PAIRS
Scaling of Historical Scenarios

SwapClear Quantitative Analytics

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1 Introduction

This document is intended as an addendum to the updated PAIRS model[1]. The PAIRS model is the quantitative model that calculates the SwapClear initial margin requirement of members and clients. PAIRS is a filtered historical model, i.e. it looks at historical events that occurred within the look-back period and from these infer a suitable initial margin which, using a statistical measurement, limit future losses and/or frequency of breaches of a specific portfolio over the holding period. PAIRS treats each historic event over overlapping holding periods, in this case 5 days, as a return. The returns are then scaled before they are applied to the rate environment of today, simulating potential future outcomes which are sorted in order of increasing impact to the portfolio in question.

To clarify, each historical return time series, $\mathbf{R}_N = \{R_t : t \in T\}$ is scaled according to the following equation,

$$\mathbf{S}_N = \left\{ R_t \cdot \left(\frac{\sigma_N}{\sigma_t} + 1 \right) \Big/ 2 : t \in T \right\}, \tag{1}$$

where $\mathbf{S}_N$ is the scaled time series up to date N.

For reference in this document, using the notation from Eq. 1, we denote the scaling factors $\mathbf{SF}_N(i)$ for each risk factor as

$$\mathbf{SF}_N(i) = \left\{ \left(\frac{\sigma_N^i}{\sigma_t^i} + 1 \right) \Big/ 2 : t \in T \right\}, \tag{2}$$

where N is the date for which initial margin is calculated, t is the scenario date and i is the index of the risk factor. One can think of the scaling factors as a scalar field over three dimensions. A popular equivalent form of the above equation is:

$$\mathbf{SF}_N(i) = \left\{ \left(\frac{\sigma_N^i + \sigma_t^i}{2\sigma_t^i} \right) : t \in T \right\}. \tag{3}$$

This document tries to provide clarity on the topic of scaling the historical scenarios. It also addresses whether the scaling is appropriate for preserving the historical 'correlation' prevalent in the markets at the time of the scenario. Potentially if there is a large asymmetry in how rates are scaled – i.e. some rates are scaled differently from other rates – the 'same correlation' would not exist between scaled risk factors and unscaled risk factors. In this document the notion of correlation being a relevant metric is challenged and a more relevant metric is proposed.

1.1 Mid-Volatility Scaling vs Hull-White Scaling

The scaling factors in Eq.2 that are used in the PAIRS model employ a methodology termed 'Mid-Volatility Scaling', which is the average of a methodology called 'Hull-White' or 'Full' Scaling and unity. The Hull-White Scaling Factors can be denoted as:

$$\mathbf{HWSF}_N(i) = \left\{ \left(\frac{\sigma_N^i}{\sigma_t^i} \right) : t \in T \right\}, \tag{4}$$

where N is the date for which initial margin is calculated, t is the scenario date and i is the index of the risk factor.

1.1.1 Graphical Comparison of Mid-Volatility Scaling vs Hull-White Scaling

Figure 1 below shows some time series of the Mid-Volatility scaling factors (in black) and Hull-White scaling factors (in red) under the IMv2 methodology, where absolute returns are used. For each scaling methodology, five graphs are shown that are composed of respectively the largest, 90th percentile, median, 10th percentile and smallest scaling factors across all risk factors on each particular date. The graphs are labelled respectively as quantile 1, 0.9, 0.5, 0.1 and 0.0. These graphs give an indication of how the distribution of scaling factors (from smallest to largest, across all risk factors) changes over time under the two methodologies.

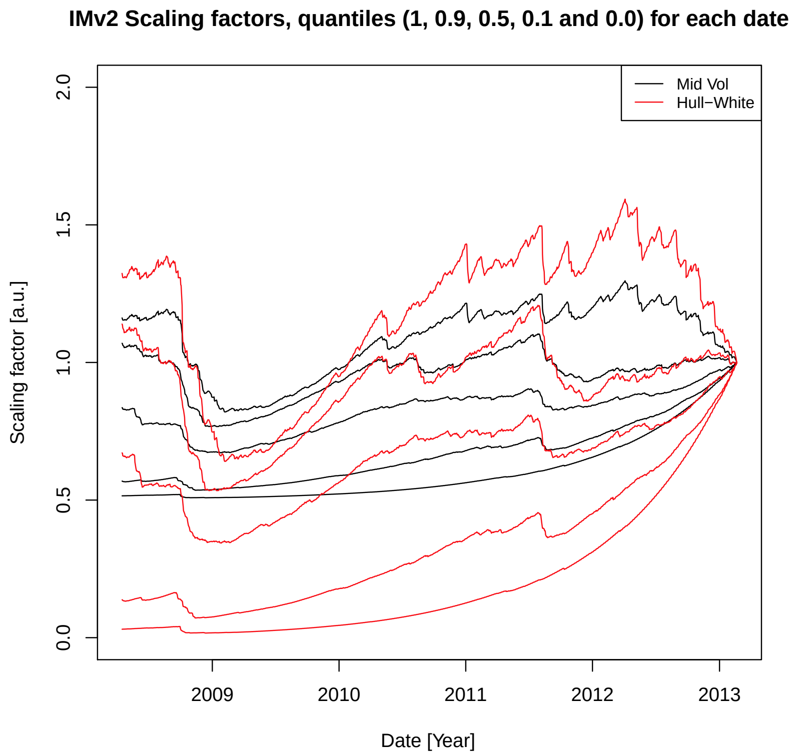

Figure 1: IMv2 Mid Volatility vs Hull-White scaling factors

One can see that in the present market enviroment for most of the quantiles, on most dates, the mid-volatility scaling factors are higher than the Hull-White scaling factors. In fact, below the line $y = 1$, the Mid-Volatility scaling factors are always higher than the Hull-White scaling factors for each quantile. This is expected because for $y < 1$,

$$\left(\frac{\sigma_N^i}{\sigma_t^i}\right) < 1 \tag{5}$$

$$\Leftrightarrow \quad \left(1 - \frac{\sigma_N^i}{\sigma_t^i}\right) \Big/ 2 > 0 \tag{6}$$

$$\Leftrightarrow \quad \left(\frac{\sigma_N^i}{\sigma_t^i} + 1\right) \Big/ 2 - \frac{\sigma_N^i}{\sigma_t^i} > 0 \tag{7}$$

$$\Leftrightarrow \quad \left(\frac{\sigma_N^i}{\sigma_t^i} + 1\right) \Big/ 2 > \frac{\sigma_N^i}{\sigma_t^i} \tag{8}$$

$$\Leftrightarrow \quad SF_t(i) > HWSF_t(i), \tag{9}$$

where $SF_t(i), HWSF_t(i)$ are the mid volatility and Hull-White scaling factors respectively, on date t for risk factor i. One can also see this graphically in figure 2 below, which shows the graph of mid volatility vs Hull-White scaling factors in red. The grey dotted line is the line $y = x$. The graph shows similarly that $SF_t(i) > HWSF_t(i) \Leftrightarrow \left(\frac{\sigma_N^i}{\sigma_t^i}\right) < 1$.

