SwapClear

Risk Analytics

Version 2.1

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Notation

Notation	Definition
$Flow(i)$	Flow for the period between calculation start date and calculation end date
$\emptyset(Discount\ curve, Flow\ Payment\ Date, VD)$	Discount factor computed on the discount curve between Value date and flow Payment date.
Ntl	Notional
R	Fixed Rate
$\delta(SD_i^P, ED_i^P)$	Year fraction between Calculation Start Payment Date (SD_i^P) and Calculation End Payment Date (SD_i^P) for the period.
$\delta(SD_i^I, ED_i^I)$	Year fraction between Start Date and End Date of the estimation period of the Index.
$\theta(SD^P, ED^P, VD)$	Accrual ratio or period fraction. It is the ratio of number of days between Value date and Start date of the period over the number of days between End date and Start date of the period
f_i	Forward rate applying onto the period starting at date "i" using the floating index convention and calendars attached to the index currency and fixing place. This forward rate is computed on the estimation curve which is assigned to the floating leg.
s	Spread for Forward Rate
NPV	Net Present Value of one or more Legs of a Swap
AI	Accrued Interest for a period or sub-period of a Swap

1 Introduction

The purpose of this document is to describe the risk analytics used by LCH SwapClear. This involves the valuation and sensitivities methodology adopted by SwapClear.

2 Eligible Trade Criteria

The following table is a summary of eligible Trade Types for Clearing within LCH SwapClear.

Trade Type	Description
Plain Vanilla Swaps	Fixed Rate Leg versus Floating Rate Leg within a single currency.
Compounding Swaps	Straight / Flat compounding Floating Rate Leg versus Fixed Rate Leg within a single currency.
Single Currency Basis Swap	Floating Rate Leg versus Floating Rate Leg of a different Index within a single currency.
Zero Coupon on the Fixed Leg only	Single terminal payment on the Fixed Leg versus Annual or higher payment frequencies on the Float Leg.
Zero Coupon on the Float Leg only	Compounding Float Leg with a single terminal payment versus Annual or higher payment frequencies on the Fixed Leg.
Zero Coupon on both the Fixed and the Float Legs	Single payment on Fixed Leg versus Compounding Float Leg with a single terminal payment.
Overnight Index Swaps (OIS)	Fixed Rate Leg versus Floating Overnight Index Rate Leg within a single currency.
Forward Rate Agreement (FRA)	Fixed Rate Tenor versus Floating Rate Tenor within a single currency.

3 Valuation

All eligible trade types are valued using the discount cash flow method. This method relies on the discounting of the trade flows (fixed and floating flows) using the discount factors inherited from the rate curve assigned to the discount of the cash flows.[1]

3.1 General Concept of the Discount Cash flow Method

The Net Present Value of each swap leg is computed as below:

$$NPV = \sum_{i=1}^{n} Flow(i)\emptyset(Discount\ curve, Flow\ Payment\ Date, Settlement\ Date)$$

The Market Value is the Net Present Value discounted to the current business date:

$$MV = \emptyset(Discount\ curve, Settlement\ Date, Business\ Date)\left(NPV_{leg\ 1} + NPV_{leg\ 2}\right)$$

3.2 Fixed Rate Flows

The NPV of a fixed leg is computed as below:

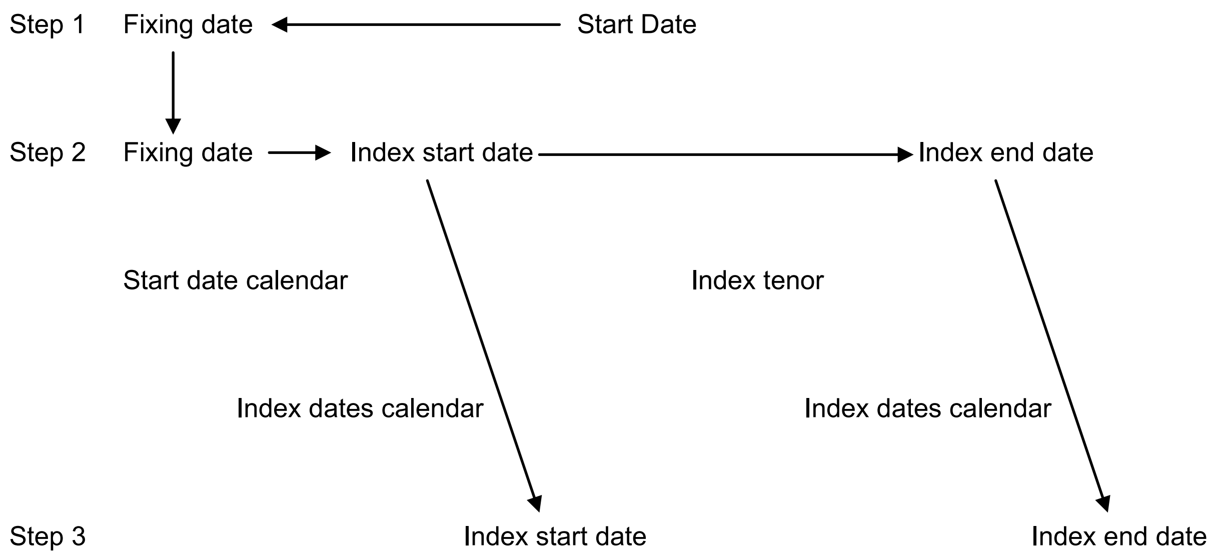
$$NPV_{Fix} = \sum_{i=1}^{m} Ntl * R_{Fix} * \delta(SD_i^P, ED_i^P) * \emptyset(Discount\ Curve, PD_i, VD)$$

