SwapClear

Risk Analytics

Version 1.2

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Notation

Notation	Definition
$Flow(i)$	Flow for the period between calculation start date and calculation end date
$\emptyset(Discount\ curve, Flow\ Payment\ Date, VD)$	Discount factor computed on the discount curve between Value date and flow Payment date.
Ntl	Notional
R	Fixed Rate
$\delta(SD_i^P, ED_i^P)$	Year fraction between Calculation Start Date (SD_i^P) and Calculation End Date (SD_i^P) for the period.
$\delta(SD_i^I, ED_i^I)$	Year fraction between Start Payment Date and End Payment Date of the estimation period of the Index.
f_i	Forward rate applying onto the period starting at date "i" using the floating index convention and calendars attached to the index currency and fixing place. This forward rate is computed on the estimation curve which is assigned to the floating leg.
s	Spread for Forward Rate
NPV	Net Present Value of one or more Legs of a Swap
AI	Accrued Interest for a period or sub-period of a Swap

1 Introduction

The purpose of this document is to describe the risk analytics used by LCH.Clearnet SwapClear. This involves the valuation and sensitivities methodology adopted by SwapClear.

2 Eligible Trade Criteria

The following table is a summary of eligible Trade Types for Clearing within LCH.Clearnet SwapClear.

Trade Type	Description
Plain Vanilla Swaps	Fixed Rate Leg versus Floating Rate Leg within a single currency.
Compounding Swaps	Straight / Flat compounding Floating Rate Leg versus Fixed Rate Leg within a single currency.
Single Currency Basis Swap	Floating Rate Leg versus Floating Rate Leg of a different Index within a single currency.
Zero Coupon on the Fixed Leg only	Single terminal payment on the Fixed Leg versus Annual or higher payment frequencies on the Float Leg.
Zero Coupon on the Float Leg only	Compounding Float Leg with a single terminal payment versus Annual or higher payment frequencies on the Fixed Leg.
Zero Coupon on both the Fixed and the Float Legs	Single payment on Fixed Leg versus Compounding Float Leg with a single terminal payment.
Overnight Index Swaps (OIS)	Fixed Rate Leg versus Floating Overnight Index Rate Leg within a single currency.
Forward Rate Agreement (FRA)	Fixed Rate Tenor versus Floating Rate Tenor within a single currency.

3 Valuation

All eligible trade types are valued using the discount cash flow method. This method relies on the discounting of the trade flows (fixed and floating flows) using the discount factors inherited from the rate curve assigned to the discount of the cash flows.[1]

3.1 General Concept of the Discount Cash flow Method

The Net Present Value of each swap leg is computed as below:

$$NPV = \sum_{i=1}^{n} Flow(i)\, \emptyset(Discount\ curve, Flow\ Payment\ Date, Settlement\ Date)$$

The Market Value is the Net Present Value discounted to the current business date:

$$MV = \emptyset(Discount\ curve, Settlement\ Date, Business\ Date)\left(NPV_{leg\ 1} + NPV_{leg\ 2}\right)$$

3.2 Fixed Rate Flows

The NPV of a fixed leg is computed as below:

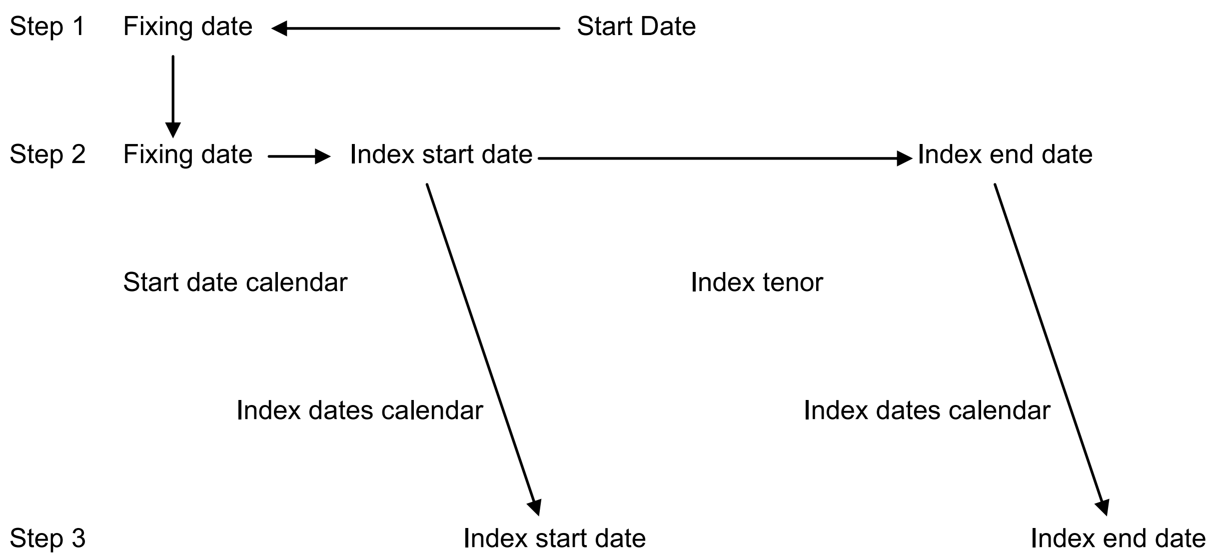
$$NPV_{Fix} = \sum_{i=1}^{m} Ntl * R_{Fix} * \delta(SD_i^P, ED_i^P) * \emptyset(Discount\ Curve, PD_i, VD)$$

[1] $\emptyset(Discount\ curve, VD, VD) = \begin{cases} 1, ITD \\ 0, EOD \end{cases}$

3.3 Floating Rate Flows

To estimate the floating leg rate period the steps are adopted:

1. The fixing date is derived from the fixing calendar of the floating index and the deal start date
2. The fixing date, the floating index offset along with the start date calendar returns the floating index estimation start period. This date is checked against the floating index date calendar to be an open day.
3. Once the floating index start date is determined, the floating index tenor rule (for instance 3M Modified Following) is applied and the floating index end date is derived. This date is checked against the floating index date calendar to be an open day.



The forward rate is given by:

$$f_j = \left(\frac{\emptyset(Estimation\ Curve,\ SD_j^I, Settlement\ Date)}{\emptyset(Estimation\ Curve,\ ED_j^I, Settlement\ Date)} - 1 \right) / \delta\left(SD_j^I, ED_j^I\right)$$

The NPV of a float leg is computed as below:

$$NPV_{Flt} = \sum_{j=1}^{n} Ntl * (f_j + s) * \delta\left(SD_j^P, ED_j^P\right) * \emptyset\left(Discount\ curve, PD_j, VD\right)$$

When there is a broken period, the floating index rate is linearly weighted with the indices that have tenors that surround the number of days of the required period.

3.4 Plain Vanilla Swaps

Plain Vanilla Swaps can have three types of payment schedule:

- Regular Payments: All payments of a leg have the same tenor. Note that they don't need to be of the same tenor from one leg to another.
- Stub (Front or Back): When the term of the Swap is not a multiple of the tenors of the legs of the Swap, a Stub Period exists in order to have a continuous accrual schedule on the term of the Swap. This Stub Period is either in the beginning of the Swap (Front) or at the end (Back). A Stub can be greater than the regular period (Long Stub) or smaller than the regular period (Short Stub).
- IMM Dates: IMM Swaps have period dates that follow the Calendar for the Interest Rates Future for a currency. In most cases they follow the IMM (International Monetary Market) Dates.

3.5 Compounding Swaps

There are two methods in which cash flows can be compounded:

3.5.1 Flat Compounding

On a Flat Compounding Leg, the floating rate is applied to the notional amount plus accrued interest caused by the previous floating rate(s). The spread, however, is only ever applied to the original constant notional amount. On the payment date the accrued amounts are summed.

The value of a Flat Compounding Float Leg is

$$NPV_{CompFlat} = \sum_{i=1}^{n} P_{CompFlat_i} * \emptyset(Discount\ curve, PD_i, VD)$$

where,

$$P_{CompFlat_i} = \sum_{j=1}^{sp} AI_{CompFlat_j}$$

where,

$$AI_{CompFlat_j} = Ntl * (f_j + s) * \delta(SD_j^P, ED_j^P) + \left(\sum_{k=1}^{j-1} AI_{CompFlat_k}\right) * (f_j) * \delta(SD_j^P, ED_j^P)$$

where,

sp = Number of sub-periods on period *i*

SD_j^P= Start Date for sub-period *j*

ED_j^P = End Date for sub-period *j*

3.5.2 Straight Compounding

On a Straight Compounding Leg, the floating rate plus spread are applied to the notional amount plus accrued interest caused by the previous floating rate(s). On the payment date the accrued amounts are summed.

