \partial Z_2}$$

6.2.6 Par Gamma

Par Gamma for linear indices is as complex as Par Delta because of the index level dependency to previous Par rates when stripping the Inflation curve. A way to calculate Gamma is to use the chain rule up to the CPI rate Gamma and then compose it with the Hessian matrix to translate CPI rate Gamma into Par Gamma.

$$\frac{\partial^2 PV(p)}{\partial R_n^2} = \sum_{k=1,2} \frac{\partial^2 PV}{\partial Z_k^2} \cdot \left(\frac{\partial Z_k}{\partial R_n} \right)^2 + \frac{\partial PV}{\partial Z_k} \cdot \frac{\partial^2 Z_k}{\partial R_n^2}$$

$$\frac{\partial^2 PV(p)}{\partial R_i \partial R_j} = \sum_{k=1,2} \frac{\partial PV}{\partial Z_k} \cdot \frac{\partial^2 Z_k}{\partial R_i \partial R_j} + \sum_{k,l=1,2} \frac{\partial^2 PV}{\partial Z_k \partial Z_l} \cdot \frac{\partial Z_k}{\partial R_i} \cdot \frac{\partial Z_l}{\partial R_j}$$

Starting from the PV formula, we can deduce that in the case of a Linear interpolation:

$$\frac{\partial^2 PV}{\partial R_n^2} = \frac{\partial^2 PV(P - L)}{\partial R_n^2} + \frac{\partial^2 PV(P - L + 1)}{\partial R_n^2}$$

And

$$\frac{\partial^2 PV}{\partial R_i \partial R_j} = \frac{\partial^2 PV(P - L)}{\partial R_i \partial R_j} + \frac{\partial^2 PV(P - L + 1)}{\partial R_i \partial R_j}$$

From the Par Gamma chain rule above, we need the following terms to be able to calculate it:

- $\frac{\partial PV}{\partial Z_p}$: First part of the chain rule in CPI rate Delta calculation
- $\frac{\partial Z_p}{\partial Z_n}$: Second part of the CPI rate Delta calculation; projection of CPI rate delta on the curve
- $\frac{\partial^2 PV}{\partial Z_p^2}$: Second order derivative of the PV to its CPI rate
- $\frac{\partial^2 Z_p}{\partial Z_n^2}$: Second order derivative of CPI rate on the curve
- $\frac{\partial^2 Z_p}{\partial Z_1 \partial Z_2}$: First cross order derivative of CPI rate on the curve
- $\frac{\partial Z_n}{\partial R_n}$: Last part of the chain rule in Par Delta calculation
- $\frac{\partial^2 Z_n}{\partial R_n^2}$: New term; second order derivative of the CPI rate with respect to the par rate
- $\frac{\partial^2 Z_n}{\partial R_j \partial R_i}$: New term; first order cross derivative of the CPI rate with respect to the par rates

The two new terms can't be calculated directly but the inverse derivatives can.

Hessian Matrix:

Second derivative of CPI rate 1 to Par rates	Second derivative of CPI rate 2 to Par rates	Second derivative of CPI rate 3 to Par rates
$\text{Date}_1 \begin{pmatrix} \text{Date}_1 & \text{Date}_2 & \text{Date}_3 \\ \frac{\partial^2 Z_1}{\partial R_1^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\text{Date}_1 \begin{pmatrix} \text{Date}_1 & \text{Date}_2 & \text{Date}_3 \\ \frac{\partial^2 Z_2}{\partial R_1^2} & \frac{\partial^2 Z_2}{\partial R_1 \partial R_2} & 0 \\ \frac{\partial^2 Z_2}{\partial R_1 \partial R_2} & \frac{\partial^2 Z_2}{\partial R_2^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\text{Date}_1 \begin{pmatrix} \text{Date}_1 & \text{Date}_2 & \text{Date}_3 \\ \frac{\partial^2 Z_3}{\partial R_1^2} & \frac{\partial^2 Z_3}{\partial R_1 \partial R_2} & \frac{\partial^2 Z_3}{\partial R_1 \partial R_3} \\ \frac{\partial^2 Z_3}{\partial R_1 \partial R_2} & \frac{\partial^2 Z_3}{\partial R_2^2} & \frac{\partial^2 Z_3}{\partial R_2 \partial R_3} \\ \frac{\partial^2 Z_3}{\partial R_1 \partial R_3} & \frac{\partial^2 Z_3}{\partial R_2 \partial R_3} & \frac{\partial^2 Z_3}{\partial R_3^2} \end{pmatrix}$

6.3 Murex Test Cases

The same tests run for pricing validation have been performed to validate IM and VM sensitivities.

Tests have shown some bugs in Murex:

- Bucketing of Inflation sensitivities is not done correctly when the inflation curve is extrapolated. This will only happen for non standard interpolated UK RPI and EU HICPxT 50Y trades. This issue has been raised to Murex with a medium priority as it will only happen for less than 30 days of the trade life and the difference in bucketing is small (less than 1% error).
- CPI Delta is not projected on the Inflation tenors correctly. Murex only calculates CPI Delta at the level of the trade fixing and applies a linear projection in time instead of a log linear projection in index levels. This is not an issue as CPI Delta is not used by LCH.
- CPI Gamma is null all the time. This is not an issue as CPI Gamma is not used by LCH.
- Par Gamma is not correctly calculated. Hessian matrices used to convert zero sensitivity to par are correct but the CPI Rate Gamma projection from trade level to the curve is not correct. The projection error is not material and can be ignored.
- CPI Rate Gamma is not correctly projected on the curve. CPI Rate Gamma is correctly calculated at trade level but linearly projected on the curve. This is not an issue as Murex uses it in the VaR calculation as a vector. When Inflation shock correlation is low (short term), Gamma is low and vice versa.

VM Results

Threshold	Par Delta			CPI Rate Delta				
	Min Error	Max Error	Pass/Fail	Min Error	Max Error	Pass/Fail		
0.01								
GB RPI AI 2M	-	0.00	0.00	PASS	-	0.00	0.00	PASS
HICP CPI XT NR	-	0.00	0.00	PASS	-	0.00	0.00	PASS
FR CPI XT NR	-	0.00	0.00	PASS	-	0.00	0.00	PASS
US CPI AI NSA	-	0.00	0.00	PASS	-	0.00	0.00	PASS

Threshold	Par Gamma			CPI Rate Gamma				
	Min Error	Max Error	Pass/Fail	Min Error	Max Error	Pass/Fail		
0.01								
GB RPI AI 2M	-	0.09	30.37	FAIL	-	0.00	0.00	PASS
HICP CPI XT NR	-	118.94	829.85	FAIL	-	0.00	-	PASS
FR CPI XT NR	-	193.16	1,503.31	FAIL	-	0.00	0.00	PASS
US CPI AI NSA	-	4,495.16	544.01	FAIL	-	0.00	0.00	PASS

Threshold	ZC Delta			ZC Gamma				
	Min Error	Max Error	Pass/Fail	Min Error	Max Error	Pass/Fail		
1								
EUR EONIA	-	0.03	0.00	PASS	-	0.00	0.00	PASS
GBP SONIA	-	0.00	0.00	PASS	-	0.00	0.00	PASS
USD FEDFUND	-	0.00	0.00	PASS	-	0.00	0.00	PASS

IM Results

Sensitivity not used by LCH

Threshhold	Par Delta			CPI Rate Delta		
	Min Error	Max Error	Pass/Fail	Min Error	Max Error	Pass/Fail
0.1	-	0.000000	-	Pass	-	0.000000
GB ZERO AI 2M - IM	-	0.000000	-	Pass	-	0.000000
HICP ZERO CPI XT NR - IM	-	0.000000	0.000000	Pass	-	0.000000
FR ZERO CPI XT NR - IM	-	0.000000	0.000001	Pass	-	0.000004
US ZERO CPI AI NSA - IM	-	0.000000	-	Pass	-	0.000000

CPI Delta as Implemented		
Min Error	Max Error	Pass/Fail
- 1,227,159.269733	25,159.320858	Fail
- 4,374,551.691278	44,662.185022	Fail
- 245,479.913550	231,594.812765	Fail
- 21,671,490.231510	13,906.085103	Fail

Threshhold	Par Gamma as Implemented			CPI Rate Gamma as Implemented		
	Min Error	Max Error	Pass/Fail	Min Error	Max Error	Pass/Fail
0.1	0.034290	30.378859	Fail	- 0.000000	0.000000	Pass
GB ZERO AI 2M - IM	- 9.978175	0.213554	Fail	- 0.000000	-	Pass
HICP ZERO CPI XT NR - IM	- 0.330295	0.280680	Fail	- 0.000000	0.000000	Pass
FR ZERO CPI XT NR - IM	- 0.035734	0.030277	Pass	- 0.000000	-	Pass
US ZERO CPI AI NSA - IM	-	-	-	-	-	-

CPI Gamma as Implemented		
Min Error	Max Error	Pass/Fail
- 0.695236	5,727.670842	Fail
- 322,829858	2,242.007450	Fail
- 320,580919	1,249.852042	Fail
- 8,586507	918.496783	Fail

Threshhold	Par Cross Gamma as Implemented			CPI Rate Cross Gamma as Implemented		
	Min Error	Max Error	Pass/Fail	Min Error	Max Error	Pass/Fail
0.001	-	-	-	-	-	-
GBP ZERO SONIA - GB ZERO AI 2M - IM	- 0.000056	0.000019	Pass	- 0.000000	0.000000	Pass
USD ZERO FEDFUNDS - US ZERO CPI AI NSA - IM	- 0.000020	0.000004	Pass	-	-	Pass
EUR ZERO EONIA - HICP ZERO CPI XT NR - IM	- 0.000004	0.000017	Pass	- 0.000000	0.000000	Pass
EUR ZERO EONIA - FR ZERO CPI XT NR - IM	- 0.000025	0.000023	Pass	- 0.000000	0.000000	Pass

CPI Cross Gamma as Implemented		
Min Error	Max Error	Pass/Fail
- 0.000000	0.000000	Pass

Threshhold	Par Gamma versus LogLinear Interp			CPI Rate Gamma versus LogLinear Interp		
	MX Total	TheoTotal	Pass/Fail	MX Total	TheoTotal	Pass/Fail
2.00%	-	-	-	-	-	-
GB ZERO AI 2M - IM	10,417.393	10,484.155	Pass	10,397.36	10,477.32	Pass
HICP ZERO CPI XT NR - IM	2,511.528	2,501.117	Pass	2,488.33	2,511.22	Pass
FR ZERO CPI XT NR - IM	1,926.825	1,926.802	Pass	1,908.03	1,908.03	Pass
US ZERO CPI AI NSA - IM	2,215.085	2,215.073	Pass	2,198.66	2,198.66	Pass

CPI Gamma versus LogLinear Interp		
MX Total	TheoTotal	Pass/Fail
- 3,865.18	-	Pass
- 6,766.80	-	Pass
- 15,053.50	-	Pass
- 6,502.09	-	Pass

Threshhold	CPI Rate Cross Gamma versus LogLinear Interp		
	MX Total	TheoTotal	Pass/Fail
1.00%	-	-	-
GBP ZERO SONIA - GB ZERO AI 2M - IM	- 11,118.462	- 11,118.462	Pass
USD ZERO FEDFUNDS - US ZERO CPI AI NSA - IM	- 2,477.131	- 2,477.132	Pass
EUR ZERO EONIA - HICP ZERO CPI XT NR - IM	- 2,666.383	- 2,666.383	Pass
EUR ZERO EONIA - FR ZERO CPI XT NR - IM	- 2,167.845	- 2,167.845	Pass

Threshhold	IRS ZC Delta			IRS ZC Gamma		
	Min Error	Max Error	Pass/Fail	Min Error	Max Error	Pass/Fail
0.1	-	-	-	-	-	-
EUR ZERO EONIA	- 0.021174	0.000958	Pass	- 0.000000	0.000000	Pass
GBP ZERO SONIA	- 0.002197	0.002477	Pass	- 0.000000	0.000000	Pass
USD ZERO FEDFUNDS	- 0.001390	0.002941	Pass	- 0.000000	0.000000	Pass

7 Historical VAR Theory

7.1 Background

The Initial Margining methodology is central to SwapClear and is periodically reviewed to ensure that it is efficient, fair, transparent and prudent. An Initial Margining methodology already exists for IR Swaps, and this is based on Historic VaR calculation. It is proposed that this methodology is extended to incorporate inflation swaps, by looking at whole market scenarios that include shifts to both nominal rates and inflation. This will allow inflation products and pure rates products such as IRS and FRAs to be margined in a single framework and allow margin offset where risks offset. FX conversion to a single reference currency will be handled entirely in the current swaps framework.

7.2 Calculation of Returns

In order to turn a series of historic market changes into a set of scenarios to apply to the present market conditions, one must select a method to measure the change or return. The return provides the measure of interest rate and inflation rate change from one point in time to another. The choice of return measure should be consistent across the time series, and be robust across a wide range of market environments.

In the most general case the scenario is a function of the market at the start of the holding period, the market at the end of the holding period and, in some cases, the volatility.

$$r = f(p_{start}, p_{end}, \sigma)$$

Historically, interest rates have often been assumed to follow a lognormal mean reverting process. For such a process the relevant change measure could be relative returns:

$$r_{relative} = \frac{p_{end} - p_{start}}{p_{start}}$$

An alternative to relative return change measure could be absolute returns:

$$r_{absolute} = p_{end} - p_{start}$$

As a consequence of the financial crisis of 2008, Inflation Rates have moved from positive to negative values and Interest Rates of major economies are at unprecedented low levels, in many cases approaching zero and in some cases reaching negative values. Low and negative values pose issues if the Initial Margin VaR methodology is based on relative returns, as they can generate explosive returns that are not meaningful. For example, a rate move of a couple of basis points has the potential to generate infinitely large returns. This observation highlights the drawback of using relative returns in such an environment.

LCH.SwapClear has moved its methodology from using relative returns to use absolute returns. Inflation scenarios using this methodology will be tested for validity.

7.3 Scenario Scaling

Empirical evidence demonstrates that the interest rate market operates under neither a clean absolute nor relative change measure. The implication is that any efficient IM model will have to include some sort of scaling to compensate for one scenario obtained under one interest rate regime applied to another regime. The natural way of compensating the scenarios is by using volatility or some other dispersion measure.

In a VaR Model, a crucial point of implementation is to make sure that returns generated in a specific environment can be properly implemented in a different environment. The way SwapClear translates returns from one environment into another one is through the scaling process. Two ways of scaling are considered, mid volatility scaling and full volatility scaling.

7.3.1 Full Volatility Scaling

The full volatility scaling approach would be to scale the return with respect to the pure relationship of volatility, as follows:

$$r_t^* = r_t \frac{\sigma_N}{\sigma_t}$$

Scaled returns using the pure relationship of volatility are translated more effectively from different volatility regimes. In this case, VaR results might, depending on market data and portfolio, present cyclical patterns. By cyclical pattern, we mean that the present IM is a function of the present volatility. So in a benign environment less IM would be charged, and in a malign environment more IM would be charged. Although one could argue that the IM increases with market volatility is a desired property of a model, regulators have historically pointed out that this property could act as a feedback loop on the financial system as a whole.

7.3.2 Mid Volatility Scaling

In mid volatility scaling, each scenario return r_t is weighted as follows:

$$r_t^* = r_t \left(\frac{\sigma_t + \sigma_N}{2\sigma_t} \right)$$

Where:

r_t^* = Scaled return

r_t = Raw return

σ = Dispersion measure of the change measure

t = Scenario day

N = Most recent day (today)

By scaling using the mid volatility, scaled returns are translated into half way between the volatility environment of the scenario day and the most recent day. This reduces the pro-cyclical effect described above for full volatility scaling.

7.3.3 Dispersion Measurement

The assumption that a time series of interest change measures can be ascribed a dispersion measurement is often made. The dispersion measurement is meant to describe how wildly the time series fluctuate.

In the historical SwapClear Initial Margining model, Standard Deviation (SD) is used as the dispersion measurement.

One other dispersion measurement which we have investigated is the Mean Absolute Deviation (referred to as MAD in this document).

$$\sigma_n^{STD} = \sqrt{\frac{\sum_{i=1}^n (r_i - \bar{r})^2}{n-1}}$$

$$\sigma_n^{MAD} = \frac{\sum_{i=1}^n |r_i - \bar{r}|}{n-1}$$

7.3.4 Exponentially Weighted Moving Average: EWMA

In practice the dispersion measurement is calculated on an exponential weighted moving average:

$$\sigma_n^{STD} = \sqrt{\sigma_{n-1}^{STD}^2 \lambda + r_n^2(1-\lambda)}$$

$$\sigma_n^{MAD} = \sigma_{n-1}^{MAD} \lambda + |r_n|(1-\lambda)$$

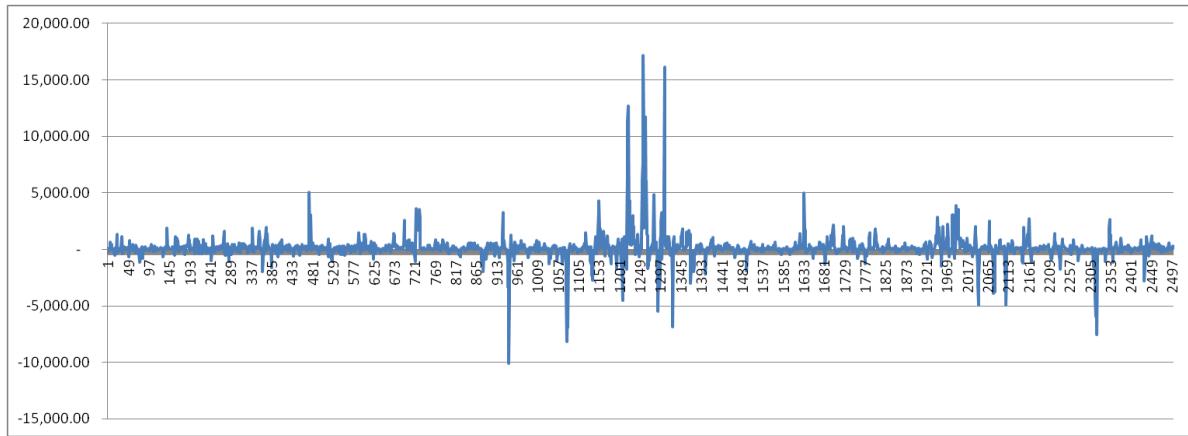
7.4 Taylor versus Full Revaluation

IM calculation has been compared in between Taylor (TDG) and Full revaluation for a UK RPI member portfolio on different days.

Differences are small (typically less than 0.1%), the top 20 scenarios from TDG and full revaluation are always matching as long as Cross Gamma is used in TDG.

	Full Reval	TDG	
WCL	8,889,691.45	8,882,114.17	-0.09%
ES6	8,005,625.75	8,001,126.47	-0.06%

	TDG (Scenario nb)	FULL (Scenario nb)	FULL	TDG	Diff (abs)	Diff (Rel)
1	2321	2321	-8,889,691.45	-8,882,114.17	- 7,577.28	0.09%
2	2317	2317	-8,392,638.04	-8,388,316.13	- 4,321.91	0.05%
3	364	364	-8,283,848.86	-8,281,906.45	- 1,942.41	0.02%
4	2042	2042	-7,652,818.08	-7,649,598.88	- 3,219.20	0.04%
5	2320	2320	-7,483,178.91	-7,479,191.60	- 3,987.31	0.05%
6	2318	2318	-7,331,579.17	-7,325,631.57	- 5,947.60	0.08%
7	2319	2319	-6,965,683.75	-6,959,935.16	- 5,748.59	0.08%
8	19	19	-6,937,521.90	-6,937,513.74	- 8.16	0.00%
9	1372	1372	-6,883,896.08	-6,883,781.57	- 114.50	0.00%
10	1209	1209	-6,852,263.66	-6,847,748.23	- 4,515.43	0.07%
11	1760	1760	-6,820,525.16	-6,819,748.12	- 777.04	0.01%
12	1761	1761	-6,647,810.22	-6,647,251.70	- 558.52	0.01%
13	1270	1270	-6,460,454.75	-6,459,893.07	- 561.67	0.01%
14	83	83	-6,193,767.42	-6,192,992.76	- 774.66	0.01%
15	1078	1078	-6,178,395.00	-6,170,235.36	- 8,159.63	0.13%
16	1293	1293	-6,152,711.65	-6,151,623.58	- 1,088.07	0.02%
17	363	363	-5,982,017.58	-5,981,557.80	- 459.79	0.01%
18	942	942	-5,972,769.67	-5,962,673.60	- 10,096.07	0.17%
19	1403	1403	-5,747,563.96	-5,745,377.99	- 2,185.97	0.04%
20	1294	1294	-5,722,918.84	-5,719,496.58	- 3,422.26	0.06%



8 Historical VaR Market Data Sets

8.1 Introduction

In-line with regulatory requirements - the Estimated Shortfall (ES) computations undertaken by CCPs requires ten years of historic time-series. In the context of Zero Coupon Inflation Swaps – this means the par zero rates.

The data has been sourced from a number of members. In addition – to widen the distribution of data – contributions have been gathered from market data vendors. Given this dataset – a process has been undertaken to create a single, cleansed, set of information which may be used going forward in the inflation swaps project.

This document describes the methodology / process which has been used to arrive at the single set of rates for each index in scope.

8.2 Member data

The table below describes the data which has been received directly from each member, for each index.

Contributor	Ccy	Index	Tenors	Coverage start	Coverage end
Barclays	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40, 50 [Y]	01/01/2007	14/12/2012
Deutche Bank	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40, 50 [Y]	01/10/2002	11/01/2013
Morgan Stanley	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40*, 50* [Y]	25/11/2003, 40Y/50Y=06/07/2006	03/06/2013
RBoS	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40* [Y]	27/06/2003, *40Y = 02/06/2005	03/06/2013
UBS	GBP	UK-RPI	2, 5, 7, 10, 15, 20, 30 [Y]	27/04/2004	31/12/2012
Barclays	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	01/01/2007	14/12/2012
Deutche Bank	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	01/10/2002	11/01/2013
Morgan Stanley	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	25/11/2003	03/06/2012
RBoS	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	18/08/2003	03/06/2012
UBS	EUR	HICPxT	2, 5, 7, 10, 15, 20, 30 [Y]	26/04/2004	31/12/2012
Barclays	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	01/01/2007	14/12/2012
Deutche Bank	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	01/10/2002	11/01/2013
Morgan Stanley	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	06/08/2004	03/06/2013
RBoS	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	18/08/2003	03/06/2013
UBS	EUR	FR-CPI	2, 5, 7, 10, 15, 20, 30 [Y]	22/06/2004	31/12/2012
Barclays	USD	US-CPI	1-10, 12, 15, 20, 25, 30 [Y]	01/01/2007	14/12/2012
Deutche Bank	USD	US-CPI	1-10, 12, 15, 20, 25, 30 [Y]	02/08/2004	11/01/2013
Morgan Stanley	USD	US-CPI	1-10, 12, 15, 20, 25, 30 [Y]	21/07/2004	03/06/2013
RBoS	USD	US-CPI	1-10, 12, 15, 20, 25, 30 [Y]	11/10/2004	03/06/2013
UBS	USD	US-CPI	2, 5, 7, 10, 15, 20, 30 [Y]	21/07/2004	31/12/2012

8.3 Market data vendor time-series

The table below describes the data which has been gathered from the market data vendors.

Contributor	Ccy	Index	Tenors	Coverage start	Coverage end
BGC	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40, 50 [Y]	25/03/2013 10/12/2003, *40Y = 22/06/2004, 50Y= 19/10/2005	03/06/2013
Bloomberg	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40*, 50* [Y]		03/06/2013
GFI	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40, 50 [Y]	19/04/2011	03/06/2013
HSBC	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40, 50 [Y]	08/11/2005 21/07/2004,	03/06/2013
ICAP	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40*, 50* [Y]	40Y/50Y = 05/03/2008. NB - gap mid-section	03/06/2013
Santander	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30, 40, 50 [Y]	12/10/2006	20/07/2008
Barclays BBG	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	08/07/2004	24/08/2010
Bloomberg	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	26/04/2004	11/01/2013
GFI	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	16/04/2012	03/06/2012
NATIXIS	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	08/03/2010	06/05/2013
ICAP	EUR	HICPxT	1-10, 12, 15, 20, 25, 30 [Y]	21/07/2004 * NB gap mid-section	03/06/2012
Santander	GBP	UK-RPI	1-10, 12, 15, 20, 25, 30 [Y]	12/10/2006	03/06/2012
Barclays BBG	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	08/07/2004	03/06/2013
Bloomberg	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	22/06/2004	25/03/2013
GFI	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	16/04/2012	03/06/2013
ICAP	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	21/07/2004	03/06/2013
Santander	EUR	FR-CPI	1-10, 12, 15, 20, 25, 30 [Y]	27/02/2008	03/06/2013
Bloomberg	USD	US-CPI	1-10, 12, 15, 20, 25, 30 [Y]	22/06/2004	25/03/2013
ICAP	USD	US-CPI	1-10, 12, 15, 20, 25, 30 [Y]	21/07/2004	03/06/2013
Santander	USD	US-CPI	1-10, 12, 15, 20, 25, 30 [Y]	12/10/2006	24/04/2013

8.4 Cleansing Methodology

Business day gaps

Initially, prior to aggregation, a process was run to determine missing dates from each contributor's time-series. This involved determining all valid business days, within the 10y period, from an LCH.C perspective and overlaying those dates upon the available time-series from each contributor. In the event that a business date was found to be valid for LCH.C but for which there was no contributor data – the data was simply copied from the previous “good” business day. This is in-line with the existing process utilised for the existing SwapClear services.