[1] $\emptyset(Discount\ curve, VD, VD) = \begin{cases} 1, ITD \\ 0, EOD \end{cases}$

3.3 Floating Rate Flows

To estimate the floating leg rate period the steps are adopted:

1. The fixing date is derived from the fixing calendar of the floating index and the deal start date
2. The fixing date, the floating index offset along with the start date calendar returns the floating index estimation start period. This date is checked against the floating index date calendar to be an open day.
3. Once the floating index start date is determined, the floating index tenor rule (for instance 3M Modified Following) is applied and the floating index end date is derived. This date is checked against the floating index date calendar to be an open day.



The forward rate is given by:

$$f_j = \left(\frac{\emptyset\left(Estimation\ Curve,\ SD_j^I, Settlement\ Date\right)}{\emptyset\left(Estimation\ Curve,\ ED_j^I, Settlement\ Date\right)} - 1\right) / \delta\left(SD_j^I, ED_j^I\right)$$

The NPV of a float leg is computed as below:

$$NPV_{Flt} = \sum_{j=1}^{n} Ntl * (f_j + s) * \delta\left(SD_j^P, ED_j^P\right) * \emptyset\left(Discount\ curve, PD_j, VD\right)$$

When there is a broken period, the floating index rate is linearly weighted with the indices that have tenors that surround the number of days of the required period.

3.4 Plain Vanilla Swaps

Plain Vanilla Swaps can have three types of payment schedule:

- Regular Payments: All payments of a leg have the same tenor. Note that they don't need to be of the same tenor from one leg to another.
- Stub (Front or Back): When the term of the Swap is not a multiple of the tenors of the legs of the Swap, a Stub Period exists in order to have a continuous accrual schedule on the term of the Swap. This Stub Period is either in the beginning of the Swap (Front) or at the end (Back). A Stub can be greater than the regular period (Long Stub) or smaller than the regular period (Short Stub).
- IMM Dates: IMM Swaps have period dates that follow the Calendar for the Interest Rates Future for a currency. In most cases they follow the IMM (International Monetary Market) Dates.

3.5 Compounding Swaps

There are two methods in which cash flows can be compounded:

3.5.1 Flat Compounding

On a Flat Compounding Leg, the floating rate is applied to the notional amount plus accrued interest caused by the previous floating rate(s). The spread, however, is only ever applied to the original constant notional amount. On the payment date the accrued amounts are summed.

The value of a Flat Compounding Float Leg is

$$NPV_{CompFlat} = \sum_{i=1}^{n} P_{CompFlat_i} * \emptyset(Discount\ curve, PD_i, VD)$$

where,

$$P_{CompFlat_i} = \sum_{j=1}^{sp} AI_{CompFlat_j}$$

where,

$$AI_{CompFlat_j} = Ntl * (f_j + s) * \delta(SD_j^P, ED_j^P) + \left(\sum_{k=1}^{j-1} AI_{CompFlat_k}\right) * (f_j) * \delta(SD_j^P, ED_j^P)$$

where,

sp = Number of sub-periods on period *i*

SD_j^P = Start Date for sub-period *j*

ED_j^P = End Date for sub-period *j*

3.5.2 Straight Compounding

On a Straight Compounding Leg, the floating rate plus spread are applied to the notional amount plus accrued interest caused by the previous floating rate(s). On the payment date the accrued amounts are summed.