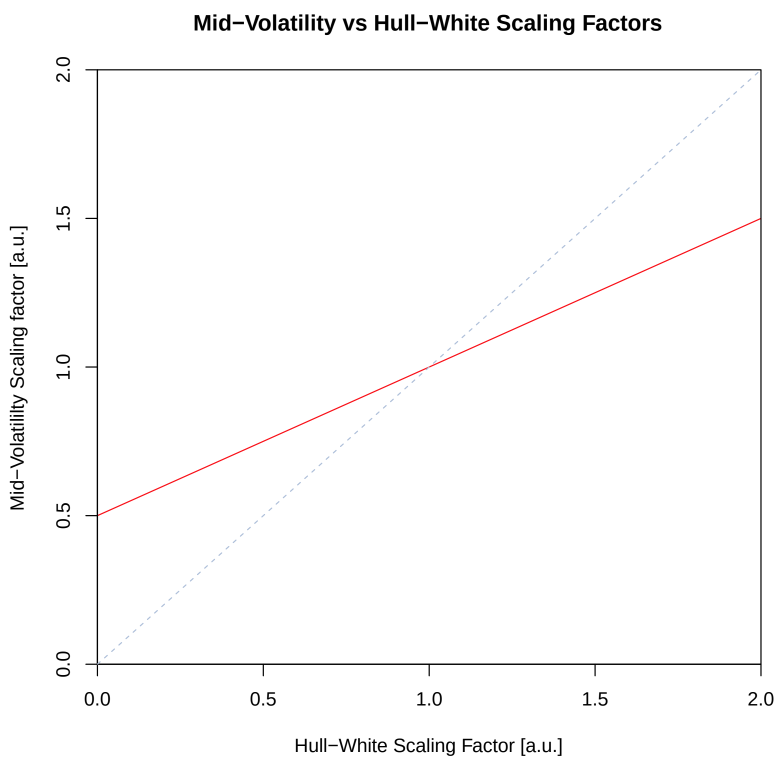

Figure 2: Mid-Volatility Scaling Factor vs Hull-White Scaling Factor

We note further that we can deduce the following from equation 5:

$$\sigma_N^i < \sigma_t^i \Leftrightarrow SF_t(i) > HWSF_t(i). \tag{10}$$

This means that whenever the dispersion on date t in the history is greater than the volatility on the last date in the time series N, the mid volatility scaling factor will be higher than the Hull-White scaling factor. Similarly, whenever the dispersion on date t in the history is less than the volatility on the last date in the time series N, the mid volatility scaling factor will be lower than the Hull-White scaling factor.

Figure 3 shows some time series of the scaling factors under the IMv1 methodology, where relative returns are used. As in the IMv2 case, five graphs are shown that are composed of respectively the largest, 90th percentile, median, 10th percentile and smallest scaling factors across all risk factors on each particular date. The graphs are labelled respectively as quantile 1, 0.9, 0.5, 0.1 and 0.0.

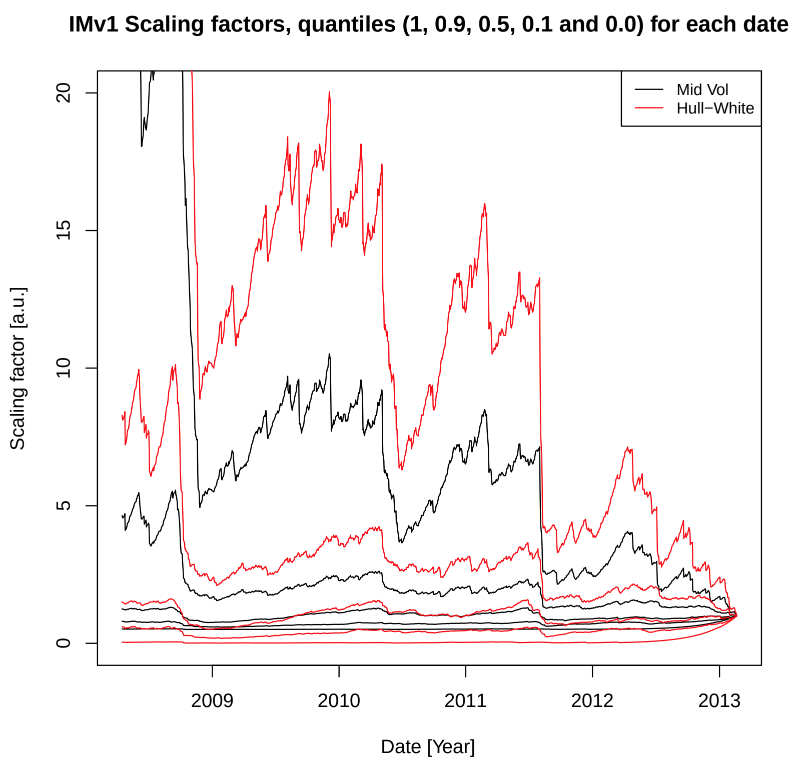

Figure 3: IMv1 Mid Volatility vs Hull-White scaling factors

One can see that above the line $y = 1$, the Hull-White scaling factors are always higher than the Mid-Volatility scaling factors for each quantile, which also follows from Eq. 9 and figure 2.

1.1.2 Half Life Comparison of Mid-Volatility Scaling vs Hull-White Scaling

The responsiveness of the EWMA estimation of standard deviation to changes in underlying volatility can be established using generated data.

As the EWMA estimator of the standard deviation is non-linear we would expect it to behave differently towards increasing and decreasing underlying volatility. Furthermore, the shape of the underlying distribution affects the responsiveness, as does the relative level of change.

A common and useful metric for measuring responsiveness is the half-life. The half-life is defined as the time the estimator takes to capture half of the change in volatility in response to a step-change in the dispersion of the underlying distribution.