Other gaps

Financial time-series commonly contain gaps in the data. This is realised in the contributors time-series data. The gaps, intra-tenor, range from single days to weeks. Depending upon the duration of the gaps different methods are employed:

Up to two days

This constitutes the majority of the cases. In the event of gaps of this nature the methodology was to fill the gaps using linear interpolation.

Beyond two days

A simple filter based on the Kalman filter approach was put in place to “fill-in” these gaps. The methodology is described below.

The readings of the time-series are denoted by $x_1, x_2 \dots x_n$. Primarily we take the mean, μ_n , of the time-series given n points.

$$\mu_n = \frac{1}{n} \sum_{i=1}^n x_i$$

When the next point is read, we can re-compute μ_n . But the more efficient method is to re-use the old estimate of μ_n and the new value of x (x_{n+1}). The new mean value is simply a weighted average of the old estimate μ_n and the new value x_{n+1} . We trust the mean value more than the new value and therefore weight accordingly.

$$\mu_{n+1} = \frac{n}{n+1} \mu_n + \frac{1}{n+1} x_{n+1}$$

As the time-series is dynamic it makes sense to maintain a rolling average. The selection of the appropriate window depends upon the gaps in the time-series. But is typically in the order of 10-30 days. Note that x_{n+1} with zero values are replaced with the μ_{n+1} value and the sample re-computed with this new average.

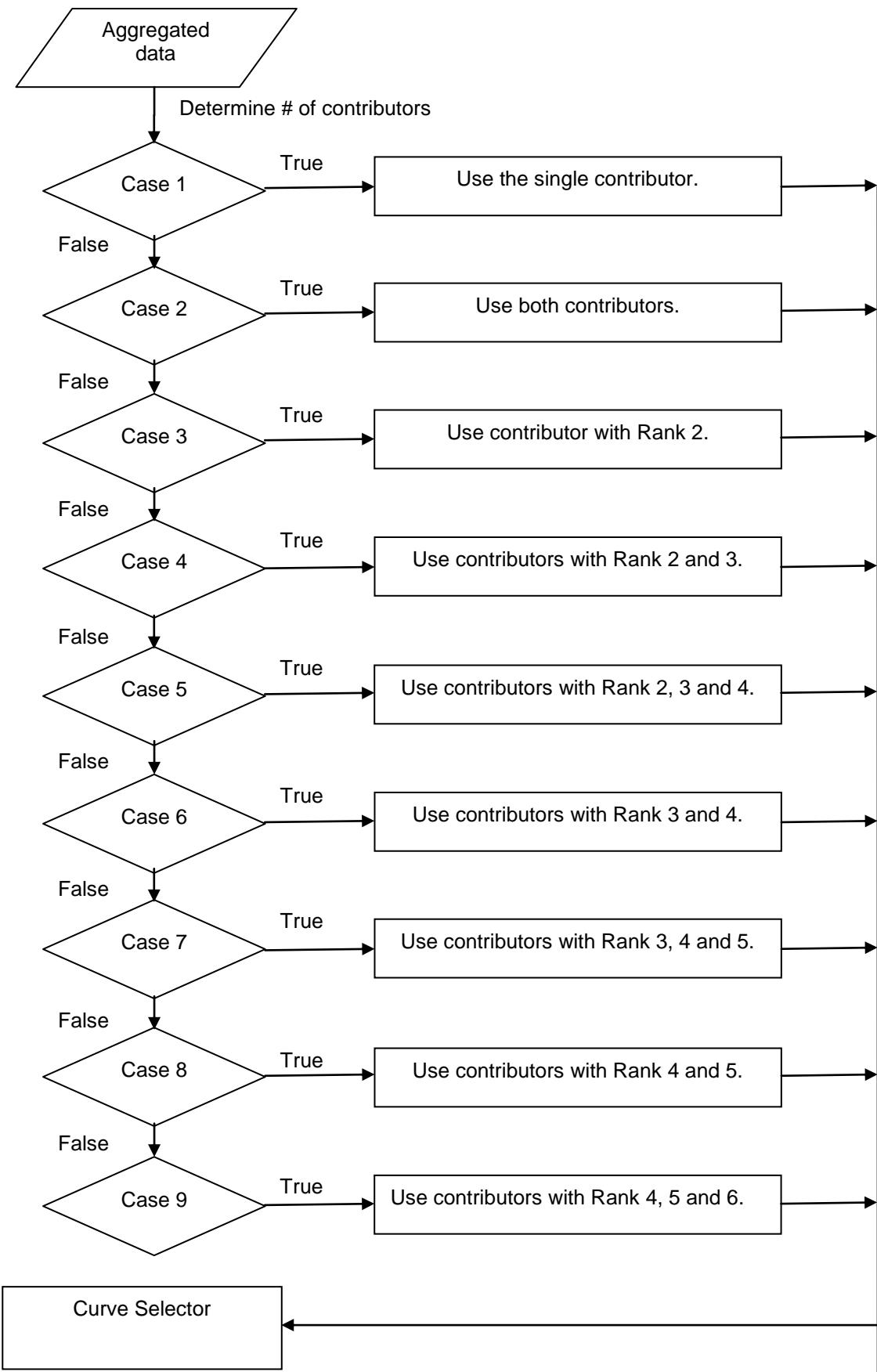
8.5 Series Aggregation

To be consistent with the methodology employed by the market data infrastructure for the existing, SwapClear, service the following methodology was employed:

1. Collate all contributions, for a single date – for all tenors.
 - a. Compute the mean of the contributions (where the contributions are not zero).

Comment: The data is taken contributor-wise. For those contributors who do not contribute a full curve, on a given date, their contribution will not be used in the aggregation process for that day. A full curve is defined as no gaps for any tenor, on a given date.

2. For each valid contributor - compute the squared error (i.e. the difference, squared, between the mean and the contributors quotes (i.e. for 1Y...nY) – on a given day).
 - a. Sum the squared error for each contributor, across all tenors, for the given date.
3. For each contributor – Rank that contributor's "Sum of Squared Errors" as produced in step 2a.
4. Select the contributors who are valid dependent upon their ranking. The rules for contributor selection are described in the flowchart that follows.
5. To produce the derived rate the contributions, as selected in the previous step, are averaged across tenors for a given date.



9 Portfolio Margining

9.1 Introduction

To be able to include Zero Coupon Inflation Swaps in the same default management fund as Interest Rate Swaps, we have to prove that these two products have:

- An economic relationship with each other;
- This relationship is statistically reliable; and
- The portfolio can be sold as one.

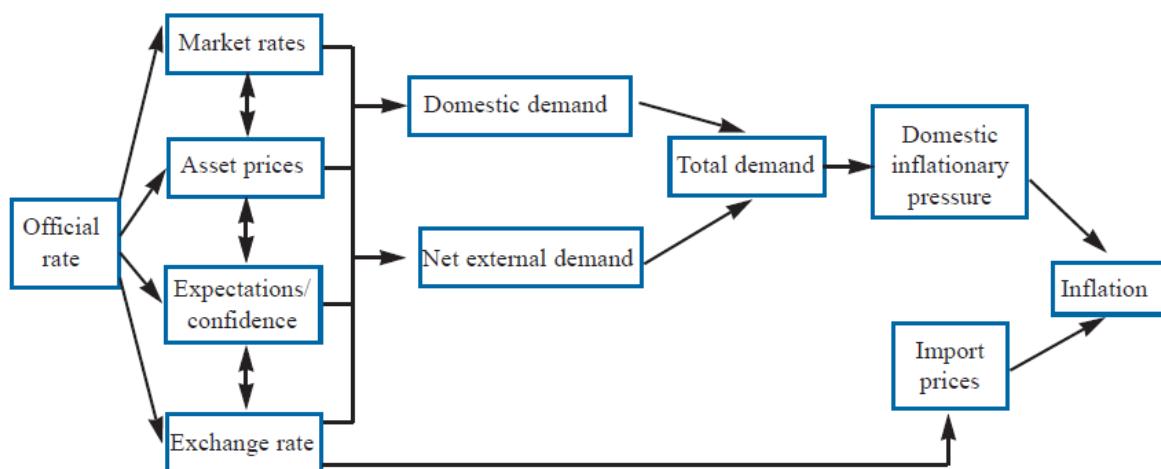
9.2 Economic Relationship

- Interest rates can influence the rate of inflation and the rate of economic growth.
- The Bank of England changes the 'base' interest rate to target the government's desired inflation rate of 2% +/-1%
- Generally, an increase in inflation leads to higher interest rates.
- A fall in inflation and lower growth leads to lower interest rates.

Central banks typically use interest rates as a major tool in monetary policy to manage inflation with the main aim of price stability. Neither high nor low inflation rates are desirable from the point of view of central banks.

The Bank of England Monetary Policy Committee's (MPC) view of the transmission mechanism from short term interest rates to inflation is as follows:

The transmission mechanism of monetary policy



Note: For simplicity, this figure does not show all interactions between variables, but these can be important.

Source: Bank of England

Typically, we would expect to see some correlation between inflation and interest rates. However, we do not expect a very strong correlation due to the lag between monetary policy implementation and its transmission into the real economy and prices. This is acknowledged by the Bank's comment below:

"We cannot be precise about the size or timing of all these channels. But the maximum effect on output is estimated to take up to about one year. And the maximum impact of a change in interest rates on consumer price inflation takes up to about two years. So interest rates have to be set based on judgments about what inflation might be – the outlook over the coming few years – not what it is today."

As noted above, the lag between the implementation of monetary policy and the transmission of that into inflation varies depending on the current economic environment. For example, in the current environment forward guidance is used as a tool for monetary policy with no explicit and therefore quantifiable measure of the impact to inflation.

It should be noted that the use of forward inflation rates derived from market inflation swaps render no significant statistical difference from the use of spot inflation rates while assessing correlation with interest rates.

In the instances where the central bank's sole mandate is to control inflation, we can see that there is a more apparent correlation of the trend between the **level** of inflation and interest rates. Within the G4, only the ECB has the sole mandate of controlling inflation. The Federal Reserve, Bank of England, and Bank of Japan are mandated - explicitly for the Federal Reserve and implicitly for the others - to control both inflation and growth. For a summary of this please see the below table:

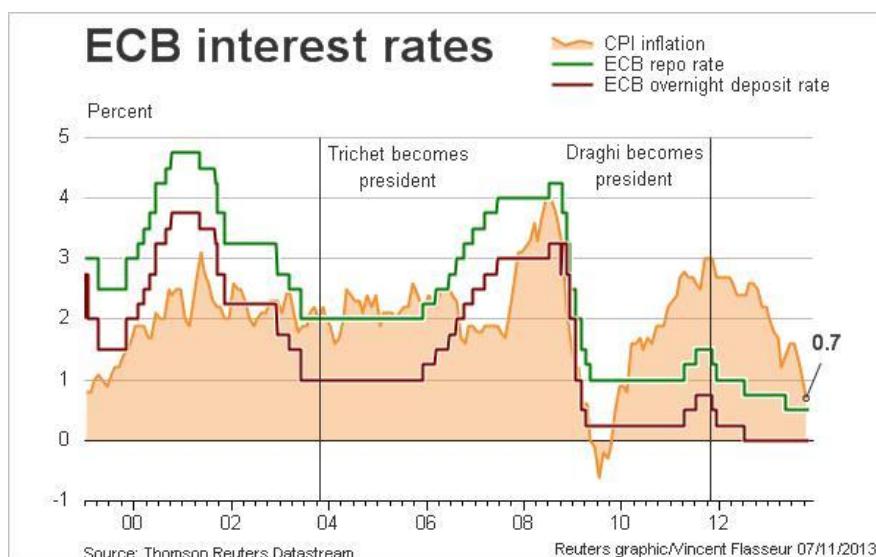
Summary of inflation objectives of central banks

Country	Explicit inflation objective	Tolerance bounds	Target changed /implemented	Target flexibility	Average long term bias between main inflation market and target measure
US	2% PCE in longer run	Not stated	Jan 2012	Inflation aim set by FOMC in context of dual mandate, reconsidered annually	+30bp due to PCE chain weighting, +10bp due to OER weight
Euro area	Close to but below 2% euro HICP	Not stated	May 2003	Primary mandate for price stability from Treaty, ECB governing council definition	-9bp due to excluding tobacco
UK	2% CPI	+/-1%	Dec 2003	Primary aim set annually by government	Unstable but historical average +85bp

Source: Barclays Research based on national central bank information

Please note that the central banks' measure of inflation differs from the market measure. The estimate of the bias is noted in the far right column in the table above.

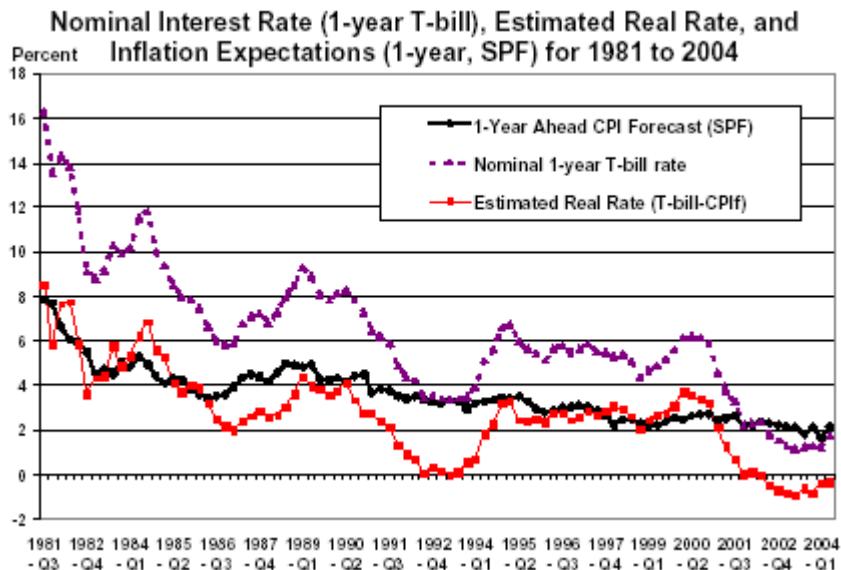
The trend of central bank interest rates against inflation can be seen below for the Eurozone:



It can be seen from this that the level of inflation has an influence on where the ECB sets interest rates. This is particularly true in the period from 2008 to the present. However, note that ECB interest rates are

more stable than inflation, as the decided interest rate is a discretionary number. However, inflation is calculated using empirical data and is therefore a more volatile time series which adds statistical noise in the attempt to show a strong statistical relationship between interest rates and inflation.

In the US, we can see that there is also a relationship between the trend of inflation and interest rates, as demonstrated in the chart below:



SPF: Survey of Professional Forecasters

Source: San Francisco Federal Reserve

Further anecdotal evidence on the relationship between interest rates and inflation can be seen in a comment extracted from the Bank of England's November 2013 inflation report:

"In August, the Committee agreed that its policy guidance would cease to hold either if medium-term inflation expectations were judged to be insufficiently well anchored, or if, in the MPC's view, inflation was more likely than not to be above 2.5% 18 to 24 months ahead."

Policy guidance referred to above references the forward guidance policy of Bank rates to the public.

9.3 Statistical Analysis

9.3.1 Correlation

A 5 day overlapping time series on the changes in interest rates and inflation is constructed. Market swap rates are used as opposed to the official rates, as these are the inputs used for the valuation of products cleared by SwapClear. A 5 day overlapping time series is implied in order to be consistent with SwapClear Initial Margin methodology.

The time horizon runs from January 2005 for GBP and June 2005 for EUR and USD, to October 2014. Inflation and interest rate swap data is compiled from a consistent set of externally sourced data.

Results of the analysis are as follows:

EUR	HICPx	Nominal Swap Rates		
		5Y	10Y	30Y
Zero Coupon Inflation Rates	5Y	0.32	0.35	0.41
	10Y	0.33	0.37	0.44
	30Y	0.31	0.37	0.46

EUR	FRCPix	Nominal Swap Rates		
		5Y	10Y	30Y
Zero Coupon Inflation Rates	5Y	0.30	0.32	0.40
	10Y	0.31	0.35	0.43
	30Y	0.29	0.35	0.44

USD	USCPI	Nominal Swap Rates		
		5Y	10Y	30Y
Zero Coupon Inflation Rates	5Y	0.33	0.40	0.48
	10Y	0.39	0.49	0.58
	30Y	0.30	0.43	0.56

GBP	UKRPI	Nominal Swap Rates		
		5Y	10Y	30Y
Zero Coupon Inflation Rates	5Y	0.32	0.32	0.30
	10Y	0.35	0.37	0.36
	30Y	0.31	0.36	0.39

The tables above show the correlations from 2005 to 2013 between interest rates and inflation. These range between 0.29 – 0.6 for 5Y, 10Y and 30Y maturities. The structural correlation between interest rates and inflation as presented in the Appendix is stable and within the 0.3 – 0.5 range, with correlations mean reverting back to their structural levels.

The Appendix includes EWMA correlation charts of the above 4 indices using the same model employed in the IM methodology. The charts show the correlation distributions and time dynamic, including the average correlation level in black and two distinctive stressed periods over the last 10 years: Lehman default (15-Sep-2008) in red and Euro Crisis (01-Jul-2012, where Spain & Cyprus seeks financial support) in blue.

Results consistently show that the bodies of these correlation distributions have tight ranges and are located at significant levels with averages close to 40%. Importantly, correlations during stressed periods are close to the average level, evidencing the stability and resilience of these dependencies among risk factors.

The previous correlations are now compared against tables describing the correlations between nominal swap rates of given tenors for EUR, USD, GBP and JPY across the same period:

5Y	EUR	USD	GBP	JPY	10Y	EUR	USD	GBP	JPY	30Y	EUR	USD	GBP	JPY
EUR	1.00	0.66	0.76	0.44	EUR	1.00	0.66	0.82	0.45	EUR	1.00	0.57	0.74	0.35
USD		1.00	0.65	0.42	USD		1.00	0.67	0.42	USD		1.00	0.61	0.39
GBP			1.00	0.41	GBP			1.00	0.41	GBP			1.00	0.41
JPY				1.00	JPY				1.00	JPY				1.00

The absolute level of correlation between inflation and rates is similar to that of JPY with EUR, USD and GBP.

Furthermore, the correlation behaviour of inflation and rates is similar to that of correlation between different currencies for nominal swap rates, where there will be periods of de-correlation followed by mean reversion. See Appendix for details of nominal swap rate correlation between EUR, USD, GBP and JPY.

Thus, the management of inflation risk is not dissimilar to that of managing an interest rate swap portfolio comprising different currencies.

9.4 Conclusion

The correlation between interest rates is positive, albeit not strongly positive. However, the positive correlation appears to be structural, which is as expected given the monetary policy of the BOE, Fed, and ECB. This is evident as we observe that correlation quickly reverts back to its structural average following an exogenous shock.

It is evident that interest rates and inflation are economically linked, given that almost all central banks use interest rates to control inflation in the economy. Furthermore, the nature of inflation being correlated demonstrates a reliable statistical relationship between the 2 factors. Hence, in case of default, a portfolio of inflation and interest rate swaps could be risk managed as one. Once the risk is neutralised, the portfolio could be sold as one or split into separate products for practical reasons, and depending on the risk appetite of the bidders of the portfolio.

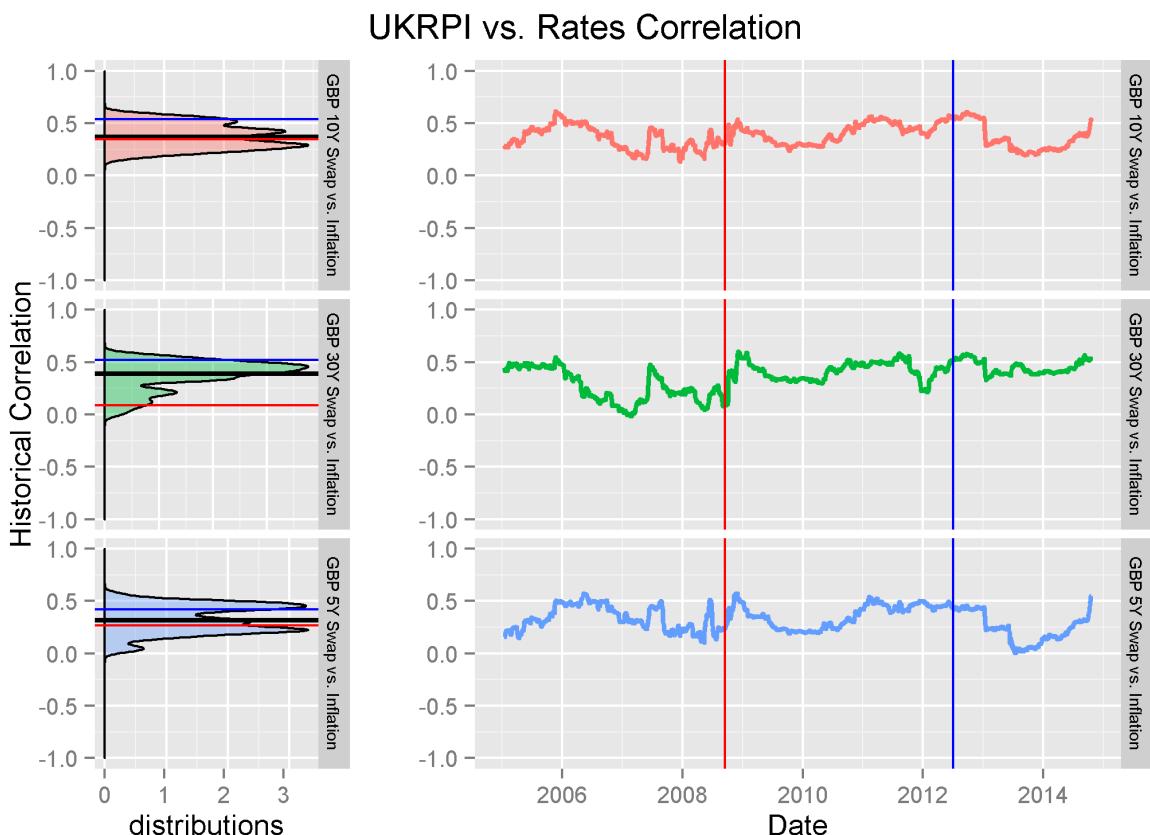
It should be emphasised that this is no different from managing a multi-currency portfolio, where the correlation between interest rates of different currencies varies.

9.5 Appendix

EWMA correlation of 5 day changes in inflation and nominal rates

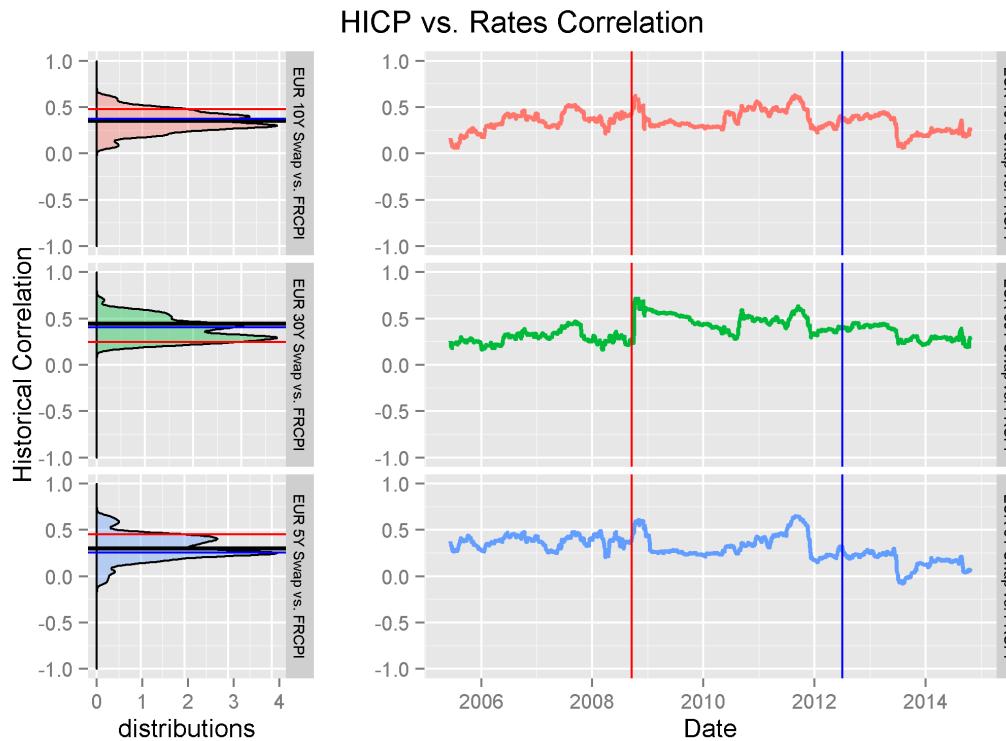
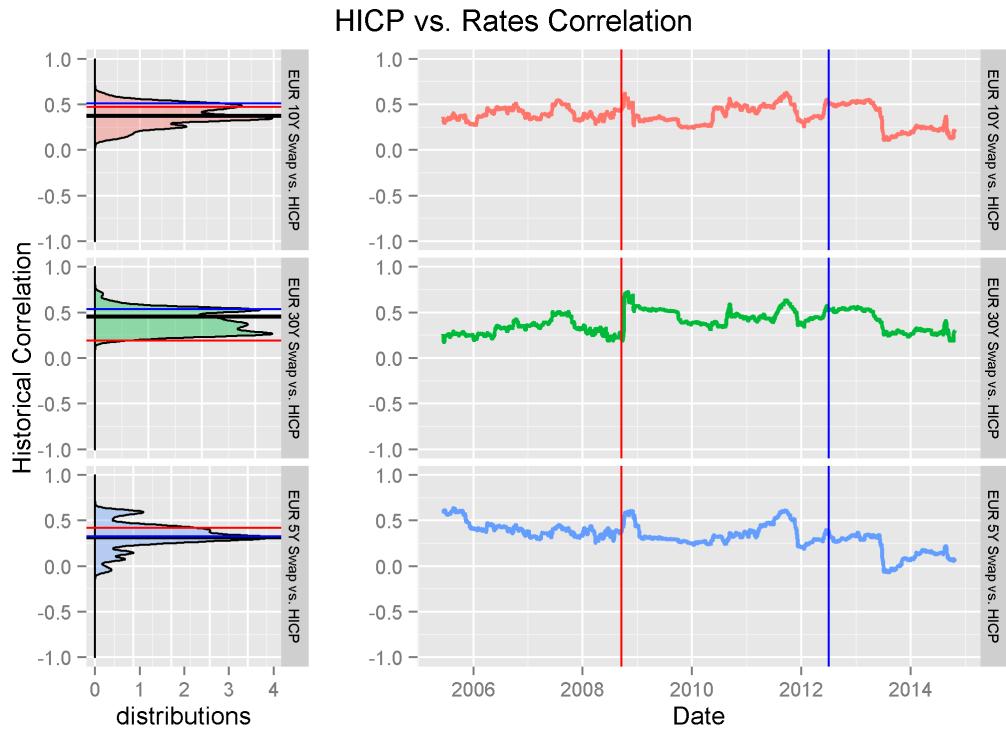
9.5.1 UKRPI – Interest Rates

The exogenous shocks to UKRPI aside from the Lehman default following September 2008 (red line) include the debate in January 2013 to change the way UKRPI was calculated. In each case, the correlation metric reverts back to its long term average.