The value of a Straight Compounding Float Leg is

$$NPV_{CompStraight} = \sum_{i=1}^{n} P_{CompStraight_i} * \emptyset(Discount\ curve, PD_i, VD)$$

where

$$P_{CompStraight_i} = \sum_{j=1}^{sp} AI_{CompStraight_j}$$

where

$$AI_{CompStraight_j} = Ntl * (f_j + s) * \delta(SD_j^P, ED_j^P) + \left(\sum_{k=1}^{j-1} AI_{CompStraight_k}\right) * (f_j + s) * \delta(SD_j^P, ED_j^P)$$

3.6 Single Currency Basis Swaps

A Basis Swap consists of two Float Legs. Both Legs have the same Index, but with different Tenors.

3.7 Zero Coupon Swaps

A Zero Coupon Swap has one or the two legs with payment only at maturity.

A Zero Coupon Leg compounds at the Tenor of the Swap. The structure of a Zero Coupon Leg is similar to a Straight Compound Leg, but instead of having sub-periods compounding on each period, Zero Coupon Leg have periods compounding on the Term of the Swap.

There are three types of Zero Coupon Swap eligible for SwapClear service:

1. Zero Coupon Fixed Leg and a Floating Leg

$$NPV_{ZCS} = NPV_{ZCfix} + NPV_{Flt}$$

where,

$$NPV_{ZCfix} = \left(\sum_{i=1}^{sp} AI_{ZCfix_i}\right) * \emptyset(Discount\ curve, T, VD)$$

where,

$$AI_{ZCfix_i} = Ntl * R_{Fix} * \delta(SD_i^P, ED_i^P) + \left(\sum_{j=1}^{i-1} AI_{ZCfix_j}\right) * R_{Fix} * \delta(SD_i^P, ED_i^P)$$

where,

T = Termination date of the Swap

2. Zero Coupon Float Leg and a Fixed Leg

$$NPV_{ZCS} = NPV_{ZCflt} + NPV_{Fix}$$

where,

$$NPV_{ZCflt} = \left(\sum_{i=1}^{sp} AI_{ZCflt_i}\right) * \emptyset(Discount\ curve, T, VD)$$

where,

$$AI_{ZCflt_i} = Ntl * (f_i + s) * \delta(SD_i^P, ED_i^P) + \left(\sum_{j=1}^{i-1} AI_{ZCflt_j}\right) * (f_i + s) * \delta(SD_i^P, ED_i^P)$$

3. Zero Coupon Fixed Leg and a Zero Coupon Float Leg

$$NPV_{ZCS} = NPV_{ZCfix} + NPV_{ZCflt}$$

3.8 Overnight Index Swaps

An OIS consists of a Fix Leg, plus a Float Leg. The float Leg of an OIS trade compounds on a daily basis as per the correspondent Over Night Rate.

The NPV of an OIS is then the sum of the NPV of the Fix Leg and the Float Leg.

$$NPV_{OIS} = NPV_{OISfix} + NPV_{OISflt}$$

3.8.1 Fix Leg

The value of the Fix Leg of an OIS is calculated in the same way as a standard vanilla Interest Rate Swap.

$$NPV_{OISfix} = \sum_{i=1}^{m} Ntl * R_{Fix} * \delta(SD_i^P, ED_i^P) * \emptyset(Discount\ curve, PD_i, VD)$$

3.8.2 Float Leg

On an OIS Floating leg, interest is accrued on a daily basis. In order to calculate the value of the interest payments, we use Accrual Factors for the relevant period 0 to *t*.

For valuation of cash flows where:

i) interest period start date ≤ valuation date:

$$PV_n = Ntl * \left[\left(AF_{SD_n,VD} * AF_{VD,ED_n}\right) - 1\right] * \emptyset(OIS, PD_n, VD)$$

where,

$$AF_{SD_n,VD} = \prod_{i=SD_n}^{VD-1} 1 + ONr_{i+1} * \delta(i, i+1)$$

$$AF_{VD,ED_n} = \frac{1}{\emptyset(OIS, ED_n^I, VD)}$$

where,

ONr_i = Overnight Rate at date i

ii) interest period start date > valuation date:

$$PV_n = Ntl * AF_{SD_n,ED_n} * \emptyset(OIS, PD_n, VD)$$

where,

$$AF_{SD_n,ED_n} = \left(\frac{\emptyset(OIS, SD_n^I, VD)}{\emptyset(OIS, ED_n^I, VD)} - 1\right)$$

The value of the OIS floating leg is:

$$NPV_{OISflt} = \sum_{n=1}^{N} PV_n$$

3.9 FRAs

There are two models for pricing FRAs: Forward Discounting and Dual Discounting. FRAs eligible for SwapClear are for currencies which adopt the Forward Discounting method.