It should be pointed out that the half-life of Hull-White scaling is identical to that of mid volatility scaling, but the steady state levels before and after the step change are different for Hull-White and mid volatility scaling. To see this, consider the definition of half-life for Hull-White scaling, where $\sigma_t, \sigma_S, \sigma_E, \sigma_H$ are the dispersion on a given scenario date, the start date of the change from the initial steady state, the end date of the change to the new steady state and the date at which the change is half as much as the change from initial to new steady state (i.e. the half-life date), respectively :

$$\frac{1}{2}\left(\frac{\sigma_S}{\sigma_t} - \frac{\sigma_E}{\sigma_t}\right) = \left(\frac{\sigma_S}{\sigma_t} - \frac{\sigma_H}{\sigma_t}\right) \tag{11}$$

$$\Leftrightarrow \quad \frac{1}{2}\left(\left(\frac{\sigma_S}{\sigma_t} - \frac{\sigma_E}{\sigma_t}\right)\Big/ 2\right) = \left(\frac{\sigma_S}{\sigma_t} - \frac{\sigma_H}{\sigma_t}\right)\Big/ 2 \tag{12}$$

$$\Leftrightarrow \quad \frac{1}{2}\left(\left(\frac{\sigma_S}{\sigma_t} + 1\right)\Big/ 2 - \left(\frac{\sigma_E}{\sigma_t} + 1\right)\Big/ 2\right) = \left(\frac{\sigma_S}{\sigma_t} + 1\right)\Big/ 2 - \left(\frac{\sigma_H}{\sigma_t} + 1\right)\Big/ 2 \tag{13}$$

Eq. 13 is the definition of half-life for mid volatility scaling. Hence we see that the half-life date, H, is identical for Hull-White and mid volatility scaling.

In Table 1, the half life (in terms of number of days) is displayed for step changes of different factors (multiply by 2, divide by 2, multiply by 1.5, divide by 1.5) for the cases of an underlying distribution being Normal and Student-T with 4 degrees of freedom. From the table one can see that half life (which as described above is identical for Hull-White and mid volatility scaling) is dependent on the type of underlying distribution and the factor applied in the step change. One can see that the half life when the step change is an increase is shorter than the half life when the step change is a decrease. This is a demonstration of the asymmetrical nature of Standard Deviation as a measure for dispersion, when used in scaling factors.

Step-change in Dispersion	$*2$	$/2$	$*1.5$	$/1.5$
Normal	37	167	56	130
Student-T, 4 DOF	41	170	55	133

Table 1: Half-life (in days) for an underlying Normal Distribution vs an underlying Student-T distribution with 4 Degrees of Freedom, when different factors for a step-change in the distributions' dispersion are applied

If one adopts the highly theoretical, and perhaps somewhat simplistic view, whereby the market is viewed as a generator of returns which is coming from a distribution with a slowly changing volatility, one will of course favour full Hull-White scaling. A more complex, and perhaps more realistic, view of the markets is where the returns are only partly explained by a slowly changing underlying volatility but the majority of the explanation of the returns is due to 'something else'. Here 'something else' could potentially be a jump model of the rates themselves or even the underlying volatility.

If the view is taken that 'something else' could be a large contributing factor, it is not clear that full Hull-White scaling is the best alternative. Mid volatility scaling emerges as a suitable alternative if we do believe that the markets are to some extent, but not fully, explained by a slowly varying underlying volatility.

1.1.3 IM performance using Hull-White, Mid Volatility and No Scaling

In the following two plots (4 and 5), we compare the performance of IMv2 when different scaling methodologies are used: Mid Volatility, Hull-White and No Scaling. Business backtesting was done on four currencies, USD, JPY, CAD and SEK (because these were the currencies for which the data series was long enough to give meaningful results). 100 synthetic portfolios were used in this simulation. The synthetic portfolios were generated by randomly picking each curve sensitivity profile from one of the present production portfolios, normalizing and then applying a random factor to the curve.

In figure 4, one can see that IM starts high, with a large drop in 1997 due to the Swedish interest rate crisis of 1992 dropping out of the series. This

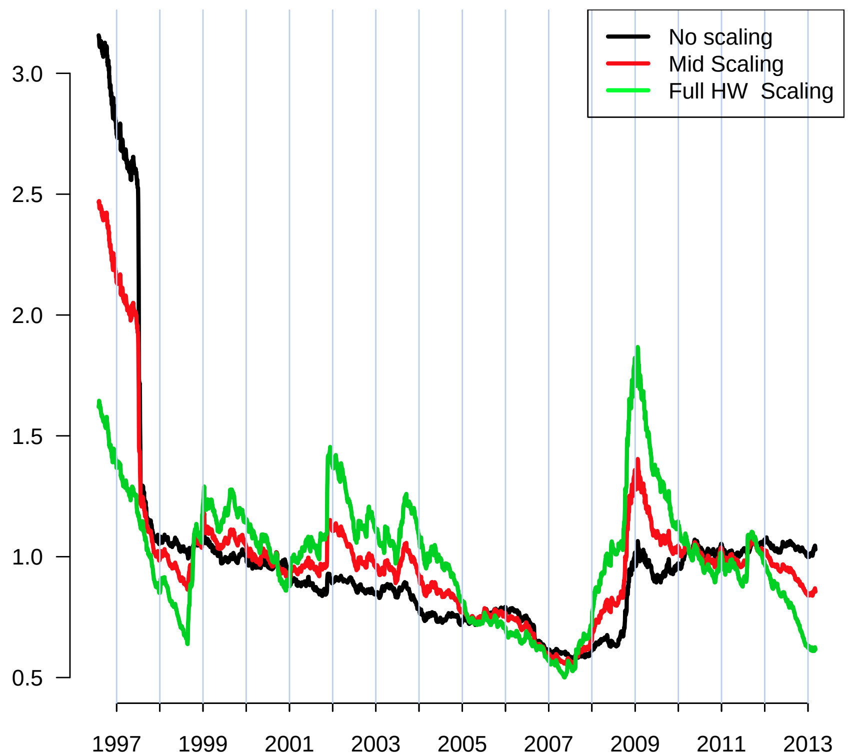

Figure 4: IMv2 using Hull-White, Mid-Volatility and No Scaling

crisis took place in a high volatility environment relative to 1997. Hence, by Eq. 10, the mid volatility scaling factor would have been higher than the Hull-White scaling factor at the time of the crisis, resulting in the higher IM observed under Mid Volatility scaling compared to Hull-White scaling before the Swedish crisis drops out. At the end of 1998 there was a new crisis (in fact two crises which almost happend at the same time: LTCM and Canadian rate spike - see Table 2), which drives a series of breaches, causing lower business confidence in figure 5. This event took place in a relatively low volatility environment, hence the performance of Hull-White scaling is worse of than that of the mid volatility scaling. After the 9/11