9.5.2 HICPx and FRCPIx – interest rates

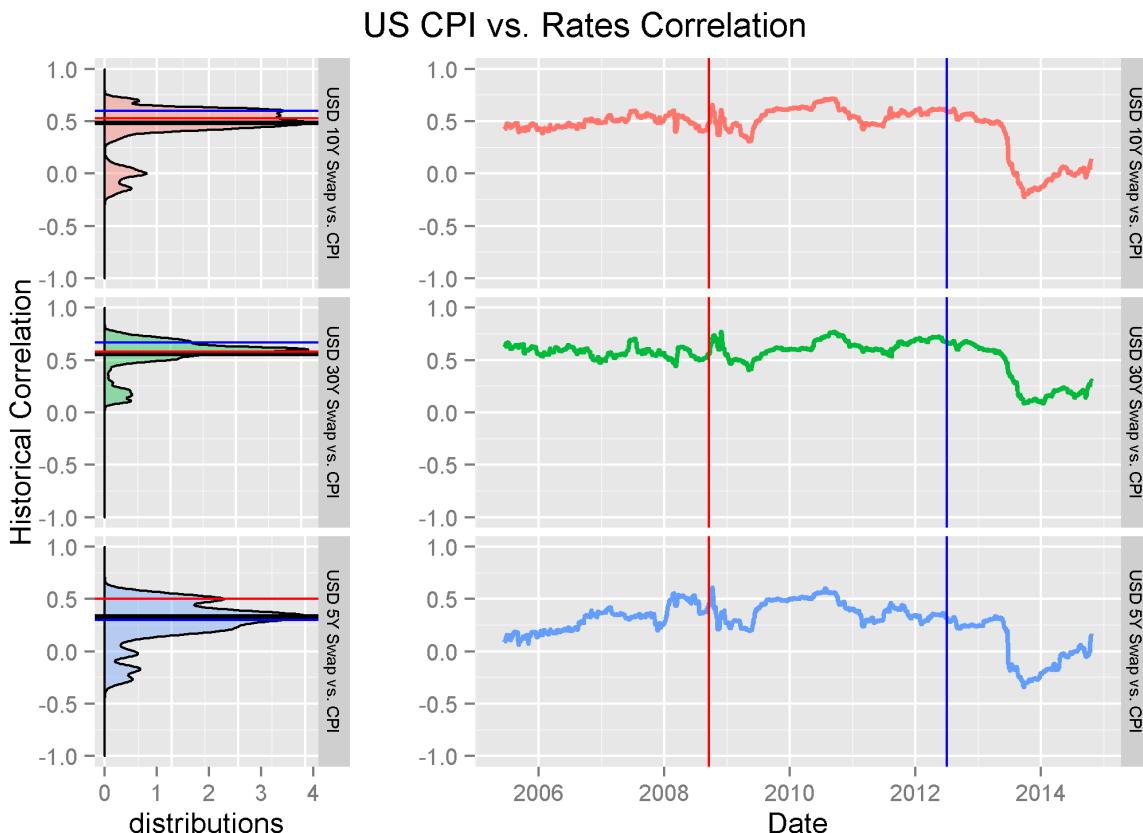
Eurozone HICPx and French CPIx correlation with interest rates demonstrate very strong mean reversion, even after periods of extreme stress such as the Lehman default in September 2008 (red line) and the European debt crisis in 2011 (blue line).



9.5.3 USCPI – Interest Rates

US CPI correlation with interest rates demonstrate very strong mean reversion, even after periods of extreme stress such as the Lehman default in September 2008 (red line) and the European debt crisis in 2011 (blue line).

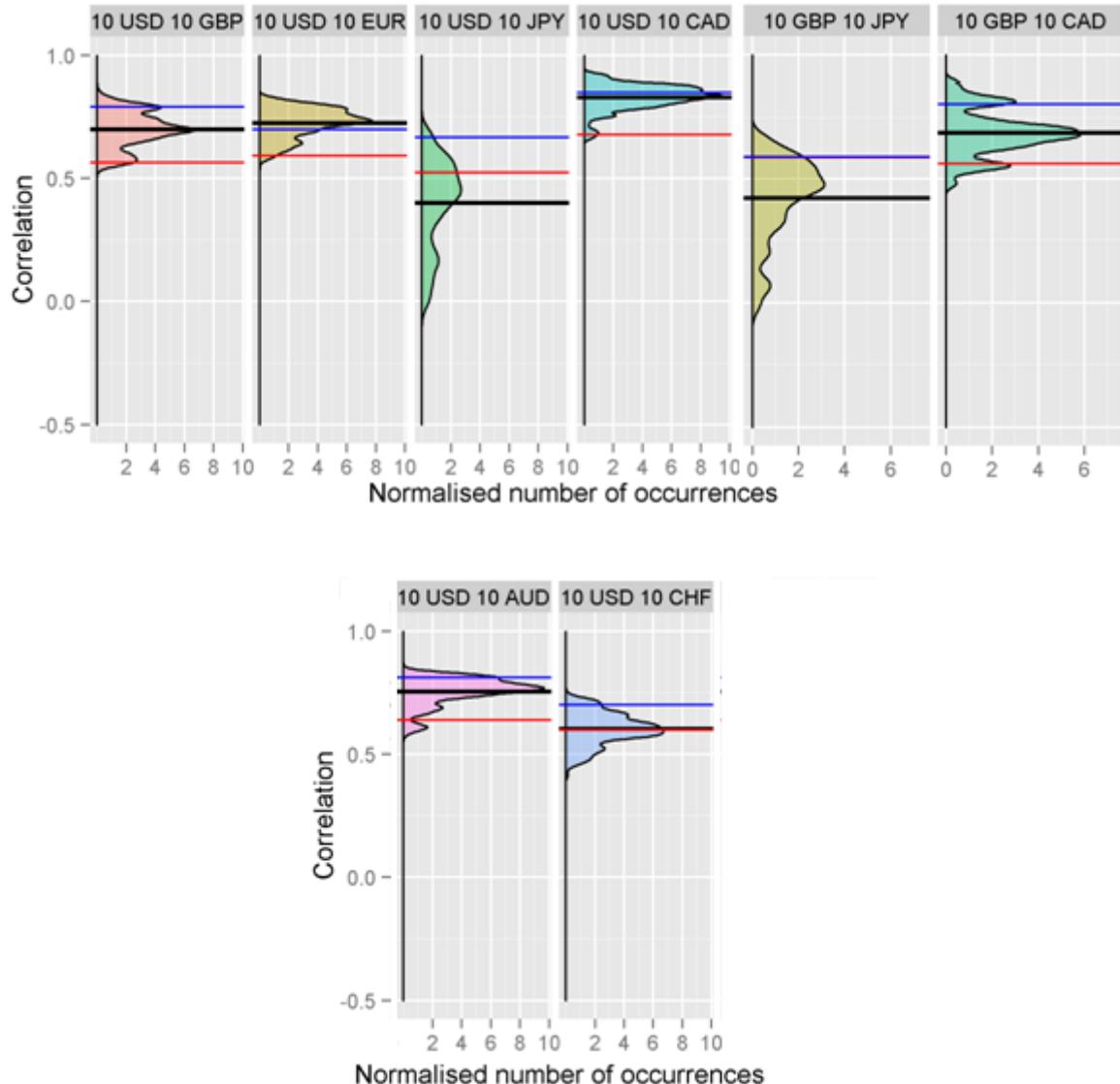
Correlation experienced a sharp decrease in 2013 mainly attributed tapering talks by the Fed, where QE in the US includes TIPS. It is unlikely that the drop in correlation is structural, and there are signs of the correlation returning to its structural range.



9.5.4 Inter-currency Correlation

The 3 currency swap rates (EUR / USD / GBP) 10-year rate currency pair correlation distributions including the average correlation levels (black) and two distinctive stressed periods over the last 12 years: Lehman (red) and Euro Crisis (blue) across 10y maturities demonstrate similar correlation behaviour to inflation and rates, where an exogenous shock causes de-correlation. The correlation demonstrates strong mean reversion attributes, as seen below.

For further information regarding inter-currency and SwapClear product correlations refer to "SwapClear Portfolio Margin Justification v 1.3".



Major currencies excluding G7 vs. G7 currencies

10 IM Backtesting Methodologies

See separate paper from SwapClear Risk Change.

11 Backtesting Results and Proposed IM Model

See separate paper from SwapClear Risk Change.

12 First Principles Margin Add-on

12.1 Purpose

IM measures are based on closing out portfolios within a market's trading capacity. However, should a position be very large and beyond the normal market capacity, additional measures are required to deal with the resulting liquidity risk. IMM First Principle Liquidity Multipliers are additional margin that is called due to concentrated currency exposure. It mitigates the risk that a position may not be able to be closed in 5 days and/or extra hedging costs may be incurred. Where a member has an exposure above set thresholds in a particular currency and maturity, an add-on is computed to reflect the expected cost of mitigating the oversize position, a multiplier is applied to the balance and an additional call is made. These thresholds are reviewed on a regular and take into account prevailing perceptions of market conditions from the SwapClear dealer community.

In addition, IMM First Principles will incorporate a bid offer spread charge, regardless of the size of a position. This is an undiversified add-on above IM and is a direct charge on inflation risk in the CCP. In its original guise, this was calibrated to a market survey to establish the standard market bid-offer for each index at certain key tenors. On recommendation from ERCO, the add-on has been extended to ensure that the total margin on a trade successfully backtests to the LCH.C risk appetite (99.7%) including over the Lehman crisis period.

12.2 Inflation Buckets

For the purposes to inflation liquidity add-ons, risk is measured to a set of buckets agreed with the Design Authority as being the key points at which risk would be mitigated in event of a default. The set of points are set as 2y, 5y, 10y, 20y, 30y (and 50y in GBP only).

In IRS some offset is allowed between the 2y and 5y buckets and the 10y and 30y buckets. Where positions in these areas represent spreads, the IMMFP addon is computed as the maximum of the two. For the inflation product, no bucket offset is given, so large spread positions would be impacted by the addon on both sides. No offset is given between indices or against IRS positions.

12.3 Market Survey

A market survey was performed to establish the depth of the market, and the impact on the market of trades larger than this. Survey respondents were Deutsche, Commerzbank, Lloyds, JP Morgan, RBS, Nomura, BAML, Barclays, Societe Generale, Citibank, Goldman Sachs and Morgan Stanley. Data was averaged from all members and this is reported below. The aggregated data was presented back to the Design Authority and also to members in bilateral meetings to ensure its validity.

12.4 Market Size

Members were asked how much inflation risk (IE01) could be done in a two day period within the prevailing bid offer spread. The figures are in currency units. Survey means are presented below:

		EUR	FRF	USD	GBP
IO1 2d	2y	121,818	91,364	115,000	79,583
	5y	228,636	172,955	195,714	159,583
	10y	386,364	280,455	342,857	285,417
	20y	313,636	183,636	275,000	419,167
	30y	297,273	169,545	295,714	569,583
	50y				487,083

For conservatism, all figures were then rounded down to the 50,000 below, and the following set represents the measure of market size used at commencement of the service:

		EUR	FRF	USD	GBP
IO1 2d	2y	100,000	50,000	100,000	50,000
	5y	200,000	150,000	150,000	150,000
	10y	350,000	250,000	300,000	250,000
	20y	300,000	150,000	250,000	400,000
	30y	250,000	150,000	250,000	550,000
	50y				450,000

12.5 Bid-Offer Add-on

The survey also was used to establish the bid offer spread of a normal market size, and this is used to compute an adjustment for closing out a position of any size. The bid offer add-on has been extended to ensure that all tenors backtest to at least 99.7% confidence including over the Lehman period. As such, the bid-offer add-on can now be considered as non-diversifiable stress add-on specific to inflation risk.

The size of the addon is the delta of the bucket multiplied by half the bid offer spread (i.e. bid-mid or mid-offer). Full Bid offer shown below.

		EUR	FRF	USD	GBP
B/O	2y	15.00	15.00	75.00	45.00
	5y	20.00	20.00	50.00	45.00
	10y	20.00	20.00	50.00	25.00
	20y	20.00	20.00	50.00	15.00
	30y	20.00	20.00	50.00	15.00
	50y				15.00

12.6 Cost for Larger Positions

Members were asked how much the market would be moved by trading a position in inflation risk (IE01) that was twice, five times and ten times the position sizes in the table above. Figures are in basis points on the Par rate. The left column shows the average of the market survey. All values were then rounded up to the next whole basis point to give the IMMFP charge for this size of position.

EUR Raw	2x	5x	10x
2y	2.99	6.66	12.23
5y	2.58	5.66	10.59
10y	2.39	5.36	9.91
20y	2.60	5.77	10.55
30y	2.66	5.80	10.50
50y			

EUR	2x	5x	10x
2y	3.00	7.00	13.00
5y	3.00	6.00	11.00
10y	3.00	6.00	10.00
20y	3.00	6.00	11.00
30y	3.00	6.00	11.00
50y			

FRF Raw	2x	5x	10x
2y	3.71	7.80	12.95
5y	2.98	6.16	10.59
10y	2.77	5.76	9.91
20y	3.18	6.73	11.39
30y	3.31	6.86	11.34
50y			

FRF	2x	5x	10x
2y	4.00	8.00	13.00
5y	3.00	7.00	11.00
10y	3.00	6.00	10.00
20y	4.00	7.00	12.00
30y	4.00	7.00	12.00
50y			

USD Raw	2x	5x	10x
2y	4.68	10.21	17.43
5y	3.39	7.57	15.43
10y	2.96	6.79	15.14
20y	3.18	7.07	15.00
30y	3.18	6.93	14.86
50y			

USD	2x	5x	10x
2y	5.00	11.00	18.00
5y	4.00	8.00	16.00
10y	3.00	7.00	16.00
20y	4.00	8.00	16.00
30y	4.00	7.00	15.00
50y			

GBP Raw	2x	5x	10x
2y	4.13	9.88	14.67
5y	3.46	7.86	11.82
10y	2.70	6.70	10.23
20y	2.50	5.95	8.83
30y	2.26	5.46	8.13
50y	2.33	5.61	8.28

GBP	2x	5x	10x
2y	5.00	10.00	15.00
5y	4.00	8.00	12.00
10y	3.00	7.00	11.00
20y	3.00	6.00	9.00
30y	3.00	6.00	9.00
50y	3.00	6.00	9.00

12.7 Extrapolation

As the market survey only covers as far out as 10 times market size, it is necessary to define how larger positions would impact the market and this requires an extrapolation scheme. 3 approaches were considered. S_x here is used to mean the additional spread for a position of x times market size.

12.7.1 Flat Extrapolation

The simplest scheme is to use flat extrapolation where the assumed basis point cost (bpc) for trades larger than 10 times market size is set to the same level as the bpc for 10 times.

$$S_x = S_{10} \text{ where } x \geq 10$$

When this is multiplied by the bucket deltas, the addon is seen to be a linear function beyond 10 times. This is currently implemented in the existing rate products within SwapClear.

As the market is less liquid than the IRS market, it was deemed not to be conservative enough to cover oversize positions.

12.7.2 Linear Extrapolation

A simple alternative is to extrapolate the bpc from 5 and 10 times in a linear fashion

$$S_x = a.x + b \text{ where } x \geq 5$$

$$S_x = \frac{x - 5}{5}(S_{10} - S_5) + S_5 \text{ where } x \geq 5$$

When this is multiplied by the bucket deltas, the addon charge is seen to be a parabolic function beyond 10 times.

$$\text{Charge} = x.S_x = a.x^2 + b.x \text{ where } x \geq 5$$

$$\text{Charge} = x.S_x = x \left(\frac{x - 5}{5}(S_{10} - S_5) + S_5 \right) \text{ where } x \geq 5$$

$$\text{Charge} = x.S_x = \frac{x^2}{5}(S_{10} - S_5) + x(2.S_5 - S_{10})$$

12.7.3 Parametric Function Approach

A number of parametric functions were considered that would allow more control on extrapolation and how fast the addons should grow. One particular set of functions were selected it contains both the above alternatives a specific limiting cases, which would allow existing behaviour to be replicated if desired.

The specific functional form is

$$S_x = ax^n + b$$

This function is linear in the case $n = 1$ and flat when $n = 0$. For values of $n > 1$, the bpc will rise faster than linear.

For any given $n (>= 1)$, which is an exogenous parameter selected to reflect risk views, the function can be calibrated to two limiting conditions to find the values of a and b , where the conditions for the function are bpc is both continuous and first order smooth at the knot point:

Condition 1: Continuous: The value of the function for a 10 times position should match that given from the survey.

$$S_{10} = a10^n + b$$

Condition 2 : Smooth : The slope of the function at the 10 times position should match the linear slope between the survey points for 5 and 10 times.

$$\frac{dS_x}{dx} = anx^{n-1} = \frac{S_{10} - S_5}{5}$$

This system of equations can solved to give the following results:

$$a = \frac{S_{10} - S_5}{5} \cdot \frac{1}{n10^{n-1}}$$

$$b = S_{10} - a10^n$$

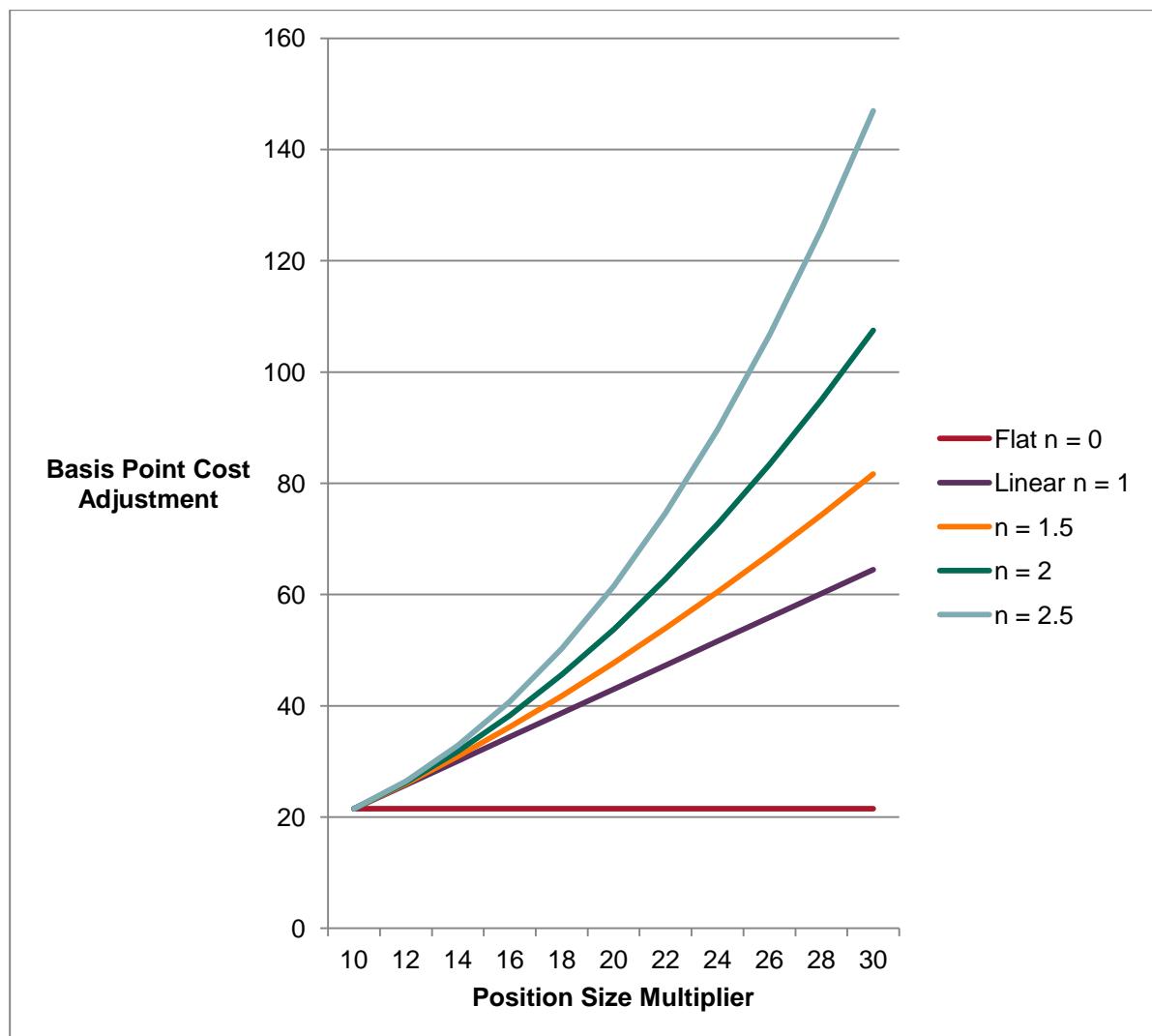
In the case of $n = 0$, the above functions do not hold, but simply collapse to

$$S_x = S_{10} \text{ where } x > 10$$

Using the following example data:

EUR	2X	5X	10X
2y	4.08	10.75	21.50

The following extrapolation curves are constructed for various given values of n , where the chart shows the basis point cost for different position sizes ranging from 10 to 30 times market size.



This is the proposed functional form to perform the extrapolation for inflation products in production.

12.8 Calibration of N Parameter

For each of the indices and tenors, an optimisation routine was performed to find the best fit level of N. The optimisation looked to minimise the sum of square errors between the projected addon level for a given N with the survey value. The bid-offer addon above was added in all cases, including for a 1x market risk level. The optimisation was constrained firstly with a positivity constraint on N and then secondly with a minimum value of N set to 1. Input values and results for N in these two cases are shown below:

		1	2	5	10	N>0	N>=1
EUR	2y	7.50	10.50	14.50	20.50	0.8184	1.0000
	5y	10.00	13.00	16.00	21.00	0.8232	1.0000
	10y	10.00	13.00	16.00	20.00	0.6724	1.0000
	20y	10.00	13.00	16.00	21.00	0.8232	1.0000
	30y	10.00	13.00	16.00	21.00	0.8232	1.0000
FRF	2y	7.50	11.50	15.50	20.50	0.6274	1.0000
	5y	10.00	13.00	17.00	21.00	0.5406	1.0000
	10y	10.00	13.00	16.00	20.00	0.6724	1.0000
	20y	10.00	14.00	17.00	22.00	0.7444	1.0000
	30y	10.00	14.00	17.00	22.00	0.7444	1.0000
USD	2y	37.50	42.50	48.50	55.50	0.6249	1.0000
	5y	25.00	29.00	33.00	41.00	0.9395	1.0000
	10y	25.00	28.00	32.00	41.00	1.0665	1.0665
	20y	25.00	29.00	33.00	41.00	0.9395	1.0000
	30y	25.00	29.00	32.00	40.00	1.0426	1.0426
GBP	2y	22.50	27.50	32.50	37.50	0.4683	1.0000
	5y	22.50	26.50	30.50	34.50	0.4683	1.0000
	10y	12.50	15.50	19.50	23.50	0.5406	1.0000
	20y	7.50	10.50	13.50	16.50	0.4683	1.0000
	30y	7.50	10.50	13.50	16.50	0.4683	1.0000
	50y	7.50	10.50	13.50	16.50	0.4683	1.0000

In all but two cases, the optimal value of N with limited constraint is below unity, and the two cases that exceed this value are only marginally over the level.