The value of a Straight Compounding Float Leg is

$$NPV_{CompStraight} = \sum_{i=1}^{n} P_{CompStraight_i} * \emptyset(Discount\ curve, PD_i, VD)$$

where

$$P_{CompStraight_i} = \sum_{j=1}^{sp} AI_{CompStraight_j}$$

where

$$AI_{CompStraight_j} = Ntl * (f_j + s) * \delta(SD_j^P, ED_j^P) + \left(\sum_{k=1}^{j-1} AI_{CompStraight_k}\right) * (f_j + s) * \delta(SD_j^P, ED_j^P)$$

3.6 Single Currency Basis Swaps

A Basis Swap consists of two Float Legs. Both Legs have the same Index, but with different Tenors.

3.7 Zero Coupon Swaps

A Zero Coupon Swap has one or the two legs with payment only at maturity.

A Zero Coupon Leg compounds at the Tenor of the Swap. The structure of a Zero Coupon Leg is similar to a Straight Compound Leg, but instead of having sub-periods compounding on each period, Zero Coupon Leg have periods compounding on the Term of the Swap.

There are three types of Zero Coupon Swap eligible for SwapClear service:

1. Zero Coupon Fixed Leg and a Floating Leg

$$NPV_{ZCS} = NPV_{ZCfix} + NPV_{Flt}$$

where,

$$NPV_{ZCfix} = \left(\sum_{i=1}^{sp} AI_{ZCfix_i}\right) * \emptyset(Discount\ curve, T, VD)$$

where,

$$AI_{ZCfix_i} = Ntl * R_{Fix} * \delta(SD_i^P, ED_i^P) + \left(\sum_{j=1}^{i-1} AI_{ZCfix_j}\right) * R_{Fix} * \delta(SD_i^P, ED_i^P)$$

where,

T = Termination date of the Swap

2. Zero Coupon Float Leg and a Fixed Leg

$$NPV_{zcs} = NPV_{zcflt} + NPV_{Fix}$$

where,

$$NPV_{zcflt} = \left(\sum_{i=1}^{sp} AI_{zcflt_i}\right) * \emptyset(Discount\ curve, T, VD)$$

where,

$$AI_{zcflt_i} = Ntl * (f_i + s) * \delta(SD_i^P, ED_i^P) + \left(\sum_{j=1}^{i-1} AI_{zcflt_j}\right) * (f_i + s) * \delta(SD_i^P, ED_i^P)$$

3. Zero Coupon Fixed Leg and a Zero Coupon Float Leg

$$NPV_{zcs} = NPV_{zcfix} + NPV_{zcflt}$$

3.8 Overnight Index Swaps

An OIS consists of a Fix Leg, plus a Float Leg. The float Leg of an OIS trade compounds on a daily basis as per the correspondent Over Night Rate.

The NPV of an OIS is then the sum of the NPV of the Fix Leg and the Float Leg.

$$NPV_{OIS} = NPV_{OISfix} + NPV_{OISflt}$$

3.8.1 Fix Leg

The value of the Fix Leg of an OIS is calculated in the same way as a standard vanilla Interest Rate Swap.

$$NPV_{OISfix} = \sum_{i=1}^{m} Ntl * R_{Fix} * \delta(SD_i^P, ED_i^P) * \emptyset(Discount\ curve, PD_i, VD)$$

3.8.2 Float Leg

On an OIS Floating leg, interest is accrued on a daily basis. In order to calculate the value of the interest payments, we use Accrual Factors for the relevant period 0 to *t*.

