DSF Sensitivities

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Abstract

1 Delta

Let:
t_d be the delivery date.
$x = reftime(t_d)$ be the reference time to delivery on OIS curve.
x_0 and x_1 be the reference times for the two adjacent pillars to x on the OIS curve.
Z_x be the zero rate on OIS curve for reference time x.
$Df_x^{OIS} = e^{-Z_x x}$ be the discount factor at time x on OIS curve.
PV be the NPV of the underlying swap instrument.
Z_i be the zero rate a a particular pillar on the zero curve.

$$F = 100 \cdot (1 + \frac{PV}{Df_x^{OIS}}) \tag{1}$$

$$F = 100 \cdot (1 + PV \cdot e^{Z_x x}) \tag{2}$$

$$\frac{\partial F}{\partial Z_i} = \frac{\partial}{\partial Z_i}(100 \cdot (1 + PV \cdot e^{Z_x x})) \tag{3}$$

$$\frac{\partial F}{\partial Z_i} = \frac{\partial}{\partial Z_i}(100 + 100 \cdot PV \cdot e^{Z_x x}) \tag{4}$$

$$\frac{\partial F}{\partial Z_i} = 100 \cdot \frac{\partial}{\partial Z_i}(PV \cdot e^{Z_x x}) \tag{5}$$

$$\frac{\partial F}{\partial Z_i} = 100 \cdot (\frac{\partial}{\partial Z_i}(PV) \cdot e^{Z_x x} + PV \cdot \frac{\partial}{\partial Z_i}(e^{Z_x x})) \tag{6}$$

$$\frac{\partial F}{\partial Z_i} = 100 \cdot (\frac{\partial}{\partial Z_i}(PV) \cdot e^{Z_x x} + PV \cdot \frac{\partial}{\partial Z_i}(e^{Z_x x} \cdot (\frac{x_1 - x}{x_1 - x_0} + \frac{x - x_0}{x_1 - x_0}))) \tag{7}$$

2 Gamma

$$\frac{\partial^2 F}{\partial Z_i \partial Z_j} = \frac{\partial}{\partial Z_j}(100 \cdot (\frac{\partial}{\partial Z_i}(PV) \cdot e^{Z_x x} + PV \cdot \frac{\partial}{\partial Z_i}(e^{Z_x x}))) \tag{8}$$

$$\frac{\partial^2 F}{\partial Z_i \partial Z_j} = 100 \cdot (\frac{\partial}{\partial Z_j}(\frac{\partial}{\partial Z_i}(PV) \cdot e^{Z_x x}) + \frac{\partial}{\partial Z_j}(PV \cdot \frac{\partial}{\partial Z_i}(e^{Z_x x}))) \tag{9}$$

$$\frac{\partial^2 F}{\partial Z_i \partial Z_j} = 100 \cdot (\frac{\partial^2}{\partial Z_i \partial Z_j}(PV) \cdot e^{Z_x x} + \frac{\partial}{\partial Z_i}(PV) \cdot \frac{\partial}{\partial Z_j}(e^{Z_x x}) + \frac{\partial}{\partial Z_j}(PV) \cdot \frac{\partial}{\partial Z_i}(e^{Z_x x}) + PV \cdot \frac{\partial^2}{\partial Z_i \partial Z_j}(e^{Z_x x})) \tag{10}$$