Internal risk opinion is that the N parameter should be set at a somewhat higher level to ensure that the service is margined conservatively and that large risk positions are discouraged. For inception of the service, the value of n has been set at 1.5 which is seen as highly conservative across all tenors and all indices.

12.9 Bucket Projections for IMMFP

Inflation deltas are available for all buckets on the IM curve. For the purposes of IMM FP these need to be re-bucketed to specific key tenor points (2y, 5y, 10y, 20y, 30y and 50y).

IM Buckets that match the IMMFP pillars are allocated on a 1:1 basis.

For all points beyond 2Y that are not core pillar points, the risk will be bucketed into the two surrounding pillars (Z_m and Z_n where $m < n$). Subscript i is used to denote the date that needs re-bucketing. In this context, i , m and n can all be considered integers representing the year length of the point. For example, rebucketing of 7y risk to 5y and 10y pillars, $i = 7$, $m = 5$ and $n = 10$.

Chain rule is the basis for the re-allocation:

$$1 \quad \frac{\partial PV}{\partial Z_m} = \frac{\partial PV}{\partial Z_i} \cdot \frac{\partial Z_i}{\partial Z_m}$$

$$2 \quad \frac{\partial PV}{\partial Z_n} = \frac{\partial PV}{\partial Z_i} \cdot \frac{\partial Z_i}{\partial Z_n}$$

$\frac{\partial PV}{\partial Z_i}$ = the delta to bucket i , either from the backbook or directly input in Smart.

The curve construction interpolation chosen is log linear in CPI levels.

$$3 \quad \ln CPI_i = \ln CPI_m + \frac{i-m}{n-m} \cdot (\ln CPI_n - \ln CPI_m)$$

This translates directly into par rate space in the case of the piecewise indices, and can be applied in the linear index cases with minimal error.

$$4 \quad CPI_a = (1 + Z_a)^a \cdot CPI_0$$

Direct substitution of 4 into 3 (CPI0 all cancel out)

$$5 \quad i \ln(1 + Z_i) = m \ln(1 + Z_m) + \frac{i-m}{n-m} \cdot (n \ln(1 + Z_n) - m \ln(1 + Z_m))$$

Combining like terms gives

$$6 \quad \ln(1 + Z_i) = \frac{m(n-i)}{i(n-m)} \ln(1 + Z_m) + \frac{n(i-m)}{i(n-m)} \ln(1 + Z_n)$$

Exponentiate

$$7 \quad Z_i = (1 + Z_m)^{\frac{m(n-i)}{i(n-m)}} \cdot (1 + Z_n)^{\frac{n(i-m)}{i(n-m)}} - 1$$

Differentiate to obtain the two required formulae for 1 and 2

$$8 \quad \frac{\partial Z_i}{\partial Z_m} = \frac{m(n-i)}{i(n-m)} \cdot (1 + Z_m)^{\frac{m(n-i)}{i(n-m)}} \cdot (1 + Z_n)^{\frac{n(i-m)}{i(n-m)}}$$

$$9 \quad \frac{\partial Z_i}{\partial Z_n} = \frac{n(i-m)}{i(n-m)} \cdot (1 + Z_n)^{\frac{m(i-n)}{i(n-m)}} \cdot (1 + Z_m)^{\frac{m(n-i)}{i(n-m)}}$$

These both simplify a little

$$10 \quad \frac{\partial Z_i}{\partial Z_m} = \frac{m(n-i)}{i(n-m)} \cdot \left(\frac{1+Z_n}{1+Z_m} \right)^{\frac{n(i-m)}{i(n-m)}}$$

$$11 \quad \frac{\partial Z_i}{\partial Z_n} = \frac{n(i-m)}{i(n-m)} \cdot \left(\frac{1+Z_n}{1+Z_m} \right)^{\frac{m(i-n)}{i(n-m)}}$$

Bucketing from 1y to 2Y pillar follows a similar pattern but is complicated by the fact that there is no 0y rate, and the 1y point is interpolated from the last known level, which may not be CPI0.

Following is reasonable projection.

Define CPI0 as the CPI0 fixing for a piecewise trade.

Define CPIL as the **Last know CPI level**

Define L as the lag in **months** from CPI0 to CPIL (this may not be the trade lag).

$$\frac{\partial Z_{1y}}{\partial Z_{2y}} = \left(\frac{24 - 2L}{24 - L} \right) \cdot (1 + Z_{2y})^{\left(\frac{-L}{24-L} \right)} \cdot \left(\frac{CPI_L}{CPI_0} \right)^{\left(\frac{12}{24-L} \right)}$$

For linear indices (US and French) the true formula is extremely complicated due to the nature of the interpolated fixing rate. However, comparison of the above formula with the correct methodology applied numerically has show that the above is a very accurate approximation and is suitable for use in productions.

More generally, where it is required to move a delta to the initial IMMFP pillar, the following is required.
Note that here ALL times are in months (2y = 24)

$$\frac{\partial Z_i}{\partial Z_n} = \left(\frac{i - L}{n - L} \right) \frac{n}{12} \cdot (1 + Z_n)^{\left(\frac{n(i-12) - L(n-12)}{12(n-L)} \right)} \cdot \left(\frac{CPI_L}{CPI_0} \right)^{\left(\frac{n-i}{n-L} \right)}$$

12.10 IMMFP Test Cases

In order to validate that the interpolation\extrapolation scheme was implemented correctly in Murex we defined a set of trades consisting of trades on different indices, maturities and rates. Given the bucket sensitivities these trades generate, we were modifying the maximum bucket IO1's that the market can absorb so that we test the multiplier extrapolation interpolation scheme in different regimes i.e. when an interpolation is needed and when extrapolation is needed.

Note that these calculations result in some extremely large add-ons. These do not represent true levels, but are highly exaggerated to ensure that the extrapolation works in even the most extreme theoretical case.

12.10.1 Case 1

In what follows we present in sequence the portfolio profile in terms of bucketed sensitivities for different indices and the liquidity cost comparison between the excel \ VBA tool and Murex for every tenor.

We start by presenting the results for EUR HICPxT. It is obvious that the liquidity extrapolation is performed appropriately as well as that the calculation of the liquidity cost is done correctly. Summation of the liquidity cost across tenor and conversion to the local currency is correct.

Case1: Test Inputs

EUR	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	9,500	- 10,431.00	- 4.00	1.10
5Y	13,400	- 179,774.00	- 64.00	13.42
10Y	19,500	- 217,461.00	- 283.00	11.15
20Y	19,200	- 325,891.00	- 425.00	16.97
30Y	17,000	-	-	-
50Y				

Case1: Test Results

EUR	Theoretical		Murex		Multiplier Diff	Dif Liq Cost	Status
	Multiplier	Liq Cost	Multiplier	Liq Cost			
2Y	0.40	4,174.46	0.40	4,174.46	-	0.00%	PASS
5Y	23.11	4,171,898.46	23.11	4,171,898.40	-	0.00%	PASS
10Y	15.72	3,452,874.11	15.72	3,452,874.11	-	0.00%	PASS
20Y	28.42	9,433,803.92	28.42	9,433,803.92	-	0.00%	PASS
30Y	-	-	-	-	-	-	PASS
50Y							
Total(EUR)		17,062,750.95		17,062,750.89		0.00%	PASS
Total(GBP)		14,483,144.22		14,483,144.17		0.00%	PASS

We continue presenting the results for UK RPI. Again, liquidity extrapolation is performed appropriately and the calculation of the liquidity cost is done correctly. Summation of the liquidity const across tenor and conversion to the local currency is correct.

Case1: Test Inputs

GBP	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	6,500	- 66,089.00	- 4.00	10.17
5Y	11,000	7.00	-	0.00
10Y	18,000	1,342.00	1.00	0.07
20Y	26,500	2.00	-	0.00
30Y	28,500	-	-	-
50Y	23,000	-	-	-

Case1: Test Results

GBP	Theoretical		Murex		Multiplier Diff	Dif Liq Cost	Status
	Multiplier	Liq Cost	Multiplier	Liq Cost			
2Y	24.53	1,622,444.90	24.53	1,622,444.93	-	0.00%	PASS
5Y	-	-	-	-	-	-	PASS
10Y	-	-	-	-	-	-	PASS
20Y	-	-	-	-	-	-	PASS
30Y	-	-	-	-	-	-	PASS
50Y	-	-	-	-	-	-	PASS
Total		1,622,444.90		1,622,444.93		0.00%	PASS
Total(GBP)		1,622,444.90		1,622,444.93		0.00%	PASS

Next we present the results for FR CPI. Liquidity extrapolation is performed appropriately and the calculation of the liquidity cost is done correctly. Summation of the liquidity cost across tenor and conversion to the local currency is correct.

Case1: Test Inputs

EUR /FR	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	6,500	- 91,736.00	- 2.00	14.11
	9,000	- 161,552.00	- 53.00	17.95
	11,000	1,614.00	4.00	0.15
	19,200	- 746,140.00	- 1,726.00	38.86
	17,000	- 3,101,979.00	- 8,306.00	182.47

Case1: Test Results

EUR/ FR	Theoretical		Murex		Multiplier Diff	Dif Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	34.51	3,166,926.78	34.51	3,166,926.73	- 0.00	0.00%	PASS
5Y	36.34	5,905,499.45	36.34	5,905,499.61	- 0.00	0.00%	PASS
10Y	-	-	-	-	-	-	PASS
20Y	103.04	86,042,920.87	103.04	86,042,921.15	- 0.00	0.00%	PASS
30Y	1,165.95	9,262,502,986.89	1,165.95	9,262,502,112.96	- 0.00	0.00%	PASS
50Y					-	-	
Total		9,357,618,334.00		9,357,617,460.45		0.00%	PASS
Total(GBP)		7,942,900,670.41		7,942,899,928.93		0.00%	PASS

Finally we present the results for US CPI. Liquidity extrapolation is performed appropriately and the calculation of the liquidity cost is done correctly. Summation of the liquidity const across tenor and conversion to the local currency is correct.

Case1: Test Inputs

USD	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	7,750	- 72,804.00	4.00	9.39
	16,600	1,716.00	2.00	0.10
	23,600	- 392,334.00	- 348.00	16.62
	24,500	- 3,040.00	- 3.00	0.12
	22,500	1.00	-	0.00

Case1: Test Results

USD	Theoretical		Murex		Multiplier Diff	Dif Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	19.82	1,442,078.14	19.82	1,442,078.14	-	0.00%	PASS
5Y	-	-	-	-	-	-	PASS
10Y	22.24	8,812,682.87	22.24	8,812,682.87	-	0.00%	PASS
20Y	-	-	-	-	-	-	PASS
30Y	-	-	-	-	-	-	PASS
50Y					-	-	
Total		10,254,761.01		10,254,761.01		0.00%	PASS
Total(GBP)		6,413,033.37		6,413,033.37		0.00%	PASS

Our portfolio also has OIS risk generated from the Inflation swaps. In this case the extrapolation method used is flat above 10 times the threshold. Below we present the results:

Case1: Test Inputs

GBP	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	96,191	2,615.00	-	137.09
	42,438	- 40.00	-	1.90
	27,645	740.00	-	29.07
	15,299	-	-	-

Case1: Test Results

GBP	Theoretical		Murex		Multiplier Diff	Dif Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	6.97	18,226.55	6.97	18,226.55	-	0.00%	PASS
5Y	1.32	52.83	1.32	52.83	- 0.00	0.00%	PASS
10Y	6.33	4,684.20	6.33	4,684.20	-	0.00%	PASS
30Y	-	-	-	-	-	-	PASS
Total		22,963.58		22,963.58		0.00%	PASS
Total(GBP)		22,963.58		22,963.58		0.00%	PASS

Case1: Test Inputs

USD	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	279,464	1,444.00	-	25.54
	117,321	927.00	-	15.78
	75,481	- 15,795.00	1.00	226.85
	41,500	- 102.00	-	1.48

Case1: Test Results

USD	Multiplier	Liq Cost	MX Mult	MX Liq Cost	Multiplier Diff	Dif Liq Cost	Status
2Y	4.43	6,396.92	4.43	6,396.92	-	0.00%	PASS
	5.71	5,293.17	5.71	5,293.17	-	0.00%	PASS
	6.27	99,014.99	6.27	99,014.99	-	0.00%	PASS
	1.02	104.21	1.02	104.21	-	0.00%	PASS
Total		110,809.29		110,809.29		0.00%	PASS
Total(GBP)		69,296.95		69,296.95		0.00%	PASS

Case1: Test Inputs

EUR	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	219,630	2,597.00	- 1.00	57.50
	112,626	7,021.00	- 5.00	115.43
	72,867	20,387.00	- 26.00	254.52
	64,610	- 896,226.00	- 723.00	6,427.07

Case1: Test Results

EUR	Multiplier	Liq Cost	MX Mult	MX Liq Cost	Multiplier Diff	Dif Liq Cost	Status
2Y	5.80	15,045.78	5.80	15,045.78	-	0.00%	PASS
	5.88	41,197.04	5.88	41,197.04	-	0.00%	PASS
	6.07	123,270.11	6.07	123,270.11	-	0.00%	PASS
	6.00	5,390,370.00	6.00	5,390,370.00	-	0.00%	PASS
Total		5,569,882.93		5,569,882.93		0.00%	PASS
Total(GBP)		4,727,808.43		4,727,808.43		0.00%	PASS

For all the above we see that the extrapolation of the liquidity multiplier is done correctly the calculation of the liquidity cost is performed correctly and the conversion to local currency is successful.

In the table below we validate that the aggregation of the liquidity cost across the different indices, inflation and OIS, is correct.

CCY	Liq Cost	MX Liq Cost	Diff	Status
EUR	7,962,111,623.06	7,962,110,881.53	0.00%	PASS
GBP	1,645,408.49	1,645,408.51	0.00%	PASS
USD	6,482,330.32	6,482,330.32	0.00%	PASS
Total	7,970,239,361.87	7,970,238,620.36	0.00%	PASS

12.10.2 Case 2

Again in this case we do not do any validation of the actual sensitivities that are generated by Murex. Our purpose is to validate the interpolation \ extrapolation of the liquidity multipliers for the different currencies and tenors as well as the aggregation on the index, currency and account level across inflation and OIS. The analysis follows the same structure as in case 1.

Extrapolation is performed appropriately as well as that the calculation of the liquidity cost is done correctly. Summation of the liquidity const across tenor and conversion to the local currency is correctly.

Case2: Test Inputs

EUR	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	9,500	15,642.00	4.00	1.65
	13,400	376,212.00	148.00	28.08
	19,500	- 186,150.00	- 270.00	9.55
	19,200	1,050,394.00	2,608.00	54.71
	17,000	1,168,946.00	2,612.00	68.76

Case2: Test Results

EUR	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	2.64	41,308.54	2.64	41,308.54	-	0.00%	PASS
5Y	58.82	22,382,890.71	58.82	22,382,890.58	-	0.00%	PASS
10Y	13.34	2,507,629.91	13.34	2,507,629.91	-	0.00%	PASS
20Y	141.32	174,491,000.54	141.32	174,491,000.54	-	0.00%	PASS
30Y	197.16	281,241,912.00	197.16	281,241,912.00	-	0.00%	PASS
50Y							
Total		480,664,741.70		480,664,741.58		0.00%	PASS
Total(GBP)		407,996,154.88		407,996,154.77		0.00%	PASS

In the case of UK RPI index, as we can see from the table below, extrapolation is done correctly and liquidity cost calculation is done correctly.

Case2: Test Inputs

GBP	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	6,500	75,416.00	10.00	11.60
	11,000	26,832.00	4.00	2.44
	18,000	153,668.00	230.00	8.54
	26,500	1,095,880.00	2,028.00	41.35
	28,500	1,620,026.00	4,832.00	56.84
	23,000	3,171,132.00	14,004.00	137.88

Case2: Test Results

GBP	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	27.77	2,098,480.06	27.77	2,098,480.10	-	0.00%	PASS
5Y	4.99	133,841.67	4.99	133,841.66	-	0.00%	PASS
10Y	14.89	2,313,799.01	14.89	2,313,799.00	-	0.00%	PASS
20Y	96.68	115,430,640.70	96.68	115,430,644.95	-	0.00%	PASS
30Y	138.63	271,013,562.43	138.63	271,013,575.19	-	0.00%	PASS
50Y	617.94	4,633,318,358.51	617.94	4,633,318,358.51		0.00%	PASS
Total		5,024,308,682.37		5,024,308,699.40		0.00%	PASS
Total(GBP)		5,024,308,682.37		5,024,308,699.40		0.00%	PASS

The same applies for FR CPI index.

Case2: Test Inputs

EUR/ FR	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	6,500	2,496,886.00	324.00	384.14
	9,000	459,422.00	28.00	51.05
	11,000	192.00	-	0.02
	19,200	-	88,582.00	4.61
	17,000	1,241,872.00	3,344.00	73.05

Case2: Test Results

EUR/ FR	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	3,764.47	11,695,204,285.43	3,764.47	11,695,203,896.51	-	0.00%	PASS
5Y	147.91	68,258,050.02	147.91	68,258,052.32	-	0.00%	PASS
10Y	-	-	-		-		PASS
20Y	8.50	754,650.27	8.50	754,650.29	-	0.00%	PASS
30Y	299.42	521,729,300.93	299.42	521,729,263.03	-	0.00%	PASS
50Y							
Total		12,285,946,286.66		12,285,945,862.15		0.00%	PASS
Total(GBP)		10,428,513,700.15		10,428,513,339.82		0.00%	PASS

The calculations on the US indices is matching the excel \ VBA tool.

Case2: Test Inputs

USD	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	7,750	417,228.00	24.00	53.84
	16,600	196,292.00	114.00	11.82
	23,600	1,480,208.00	1,312.00	62.72
	24,500	689,446.00	1,218.00	28.14
	22,500	522,000.00	1,238.00	23.20

Case2: Test Results

USD	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	195.87	82,181,879.59	195.87	82,181,879.59	-	0.00%	PASS
5Y	17.12	3,376,336.90	17.12	3,376,336.90	-	0.00%	PASS
10Y	127.14	198,798,483.37	127.14	198,798,483.37	-	0.00%	PASS
20Y	42.17	30,159,136.24	42.17	30,159,136.24	-	0.00%	PASS
30Y	32.99	17,896,765.37	32.99	17,896,765.37	-	0.00%	PASS
50Y							
Total		332,412,601.46		332,412,601.46		0.00%	PASS
Total(GBP)		207,881,305.44		207,881,305.44		0.00%	PASS

Our portfolio also has OIS risk generated from the Inflation swaps. In this case the extrapolation method use is flat above 10 times the threshold. Below we present the results:

Case2: Test Inputs

GBP	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	96,191	8,093.00	-	454.60
	42,438	35,139.00	-	1,836.60
	27,645	7,470.00	-	1,735.77
	15,299	- 1,223,584.00	-	40,908.80

Case2: Test Results

GBP	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	6.97	56,383.92	6.97	56,383.92	-	0.00%	PASS
5Y	5.46	191,873.85	5.46	191,873.85	-	0.00%	PASS
10Y	6.33	47,806.00	6.33	47,806.00	-	0.00%	PASS
30Y	8.14	9,983,529.05	8.14	9,983,529.05	-	0.00%	PASS
Total		10,279,592.81		10,279,592.81		0.00%	PASS
Total(GBP)		10,279,592.81		10,279,592.81		0.00%	PASS

Case2: Test Inputs

USD	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	279,464	- 16.00	-	1.08
	117,321	821.00	- 7.00	26.98
	75,481	- 107,011.00	- 12.00	1,437.78
	41,500	- 237,854.00	- 28.00	3,020.05

Case2: Test Results

USD	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	0.08	1.36	0.08	1.36	-	0.00%	PASS
5Y	5.71	4,573.80	5.71	4,573.80	-	0.00%	PASS
10Y	6.27	670,723.09	6.27	670,723.09	-	0.00%	PASS
30Y	7.20	1,713,274.56	7.20	1,713,274.56	-	0.00%	PASS
Total		2,388,572.81		2,388,572.81		0.00%	PASS
Total(GBP)		1,493,744.92		1,493,744.92		0.00%	PASS

Case2: Test Inputs

EUR	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	219,630	- 25,246.00	2.00	567.58
5Y	112,626	6,609.00	4.00	118.03
10Y	72,867	- 109,624.00	148.00	1,446.75
30Y	64,610	- 609,603.00	699.00	4,436.15

Case2: Test Results

EUR	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	5.80	146,393.16	5.80	146,393.16	-	0.00%	PASS
5Y	5.88	38,791.77	5.88	38,791.77	-	0.00%	PASS
10Y	6.07	668,144.20	6.07	668,144.20	-	0.00%	PASS
30Y	6.00	3,670,200.00	6.00	3,670,200.00	-	0.00%	PASS
Total		4,523,529.13		4,523,529.13		0.00%	PASS
Total(GBP)		3,839,646.08		3,839,646.08		0.00%	PASS

In the table below we can see the overall comparison between the excel \ VBA tool and Murex results. In this case the results are identical.

CCY	Liq Cost	MX Liq Cost	Diff	Status
EUR	10,840,349,501.11	10,840,349,140.68	0.00%	PASS
GBP	5,034,588,275.18	5,034,588,292.21	0.00%	PASS
USD	209,375,050.36	209,375,050.36	0.00%	PASS
Total	16,084,312,826.65	16,084,312,483.25	0.00%	PASS

12.10.3 Case 3

Again in this case we do not do any validation of the actual sensitivities that are generated by Murex. Our purpose is to validate the interpolation \ extrapolation of the liquidity multipliers for the different currencies and tenors as well as the aggregation on the index, currency and account level across inflation and OIS. In this case our portfolio includes OIS trades so that we can validate that the aggregation of the OIS risk generated by inflation swaps and OIS swaps is correct as well as the calculation of the overall liquidity cost is done appropriately.

Since the inflation trade population was not altered we are going to present below the results of the OIS risk only.

Case3: Test Inputs

GBP	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	96,191	5,081.00	-	1.00
	42,438	26,444.00	-	5.00
	27,645	-	477,523.00	219.00
	15,299	-	56,136.00	9.00
				2,010.25

Case3: Test Results

GBP	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	6.97	35,390.28	6.97	35,390.28	-	0.00%	PASS
	5.46	144,309.71	5.46	144,309.71	-	0.00%	PASS
	6.33	3,027,108.13	6.33	3,027,108.13	-	0.00%	PASS
	8.14	457,245.21	8.14	457,245.21	-	0.00%	PASS
Total		3,664,053.33		3,664,053.33		0.00%	PASS
Total(GBP)		3,664,053.33		3,664,053.33		0.00%	PASS

Case3: Test Inputs

USD	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	279,464	2,556.00	-	1.00
	117,321	11,597.00	-	10.00
	75,481	-	471,588.00	201.00
	41,500	-	27,210.00	11.00
				45.60
				202.06
				6,708.98
				393.13

Case2: Test Results

USD	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	4.43	11,313.27	4.43	11,313.27	-	0.00%	PASS
	5.71	66,055.85	5.71	66,055.85	-	0.00%	PASS
	6.27	2,960,807.71	6.27	2,960,807.71	-	0.00%	PASS
	7.20	196,197.12	7.20	196,197.12	-	0.00%	PASS
Total		3,234,373.94		3,234,373.95		0.00%	PASS
Total(GBP)		2,022,684.68		2,022,684.69		0.00%	PASS

Case3: Test Inputs

EUR	Liq Notional	Tenor Delta	Tenor Gamma	Mkt Size
2Y	219,630	5,722.00	- 4.00	124.29
	112,626	20,422.00	- 32.00	350.69
	72,867	- 1,419,374.00	- 605.00	20,999.93
	64,610	- 980,359.00	- 755.00	7,099.70

Case3: Test Results

EUR	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	5.80	33,120.32	5.80	33,120.32	-	0.00%	PASS
	5.88	119,528.17	5.88	119,528.17	-	0.00%	PASS
	6.07	8,626,745.76	6.07	8,626,745.76	-	0.00%	PASS
	6.00	5,895,744.00	6.00	5,895,744.00	-	0.00%	PASS
Total		14,675,138.25		14,675,138.25		0.00%	PASS
Total(GBP)		12,456,499.22		12,456,498.79		0.00%	PASS

From the table below we can see that Murex implementation is consistent with the Excel \ VBA model.

CCY	Liq Cost	MX Liq Cost	Diff	Status
EUR	7,969,840,305.56	7,969,839,564.02	0.00%	PASS
GBP	13,836,739.36	13,836,739.47	0.00%	PASS
USD	8,435,718.05	8,435,718.05	0.00%	PASS
Total	7,992,112,762.97	7,992,112,021.54	0.00%	PASS

12.10.4 Case 4

In this case we are validating the use of the bid-offer spread in the calculation of the liquidity cost per currency and tenor, as well as, the overall liquidity charge a member will be charged. To do so we modified the input matrix that members provide with the spread cost that a trade of a given magnitude will cause the market to move. In this validation exercise we are going to use a bid-offer spread of 3 bps.

In the current case we used one of the portfolios that members provided (Barclays) and the risk profile as well as, the liquidity cost calculated per currency and tenor is provided below.