For valuation of cash flows where:

i) interest period start date ≤ valuation date:

$$PV_n = Ntl * \left[\left(AF_{SD_n,VD} * AF_{VD,ED_n}\right) - 1\right] * \emptyset(OIS, PD_n, VD)$$

where,

$$AF_{SD_n,VD} = \prod_{i=SD_n}^{VD-1} 1 + ONr_i * \delta(i, i+1)$$

$$AF_{VD,ED_n} = \frac{1}{\emptyset(OIS, ED_n^I, VD)}$$

where,

ONr_i = Overnight Rate at date i

ii) interest period start date > valuation date:

$$PV_n = Ntl * AF_{SD_n,ED_n} * \emptyset(OIS, PD_n, VD)$$

where,

$$AF_{SD_n,ED_n} = \left(\frac{\emptyset(OIS, SD_n^I, VD)}{\emptyset(OIS, ED_n^I, VD)} - 1\right)$$

The value of the OIS floating leg is:

$$NPV_{OISflt} = \sum_{n=1}^{N} PV_n$$

3.9 FRAs

There are two models for pricing FRAs: Forward Discounting and Dual Discounting. FRAs eligible for SwapClear are for currencies which adopt the Forward Discounting method.

3.9.1 Forward Discounting

The NPV of a FRA is computed as below:

$$NPV_{FRA} = Ntl * (R_{Fix} - f) * \delta(SD^P, ED^P) * \emptyset(Estimation\ curve, ED^P, PD) * \emptyset(Discount\ curve, PD, VD)$$

Note that $SD^P = PD$.

4 Sensitivities

SwapClear calculates the sensitivity of the value of a Swap for a change in Interest Rates. On a Swap, the first derivative of NPV with respect to the Interest Rate is known as Delta. The second derivative of NPV with respect to the Interest Rate is known as Gamma. These sensitivities can be calculated with respect to the Par Curve and with respect to the Zero Coupon Rate Curve.

A Swap has two categories of sensitivities:

1. Forward Sensitivity is the sensitivity of the NPV of a Swap with respect to a change in the forward rates.
2. Discount Sensitivity is the sensitivity of the NPV of a Swap with respect to a change in the discount rate applied on the cashflows of a Swap.

SwapClear calculates Delta and Gamma for both Forward and Discount sensitivities.

SwapClear calculates the sensitivities with respect to the Zero Coupon Curve using an analytical approach.

The *n*-th derivative of $\emptyset_i$ is:

$$\frac{\partial^n \emptyset_i}{\partial z_i^n} = (-1)^n * \delta(VD, i)^n * \emptyset_i$$

In order to the calculate the sensitivities with respect to the Par Curve, SwapClear uses the Jacobian Transformation to convert the Zero Delta into Par Delta, and in addition of the Jacobian Transformation, it uses the Hessian Matrix to convert the Zero Gamma into Par Gamma.

The Jacobean transformation is applied as follows:

$$\begin{bmatrix} \frac{\partial NPV}{\partial m_1} \\ \frac{\partial NPV}{\partial m_2} \\ \dots \\ \frac{\partial NPV}{\partial m_n} \end{bmatrix} = [dMdZ^{-1}]^T \begin{bmatrix} \frac{\partial NPV}{\partial z_1} \\ \frac{\partial NPV}{\partial z_2} \\ \dots \\ \frac{\partial NPV}{\partial z_n} \end{bmatrix}$$

Where,

$$dMdZ = \begin{bmatrix} \frac{\partial m_1}{\partial z_1} & \frac{\partial m_1}{\partial z_2} & \dots & \frac{\partial m_1}{\partial z_n} \\ \frac{\partial m_2}{\partial z_1} & \frac{\partial m_2}{\partial z_2} & \dots & \frac{\partial m_2}{\partial z_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial m_n}{\partial z_1} & \frac{\partial m_n}{\partial z_2} & \dots & \frac{\partial m_n}{\partial z_n} \end{bmatrix}$$

Entries in the $dMdZ$ matrix are calculated as below:

$$\frac{\partial m_i}{\partial z_j} = \frac{\partial m_i}{\partial NPV_i} * \frac{\partial NPV_i}{\partial z_j} = \frac{\partial NPV_i}{\partial z_j} \div \frac{\partial NPV_i}{\partial m_i}$$

SwapClear consolidates the Sensitivities of a Swap per Zero Curve, using the Zero Curve Knot Points. In order to consolidate the sensitivities, it is necessary to project the sensitivities from the Sensitivity Dates to the Knot Point Dates of the relevant Curve.

If the sensitivity date is in the same date of a Knot Point Date, then the whole sensitivity is allocated to this Knot Point.

If the sensitivity is between two dates of the curve, then this is linear interpolated to project it into the two neighbouring Knot Points.