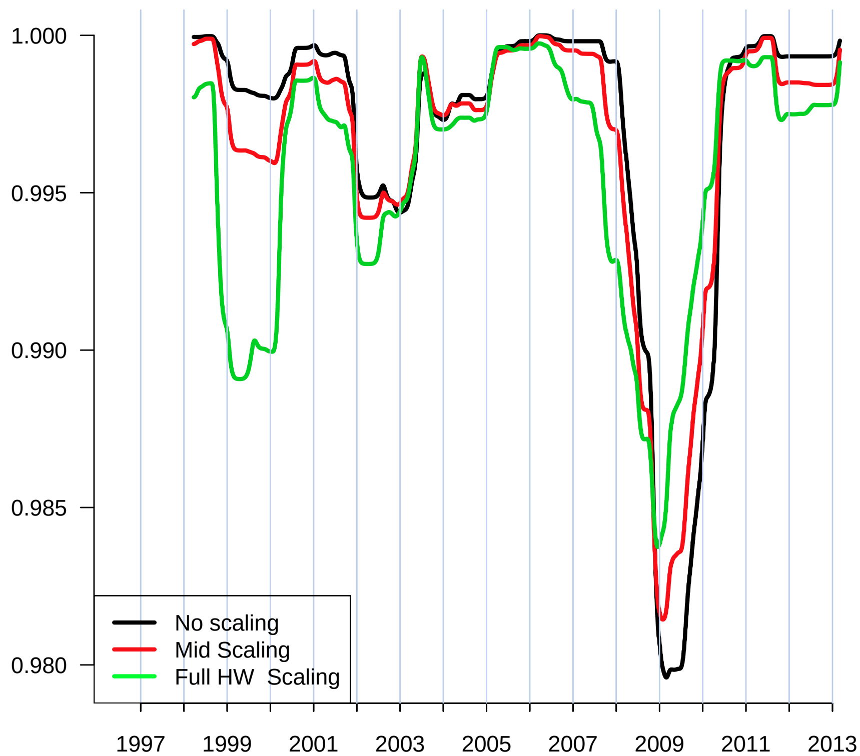

Figure 5: Business Confidence for IMv2 using Hull-White, Mid-Volatility and No Scaling

event in 2001, there was a dip in confidence in all three methodologies. In the Lehman's Event of 2008 all three scaling methodologies (no scaling, Hull-White and mid volatility) are unable to capture the PnL swings due to this event being driven by unprecedented market moves. We also note that this event happened in a high volatility market compared to the volatility of the driving scenarios in the history.

One can see that over the whole time series in both plots 4 and 5, none of the scaling methodologies clearly outperforms the others. Rather, depending on the levels of volatility on the dates of driving scenarios, relative to the levels of volatility on the current date, one methodology may outperform the other at that particular point in time (again by Eq. 10).

Date	Rate
1998-08-20	5.5088
1998-08-21	5.5288
1998-08-24	5.5387
1998-08-25	5.5287
1998-08-26	5.6477
1998-08-27	6.2737
1998-08-28	5.9956
1998-08-31	5.9458
1998-09-01	5.8167

Table 2: CAD ZERO CDOR STD 2 Year Pillar rates in August 1998, showing large move on 27th August

1.2 Correlation Analysis

In this section, we analyse the effect scaling has on the correlation among risk factor return series $\mathbf{R}_N$. i.e. We analyse the correlations among scaled risk factor return series $\mathbf{S}_N$ compared to the correlations among the unscaled risk factor return series $\mathbf{R}_N$.

One should note that 'correlation' among time series can be analysed in several different ways, including PCA analysis and correlation matrices on the whole time series for each risk factor, or correlation analyses on subsets of the time series for each risk factor, representing snapshots of correlation at particular points in time. Performing any kind of correlation analysis on the risk factors' whole time series, would provide a picture of correlation of risk factors 'on average' over the whole history up to the most recent date N and would not provide insight into where in the time series the scaling has a significant effect on the relationship between risk factors. We therefore perform instead a correlation computation on a rolling window of 100 days for the unscaled and scaled return series of pairs of risk factors.

To clarify, we introduce the following notation. For two risk factors A and B, let us denote the unscaled return series as $\mathbf{R}_N^A = \{R_t^A : t \in T\}$ and $\mathbf{R}_N^B = \{R_t^B : t \in T\}$ respectively. We denote the corresponding scaled return series as $\mathbf{S}_N^A = \{S_t^A : t \in T\}$ and $\mathbf{S}_N^B = \{S_t^B : t \in T\}$ respectively. Now, for a given date t, we define the subsets of the unscaled return series for the previous consecutive 100 days up to t as $\mathbf{R}_t^A = \{R_i^A : i \in [t-99, t] \subset T\}$ and $\mathbf{R}_t^B = \{R_i^B : i \in [t-99, t] \subset T\}$. Similarly, for a given date t, we define the subsets of the scaled return series for the previous consecutive 100 days up to t as $\mathbf{S}_t^A = \{S_i^A : i \in [t-99, t] \subset T\}$ and $\mathbf{S}_t^B = \{S_i^B : i \in [t-99, t] \subset T\}$.

We calculate Pearson's Product-Moment Correlation Coefficient for consecutive 100 day subsets of the pair of risk factor time series A and B up to date t for the scaled return series as:

$$Corr_t^{\mathbf{R}} = \frac{Cov(\mathbf{R}_t^A, \mathbf{R}_t^B)}{\sqrt{Var(\mathbf{R}_t^A) \cdot Var(\mathbf{R}_t^B)}},$$

and for the scaled return series as:

$$Corr_t^{\mathbf{S}} = \frac{Cov(\mathbf{S}_t^A, \mathbf{S}_t^B)}{\sqrt{Var(\mathbf{S}_t^A) \cdot Var(\mathbf{S}_t^B)}},$$

We present below graphs where $Corr_t^{\mathbf{R}}$ and $Corr_t^{\mathbf{S}}$ are plotted on the same axes against the dates, t, in the history T from date 100 to the most recent date N.

In Fig. 6, we compare the scaled and unscaled correlations for the pillars AUD ZERO BBSW STD 1W and AUD ZERO BBSW STD 15Y (as an example of intra-curve correlation).