Case 4: Test Inputs

EUR	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	950,000	- 54,360.00	- 5.00	0.06
	1,340,000	- 261,264.00	- 143.00	0.19
	1,950,000	- 712,141.00	- 607.00	0.37
	1,920,000	- 1,909,014.00	- 3,645.00	0.99
	1,700,000	- 350,979.00	- 618.00	0.21

Case 4: Test Results

EUR	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	1.50	81,545.63	1.50	81,545.63	-	0.00%	PASS
	1.50	392,056.88	1.50	392,056.88	-	0.00%	PASS
	1.50	1,068,894.38	1.50	1,068,894.38	-	0.00%	PASS
	1.50	2,867,621.63	1.50	2,867,621.63	-	0.00%	PASS
	1.50	527,163.75	1.50	527,163.75	-	0.00%	PASS
Total		4,937,282.25		4,937,282.27		0.00%	PASS
Total(GBP)		4,190,846.55		4,190,846.57		0.00%	PASS

Case 4: Test Inputs

GBP	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	650,000	- 288,128.00	- 37.00	0.44
	1,100,000	- 1,302,006.00	- 617.00	1.18
	1,800,000	- 5,410,934.00	- 5,338.00	3.01
	2,650,000	- 9,936,433.00	- 17,604.00	3.75
	2,850,000	- 10,438,692.00	- 31,810.00	3.66
	2,300,000	- 7,541,758.00	- 28,296.00	3.28

Case 4: Test Results

GBP	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	1.50000000000000	432,233.62500	1.50000000	432,233.6300	-	0.00%	PASS
5Y	2.25000000000000	2,930,906.74764	2.25000000	2,930,906.7500	-	0.00%	PASS
10Y	6.65000000000000	36,097,203.72422	6.65000000	36,097,203.7200	-	0.00%	PASS
20Y	7.32000000000000	73,194,198.71221	7.32000000	73,194,198.7100	-	0.00%	PASS
30Y	6.65000000000000	70,138,511.26299	6.65000000	70,138,511.2600	-	0.00%	PASS
50Y	6.03000000000000	46,028,487.90453	6.03000000	46,028,487.9000	-	0.00%	PASS
Total		228,821,541.98		228,821,541.98		0.00%	PASS
Total(GBP)		228,821,541.98		228,821,541.98		0.00%	PASS

Case 4: Test Inputs

EUR/ FR	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	650,000	175,528.00	5.00	0.27
	900,000	- 52,849.00	- 17.00	0.06
	1,100,000	- 259,107.00	- 218.00	0.24
	1,920,000	- 80,312.00	- 132.00	0.04
	1,700,000	15,981.00	50.00	0.01

Case 4: Test Results

EUR/ FR	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	1.50	263,297.63	1.50	263,297.63	-	0.00%	PASS
5Y	1.50	79,292.63	1.50	79,292.63	-	0.00%	PASS
10Y	1.50	388,905.75	1.50	388,905.75	-	0.00%	PASS
20Y	1.50	120,616.50	1.50	120,616.50	-	0.00%	PASS
30Y	1.50	24,027.75	1.50	24,027.75	-	0.00%	PASS
50Y							
Total		876,140.25		876,140.26		0.00%	PASS
Total(GBP)		743,682.28		743,682.29		0.00%	PASS

Case 4: Test Inputs

USD	max IO1	Tenor Delta	Tenor Gamma	Mkt Size
2Y	775,000	- 302,047.00	- 39.00	0.39
	1,660,000	- 677,497.00	- 213.00	0.41
	2,360,000	- 252,576.00	- 279.00	0.11
	2,450,000	- 356,015.00	- 515.00	0.15
	2,250,000	- 21,744.00	- 23.00	0.01

Case 4: Test Results

USD	Theoretical		Murex		Multiplier Diff	Diff Liq Cost	Status
	Multiplier	Liq Cost	MX Mult	MX Liq Cost			
2Y	1.50	453,114.38	1.50	453,114.38	-	0.00%	PASS
	1.50	1,016,485.13	1.50	1,016,485.13	-	0.00%	PASS
	1.50	379,177.88	1.50	379,177.88	-	0.00%	PASS
	1.50	534,601.88	1.50	534,601.88	-	0.00%	PASS
	1.50	32,641.88	1.50	32,641.88	-	0.00%	PASS
Total		2,416,021.13		2,416,021.13		0.00%	PASS
Total(GBP)		1,510,910.31		1,510,910.31		0.00%	PASS

From the table below we can see that Murex implementation is consistent with the Excel \ VBA model.

CCY	Liq Cost	MX Liq Cost	Diff	Status
EUR	4,934,528.83	4,934,528.83	0.00%	PASS
GBP	228,821,541.98	228,821,541.98	0.00%	PASS
USD	1,510,910.31	1,510,910.31	0.00%	PASS
Total	235,266,981.12	235,266,981.12	0.00%	PASS

13 Stress Testing

The following section outlines the proposals and required configuration for the extension of the stress testing framework to incorporate inflation stresses. The inflation stresses will involve shifts to the par “zero coupon” rates and will focus on the same key structures as the interest rate and fx scenarios. Namely:

- Absolute scenarios.
- Relative scenarios.
- Hypothetical scenarios.

Following a review by the Bank of England, an addendum (“ZC Inflation Stress Testing – Additional Scenarios”) was added to address the Bank’s feedback and questions.

13.1 Currencies / Indices in-scope

Per the related [Business Requirements document](#) - the following currencies / indices are in considered in-scope:

Currency	Index
GBP	UK RPI
USD	US CPI
EUR	Eurozone HICPxT French CPIxT

13.2 Dataset

The dataset used for analysis was amalgamated from design authority member’s output – along with data gathered from Bloomberg. The dataset begins to become sparse at the beginning of the dataset (circa late 2002). Thus, the swaps dataset was augmented using linker data where available. The dataset contains zero coupon inflation-indexed rates encompassing 1y through to 30y (and up to 50y for some currencies). These zero coupon rates have been cleansed (removal of outliers deemed spurious and non-business day values). The 5 day return (or change) was subsequently computed for each pillar for each date in the time-series. The analysis has subsequently focused on these returns and generating plausible but extreme scenarios based upon them.

13.3 Absolute scenarios

The scenarios described in this section are based on absolute returns. As per the underlying Initial Margin methodology – the returns are based on a 5 day change.

$$r_{absolute} = p_{end} - p_{start}$$

Figure 7 - Absolute Scenario definition

At present there are 56 absolute scenarios configured to run on daily basis as part of the default fund sizing process. The earliest scenario details the IR and FX changes, over a 5 day period, as-of the 21st October 1992. The most recent scenario details the IR and FX changes, over a 5 day period, as-of the 10th August 2011. As part of the Inflation project the existing scenarios will need to be updated, new absolute scenarios specified and finally a strategy put in place for those scenarios for which there is no inflation data available.

13.3.1 Augmenting existing scenarios

The existing stress test scenarios attempt to capture particular periods of distress in the financial markets. It is therefore appropriate to extend the existing set of scenarios to incorporate the changes in zero coupon inflation rates. This ensures that:

1. The full set of risk factors, to which the service is exposed, is incorporated for the purposes of default fund sizing.
2. The actual market shocks on the stress dates are accounted for.

Government inflation linked bonds in the Eurozone and the US did not exist prior to 2001 and 1999 respectively. Thus, where data was not available, EUR HICPx was proxied to FRxT if only OATi existed during that period.

Key point:

At the time of writing, of the 56 existing, absolute, scenarios – the available inflation dataset has coverage for 49 of them. The details of the extensions required are detailed in Appendix 1.

13.3.2 Gaps for existing absolute scenarios

Historical Date	Midas Id	Impacted Ccy/Idx
21/10/1992	344	ALL
01/03/1994	343	ALL
29/03/1994	342	ALL
29/07/1994	341	ALL
03/04/1995	340	ALL
27/11/1997	339	ALL
05/10/1999	338	ALL

Figure 8 - Gaps in coverage for existing absolute scenarios

To address the gaps noted above, a regression analysis was performed on interest rates against real rates as the R-square is more significant using real rates as opposed to inflation rates. The R-square for the regression across the curve ranged between 70-90% for EUR HICPx and 30-50% for US CPI, which validates the use of real rates in the regression analysis. Note that actual real rates traded in the market were used, rather than inferring a real rate from nominal and inflation rates. The lookback period used for the regression was August 2007 – August 2009, i.e. the period covering the global financial crisis. The inflation rate shock is then inferred using the Fisher equation, which is:

$$M \approx R + I, \text{ where } M = \text{nominal interest rates}; R = \text{Real interest rates}; I = \text{Inflation rates}$$

13.4 New absolute scenarios

The objective of this section is to detail the new scenarios which are proposed (over and above the existing set of scenarios).

13.4.1 Methodology to select new absolute scenarios

The following steps are prescriptive of the methodology used to capture the largest/smallest moves over the dataset.

- 1 For each date in the time-series and for each ZCIIS tenor presented for the given date, the 5 day change is computed
- 2 The dates of the largest upshifts and downshifts for the following benchmark tenors are identified – 5Y / 10Y / 30Y
- 3 For each of the dates identified, the curve shift on that date is identified. Redundant curve shifts (i.e. the shifts are similar to another date) are discarded.

13.4.2 Results by currency / index

Index	Date	Rationale	Overlap existing?
UK-RPI	03/02/2009	Up - dominates the up shifts.	No
UK-RPI	16/01/2013	Up - CPAC (Unprecedented up shifts across the curve).	No
UK-RPI	23/09/1992	Week following UK's exit from ERM	No
UK-RPI	17/05/1990	Large bear steepening of the curve	No
UK-RPI	14/04/1992	Bear steepening of the curve	No
US-CPI	20/10/2008	Front end collapse with back end rallying	No
US-CPI	09/02/2009	Bull flattening of curve	No
US-CPI	22/01/2009	Up - large move in mid-curve.	Yes
US-CPI	22/10/2008	Down - severe shock - sustained across the curve.	Yes
US-CPI	25/11/2008	Down - Sustained negative shock, sustained across the curve.	No
FR-CPI	12/12/2008	Up - Dominates the up shifts.	Yes
FR-CPI	13/01/2009	Up - large shifts between 1 and 8y buckets.	No
FR-CPI	29/10/2008	Down - Dominates the downshifts.	No
FR-CPI	27/11/2008	Down - Sustained negative shifts.	No
EU-HICP	31/05/2005	Up - Dominates the up shifts.	No
EU-HICP	12/12/2008	Up - Dominates the up shifts.	Yes
EU-HICP	31/05/2011	Down - Dominates the down shifts.	No
EU-HICP	26/11/2008	Down – almost parallel downshift of curve	No

Figure 9 - Table of new, absolute, scenarios

The full output (shifts to be applied) is available in Appendix 3.

For each new scenario specified in Figure 9 (except for the cases where Overlap existing is set to Yes) – new interest rate shifts will need to be applied.

13.5 Relative scenarios

In high interest rate environments, interest rates appeared to follow a log-normal, mean reverting, process. In such interest rate regimes – a relative change measure was typically used.

$$r_{relative} = \frac{p_{end} - p_{start}}{p_{start}}$$

Figure 10 – Relative return measure

As a consequence of the financial crisis in 2008 – interest rates (and inflation rates) have moved to historically low values (In most cases approaching zero. In some cases the rates have reached negative values).

In such a low rate environment – there is a potential that the low and negative rates observed will cause explosions in returns (or errors due to divisions by zero). Nevertheless the relative shifts from earlier periods remain.

13.5.1 Augmenting existing relative scenarios

The existing stress test scenarios attempt to capture particular periods of distress in the financial markets. It is prudent to extend the existing set of scenarios to incorporate the changes in zero coupon inflation rates. This ensures that:

1. The full set of risk factors, to which the service is exposed, is incorporated for the purposes of default fund sizing.
2. The actual market shocks on the stress dates are accounted for.

Key point

- Of the existing 26 relative scenarios – there is coverage, within the available dataset, for 22 of the scenarios. For those existing scenarios for which there is no coverage - see section 12.6

13.5.2 Important note – inflation market turbulence

The inflation market experienced a severe shock to the front end of the curve around the financial crisis in late 2008 / early 2009. This caused shorter term tenors (<5y) to breach zero and go negative. (In some cases significantly). The sudden drop in inflation rates combined with negative inflation rates caused aberrations in the returns. As a direct consequence the methodology for computing Initial Margin changed from being based on relative returns to absolute returns.

Figure 11 below shows the time-series for the [1y-3y] tenors. The effect of Lehman is very pronounced as the 1y-3y tenors went significantly negative. Further *Figure 12* indicates the effect on the relative returns, i.e. the explosions in returns, which this caused.

Whilst the changes in inflation rates are real – the choice of relative returns for situations where the inflation rate is approaching zero (or is negative) causes unrealistic returns.

UK-RPI [1-3y] ZCIIS rates

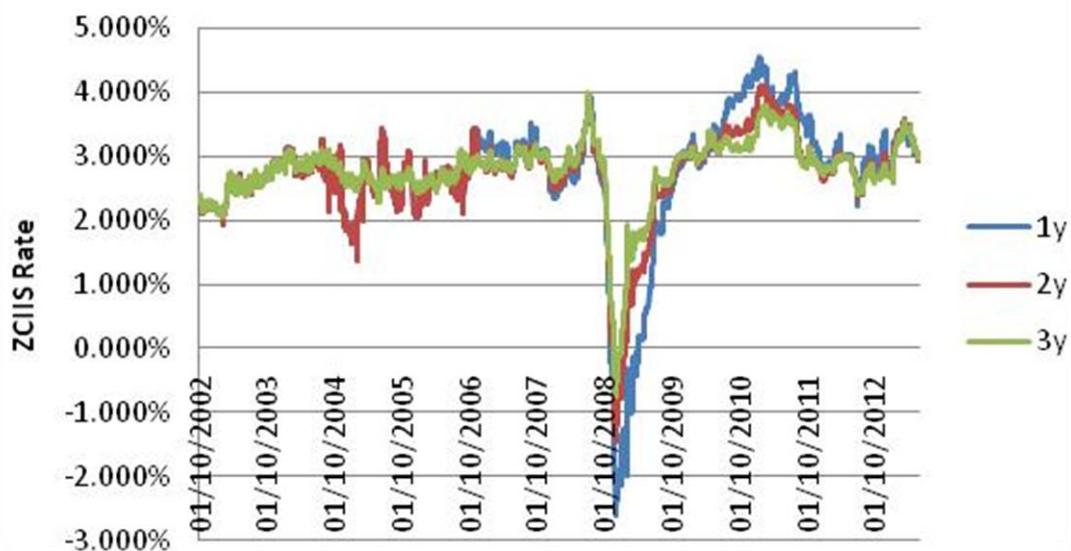


Figure 11 - UK RPI time-series

UK-RPI ZCIIS Returns (Relative)



Figure 12 - UK RPI relative returns

Key point:

For those dates/tenors which exhibited explosions in returns a simple methodology was used to correct the data:

- Values from the bucket to the right hand side (i.e. in the event that the 1y bucket has an erroneous value – the value will be replaced by the shift value in the 2y bucket).
- In the event that the erroneous values occurs in buckets >1y. The shifted value for the bucket will be created by interpolating the available buckets on either side. (i.e. 2y bucket will be interpolated from the 1y and 3y values).

Figure 13 details the changes which have been made to the relative shifts to resolve this.

Index	Scenario Date	Tenor	Observed Chg	Replaced with	Replaced value
UK RPI	20/11/2008	2y	-776.06%	Interp 1y/3y. (relative shift)	5.360%
EU HICPxT	17/12/2008	1y	1222.58%	Shift from 2y.	92.214%
EU HICPxT	20/11/2008	1y	786.96%	Shift from 2y.	65.188%
FR CPI	11/11/2008	1y	1480.77%	Shift from 2y.	-76.848%
FR CPI	12/11/2008	1y	1011.11%	Shift from 2y.	-93.939%
FR CPI	17/12/2008	1y	-789.23%	Shift from 2y.	113.913%
FR CPI	10/11/2008	1y	-400.00%	Shift from 2y.	-69.778%
FR CPI	07/11/2008	1y	-929.27%	Shift from 2y.	-66.132%
FR CPI	20/11/2008	2y	1869.23%	Interp 1y/3y. (relative shift)	-21.368%
FR CPI	10/12/2008	2y	348.05%	Interp 1y/3y. (relative shift)	0.917%
US CPI	08/06/2009	1y	461.27%	Shift from 2y.	49.892%
US CPI	22/01/2009	4y	853.70%	Interp 3y/5y. (relative shift)	-18.074%
US CPI	18/12/2008	6y	1095.62%	Interp 5y/7y. (relative shift)	21.324%

Figure 13 – Changes made to dampen relative shifts

Appendix 2 details the changes made to the data.

13.6 Gaps for existing relative scenarios

There is no coverage for the following, existing relative, stress test scenarios.

Historical Date	Midas Id	Impacted Ccy/Idx
05/10/1999	367	EUR HICPx
04/10/1999	368	EUR HICPx
01/10/1999	369	EUR HICPx
04/03/1999	370	EUR HICPx

Figure 14 - Gaps in coverage for existing relative scenarios

These gaps are now filled using the regression analysis mentioned in section 12.3.2.

13.7 New relative scenarios

13.7.1 Methodology to select new relative scenarios

Given the current low interest / inflation rate regime, and the propensity for generating explosive returns when using a relative return measure – it is proposed that no further relative scenarios are to be added. This should be re-visited when interest/inflation rates begin to return to pre-crisis levels.

13.7.2 Results by currency / index

N/A.

13.7.3 Shifts to merge

N/A.

13.8 Hypothetical scenarios

13.8.1 Inflation Only Scenarios

Hypothetical inflation only scenarios were designed to ensure comprehensive coverage of potential inflation events, even if there were no historical precedence.

The inflation curve moves were designed to reflect a typical parallel, steepening and flattening move for each index. Hence, the following scenarios are defined:

- Rally
- Sell off
- Bull steepen
- Bull flatten
- Bear steepen
- Bear flatten

The magnitude of the curve shifts are constrained by the maximum and minimum curve shifts observed over the last 30 years, where data is available, based on the most traded tenor for each respective index. This is to ensure that the scenarios are extreme but plausible. These are:

• UKRPI:	30Y	min: -62bps	max: +112bps
• USCPI:	5Y	min: -115bps	max: +70bps
• EU HICPx:	10Y	min: -40bps	max: +39bps
• EU FRxT:	10Y	min: -41bps	max: +38bps

The parameters were perturbed such that the curve reaches but does not exceed minimum and maximum levels defined above.

See Appendix 4 for details.

13.8.2 Real Rate Scenarios

Real rate scenarios were developed to cover the potential for de-correlation between inflation and nominal interest rates. Similar to the inflation scenario generation, the largest real rate shifts over the last 30 years were identified based on the most traded tenor of each respective index.

The real rate shifts identified were:

Index	Real rates up	Real rates down
UKRPI	91	(142)
HICPx	40	(74)
FRxT	41	(70)
USCPI	111	(89)

Several approaches were considered to determine the allocation of the real rate shift between inflation and nominal rates.

Historical Data

Index	Real rates up	Inflation	Nominal	Real rates down	Inflation	Nominal
UKRPI	91	(62)	29	(142)	112	(30)
HICPx	40	(7)	34	(74)	26	(48)
FRxT	41	(8)	34	(70)	22	(48)
USCPI	111	(89)	22	(89)	38	(51)

These scenarios are already covered in the suite of historical scenarios, and would not add value in ensuring that the scenarios are comprehensive.

Furthermore, the severity of the HICPx and FRxT shifts appear relatively benign vis a vis the UKRPI and USCPI scenarios.

Allocation of real rate shocks between inflation and nominal

The historical analysis above was inconclusive as to how a real rate shock might be split between inflation and nominal rates, as different historic situations have behaved differently. A regression analysis was performed to try and ascertain a rule that could be used. However this was also inconclusive (results are below). It was therefore decided that the real rate move would be divided evenly (i.e. 50:50) between the inflation and nominal rates.

Index	Real rates up	Inflation	Nominal	Real rates down	Inflation	Nominal
UKRPI	91	(28)	63	(142)	44	(98)
HICPx	40	(25)	15	(74)	46	(28)
FRxT	41	(20)	21	(70)	34	(36)
USCPI	111	(60)	51	(83)	45	(38)

Size of scenarios

Rather than replicate the historic shocks, we calibrated the hypothetical scenario shock to be consistent across the inflation indices under consideration, and to be conservative with respect to most of the historic events observed. We therefore decided on a shock size for real rates of 100bp, which is more conservative than the most severe events in US, French and European inflation in the historic dataset. The UK example shows us that a more extreme scenario could occur in the other rates; hence we have up-sized the shock to the 100bp level. We do, however, believe that any further increase would be excessively conservative, given the unique events surrounding the UK's membership and exit from the European Exchange Rate Mechanism (ERM) which led to that move in real rates. Hence, although we model this scenario with respect to UK rates, we have not applied it to the other currencies.

The scenarios have therefore been defined as:

- Inflation/interest rate decorrelation 1: Inflation +50bp, Interest Rates -50bp
- Inflation/interest rate decorrelation 2: Inflation -50bp, Interest Rates +50bp

The above shocks are to be applied to each inflation index (and the corresponding interest rate) individually. The table below illustrates the behaviour of real rates and inflation during the recent financial crisis and resulting global recession. It demonstrates that a scenario of this severity covering all relevant inflation and interest rates and occurring at the same time would not be plausible.

	Real Rates			Inflation Rates			Nominal Rates		
	5Y	10Y	30Y	5Y	10Y	30Y	5Y	10Y	30Y
October 2008 - US enters recession									
USCPI (27 October)	89	31	11	(93)	(40)	(49)	(4)	(9)	(38)
HICPx (29 October)	31	24	(10)	(44)	(27)	(14)	(14)	(3)	(24)
FRXt (29 October)	27	20	(10)	(40)	(23)	(14)	(14)	(3)	(24)
UKRPI (27 October)	(19)	(17)	(9)	(9)	(2)	0	(28)	(19)	(9)
Global Recession November 2008									
USCPI	32	(11)	38	(49)	(32)	(86)	(16)	(42)	(48)
HICPx	16	11	(30)	(39)	(37)	(36)	(23)	(26)	(67)
FRXt	14	7	(29)	(36)	(33)	(37)	(23)	(26)	(67)
UKRPI	50	15	(20)	(69)	(39)	(26)	(20)	(25)	(46)
February 2009 - Stimulus package									
USCPI	(30)	4	(10)	51	24	23	20	28	13
HICPx	16	12	9	(15)	(6)	(7)	1	6	2
FRXt	2	7	1	(1)	(0)	1	1	6	2
UKRPI	37	33	46	(28)	(25)	(34)	9	8	12

The proposed scenarios are detailed in Appendix 5

14 Default Fund Sizing

The methodology for both sizing and allocation of the default fund of SwapClear is fundamentally unaltered by the inclusion of inflation products. It is however expected that the introduction of inflation risk will impact both the size and allocation values.

In order to investigate the potential impacts of inflation on default fund size, a number of scenarios were constructed to represent plausible and realistic risk profiles in inflation and then superimposing these upon the existing rate risk profiles. The results of these tests were highly intuitive. At low levels of risk in inflation, the stress over IM is dominated by large rate positions and the default fund does not change in any significant way. As the level of inflation risk is increased, it begins to become at least a partial driver of the default fund size. When the level of inflation risk approaches and then exceeds the rates risk in the existing backbook, the stress scenarios that drive default fund size are found to be inflation dominated and the inflation risk becomes a very significant force in the default fund size.

The following chart shows the impact of adding an amount of risk (x axis) to all members and to an associated client where applicable. The risk was all added in the 30 year tenor point on GBP RPI. The y axis shows the calculated default fund size and the chart exhibits the behaviour described previously

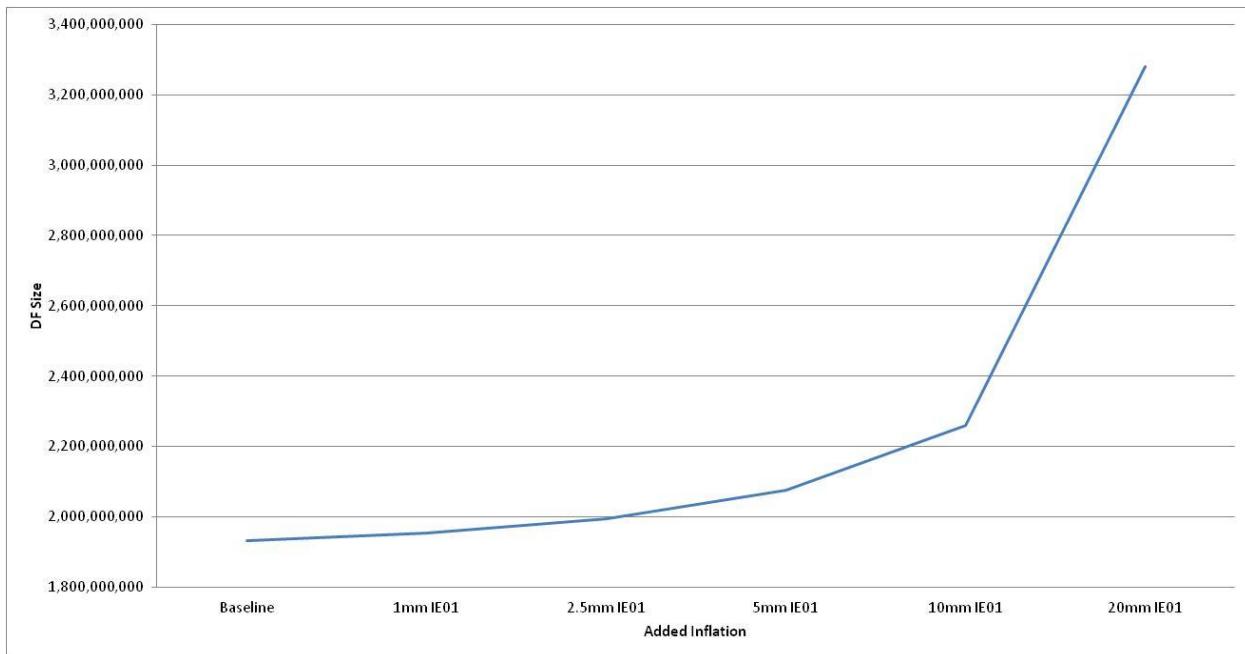


Figure 15– Simplified impact of inflation on default fund size

15 Default Management

LCH has a proven track record of handling Defaults, managing the Lehman Default (\$9 Trillion Portfolio of 66,390 Trades) and using only 35% of Lehman's Initial Margin across all assets cleared in LCH (the remaining 65% was returned to Lehman's Administrator).