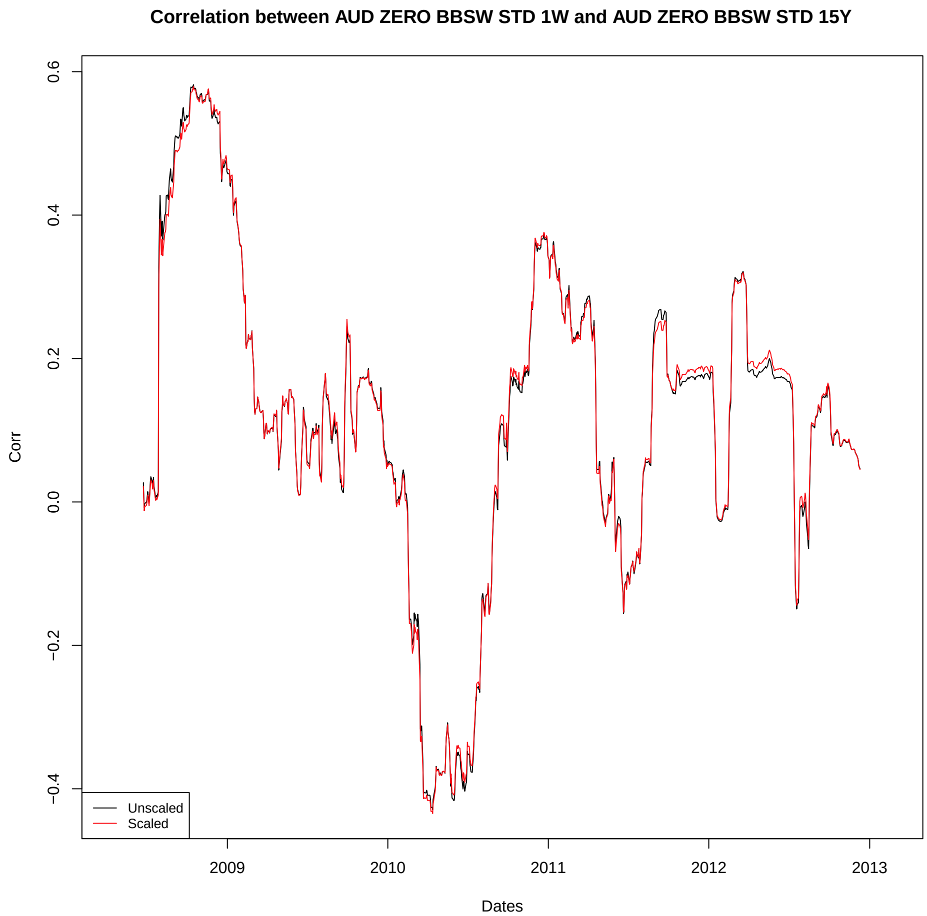

Figure 6: Intra-Curve Correlation between the pillars AUD ZERO BBSW STD 1W and AUD ZERO BBSW STD 15Y

In Fig. 7, we compare the scaled and unscaled correlations for the pillars AUD ZERO BBSW STD 1W and EUR ZERO EURIBOR STD 50Y (as an example of inter-curve correlation).

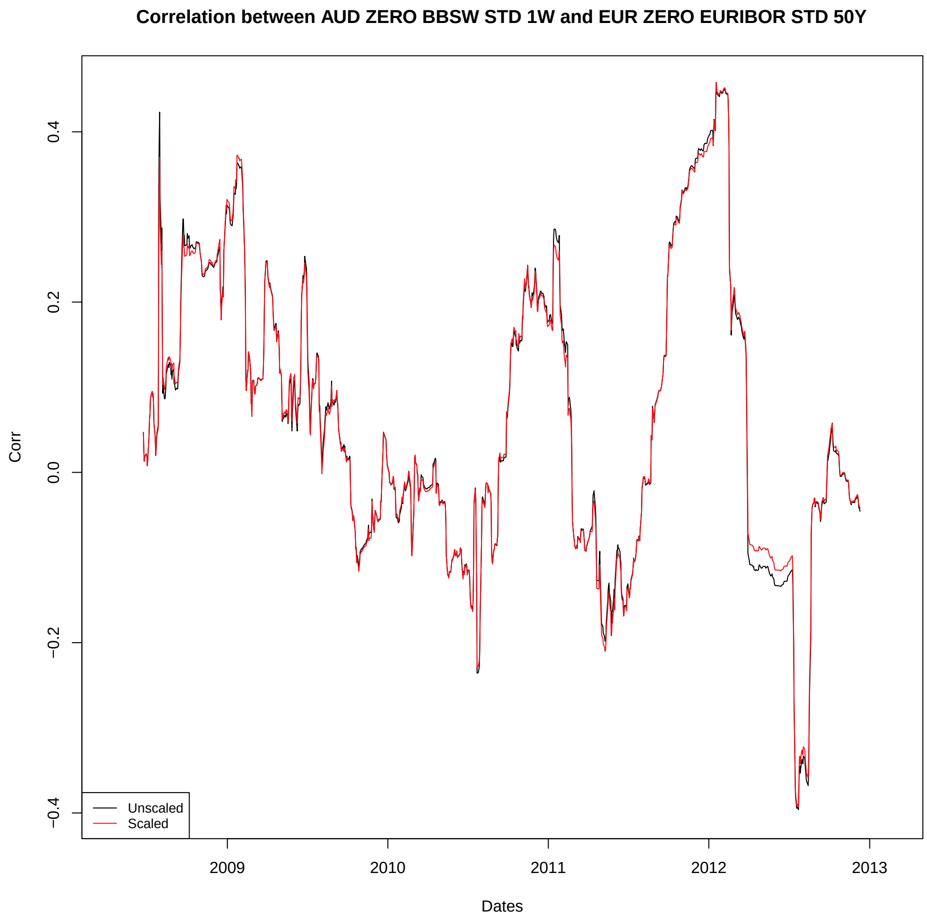

Figure 7: Inter-Curve Correlation between the pillars AUD ZERO BBSW STD 1W and EUR ZERO EURIBOR STD 50Y

In Fig. 8, we compare the scaled and unscaled correlations for the pillars USD ZERO LIBOR STD 2Y and EUR ZERO EURIBOR STD 10Y (as an example of inter-curve correlation).

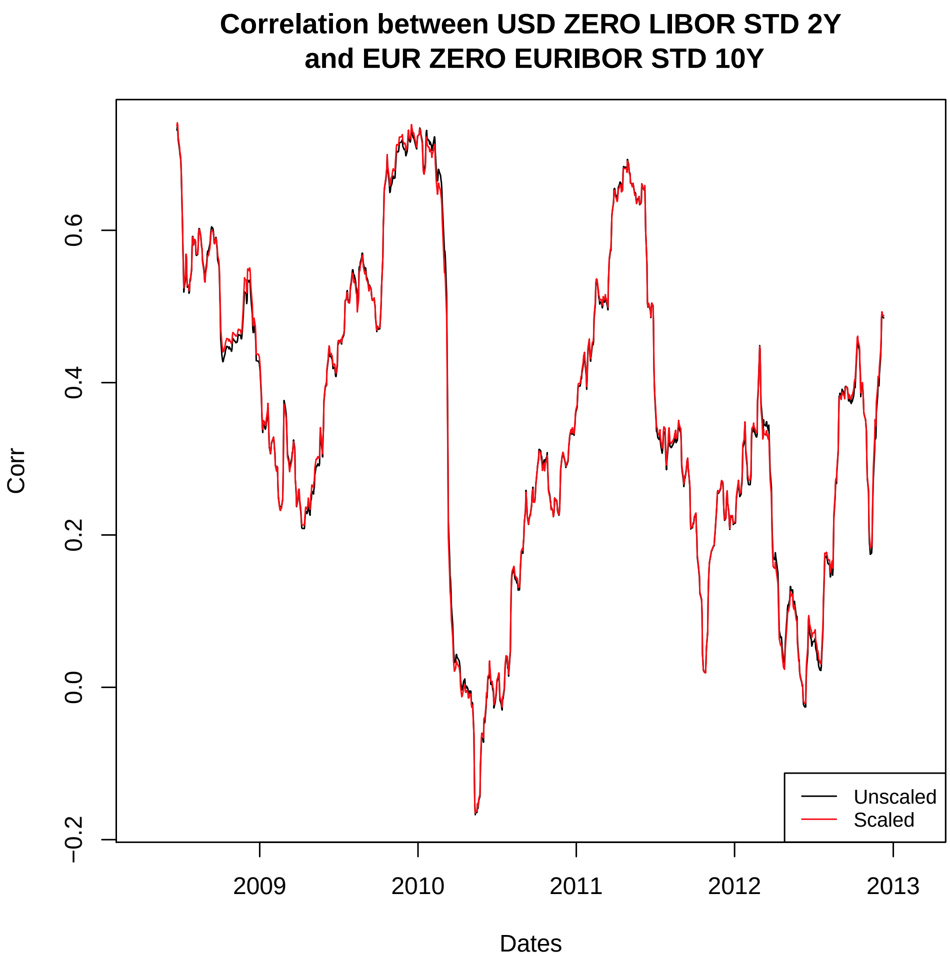

Figure 8: Inter-Curve Correlation between the pillars USD ZERO LIBOR STD 2Y and EUR ZERO EURIBOR STD 10Y

In Fig. 9, we compare the scaled and unscaled correlations for the pillars GBP ZERO LIBOR STD 30Y and USD ZERO LIBOR STD 5Y (as an example of inter-curve correlation).

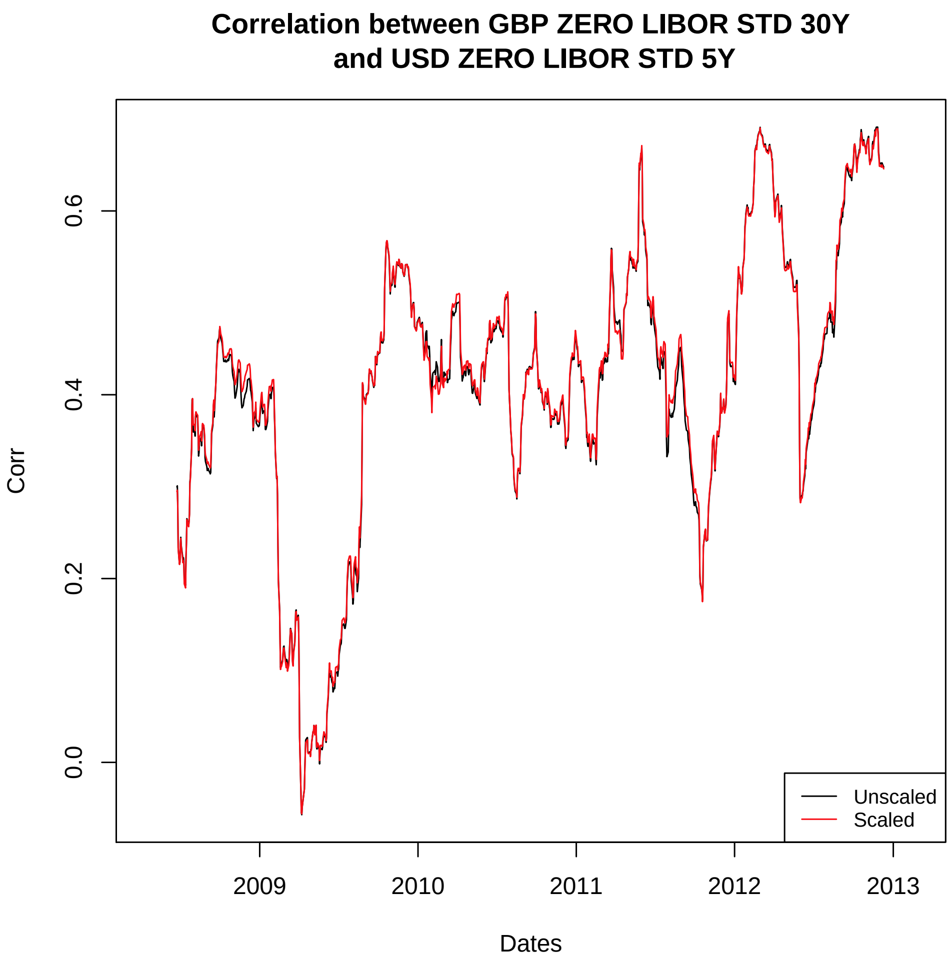

Figure 9: Inter-Curve Correlation between the pillars GBP ZERO LIBOR STD 30Y and USD ZERO LIBOR STD 5Y

In Fig. 10, we compare the scaled and unscaled correlations for the pillars JPY ZERO LIBOR STD 5Y and ZAR ZERO JIBAR STD 10Y (as an example of inter-curve correlation).