In event of a Default occurring, the Default Management Group (DMG) act on behalf of SwapClear to assess, reduce and then eliminate the Risk and Positions of the Defaulting Member or Client from its book in a swift and orderly fashion.

15.1 Default Management Group (DMG)

Once a Default occurs, a subset of Clearing Members known as the Default Management Group (DMG) will second key personnel to SwapClear. Chinese walls are erected between DMG Members and their firms to ensure total confidentiality.

With the addition of ZCIIS to the Service, the existing SwapClear DMG Panel (numbering 6 Members) will be augmented with 3 Traders who are Inflation Specialists, bringing the total number of Members on the Panel to 9.

It is important that no Member is over-represented and that more than 3 Inflation Traders form the available pool, to cover the case where the allocated Trader is from the defaulting Institution.

The requirement to provide an Inflation Specialist will operate on a rotational basis.

15.2 Default Management Process (DMP)

In event of a Default occurring, DMG Members, acting on behalf of SwapClear and using SwapClear Systems, assess, reduce and then eliminate the Risk and Positions of the Defaulting Member or Client from its book in a swift and orderly fashion.

This is achieved by taking the following key actions: -

- Risk-Neutralise and Hedge the Defaulter's Portfolio
- Auction the combined Defaulter's and Hedge Trade Portfolios
- Attribute the losses due to the Default to the 'Default Waterfall'.

The phases and steps in the Default Management Process are summarised below: -

Financial Resources	Phase	Process
Defaulter Margin and Default Fund Contribution and Proportion of £20m LCH.C Capital	Phase 1 Risk Neutralisation & Portfolio Splitting	<ul style="list-style-type: none"> • Default Management Group (DMG) is seconded into LCH.Clearnet when a Default is called to hedge and risk-manage the Defaulter's portfolio until the portfolio is auctioned and transferred • DMG neutralise and hedge Risk by executing Hedging Trades with Non-Defaulted SwapClear that are subsequently cleared • DMG splits the Defaulted Member's currency portfolios into smaller sub-portfolios in order for an orderly, efficient and competitive auction process to take place.
Non-Defaulters Default Fund Contribution	Phase 2 Portfolio Auction	<ul style="list-style-type: none"> • Members asked to bid for hedged portfolios of the Defaulting Member's trades • Members must use all reasonable efforts to make a commercially reasonable Bid for a portfolio or sub-portfolio; no obligation for members to bid in any auction. • Auction incentives include: - <ol style="list-style-type: none"> 1. SwapClear Default Fund Contribution - Each Clearing Member has a proportion of their Default Fund at risk as a result of poor DMP and auction participation 2. Auction Incentive Pools - Apportion each Member's Default Fund Contribution into Auction Incentive Pools (AIPs) linked to each individual currency portfolio auction; incentivises Clearing Members to bid more aggressively in the currencies for which they have the greater proportion of the overall risk; enables the DMG to assess bids in each auction against the financial resources apportioned to each AIP. 3. Proportional Loss Attribution Process - Rewards good auction behaviour and penalises bad or non-bidding behaviour by sequencing the usage of their contributions to cover losses in each AIP.
	Phase 3 Portfolio Transfer	<ul style="list-style-type: none"> • Winner of the auction is notified and the portfolio of trades is transferred to the winning bidder's portfolio in SwapClear.
	Phase 4 Loss Attribution	<ul style="list-style-type: none"> • For losses over and above the financial resources of the Defaulter, the funded Default Fund Contributions of the SwapClear Members will be attributed into tranches by each AIP based upon their bidding behavior during the auction: - <ul style="list-style-type: none"> Tranche 1 – Non Bidders Tranche 2 – Auction Bidders (not winner) Tranche 3 – Auction Winner (plus those with same bid as winner). • Losses over and above the financial resources available in an AIP will be further attributed as follows: - <ul style="list-style-type: none"> Tranche 4 – Surplus resources from other AIPs Tranche 5 – Any remaining resources. • Further losses that exceed the funded financial resources will in turn be attributed in the same way, i.e. Tranche 1 to 2 to ... 5, to the unfunded contributions of the SwapClear Members.

15.2.1 Portfolio Risk Neutralisation and Hedging

In relation to Zero Coupon Inflation Indexed Swaps, the Default Management Process for Book Hedging will follow the existing SwapClear framework and, in effect, treat the Inflation Index as a separate currency (EUR and FRF indices to be considered together).

Taken in isolation, the Inflation Books may contain significant Interest Rate Exposure. To extract maximum value at auction, this Interest Rate Risk needs to be eliminated. In this situation, Members will be required

to carry out ‘pass-through’ Interest Rate Swaps to transfer this Risk from the Inflation Book to the core Interest Rate Book. This will allow the Inflation and Interest Rate Risks to be hedged prior to auction in a simple way.

Legal Commitment

LCH.Clearnet Ltd will require a legal commitment from all Members clearing ZCIIS that they will execute pass-through Interest Rates Swap trades at market for no cost, in order to transfer the Interest Rate Delta from the Inflation Book to the Interest Rate Book.

15.2.2 Portfolio Auction

Once hedged, the DMG conducts a series of auctions to sell the portfolio. The DMG has discretion to split each currency portfolio into smaller sub-portfolios if it is deemed that this will lead to a more efficient and competitive auction process.

Clearing Members must make all reasonable efforts to participate and provide a commercially reasonable bid for a portfolio or sub-portfolio during the auction process.

In relation to Zero Coupon Inflation Indexed Swaps, the Default Management Process for auction of hedged portfolios will follow the existing SwapClear framework, and in effect, treat the Inflation Index as a separate currency.

The USD and GBP Inflation Books will each form an Auction Pack. The EUR and French Books will be auctioned as a combined book on the advice of membership consultation, and as such form a single AIP.

Members will be asked to bid for hedged portfolios of the Defaulting Member’s trades.

Where a group has an affiliated SwapClear Clearing Member (SCM) or Futures Commission Merchant (FCM), it can, if it chooses to do so, submit the same bid on behalf of all affiliated entities.

Bids are to be submitted for the entire currency portfolio; partial bids are not permitted.

All Members must use all reasonable efforts to make a commercially reasonable bid for a portfolio or sub-portfolio.

The Loss Attribution Process (see Section 5.2.5 below) incentivises all Members to bid as near to the winning bid in order to protect their capital which is now at risk if they either do not bid or submit an ‘off-market’ bid as well as rewarding good bidding behavior and penalising bad bidding behavior.

There is no legal obligation for Members to bid for any auction portfolio; however, Members’ bids (or lack thereof) will determine how their own Default Fund contributions are utilised in meeting losses crystallised by the auction process.

If multiple bidders submitted the same best bid, then the bidder who submitted the bid first would win the auction and be awarded the auction portfolio. In the case where they were part of the same group, the group must instruct which entity to transfer the portfolio.

If Auction Incentive Pool (AIP) funds (see Section 5.2.3 below) are not sufficient to meet the best bid in an auction, Members will be notified that the auction has failed and will be re-auctioned. The DMG may also continue to hedge the portfolio to increase the chances of a successful auction.

Each portfolio should be auctioned at least once prior to the DMG accepting a bid lower than the size of the AIP; however the DMG maintains the discretion to accept bids lower than the minimum at any point in time if they believe it to be in the best interests of the Clearing House.

As there is No Forced Allocation (NCA), there is no limit to the number of auctions that could be run.

15.2.3 Portfolio Incentivisation Process

As part of the Portfolio Auction stage (see Section 6.2.2 above), each Member's Default Fund contribution is apportioned into Auction Incentive Pools (AIPs) associated with each individual Currency Portfolio Auction. All Clearing Members will be informed of the size of their contribution to each Auction Incentive Pool and where their contribution ranks relative to all other Member Contributions in that currency Auction Incentive Pool.

What is the Purpose of the AIPs?

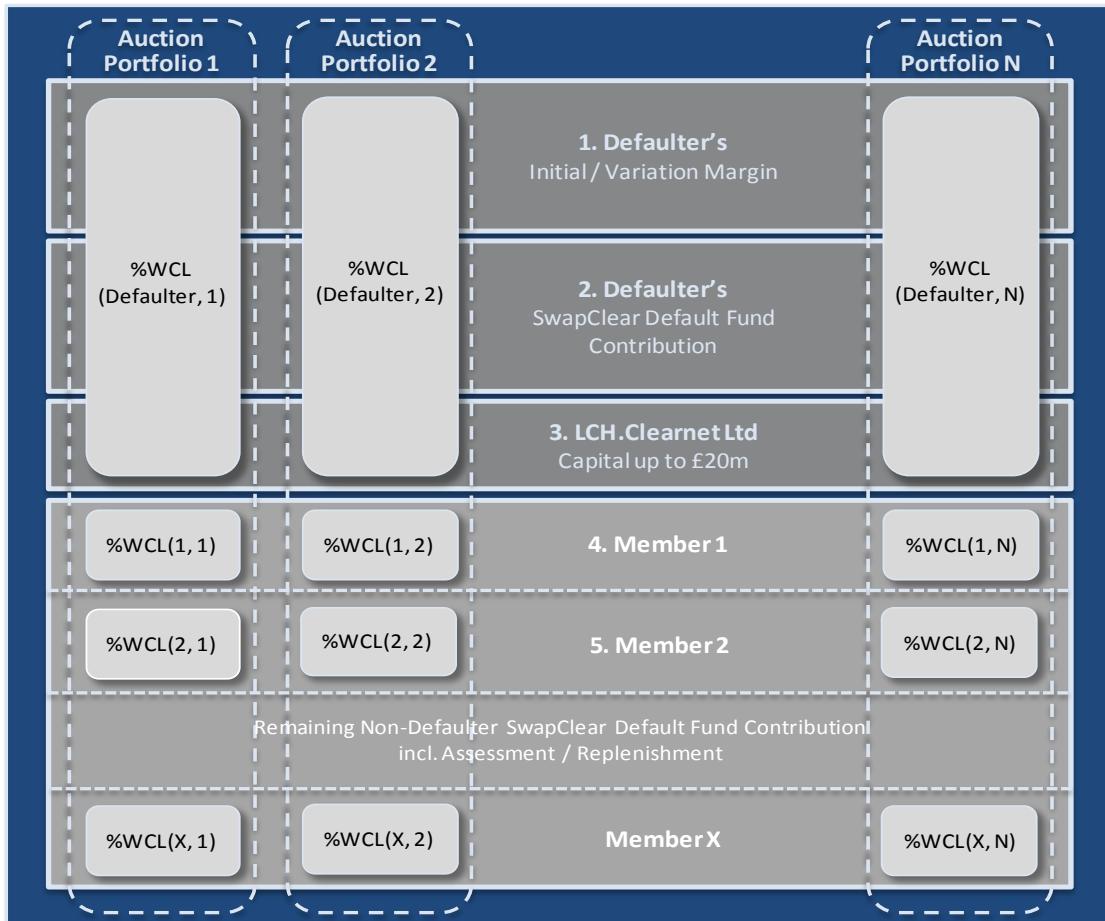
AIPs can be used by the DMG as a proxy for a reserve price to assess and determine the level of bid that could utilise an entire currency AIP, thus preventing losses in the first auction from spilling into other currency AIPs before the remaining auctions of other Currency Portfolios have had an opportunity to be auctioned; i.e., each Currency Portfolio Auction is not contingent on another.

Incentivises those Clearing Members to bid more aggressively in the currencies for which they have the greatest risks and the most to lose.

Allows SwapClear sequence the usage of each Member's contribution to each AIP relative to their bidding behaviours in each currency auction; i.e., The Loss Attribution Process (see Section 5.2.5 below).

How are the AIPs Apportioned / Calculated?

SwapClear will apportion the available SwapClear financial resources into AIPs linked to each Currency Auction Portfolio or sub-portfolio (if split). The AIPs will incorporate the following financial resources: -



Level	Financial Resources	AIP Calculation
1	Defaulter's Initial and Variation Margin (IM and VM)	<ul style="list-style-type: none"> Defaulter's Worst Case Loss (WCL) relative to the sum of the Defaulter's total WCL across all portfolios as of the Pre-Auction Margin Run.
2	Defaulter's Contribution to the SwapClear Default Fund	<p><u>Note:</u> Any Variation Margin losses or gains and hedging costs resulting from the hedging and risk neutralisation will be allocated pro-rata across all AIPs prior to the final determination of each AIP.</p>
3	Proportion of the LCH.Clearnet Ltd. Capital (up to £20m)	
4	Non-Defaulters' Funded Contribution to the SwapClear Default Fund	<ul style="list-style-type: none"> Proportion of every Clearing Member's individual WCL by currency relative to the sum of their WCL taken as an average over the previous 20 business days prior to default.
5	Non-Defaulters' Unfunded Obligations to the SwapClear Default Fund	<ul style="list-style-type: none"> As per Level 4.

All Clearing Members will be informed of the size of their contribution to each AIP (Level 4) and where their contribution ranks relative to all other Member contributions in that currency AIP - for example "You are the 3rd largest contributor to the EUR AIP". However Members will not be told the total size of each AIP.

15.2.4 Portfolio Transfer

The winner of the auction is notified and the portfolio of trades is transferred to the winning bidder's portfolio in SwapClear.

15.2.5 Loss Attribution Process

How does the Loss Attribution Process work?

All of the Defaulter's contributions via Initial / Variation Margin and their Default Fund contributions will be fully exhausted prior to any Loss Attribution of Non-Defaulting Members' Default Funds.

The Loss Attribution Process assumes the SwapClear Default has completed within the confines of the total financial resources available to SwapClear.

The following losses (or gains) will be attributed against each AIP: -

- **Variation Margin losses or gains** - The total realized VM losses or gains from the start of the auction until the end of the auction.
- **Auction losses** - The difference between the winning bid and the value of the portfolio prior to auction at the Pre-Auction Margin Run.

If the sum of the losses are greater than the financial resources across all Levels 1, 2 and 3, then the Level 4 contributions (i.e. those AIP contributions of the Non-Defaulters) will be attributed based on their bidding behavior.

The table below summarises how the resources are attributed: -

Tranche	Resource Attribution
Tranche 1 Non-Bidders	<ul style="list-style-type: none"> • Members who have not submitted a bid will have their portion of AIP funds allocated to Tranche 1 • Tranche 1 funds are utilised pro-rata to meet any auction losses / VM losses.
Tranche 2 All other Bidders (except the Winning Bidder)	<ul style="list-style-type: none"> • Members who have submitted a bid (but not the winning bid) will have their portion of AIP funds allocated to Tranche 2 • Tranche 2 funds are proportionally utilised based on the relative gap between each Member's bid and the winning bid.
Tranche 3 Winning Bidder	<ul style="list-style-type: none"> • The winning bidder (and those participants who have submitted an identical bid to the auction winner but have not necessarily won the auction) will have their portion of AIP funds allocated to Tranche 3 • Tranche 3 funds are utilised pro-rata to meet any remaining auction losses / VM losses.
Tranche 4 Surplus Resources from other AIPs	<ul style="list-style-type: none"> • The sum of all remaining AIP contributing members resources from other AIPs will be utilised to cover any losses (i.e. The resources of those members who had positions in that currency at the time of default). • The losses will be allocated based upon their aggregated pre-default WCL of each member across all the currencies for which they had exposures.
Tranche 5 Remaining Resources	<ul style="list-style-type: none"> • The remaining capital of members who did not have a position in the currency(ies) that failed an auction, pro-rata based upon each member's capital relative to total remaining Default Fund contribution.

15.2.6 Reports

The set of reports required to manage, hedge and auction a book containing ZCIIS, will be very similar to those required for the existing SwapClear Service for Interest Rates.

I01 Report Ladders in the same format as the existing Swaps PV01 will be generated for the consumption of Members to measure Risk and manage a Default situation.

Appendix 1 – Absolute stresses

This section details the additional inflation shifts which need to be added to the existing set of absolute scenarios. All numbers are in basis points.

UK-RPI

UK-RPI	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y	50Y
27/05/2009	0	0	1	2	3	3	4	4	3	1	0	(1)	(2)	(0)	(1)	(3)	(4)
10/12/2008	42	47	43	42	36	30	24	20	16	14	11	5	2	(1)	(2)	(4)	(5)
03/12/2008	(85)	(21)	(3)	0	(4)	(4)	(5)	(6)	(8)	(9)	(6)	(6)	(8)	(11)	(17)	(15)	(16)
20/11/2008	(55)	(55)	(56)	(56)	(52)	(48)	(44)	(42)	(41)	(39)	(35)	(24)	(20)	(19)	(19)	(16)	(19)
12/11/2008	(86)	(66)	(49)	(44)	(38)	(31)	(25)	(18)	(12)	(5)	(4)	(3)	(1)	1	4	4	4
10/11/2008	(108)	(88)	(66)	(57)	(45)	(40)	(34)	(28)	(22)	(16)	(13)	(10)	(7)	(4)	(0)	(0)	(0)
07/11/2008	(112)	(88)	(64)	(53)	(40)	(36)	(32)	(27)	(23)	(18)	(15)	(12)	(7)	(3)	2	2	2
22/10/2008	(43)	(43)	(41)	(39)	(36)	(32)	(28)	(24)	(20)	(16)	(12)	(11)	(12)	(18)	(20)	(13)	(14)
10/10/2008	(85)	(75)	(69)	(63)	(56)	(51)	(45)	(40)	(34)	(29)	(27)	(19)	(13)	(11)	(8)	(8)	(8)
07/10/2008	(43)	(44)	(43)	(38)	(34)	(34)	(34)	(34)	(34)	(34)	(32)	(30)	(26)	(21)	(17)	(20)	(17)
25/09/2008	3	3	3	3	3	4	4	5	6	7	6	6	5	4	3	(3)	(2)
24/09/2008	14	14	14	14	14	14	13	13	13	13	12	11	10	8	6	1	1
23/09/2008	11	11	11	11	11	12	13	14	15	16	16	15	15	15	14	8	9
22/09/2008	(1)	(1)	(1)	(1)	(1)	0	1	2	3	3	3	4	4	4	4	(1)	(1)
09/06/2008	15	15	15	14	14	13	12	13	10	9	8	8	10	11	12	11	10
03/07/2003	(9)	(9)	(9)	(10)	(10)	(10)	(10)	(10)	(10)	(9)	(9)	(7)	(7)	(7)	(7)	(6)	(6)
19/03/2003	9	9	17	18	15	14	13	12	12	12	11	11	12	10	9	8	8
27/05/2009	0	0	1	2	3	3	4	4	3	1	0	(1)	(2)	(0)	(1)	(3)	(4)
26/09/2008	(5)	(5)	(5)	(5)	(5)	(4)	(3)	(2)	(1)	1	0	(0)	(1)	(2)	(3)	(3)	(3)
17/09/2008	(17)	(14)	(13)	(12)	(13)	(12)	(11)	(9)	(8)	(6)	(6)	(3)	(1)	2	7	6	6
18/09/2008	(5)	(2)	(2)	(0)	(1)	(1)	0	1	2	3	4	5	6	8	12	11	11
08/10/2008	(55)	(56)	(53)	(47)	(41)	(41)	(41)	(40)	(40)	(40)	(38)	(34)	(29)	(23)	(17)	(20)	(17)
08/08/2011	(54)	(38)	(34)	(26)	(25)	(21)	(19)	(16)	(15)	(15)	(14)	(13)	(11)	(8)	(8)	(8)	(8)
09/10/2008	(78)	(78)	(71)	(63)	(56)	(52)	(48)	(44)	(39)	(35)	(33)	(29)	(22)	(16)	(10)	(13)	(10)
10/08/2011	(67)	(57)	(47)	(41)	(35)	(32)	(30)	(27)	(25)	(23)	(20)	(17)	(12)	(12)	(9)	(9)	(9)
30/09/2008	(9)	(9)	(9)	(9)	(9)	(8)	(7)	(6)	(6)	(5)	(5)	(5)	(5)	(5)	(5)	(1)	(5)
16/08/2007	(9)	(16)	(16)	(14)	(13)	(11)	(9)	(8)	(6)	(5)	(3)	(3)	(4)	(4)	(4)	(4)	(4)
17/12/2008	19	19	18	17	17	16	15	14	13	12	11	11	13	12	12	13	15
18/12/2008	(9)	12	14	15	13	11	10	8	7	5	4	4	6	5	5	6	8

05/11/2008	(28)	(33)	(29)	(23)	(20)	(18)	(18)	(17)	(13)	(10)	(6)	(3)	(1)	0	3	3	3
22/12/2008	(15)	7	12	11	10	9	9	8	7	6	4	4	5	5	5	5	4
09/12/2008	34	39	36	35	30	23	16	10	5	2	1	(4)	(10)	(12)	(15)	(18)	(18)
11/12/2008	34	34	32	33	29	27	24	22	21	19	16	11	10	9	9	8	7
08/12/2008	22	27	25	25	20	15	10	6	2	(1)	(2)	(6)	(11)	(17)	(20)	(20)	(20)
13/11/2008	(21)	(21)	(24)	(21)	(21)	(16)	(12)	(8)	(4)	1	3	7	8	7	5	4	7
11/11/2008	(84)	(64)	(45)	(40)	(32)	(27)	(22)	(17)	(11)	(6)	(4)	(2)	0	3	5	5	5
12/12/2008	42	42	41	40	38	35	33	31	28	27	25	22	25	24	24	24	24
21/10/1992	(27)	(27)	(27)	(27)	(25)	(22)	(16)	(11)	(4)	2	13	26	26	26	26	26	26
01/03/1994	(15)	(15)	(6)	2	7	11	14	16	18	19	20	18	12	10	10	10	10
29/03/1994	34	34	40	42	41	40	38	37	36	36	35	33	31	30	30	30	30
29/07/1994	16	16	11	9	9	8	8	7	6	5	2	(2)	(8)	(10)	(10)	(10)	(10)
03/04/1995	5	5	(1)	(3)	(3)	(3)	(3)	(2)	(1)	0	3	5	6	6	6	6	6
27/11/1997	9	9	9	9	9	9	7	6	5	3	1	(2)	(3)	(2)	(2)	(2)	(2)
05/10/1999	19	19	15	13	12	10	8	7	5	3	0	(4)	(10)	(13)	(13)	(13)	(13)
08/01/2001	1	1	(2)	(4)	(6)	(6)	(7)	(7)	(7)	(7)	(6)	(3)	1	3	3	3	3
16/11/2001	25	25	20	17	15	13	11	9	8	7	5	4	2	3	3	3	3
19/03/2003	(1)	(1)	5	5	4	3	1	0	(0)	(0)	(0)	0	2	4	4	4	4
03/07/2003	(14)	(14)	(9)	(2)	2	5	6	7	7	7	6	4	1	0	0	0	0