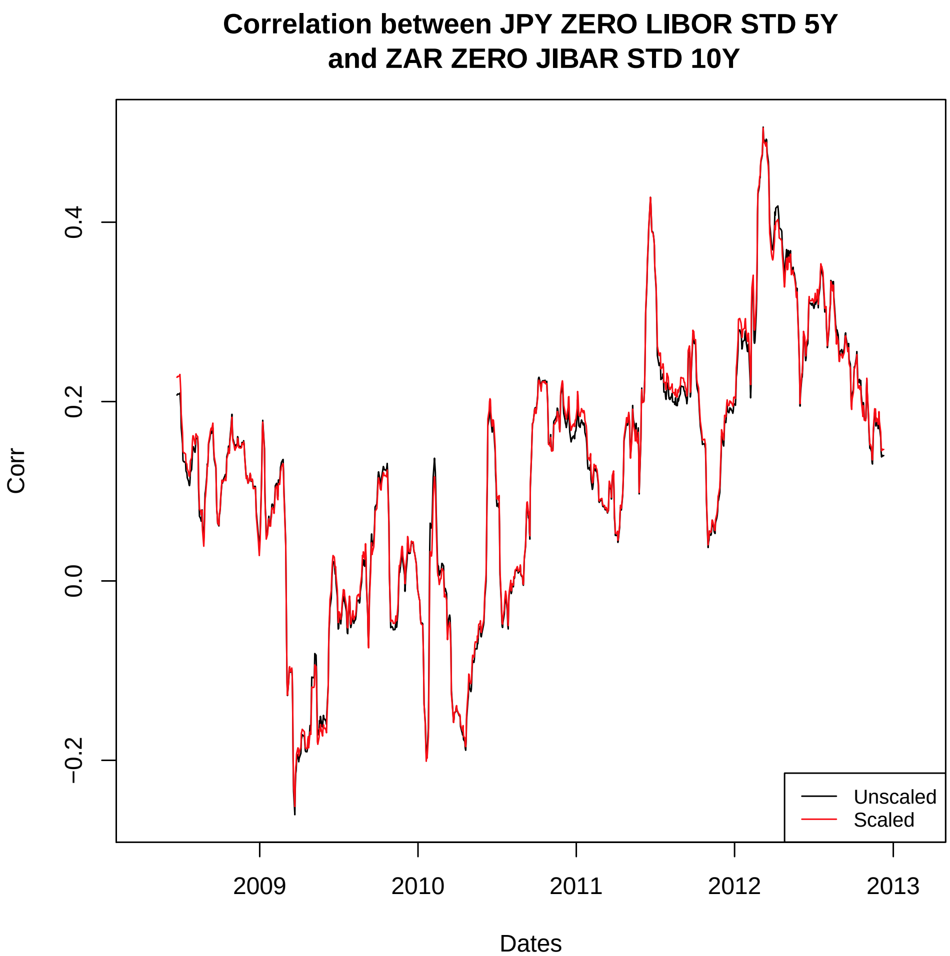

Figure 10: Inter-Curve Correlation between the pillars JPY ZERO LIBOR STD 5Y and ZAR ZERO JIBAR STD 10Y

In Fig. 11, we compare the scaled and unscaled correlations for the pillars EUR ZERO EURIBOR STD 2Y and EUR ZERO EURIBOR STD 30Y (as an example of intra-curve correlation).

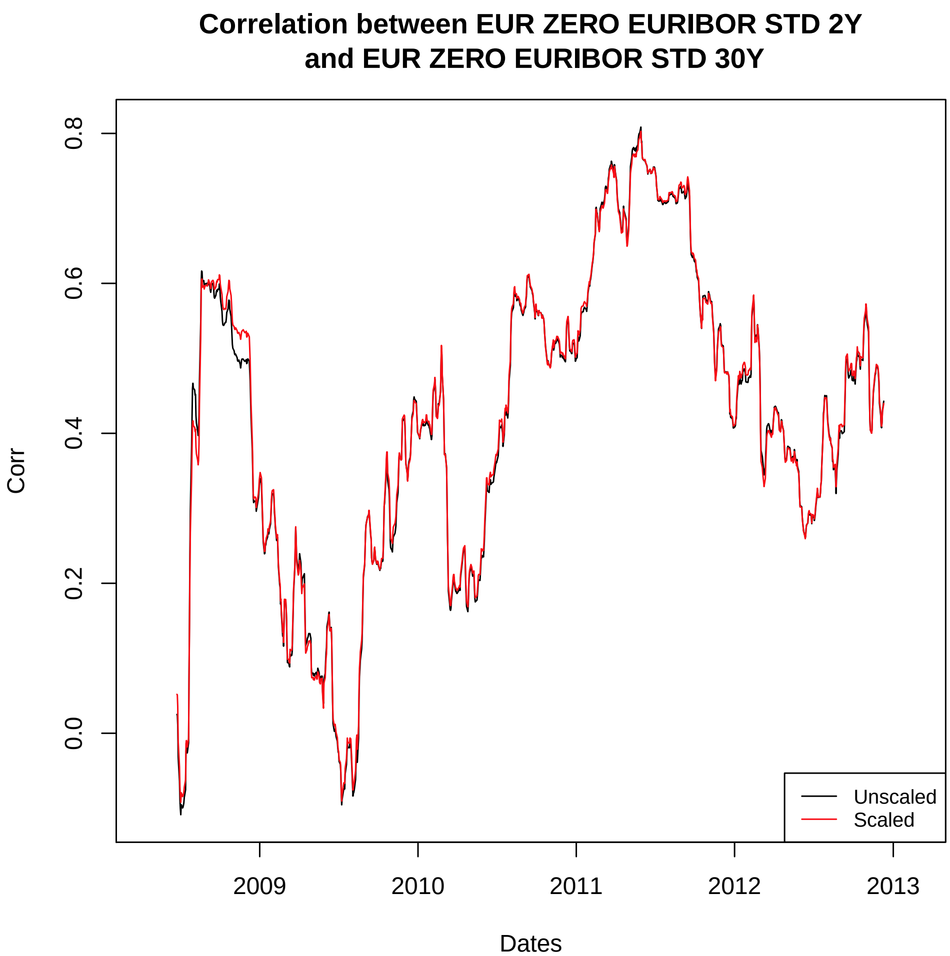

Figure 11: Intra-Curve Correlation between the pillars EUR ZERO EURIBOR STD 2Y and EUR ZERO EURIBOR STD 30Y

One can see that the scaled and unscaled correlations are almost the same and where they are different (during periods of higher volatility), the difference is still minimal. This is not surprising because the change in the scaling factors (see Eq. 1) with time is slow, and hence over the 100 day window, the scaling factors only vary marginally. Therefore the correlation of the scaled return series would be expected to be very similar to the correlation of the unscaled time series. In fact, in the limit where the moving window is very small, the scaling factors would be almost constant and hence scaled and unscaled correlation would be almost identical.

2 Other factors

FX is another risk factor that needs to be considered. FX shifts are applied to PnLs calculated by PAIRS for each separate local currency portion of the portfolio, to convert to the base currency:

$$\mathbf{PnL}_N = \sum_{ccy} \mathbf{PnL}_N^{ccy} / \mathbf{FX}_N^{ccy}, \tag{14}$$

Here $\mathbf{PnL}_N$ is the set of PnLs for each of the 2500 scenarios all expressed in the base currency, and $\mathbf{FX}_N^{ccy}$ is the set of 2500 FX rate scenarios computed using the historical FX returns for a particular currency ccy.

Figure 12 below shows five graphs that are composed of respectively the largest, 90th percentile, median, 10th percentile and smallest of (FX rate on date t)/(FX rate on date N) across all currencies on each particular date t. The graphs are labelled respectively as quantile 1, 0.9, 0.5, 0.1 and 0.0. One can see that the range of the distribution decreases somewhat from the post Lehman's event period to more recent times and there are not as strong trends as observed above for the scaling factors, although the order of magnitude of these FX factors is roughly equivalent to that of the scaling factors in the IMv2 case.

3 Summary of Findings

In this document, different methodologies for scaling the historical scenarios in the PAIRS model were investigated. In addition, the appropriateness of the scaling factors in terms of the preservation of the historical correlation prevalent in the markets at the time of the scenario was investigated.

3.1 Scaling

We have in this paper investigated three possibilities for the scaling factors: Mid Volatility Scaling, Full (Hull-White) Volatility Scaling and No Scaling. In comparing the IM levels and confidence of the three methodologies, it was shown that none of the methodologies clearly outperforms the other over the whole backtesting period. However during each of the major events that causes a large number of breaches and hence a drop in confidence, the worst

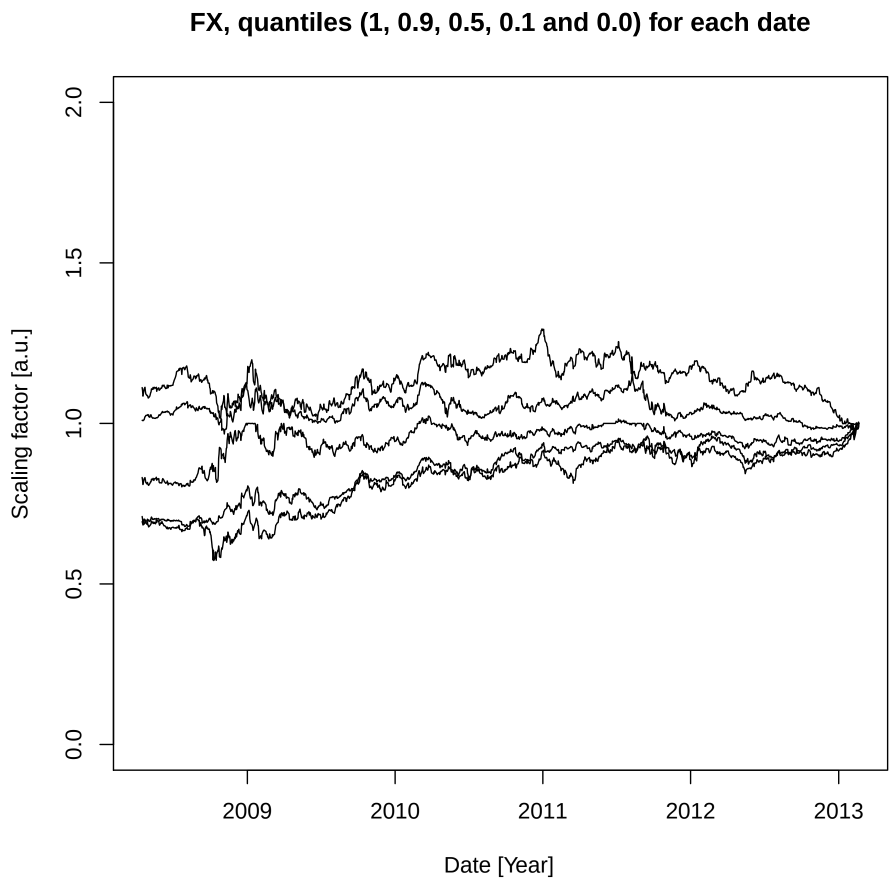

Figure 12: FX

performer was always either No Scaling or Hull-White Scaling but never Mid-Volatility. Mid-Volatility can be viewed as a good medium between Full Volatility Scaling and No Scaling. In particular, in the 1998 LTCM and Canadian rate spike events, one can see that Mid-Volatility performed much better than Full-Volatility. Furthermore we note that Mid-Volatilty has been used by SwapClear in IMv1 and the track record over time has shown it to be a good scaling methodology.

3.2 Correlation

It has been shown that correlation of the scenario risk factors is not a relevant metric since the correlation change as estimated over a very short period of time is the same, irrespective of scaling or not. On the other hand, since the magnitude of the scaling is potentially different for different risk factors, the relative importance of risk factors changes between non-scaling and scaling.

A more relevant metric would be how the scaling factors $\mathbf{SF}_N(i)$ (see Eq. 2) change over time and with risk factors.

In Figure 3 "IMv1 Mid Volatility vs Hull-White scaling factors" one can see in the case of IMv1, that from the beginning of the time series up to the middle of 2011 the distribution is quite skewed and there are some very large outliers. In particular, during the post Lehman's event period, we see some massive outliers because of some interest rate pillars being very low and having a low volatility at that point, causing the denominator of the scaling factors to be low and hence the scaling factors to blow up.

In Figure 1, "IMv2 Mid Volatility vs Hull-White scaling factors" the distribution at each point in time is far less skewed, and the range of the scaling factors is very small compared to the range for IMv1. The reason for this is that the numerator of the scaling factor (see equation 1) is driven by the volatility of the return series at the most recent point in time N, and currently since interest rates for some pillars are very low and close to zero, their relative returns time series are much more volatile than their absolute returns time series at time N.

SwapClear will monitor the behaviour of the scaling factors on a periodic basis as part of the model review process.

References

[1] Initial Margining Update, SwapClear IM - Assessment of Possible Recalibration, January 2013