EU-HICPxT

EU-HICP	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y	50Y
27/05/2009	(10)	(9)	(4)	(1)	(2)	(3)	(4)	(4)	(5)	(5)	(4)	(3)	(1)	1	1	1	1
10/12/2008	10	20	26	28	28	29	27	25	23	21	21	20	16	10	9	9	9
03/12/2008	12	12	14	18	20	20	21	21	22	24	24	22	16	10	10	11	12
20/11/2008	18	19	15	13	13	9	9	8	8	7	6	5	2	1	1	1	1
12/11/2008	(46)	(46)	(46)	(41)	(36)	(28)	(22)	(16)	(12)	(10)	(9)	(8)	(6)	(6)	(6)	(6)	(6)
10/11/2008	(52)	(50)	(47)	(40)	(32)	(25)	(19)	(14)	(10)	(8)	(6)	(5)	(4)	(3)	(2)	(2)	(2)
07/11/2008	(60)	(51)	(45)	(39)	(34)	(28)	(22)	(18)	(14)	(11)	(9)	(8)	(7)	(6)	(6)	(6)	(6)
22/10/2008	(56)	(42)	(32)	(23)	(16)	(13)	(11)	(9)	(7)	(5)	(4)	(3)	(2)	(2)	(3)	(3)	(3)
10/10/2008	(29)	(29)	(28)	(26)	(25)	(23)	(21)	(19)	(17)	(16)	(15)	(14)	(12)	(10)	(8)	(7)	(7)
07/10/2008	(39)	(39)	(39)	(39)	(38)	(37)	(36)	(35)	(34)	(34)	(32)	(31)	(31)	(30)	(29)	(29)	(29)
25/09/2008	9	9	8	10	10	10	8	7	7	6	5	3	2	2	2	2	2
24/09/2008	13	13	11	13	14	12	10	9	7	6	4	2	2	2	1	1	1
23/09/2008	18	18	17	16	16	14	12	10	8	6	4	3	3	3	3	3	3
22/09/2008	15	15	14	13	13	11	9	8	7	5	4	3	2	2	1	1	1
09/06/2008	22	10	9	8	8	8	7	7	6	4	3	2	2	2	1	1	1
03/07/2003	14	(4)	(2)	(4)	(5)	(6)	(7)	(6)	(6)	(5)	(3)	(6)	(8)	(7)	(7)	(8)	(7)
19/03/2003	(1)	4	9	10	10	10	10	10	9	8	7	5	5	3	3	3	3
27/05/2009	(10)	(9)	(4)	(1)	(2)	(3)	(4)	(4)	(5)	(5)	(4)	(3)	(1)	1	1	1	1
26/09/2008	11	11	9	10	7	6	5	4	4	4	1	(0)	(1)	(1)	(2)	(2)	(2)
17/09/2008	9	9	8	7	6	5	5	4	3	2	1	1	(1)	(3)	(5)	(6)	(6)
18/09/2008	10	10	9	8	8	7	6	5	3	2	1	(0)	(2)	(4)	(5)	(7)	(6)
08/10/2008	(30)	(30)	(31)	(32)	(33)	(31)	(30)	(29)	(28)	(27)	(26)	(25)	(24)	(22)	(21)	(20)	(19)
08/08/2011	(27)	(27)	(25)	(22)	(20)	(17)	(18)	(17)	(16)	(15)	(15)	(15)	(15)	(15)	(15)	(15)	(15)
09/10/2008	(25)	(25)	(24)	(24)	(23)	(22)	(20)	(19)	(18)	(17)	(16)	(16)	(15)	(14)	(13)	(12)	(12)
10/08/2011	(19)	(19)	(15)	(11)	(8)	(5)	(6)	(5)	(4)	(3)	(3)	(3)	(3)	(3)	(3)	(3)	(3)
30/09/2008	3	(0)	(2)	(3)	(4)	(4)	(3)	(3)	(2)	(2)	(2)	(3)	(4)	(5)	(5)	(5)	(5)
16/08/2007	(0)	(0)	(1)	(1)	(2)	(1)	(1)	(1)	(0)	0	0	0	(0)	(0)	0	0	0
17/12/2008	38	38	38	38	37	32	27	22	18	15	13	13	13	13	14	14	14
18/12/2008	11	13	17	17	18	15	11	6	3	0	(2)	(1)	0	(1)	(0)	(0)	(0)
05/11/2008	(3)	(13)	(15)	(12)	(9)	(8)	(7)	(7)	(6)	(5)	(7)	(9)	(11)	(13)	(13)	(13)	(13)
22/12/2008	(27)	(25)	(21)	(20)	(18)	(17)	(17)	(16)	(15)	(14)	(15)	(14)	(11)	(9)	(8)	(8)	(8)
09/12/2008	28	25	25	26	27	25	24	24	23	22	21	19	14	8	10	11	12
11/12/2008	16	29	35	36	35	35	30	28	25	22	22	22	21	20	18	18	18
08/12/2008	20	17	18	18	21	20	21	22	22	24	23	24	16	8	10	11	12
13/11/2008	(22)	(24)	(26)	(24)	(21)	(15)	(9)	(5)	(2)	(0)	(2)	(2)	(2)	(3)	(3)	(3)	(2)

11/11/2008	(43)	(43)	(43)	(39)	(34)	(27)	(21)	(16)	(13)	(10)	(9)	(9)	(8)	(7)	(7)	(7)	(7)
12/12/2008	40	53	60	62	62	59	50	44	39	39	31	30	28	26	24	24	24
08/01/2001	(19)	(19)	(19)	(19)	(19)	(19)	(19)	(19)	(19)	(18)	(16)	(14)	(11)	(7)	(4)	(4)	(4)
16/11/2001	16	16	16	16	16	16	16	16	16	16	15	14	14	13	13	13	13
19/03/2003	1	1	1	1	1	1	0	(1)	(2)	(4)	(4)	(4)	(5)	(5)	(5)	(5)	(5)
03/07/2003	2	2	2	2	2	2	3	4	5	6	7	8	9	10	10	10	10
05/10/1999	12	10	9	8	7	6	5	5	4	3	3	3	2	2	2	1	1
27/11/1997	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
03/04/1995	(11)	(10)	(9)	(7)	(7)	(6)	(5)	(4)	(3)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)
29/07/1994	3	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
29/03/1994	2	2	2	3	2	2	2	3	3	3	3	3	3	3	3	3	3
01/03/1994	3	12	12	11	11	10	9	8	8	7	7	7	7	7	7	7	7
10/08/2011	(21)	(23)	(20)	(17)	(15)	(13)	(11)	(10)	(8)	(7)	(7)	(7)	(7)	(7)	(7)	(7)	(7)

FR-CPIxT

FR-CPIxT	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y	50Y
27/05/2009	(4)	3	2	1	(2)	(4)	(4)	(6)	(6)	(6)	(5)	(3)	(0)	2	5	5	5
10/12/2008	21	27	24	23	30	33	32	31	29	25	25	26	19	13	13	13	13
03/12/2008	20	20	20	21	21	20	21	20	19	20	18	16	16	14	13	14	15
20/11/2008	44	24	20	15	14	13	12	11	10	10	8	6	3	0	(1)	(1)	(1)
12/11/2008	(46)	(47)	(44)	(41)	(39)	(32)	(26)	(19)	(16)	(10)	(8)	(7)	(7)	(7)	(7)	(7)	(7)
10/11/2008	(45)	(44)	(39)	(34)	(30)	(24)	(18)	(13)	(11)	(8)	(5)	(3)	(2)	(1)	(0)	(0)	(0)
07/11/2008	(38)	(37)	(33)	(32)	(29)	(24)	(20)	(17)	(16)	(14)	(11)	(9)	(9)	(7)	(6)	(6)	(6)
22/10/2008	(65)	(45)	(31)	(20)	(13)	(11)	(10)	(8)	(7)	(5)	(4)	(4)	(4)	(5)	(6)	(6)	(6)
10/10/2008	(34)	(34)	(32)	(30)	(28)	(25)	(23)	(20)	(17)	(14)	(14)	(14)	(13)	(13)	(12)	(12)	(11)
07/10/2008	(37)	(37)	(36)	(34)	(33)	(32)	(30)	(29)	(28)	(26)	(27)	(27)	(27)	(28)	(28)	(28)	(27)
25/09/2008	9	9	9	10	10	9	7	6	5	5	3	2	1	1	2	2	2
24/09/2008	15	15	15	15	16	15	13	11	9	8	5	2	1	1	1	2	2
23/09/2008	25	25	22	20	20	18	15	13	11	9	6	3	1	1	0	1	1
22/09/2008	28	28	23	20	18	16	15	12	10	8	5	3	2	(0)	(2)	(1)	(1)
09/06/2008	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1
03/07/2003	19	19	(5)	(4)	(3)	(3)	(3)	(2)	(0)	1	2	0	(3)	(4)	(5)	(7)	(7)
19/03/2003	2	2	12	14	15	13	12	12	11	11	9	7	6	4	3	3	3
27/05/2009	(4)	3	2	1	(2)	(4)	(4)	(6)	(6)	(6)	(5)	(3)	(0)	2	5	5	5
26/09/2008	13	13	12	10	9	7	6	4	3	2	1	(1)	(2)	(2)	(1)	(1)	(1)
17/09/2008	15	15	14	13	12	11	11	10	9	8	8	7	4	2	(1)	(2)	(2)
18/09/2008	17	17	16	16	18	17	16	14	12	10	8	6	3	0	(3)	(3)	(3)
08/10/2008	(33)	(33)	(31)	(30)	(28)	(26)	(25)	(23)	(22)	(20)	(20)	(19)	(18)	(17)	(16)	(16)	(15)
08/08/2011	(27)	(27)	(25)	(22)	(20)	(19)	(18)	(17)	(16)	(15)	(15)	(15)	(15)	(15)	(15)	(15)	(15)
09/10/2008	(29)	(29)	(26)	(24)	(21)	(20)	(18)	(17)	(15)	(14)	(14)	(13)	(13)	(12)	(12)	(11)	(11)
10/08/2011	(19)	(19)	(15)	(11)	(8)	(7)	(6)	(5)	(4)	(3)	(3)	(3)	(3)	(3)	(3)	(3)	(3)
30/09/2008	(3)	(3)	(2)	(2)	(3)	(3)	(3)	(4)	(4)	(4)	(4)	(5)	(5)	(5)	(5)	(5)	(5)
16/08/2007	(10)	(10)	(8)	(7)	(6)	(5)	(4)	(4)	(3)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)
17/12/2008	51	39	46	47	44	37	31	26	23	23	19	17	15	16	16	16	16
18/12/2008	17	17	23	23	20	15	12	10	8	10	7	4	3	3	2	2	2
05/11/2008	(21)	(21)	(17)	(13)	(10)	(8)	(8)	(9)	(9)	(10)	(11)	(12)	(13)	(13)	(13)	(13)	(13)
22/12/2008	(28)	(28)	(18)	(15)	(14)	(16)	(16)	(14)	(12)	(8)	(9)	(10)	(9)	(7)	(7)	(7)	(7)
09/12/2008	25	31	34	32	31	29	29	30	30	31	27	26	17	13	15	16	17
11/12/2008	50	38	34	34	39	40	37	32	30	23	25	29	27	22	20	20	20
08/12/2008	26	32	34	31	28	28	29	30	31	33	29	27	18	14	15	16	17
13/11/2008	(27)	(27)	(25)	(23)	(21)	(17)	(12)	(8)	(5)	(3)	(2)	(2)	(2)	(2)	(2)	(2)	(2)

11/11/2008	(39)	(40)	(37)	(34)	(31)	(26)	(20)	(15)	(14)	(10)	(8)	(7)	(7)	(7)	(7)	(7)
12/12/2008	74	62	59	60	65	63	56	48	41	33	34	36	34	28	26	26
08/01/2001	(19)	(19)	(19)	(19)	(19)	(19)	(19)	(19)	(19)	(18)	(16)	(14)	(11)	(7)	(4)	(4)
16/11/2001	16	16	16	16	16	16	16	16	16	16	16	15	14	14	13	13
19/03/2003	1	1	1	1	1	1	0	(1)	(2)	(4)	(4)	(4)	(5)	(5)	(5)	(5)
03/07/2003	2	2	2	2	2	2	3	4	5	6	7	8	9	10	10	10
05/10/1999	12	10	9	8	7	6	5	5	4	3	3	3	2	2	2	1
27/11/1997	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
03/04/1995	(11)	(10)	(9)	(7)	(7)	(6)	(5)	(4)	(3)	(2)	(2)	(2)	(2)	(2)	(2)	(2)
29/07/1994	3	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
29/03/1994	2	2	2	3	2	2	2	3	3	3	3	3	3	3	3	3
01/03/1994	3	12	12	11	11	10	9	8	8	7	7	7	7	7	7	7
10/08/2011	(21)	(23)	(20)	(17)	(15)	(13)	(11)	(10)	(8)	(7)	(7)	(7)	(7)	(7)	(7)	(7)

US-CPI

US-CPI	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y
27/05/2009	23	20	13	16	22	21	18	18	16	16	17	17	19	20	22
10/12/2008	34	71	(20)	(23)	(4)	(47)	(17)	(14)	(10)	(20)	(11)	(6)	(1)	4	(11)
03/12/2008	(32)	(111)	(83)	(28)	(22)	43	24	29	23	33	22	19	45	29	31
20/11/2008	(67)	(63)	(35)	(49)	(62)	(56)	(68)	(57)	(63)	(58)	(69)	(59)	(56)	(53)	(59)
12/11/2008	(1)	19	20	(7)	(12)	(28)	(21)	(22)	(9)	(4)	(12)	7	(1)	1	2
10/11/2008	(12)	12	18	33	37	38	(4)	(4)	13	26	24	36	28	48	38
07/11/2008	(16)	17	34	26	34	37	33	27	87	37	42	41	46	24	46
22/10/2008	(315)	(185)	(120)	(90)	(39)	(37)	(37)	(56)	(23)	(15)	(64)	(56)	(26)	(4)	(34)
10/10/2008	(64)	(125)	(92)	(77)	(71)	(76)	(53)	(62)	(57)	(51)	(45)	(39)	(38)	(38)	(34)
07/10/2008	(1)	(41)	(34)	(18)	(19)	(25)	(25)	(28)	(31)	(34)	(31)	(28)	(16)	(24)	(26)
25/09/2008	55	59	56	57	47	58	46	47	47	45	40	18	11	8	6
24/09/2008	31	36	28	28	18	14	31	33	34	27	32	26	25	18	23
23/09/2008	74	26	34	42	32	32	29	30	27	29	26	19	13	14	16
22/09/2008	64	28	26	33	31	29	37	28	13	28	27	31	22	21	7
09/06/2008	29	19	18	11	15	12	9	9	5	2	3	2	(8)	(7)	(6)
27/05/2009	23	20	13	16	22	21	18	18	16	16	17	17	19	20	22
26/09/2008	(76)	(25)	(16)	(0)	(7)	(9)	(11)	(15)	(18)	(12)	(13)	(14)	(12)	(12)	(13)
17/09/2008	(17)	(92)	(65)	(49)	(29)	(30)	(33)	(37)	(120)	(33)	(32)	(31)	(29)	(23)	(24)
18/09/2008	(64)	(92)	(73)	(60)	(47)	(53)	(45)	(44)	(40)	(38)	(35)	(37)	(35)	(31)	(26)
08/10/2008	(88)	(74)	(53)	(46)	(48)	(35)	(24)	(25)	(24)	(33)	(33)	(24)	(37)	(38)	(32)
08/08/2011	(87)	(58)	(43)	(33)	(27)	(28)	(25)	(20)	(17)	(16)	(17)	(14)	(13)	(12)	(11)
09/10/2008	(21)	(72)	(57)	(45)	(31)	(13)	11	5	3	(3)	(8)	(24)	(39)	(34)	(27)
10/08/2011	(52)	(25)	(16)	(3)	5	7	9	13	14	19	14	14	10	9	9
30/09/2008	(39)	9	(5)	(11)	(7)	(11)	(18)	(22)	(19)	(23)	(20)	(19)	(23)	(20)	(22)
16/08/2007	1	(8)	(3)	(2)	(5)	(6)	(6)	(7)	(5)	(1)	1	(1)	1	(5)	(1)
17/12/2008	(159)	3	40	46	15	16	25	28	27	25	16	12	7	5	1
18/12/2008	4	30	103	73	55	75	72	73	61	39	38	23	(6)	(14)	12
05/11/2008	50	55	98	83	60	57	47	46	34	31	39	30	31	32	32
22/12/2008	31	118	68	45	70	53	30	33	35	31	23	18	9	5	27
09/12/2008	75	73	64	33	39	(44)	(10)	(18)	(9)	(22)	(10)	(3)	12	9	14
11/12/2008	(5)	(0)	(41)	(36)	(43)	(41)	(44)	(43)	(37)	(38)	(33)	(21)	(7)	(2)	(8)
08/12/2008	42	(96)	(35)	(65)	(39)	(14)	(4)	0	1	(4)	7	9	29	38	24
13/11/2008	25	17	30	7	(13)	(26)	(17)	(25)	(26)	(21)	(14)	(15)	(11)	(14)	(16)
11/11/2008	(12)	15	9	1	0	(9)	2	(1)	8	2	10	11	1	20	12
12/12/2008	34	(13)	(1)	(42)	(4)	(35)	(33)	(32)	(15)	(20)	(28)	(19)	27	(3)	(8)

05/10/1999	15	15	15	15	15	15	15	14	14	13	11	10	10	10
08/01/2001	11	11	11	11	10	9	9	8	8	8	9	11	11	11
16/11/2001	23	23	23	23	23	23	24	24	25	25	24	22	22	22
19/03/2003	(11)	(11)	(11)	(11)	(11)	(9)	(7)	(6)	(4)	(3)	(2)	(1)	0	0
03/07/2003	(3)	(3)	(3)	(3)	(3)	(3)	(2)	(2)	(2)	(2)	(2)	(3)	(3)	(3)
27/11/1997	(0)	(0)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(2)
03/04/1995	7	6	5	4	4	3	3	3	2	3	3	3	3	3
29/07/1994	2	(7)	(7)	(7)	(7)	(7)	(7)	(6)	(6)	(6)	(7)	(7)	(7)	(8)
29/03/1994	(2)	6	7	8	10	10	10	10	10	10	10	11	11	12
01/03/1994	5	10	10	9	9	9	9	9	9	9	9	10	10	11
10/08/2011	9	21	20	19	18	17	16	16	15	14	15	15	16	17

Appendix 2 – Augmenting existing *relative* scenarios

Bold/yellow indicates that a change has been made to the raw data. The methodology is described in the body of the document and the attached spreadsheet within this section of the appendix details the workings. The following numbers are noted in percentage points.

UK-RPI

UK-RPI	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y	50Y
10/08/2011	(15.8%)	(15.6%)	(13.6%)	(12.0%)	(10.6%)	(9.7%)	(9.0%)	(8.2%)	(7.6%)	(6.9%)	(5.9%)	(4.8%)	(3.3%)	(3.2%)	(2.4%)	(2.4%)	(2.4%)
07/08/2009	(6.0%)	0.2%	0.3%	0.0%	(0.2%)	(0.3%)	(0.3%)	(0.3%)	(0.3%)	(0.1%)	(0.2%)	(0.2%)	(0.0%)	(0.2%)	(0.4%)	(0.3%)	(0.4%)
10/06/2009	(2.0%)	(1.1%)	(1.7%)	(2.2%)	(2.1%)	(1.8%)	(2.0%)	(1.6%)	(1.3%)	(1.1%)	(1.5%)	(2.2%)	(2.7%)	(2.4%)	(2.2%)	(2.1%)	(2.0%)
08/06/2009	5.0%	(0.8%)	(0.9%)	(0.6%)	(0.3%)	(0.6%)	(0.6%)	(0.3%)	0.1%	0.4%	0.2%	(0.3%)	(0.8%)	(1.0%)	(0.2%)	(0.3%)	(0.6%)
28/05/2009	2.2%	1.0%	1.2%	1.4%	1.6%	1.4%	1.3%	1.2%	1.2%	1.2%	1.3%	1.2%	0.8%	0.6%	0.5%	0.5%	0.5%
22/01/2009	6.9%	(159.1%)	43.5%	23.0%	16.6%	13.1%	10.9%	9.3%	8.2%	7.2%	5.9%	5.1%	4.5%	4.0%	2.8%	2.9%	3.1%
17/12/2008	(9.0%)	(21.1%)	(74.8%)	66.3%	24.1%	15.0%	10.8%	8.4%	6.8%	5.6%	4.4%	3.8%	4.1%	3.8%	4.0%	4.3%	5.1%
18/12/2008	4.2%	(14.3%)	(70.6%)	47.1%	16.9%	10.0%	7.0%	5.0%	3.5%	2.1%	1.5%	1.3%	1.9%	1.6%	1.7%	1.9%	2.6%
10/12/2008	(16.8%)	(34.5%)	(64.2%)	(268.4%)	108.2%	40.3%	22.0%	13.9%	9.4%	7.2%	4.8%	1.8%	0.7%	(0.2%)	(0.7%)	(1.2%)	(1.6%)
03/12/2008	51.4%	18.2%	3.9%	(0.6%)	(11.5%)	(4.6%)	(4.3%)	(4.2%)	(4.3%)	(4.4%)	(2.6%)	(2.1%)	(2.4%)	(3.4%)	(5.2%)	(4.6%)	(5.0%)
20/11/2008	128.4%	5.4%	(117.7%)	(65.1%)	(42.0%)	(30.1%)	(22.9%)	(18.8%)	(16.3%)	(14.4%)	(11.5%)	(7.2%)	(5.7%)	(5.3%)	(5.2%)	(4.5%)	(5.5%)
12/11/2008	(184.2%)	(86.2%)	(47.1%)	(32.6%)	(22.4%)	(15.8%)	(11.1%)	(7.4%)	(4.4%)	(1.8%)	(1.3%)	(0.9%)	(0.2%)	0.4%	1.0%	1.0%	1.1%
11/11/2008	(150.5%)	(74.5%)	(40.9%)	(28.1%)	(18.7%)	(13.5%)	(9.7%)	(6.7%)	(4.2%)	(2.1%)	(1.2%)	(0.6%)	0.1%	0.7%	1.3%	1.4%	1.4%
10/11/2008	(131.0%)	(78.4%)	(49.0%)	(34.9%)	(23.9%)	(18.2%)	(14.0%)	(10.6%)	(7.8%)	(5.4%)	(4.2%)	(3.1%)	(2.0%)	(1.0%)	(0.0%)	(0.0%)	(0.0%)
07/11/2008	(123.0%)	(75.3%)	(46.9%)	(32.7%)	(21.4%)	(16.7%)	(13.3%)	(10.5%)	(8.2%)	(6.3%)	(4.9%)	(3.5%)	(2.0%)	(0.7%)	0.6%	0.6%	0.6%
10/10/2008	(31.5%)	(26.9%)	(23.8%)	(20.8%)	(18.1%)	(15.8%)	(13.5%)	(11.5%)	(9.7%)	(7.9%)	(7.2%)	(4.8%)	(3.4%)	(2.8%)	(2.0%)	(2.1%)	(2.1%)
25/09/2008	1.1%	1.0%	1.0%	0.9%	1.1%	1.3%	1.5%	1.7%	1.9%	1.7%	1.6%	1.3%	1.0%	0.8%	(0.7%)	(0.7%)	
23/09/2008	4.3%	4.1%	3.9%	3.7%	3.6%	3.8%	3.9%	4.1%	4.2%	4.4%	4.2%	4.1%	3.9%	3.7%	3.6%	2.2%	2.4%
22/09/2008	(0.2%)	(0.2%)	(0.2%)	(0.2%)	(0.2%)	0.1%	0.3%	0.5%	0.7%	0.9%	0.9%	0.9%	1.0%	1.1%	1.1%	(0.3%)	(0.3%)
03/07/2003	(3.3%)	(3.3%)	(3.5%)	(3.7%)	(3.7%)	(3.8%)	(3.9%)	(3.8%)	(3.7%)	(3.5%)	(3.3%)	(2.6%)	(2.6%)	(2.8%)	(2.6%)	(2.4%)	(2.2%)
19/03/2003	3.5%	3.5%	6.7%	6.8%	5.8%	5.3%	5.1%	4.7%	4.7%	4.5%	4.4%	4.5%	4.5%	3.9%	3.6%	3.1%	3.1%
03/01/2000	2.7%	2.7%	2.4%	2.3%	2.3%	2.3%	2.3%	2.3%	2.4%	2.3%	2.2%	1.9%	1.5%	1.6%	1.6%	1.6%	1.6%
05/10/1999	6.7%	6.7%	4.6%	3.6%	3.0%	2.6%	2.2%	1.8%	1.4%	1.0%	0.0%	(1.6%)	(4.3%)	(5.4%)	(5.4%)	(5.4%)	(5.4%)
04/10/1999	10.9%	10.9%	8.0%	6.3%	5.2%	4.3%	3.6%	2.8%	2.1%	1.4%	0.1%	(2.2%)	(6.0%)	(7.8%)	(7.8%)	(7.8%)	(7.8%)
01/10/1999	13.6%	13.6%	10.5%	8.8%	7.8%	7.2%	6.6%	6.1%	5.7%	5.3%	4.3%	2.7%	(0.3%)	(1.8%)	(1.8%)	(1.8%)	(1.8%)
04/03/1999	6.9%	6.9%	6.4%	6.3%	6.3%	6.3%	6.2%	6.0%	5.9%	5.7%	5.3%	4.7%	4.3%	4.7%	4.7%	4.7%	4.7%
03/07/2003	(5.4%)	(5.4%)	(3.5%)	(0.9%)	0.8%	1.8%	2.4%	2.7%	2.8%	2.8%	2.3%	1.4%	0.3%	0.1%	0.1%	0.1%	0.1%
19/03/2003	(0.3%)	(0.3%)	1.9%	2.1%	1.7%	1.1%	0.5%	0.2%	0.0%	(0.1%)	(0.1%)	0.1%	0.8%	1.6%	1.6%	1.6%	1.6%

EU-HICPxT

EU-HICP	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y	50Y
10/08/2011	(10.6%)	(10.6%)	(8.1%)	(6.0%)	(3.8%)	(2.2%)	(2.7%)	(2.2%)	(1.6%)	(1.2%)	(1.1%)	(1.1%)	(1.1%)	(1.0%)	(1.0%)	(1.0%)	(1.0%)
07/08/2009	2.1%	2.3%	2.5%	2.1%	2.8%	2.2%	2.1%	1.9%	1.8%	1.5%	1.2%	1.2%	1.1%	0.9%	0.8%	0.8%	0.8%
10/06/2009	(0.6%)	0.7%	1.5%	2.7%	3.8%	3.4%	2.6%	2.2%	1.8%	1.4%	1.7%	1.8%	1.9%	2.0%	2.1%	2.1%	2.1%
08/06/2009	(2.2%)	(2.0%)	(0.6%)	0.3%	0.9%	1.1%	0.7%	0.4%	0.3%	0.3%	0.4%	0.4%	0.3%	0.2%	0.1%	0.1%	0.1%
28/05/2009	9.0%	4.0%	3.4%	2.5%	0.9%	0.8%	0.2%	0.2%	0.2%	0.1%	0.0%	1.1%	1.9%	2.5%	2.8%	2.8%	2.8%
22/01/2009	15.9%	6.4%	5.4%	5.2%	5.2%	5.4%	5.7%	5.8%	6.1%	6.0%	5.7%	4.6%	4.2%	3.6%	3.2%	3.2%	3.1%
17/12/2008	92.2%	92.2%	54.6%	38.7%	30.1%	21.9%	16.2%	12.3%	9.5%	7.5%	6.5%	6.4%	6.4%	6.8%	7.4%	7.4%	7.3%
18/12/2008	57.5%	22.4%	19.7%	14.9%	13.3%	9.4%	6.2%	3.4%	1.3%	0.1%	(0.8%)	(0.3%)	0.0%	(0.3%)	(0.2%)	(0.2%)	(0.1%)
10/12/2008	(147.0%)	92.1%	60.0%	40.4%	29.5%	25.1%	19.3%	16.3%	13.9%	12.0%	11.4%	10.5%	8.6%	5.9%	4.9%	4.9%	4.9%
03/12/2008	(64.3%)	125.3%	45.5%	35.1%	26.2%	20.7%	18.3%	16.3%	15.4%	16.3%	15.0%	13.3%	9.9%	5.9%	5.8%	6.4%	7.0%
20/11/2008	65.2%	65.2%	28.5%	16.0%	12.1%	7.4%	6.0%	5.2%	4.5%	3.9%	3.4%	2.6%	0.9%	0.3%	0.6%	0.6%	0.6%
12/11/2008	(94.4%)	(59.0%)	(44.4%)	(33.3%)	(25.4%)	(18.1%)	(12.9%)	(9.0%)	(6.4%)	(5.2%)	(4.3%)	(3.9%)	(3.1%)	(2.9%)	(2.7%)	(2.7%)	(2.7%)
11/11/2008	(83.3%)	(53.5%)	(40.7%)	(30.5%)	(23.1%)	(16.8%)	(12.3%)	(9.1%)	(6.8%)	(5.0%)	(4.5%)	(4.2%)	(3.6%)	(3.3%)	(3.2%)	(3.2%)	(3.2%)
10/11/2008	(77.5%)	(53.1%)	(40.3%)	(29.4%)	(21.1%)	(15.1%)	(11.1%)	(8.0%)	(5.5%)	(4.0%)	(2.8%)	(2.3%)	(1.7%)	(1.2%)	(1.0%)	(1.0%)	(1.0%)
07/11/2008	(80.2%)	(53.8%)	(38.8%)	(29.0%)	(22.0%)	(16.8%)	(12.9%)	(10.0%)	(7.7%)	(5.8%)	(4.5%)	(4.1%)	(3.4%)	(3.0%)	(2.9%)	(2.9%)	(2.9%)
10/10/2008	(15.9%)	(14.3%)	(13.0%)	(11.9%)	(10.9%)	(10.0%)	(9.1%)	(8.3%)	(7.5%)	(6.9%)	(6.4%)	(5.9%)	(5.0%)	(4.3%)	(3.5%)	(3.0%)	(2.8%)
25/09/2008	4.8%	4.2%	3.3%	4.2%	4.0%	3.9%	3.3%	2.9%	2.7%	2.5%	1.8%	1.2%	0.8%	0.6%	0.6%	0.6%	0.6%
23/09/2008	9.5%	8.4%	7.4%	7.0%	6.8%	5.8%	4.8%	3.9%	3.1%	2.5%	1.6%	1.0%	1.0%	1.3%	1.2%	1.2%	1.2%
22/09/2008	8.1%	7.1%	6.1%	5.6%	5.3%	4.6%	3.8%	3.3%	2.7%	2.1%	1.6%	1.1%	0.8%	0.6%	0.3%	0.3%	0.3%
03/07/2003	8.8%	(2.3%)	(0.9%)	(2.3%)	(2.8%)	(3.2%)	(3.6%)	(3.3%)	(3.0%)	(2.7%)	(1.5%)	(2.7%)	(3.8%)	(3.1%)	(2.9%)	(3.5%)	(3.1%)
19/03/2003	(0.3%)	2.1%	4.6%	4.9%	5.0%	4.8%	5.0%	4.8%	4.3%	3.8%	3.3%	2.4%	2.1%	1.4%	1.2%	1.2%	1.1%
03/01/2000	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.0%	10.2%	9.1%	8.1%	7.3%	7.3%	7.3%
03/07/2003	1.3%	1.3%	1.3%	1.3%	1.3%	1.3%	1.9%	2.6%	3.2%	3.9%	4.1%	4.3%	4.7%	5.0%	5.1%	5.1%	5.1%
19/03/2003	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.1%	(0.7%)	(1.4%)	(2.1%)	(2.3%)	(2.4%)	(2.5%)	(2.6%)	(2.6%)	(2.6%)	(2.6%)
05/10/1999	3.4%	2.6%	2.1%	1.6%	1.2%	1.2%	1.2%	0.8%	0.8%	0.7%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%
04/10/1999	3.4%	2.6%	2.5%	2.1%	1.6%	1.6%	1.2%	1.2%	0.8%	0.7%	0.7%	0.7%	0.4%	0.4%	0.4%	0.4%	0.4%
01/10/1999	2.6%	1.7%	2.1%	2.1%	2.4%	2.4%	2.0%	1.9%	1.9%	1.5%	1.5%	1.5%	1.5%	1.6%	1.2%	1.2%	1.2%
04/03/1999	3.0%	2.1%	2.1%	1.6%	1.6%	1.6%	2.0%	1.9%	1.9%	2.2%	2.2%	2.2%	2.3%	1.9%	1.9%	1.9%	1.9%

FR-CPI

FR-CPI	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y	50Y
10/08/2011	(11.3%)	(11.3%)	(8.2%)	(5.7%)	(3.5%)	(3.0%)	(2.4%)	(2.0%)	(1.5%)	(1.1%)	(1.0%)	(1.0%)	(1.0%)	(1.0%)	(1.0%)	(1.0%)	(1.0%)
07/08/2009	45.9%	7.1%	5.3%	4.9%	4.6%	3.7%	2.2%	2.0%	1.8%	1.3%	1.2%	1.2%	1.0%	0.9%	0.8%	0.8%	0.8%
10/06/2009	12.8%	5.0%	3.8%	4.3%	5.9%	5.8%	5.0%	4.1%	3.4%	2.8%	3.1%	3.3%	3.1%	2.1%	2.0%	2.0%	1.9%
08/06/2009	5.7%	2.4%	1.2%	1.5%	2.9%	2.6%	2.1%	1.5%	1.0%	1.2%	1.5%	1.5%	1.2%	0.0%	(0.2%)	(0.2%)	(0.3%)
28/05/2009	2.6%	5.8%	2.9%	1.4%	(0.7%)	(1.1%)	(1.3%)	(1.4%)	(1.6%)	(1.3%)	(1.1%)	(0.6%)	0.3%	1.2%	2.1%	2.1%	2.1%
22/01/2009	(43.9%)	3.7%	5.3%	6.0%	8.1%	7.8%	7.2%	7.2%	7.6%	7.3%	7.6%	6.1%	5.1%	5.6%	5.4%	5.3%	5.3%
17/12/2008	113.9%	113.9%	63.5%	45.7%	33.4%	23.6%	17.5%	13.5%	11.5%	11.0%	8.9%	7.9%	7.2%	8.2%	8.3%	8.2%	8.2%
18/12/2008	71.6%	32.6%	25.5%	19.0%	13.0%	8.6%	6.4%	4.8%	3.8%	4.8%	3.0%	1.7%	1.2%	1.6%	0.9%	0.9%	0.9%
10/12/2008	(76.2%)	0.9%	48.8%	29.7%	29.5%	26.4%	22.8%	19.8%	17.0%	13.8%	13.5%	13.8%	10.2%	7.3%	7.2%	7.2%	7.2%
03/12/2008	(41.8%)	(164.7%)	69.9%	37.5%	25.7%	19.5%	17.7%	14.8%	12.7%	12.4%	10.7%	9.5%	9.5%	8.0%	7.6%	8.2%	8.8%
20/11/2008	(85.7%)	(21.4%)	43.0%	19.4%	13.5%	10.1%	8.6%	6.7%	5.6%	5.6%	4.2%	2.7%	1.3%	0.2%	(0.5%)	(0.5%)	(0.5%)
12/11/2008	(93.9%)	(93.9%)	(47.7%)	(33.6%)	(26.6%)	(20.2%)	(14.9%)	(10.6%)	(8.4%)	(5.2%)	(3.8%)	(3.3%)	(3.3%)	(3.2%)	(3.3%)	(3.3%)	(3.3%)
11/11/2008	(76.8%)	(76.8%)	(39.5%)	(27.6%)	(21.6%)	(16.0%)	(11.8%)	(8.5%)	(7.2%)	(5.2%)	(3.9%)	(3.3%)	(3.4%)	(3.4%)	(3.4%)	(3.4%)	(3.4%)
10/11/2008	(69.8%)	(69.8%)	(38.1%)	(26.2%)	(19.6%)	(14.4%)	(10.3%)	(7.1%)	(5.8%)	(3.8%)	(2.3%)	(1.5%)	(1.1%)	(0.6%)	(0.2%)	(0.2%)	(0.2%)
07/11/2008	(66.1%)	(66.1%)	(34.5%)	(24.8%)	(19.4%)	(14.4%)	(11.3%)	(9.0%)	(8.2%)	(6.9%)	(5.4%)	(4.4%)	(4.2%)	(3.3%)	(2.9%)	(2.9%)	(2.9%)
10/10/2008	(19.5%)	(16.7%)	(14.8%)	(13.5%)	(12.4%)	(11.0%)	(9.6%)	(8.4%)	(7.1%)	(5.9%)	(5.8%)	(5.6%)	(5.5%)	(5.3%)	(5.1%)	(4.8%)	(4.7%)
25/09/2008	4.7%	4.1%	4.0%	4.0%	3.9%	3.4%	2.8%	2.4%	1.9%	1.8%	1.3%	0.8%	0.3%	0.4%	0.8%	0.8%	0.8%
23/09/2008	14.4%	12.3%	10.1%	9.0%	8.4%	7.5%	6.3%	5.4%	4.4%	3.6%	2.5%	1.2%	0.5%	0.2%	0.1%	0.3%	0.3%
22/09/2008	16.5%	14.0%	10.9%	8.9%	7.7%	6.7%	6.2%	5.0%	3.9%	3.1%	1.8%	1.2%	0.8%	(0.1%)	(0.6%)	(0.5%)	(0.5%)
03/07/2003	13.4%	13.4%	(2.8%)	(2.7%)	(1.7%)	(1.7%)	(1.7%)	(0.9%)	(0.1%)	0.6%	1.1%	0.2%	(1.5%)	(1.6%)	(2.1%)	(3.0%)	(3.0%)
19/03/2003	0.8%	0.8%	6.9%	7.9%	8.5%	7.4%	6.6%	6.4%	6.0%	5.5%	4.4%	3.2%	2.6%	1.8%	1.4%	1.4%	1.3%
03/01/2000	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.0%	10.2%	9.1%	8.1%	7.3%	7.3%	7.3%
03/07/2003	1.3%	1.3%	1.3%	1.3%	1.3%	1.3%	1.9%	2.6%	3.2%	3.9%	4.1%	4.3%	4.7%	5.0%	5.1%	5.1%	5.1%
19/03/2003	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.1%	(0.7%)	(1.4%)	(2.1%)	(2.3%)	(2.4%)	(2.5%)	(2.6%)	(2.6%)	(2.6%)	(2.6%)
05/10/1999	3.4%	2.6%	2.1%	1.6%	1.2%	1.2%	1.2%	0.8%	0.8%	0.7%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%
04/10/1999	3.4%	2.6%	2.5%	2.1%	1.6%	1.6%	1.2%	1.2%	0.8%	0.7%	0.7%	0.7%	0.4%	0.4%	0.4%	0.4%	0.4%
01/10/1999	2.6%	1.7%	2.1%	2.1%	2.4%	2.4%	2.0%	1.9%	1.9%	1.5%	1.5%	1.5%	1.5%	1.6%	1.2%	1.2%	1.2%
04/03/1999	3.0%	2.1%	2.1%	1.6%	1.6%	1.6%	2.0%	1.9%	1.9%	2.2%	2.2%	2.2%	2.3%	1.9%	1.9%	1.9%	1.9%

US-CPI

US-CPI	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y
10/08/2011	(40.3%)	(14.1%)	(7.7%)	(1.2%)	2.1%	2.8%	3.7%	5.0%	5.5%	7.0%	5.1%	4.8%	3.3%	3.1%	3.2%
07/08/2009	24.7%	22.2%	3.4%	13.5%	12.8%	12.9%	9.4%	7.5%	7.5%	7.6%	7.5%	7.0%	7.9%	6.6%	5.4%
10/06/2009	255.6%	66.3%	37.5%	31.9%	21.7%	19.0%	16.1%	13.6%	11.4%	10.2%	10.7%	10.0%	9.7%	8.9%	10.2%
08/06/2009	49.9%	49.9%	28.0%	24.1%	17.3%	11.4%	9.7%	8.3%	7.4%	6.9%	5.9%	5.2%	5.2%	5.1%	3.5%
28/05/2009	(34.1%)	99.4%	27.7%	14.4%	17.9%	13.7%	10.2%	8.6%	7.6%	7.5%	8.0%	10.6%	8.0%	7.8%	5.8%
22/01/2009	(23.9%)	(31.2%)	(81.0%)	(18.1%)	44.8%	10.3%	2.0%	1.0%	2.6%	4.5%	5.7%	4.6%	1.7%	1.9%	3.6%
17/12/2008	44.8%	(1.3%)	(20.0%)	(43.8%)	(41.2%)	114.7%	45.5%	36.0%	25.2%	18.8%	10.7%	7.6%	4.0%	2.8%	0.6%
18/12/2008	(1.0%)	(11.4%)	(53.3%)	(69.7%)	(123.9%)	21.3%	166.5%	108.9%	62.6%	29.2%	26.1%	14.2%	(3.1%)	(7.4%)	6.3%
10/12/2008	(8.7%)	(21.4%)	11.2%	28.6%	13.0%	(77.0%)	(23.8%)	(15.3%)	(8.9%)	(12.9%)	(7.1%)	(3.7%)	(0.7%)	2.2%	(5.7%)
03/12/2008	8.8%	50.8%	87.4%	53.5%	195.0%	239.2%	49.9%	46.0%	25.3%	26.9%	15.6%	12.7%	34.6%	19.4%	19.4%
20/11/2008	30.1%	44.2%	125.1%	(64.3%)	(50.0%)	(43.2%)	(42.0%)	(35.2%)	(33.5%)	(27.7%)	(30.6%)	(26.3%)	(23.4%)	(22.1%)	(24.7%)
12/11/2008	0.2%	(11.8%)	(36.1%)	(7.7%)	(8.2%)	(17.4%)	(12.4%)	(11.6%)	(4.1%)	(1.8%)	(5.2%)	3.0%	(0.4%)	0.3%	0.8%
11/11/2008	4.9%	(9.3%)	(21.9%)	0.8%	0.1%	(5.9%)	1.2%	(0.6%)	4.1%	0.9%	4.6%	5.0%	0.2%	8.8%	5.4%
10/11/2008	5.0%	(7.6%)	(37.3%)	64.6%	37.2%	34.4%	(2.3%)	(2.1%)	6.8%	13.0%	11.9%	18.2%	13.9%	23.8%	19.1%
07/11/2008	6.9%	(11.0%)	(55.3%)	70.4%	32.7%	30.6%	24.5%	18.0%	70.6%	19.0%	21.8%	20.7%	23.0%	11.6%	24.5%
10/10/2008	(40.2%)	(98.3%)	(58.1%)	(42.8%)	(35.3%)	(36.7%)	(25.5%)	(29.3%)	(26.5%)	(23.2%)	(20.6%)	(17.6%)	(17.0%)	(17.1%)	(15.4%)
25/09/2008	46.3%	73.8%	46.3%	39.4%	28.0%	34.4%	25.1%	24.5%	24.2%	22.4%	19.4%	8.3%	5.2%	3.7%	2.5%
23/09/2008	71.4%	28.5%	26.3%	28.5%	18.7%	17.3%	14.5%	14.6%	12.6%	13.3%	11.8%	8.4%	5.6%	6.1%	6.7%
22/09/2008	43.2%	21.9%	16.4%	18.7%	16.0%	14.5%	17.9%	13.0%	5.4%	12.5%	12.2%	13.6%	9.4%	8.7%	2.7%
03/01/2000	7.1%	7.1%	7.1%	7.1%	7.1%	6.5%	5.9%	5.4%	5.1%	4.8%	4.5%	4.3%	4.1%	4.1%	4.1%
05/10/1999	7.6%	7.6%	7.6%	7.6%	7.6%	7.4%	7.1%	6.7%	6.3%	5.9%	5.2%	4.4%	3.7%	3.7%	3.7%
04/10/1999	6.4%	6.4%	6.4%	6.4%	6.4%	6.7%	6.8%	6.6%	6.3%	6.0%	5.2%	4.2%	3.2%	3.2%	3.2%
01/10/1999	12.5%	12.5%	12.5%	12.5%	12.5%	12.4%	12.0%	11.6%	11.1%	10.5%	9.5%	8.2%	7.1%	7.1%	7.1%
04/03/1999	(3.2%)	(3.2%)	(3.2%)	(3.2%)	(3.2%)	(1.5%)	0.1%	1.4%	2.2%	2.7%	3.2%	3.1%	2.4%	2.4%	2.4%
03/07/2003	(2.4%)	(2.4%)	(2.4%)	(2.4%)	(2.4%)	(2.0%)	(1.6%)	(1.3%)	(1.1%)	(1.0%)	(0.9%)	(1.0%)	(1.3%)	(1.3%)	(1.3%)
19/03/2003	(6.5%)	(6.5%)	(6.5%)	(6.5%)	(6.5%)	(5.1%)	(3.9%)	(2.9%)	(2.2%)	(1.6%)	(0.8%)	(0.2%)	0.1%	0.1%	0.1%

Appendix 3 – New Absolute scenario definitions

This section details the new shifts that should be added to the, historical, absolute suite of scenarios.

Inflation shifts: UK-RPI

UK-RPI	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y	50Y
23/09/1992	(3)	(3)	(3)	(3)	16	35	52	66	78	87	101	112	112	112	112	112	112
17/05/1990	(126)	(126)	(126)	(126)	(106)	(91)	(79)	(70)	(62)	(55)	(45)	(36)	(36)	(36)	(36)	(36)	(36)
14/04/1992	(107)	(107)	(107)	(107)	(99)	(92)	(87)	(82)	(78)	(74)	(66)	(54)	(54)	(54)	(54)	(54)	(54)
03/02/2009	167	117	96	83	74	68	62	54	47	41	29	16	2	0	1	2	3
16/01/2013	28	41	44	47	49	48	46	44	42	40	39	35	31	30	27	28	29

Inflation shifts: US-CPI

USCPI	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y
22/01/2009	75	73	64	33	39	(44)	(10)	(18)	(9)	(22)	(10)	(3)	12	9	14
25/11/2008	(188)	(77)	(86)	(112)	(115)	(95)	(89)	(73)	(79)	(82)	(70)	(93)	(123)	(100)	(85)
22/10/2008	(315)	(185)	(120)	(90)	(39)	(37)	(37)	(56)	(23)	(15)	(64)	(56)	(26)	(4)	(34)
20/10/2008	(151)	(151)	(91)	(40)	(9)	6	13	16	17	18	23	34	52	52	52
09/02/2009	72	72	69	60	51	43	36	31	27	24	21	19	23	23	23

Inflation shifts: FR-CPI

FR-CPI	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40y	50y
12/12/2008	74	62	59	60	65	63	56	48	41	33	34	36	34	28	26	26	26
13/01/2009	30	50	53	54	46	37	31	25	22	19	17	18	20	19	18	18	18
29/10/2008	(71)	(67)	(57)	(49)	(44)	(38)	(32)	(27)	(24)	(22)	(18)	(14)	(11)	(12)	(11)	(12)	(13)
27/11/2008	(37)	(35)	(35)	(36)	(35)	(32)	(33)	(32)	(32)	(34)	(33)	(34)	(34)	(33)	(32)	(32)	(32)

Inflation shifts: EU-HICPxT

EU-HICPxT	1y	2y	3y	4y	5y	6Y	7Y	8Y	9Y	10Y	12Y	15Y	20Y	25Y	30Y	40Y	50Y
31/05/2005	82	43	26	19	14	10	7	5	4	2	(1)	2	1	(1)	(4)	(4)	(5)
12/12/2008	40	53	60	62	62	59	50	44	39	39	31	30	28	26	24	24	24
31/05/2011	(80)	(42)	(34)	(26)	(19)	(16)	(14)	(12)	(10)	(8)	(8)	(8)	(7)	(7)	(6)	(6)	(6)
26/11/2008	(46)	(46)	(46)	(46)	(37)	(38)	(39)	(41)	(42)	(41)	(40)	(39)	(47)	(48)	(48)	(48)	(48)

Appendix 4 – Hypothetical Inflation Only Scenarios

The following shifts are quoted in basis points.

UKRPI	Sell off	Rally	Bull Steepen	Bull Flatten	Bear Steepen	Bear Flatten
1Y	(62)	112	3	112	(62)	(2)
2Y	(62)	112	7	108	(60)	(4)
3Y	(62)	112	10	105	(59)	(5)
4Y	(62)	112	13	102	(57)	(7)
5Y	(62)	112	16	99	(55)	(9)
6Y	(62)	112	19	96	(53)	(11)
7Y	(62)	112	22	93	(52)	(12)
8Y	(62)	112	25	90	(50)	(14)
9Y	(62)	112	28	87	(48)	(16)
10Y	(62)	112	31	84	(47)	(17)
12Y	(62)	112	37	78	(44)	(20)
15Y	(62)	112	45	70	(39)	(25)
20Y	(62)	112	57	58	(32)	(32)
25Y	(62)	112	69	47	(26)	(38)
30Y	(62)	112	79	36	(20)	(44)
40Y	(62)	112	97	18	(10)	(54)
50Y	(62)	112	112	4	(2)	(62)

EU HICPx	Rally	Sell off	Bull steepen	Bull flatten	Bear steepen	Bear flatten
1Y	39	(38)	2	39	(38)	(2)
2Y	39	(38)	3	38	(36)	(3)
3Y	39	(38)	5	36	(35)	(5)
4Y	39	(38)	6	35	(33)	(6)
5Y	39	(38)	8	33	(32)	(8)
6Y	39	(38)	9	32	(30)	(9)
7Y	39	(38)	11	30	(29)	(11)
8Y	39	(38)	12	29	(27)	(12)
9Y	39	(38)	14	27	(26)	(13)
10Y	39	(38)	15	26	(24)	(15)
12Y	39	(38)	18	23	(22)	(17)
15Y	39	(38)	22	20	(18)	(21)
20Y	39	(38)	28	14	(12)	(27)
25Y	39	(38)	33	8	(7)	(33)
30Y	39	(38)	39	3	(2)	(38)

FRxT	Rally	Sell off	Bull Steepen	Bull Flatten	Bear Steepen	Bear flatten
1Y	38	(41)	2	37	(41)	(2)
2Y	38	(41)	3	35	(39)	(3)
3Y	38	(41)	5	34	(37)	(5)
4Y	38	(41)	6	32	(36)	(7)
5Y	38	(41)	8	31	(34)	(8)
6Y	38	(41)	9	29	(32)	(10)
7Y	38	(41)	11	28	(31)	(12)
8Y	38	(41)	12	26	(29)	(13)
9Y	38	(41)	14	25	(27)	(15)
10Y	38	(41)	15	23	(26)	(16)
12Y	38	(41)	18	21	(23)	(19)
15Y	38	(41)	22	17	(18)	(23)
20Y	38	(41)	28	11	(11)	(30)
25Y	38	(41)	33	6	(5)	(35)
30Y	38	(41)	38	1	0	(41)

US CPI	Rally	Sell off	Bull Steepen	Bull Flatten	Bear Steepen	Bear flatten
1Y	70	-115	3	69	-114	-5
2Y	70	-115	5	66	-109	-10
3Y	70	-115	8	63	-104	-15
4Y	70	-115	12	60	-99	-19
5Y	70	-115	15	58	-95	-23
6Y	70	-115	18	55	-90	-25
7Y	70	-115	21	52	-85	-32
8Y	70	-115	22	49	-82	-37
9Y	70	-115	25	47	-77	-41
10Y	70	-115	27	44	-73	-46
12Y	70	-115	33	40	-64	-53
15Y	70	-115	40	32	-53	-66
20Y	70	-115	51	21	-35	-83
25Y	70	-115	62	11	-17	-100
30Y	70	-115	70	1	-2	-115

Appendix 5 – Hypothetical Real Rate Scenarios

Tenor (Y)	UKRPI						USCPI					
	Real rates up	Inflation	Nominal	Real rates down	Inflation	Nominal	Real rates up	Inflation	Nominal	Real rates down	Inflation	Nominal
1Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
2Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
3Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
4Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
5Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
6Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
7Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
8Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
9Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
10Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
12Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
15Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
20Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
25Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
30Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
40Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
50Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)

Tenor (Y)	FRxT						HICPx					
	Real rates up	Inflation	Nominal	Real rates down	Inflation	Nominal	Real rates up	Inflation	Nominal	Real rates down	Inflation	Nominal
1Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
2Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
3Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
4Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
5Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
6Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
7Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
8Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
9Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
10Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
15Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
20Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
25Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)
30Y	100	(50)	50	(100)	50	(50)	100	(50)	50	(100)	50	(50)

Appendix 6 – Seasonality Cleansing Algorithm



Seasonality cleansing
1.0.xlsx