3.9.1 Forward Discounting

The NPV of a FRA is computed as below:

$$NPV_{FRA} = Ntl * (R_{Fix} - f) * \delta(SD^P, ED^P) * \emptyset(Estimation\ curve, ED^P, PD) * \emptyset(Discount\ curve, PD, VD)$$

Note that $SD^P = PD$.

3.10 Inflation Swaps

A zero coupon inflation indexed swap is a "pure inflation" instrument. Cash flows between the two parties in the transaction occur only at maturity and involve the exchange of a notional adjusted for inflation (that has accrued over the lifetime of the deal) against the notional capitalised with a fixed rate. The fixed rate, agreed at inception, reflects the future expectation of inflation. It can also be regarded as the market price of expected inflation over the period of the zero coupon swap.

An Inflation Swap is composed of:

- Inflation Index + Lag
- Start Date
- End Date
- Nominal
- Fixed rate
- Pay or Receive floating leg

Market convention is to refer to the fixed rate when describing the direction of the trade. A pay position means paying the fixed leg and receiving the inflation linked leg. Likewise, a receive position implies receiving the fixed leg. Long and short are generally used in an inflation rates perspective, where a long position implies paying fixed so money is made as inflation rates rise.

3.10.1 Cashflows

The ZCIIS is defined as a zero coupon, single flow swap between a known fixed rate and an indexed floating amount.

Party B pays Party A the fixed amount $N[(1+K)^M - 1]$

Party A pays Party B the index linked amount $N\left[\frac{I(T_M)}{I(T_0)} - 1\right]$

In practice, these flows are netted and a single balancing payment is made at the maturity of the swap.

Net flow paid by Party A to Party B $N\left[\frac{I(T_M)}{I(T_0)} - (1+K)^M\right]$

Where:

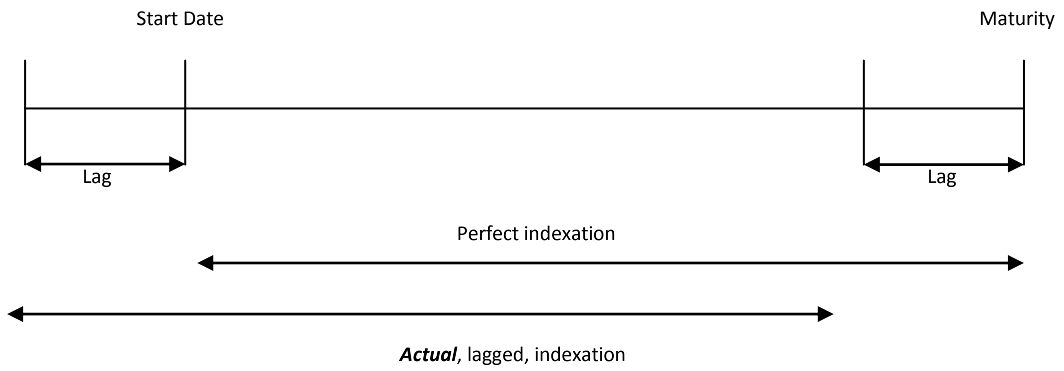
N	The notional amount
K	The fixed rate (observed in the market)
$I(x)$	Observed CPI at time x
M	The length of the swap (measured in years)

The inflation market makes use of indexation lags for the following reasons:

- The process of gathering and computing data required for inflation is time-consuming.
- Bond market participants need a known fixing to compute accrued interest in the event of bond purchase/sale.

Due to these practicalities it is typical that inflation is published around two weeks after the month under consideration. In both the bond and derivative markets the lags are index-dependent; but are usually 2-3 months in duration.

A consequence of the indexation lag is that the inflation receiver (buyer) has no "inflation protection" over the final period of the deal. By way of compensation – the inflation buyer will receive the inflation that was observed over the lagged period which precedes the start of the deal. (This is illustrated below by the "Actual, lagged, indexation line").



Three market data components are needed to price a ZCIIS:

- A Seasonality adjustment to correct the index level for monthly deviations
- An Inflation curve to calculate the index level at maturity of a swap
- An Interest Rates curve to discount the expected cash flow from payment date to pricing date

3.10.2 Seasonality

Inflation is subject to seasonality and it is therefore necessary to adjust it on a month to month basis. The type used is Multiplicative.

$$CPI_p = \frac{S_p * CPI_1}{S_1} * (1 + R)^p$$

Where:

CPI_1	Left pillar CPI of the current bucket
CPI_2	Right pillar CPI of the current bucket
S_1	Seasonality factor for start month of the bucket
S_2	Seasonality factor for end month of the bucket
R	Average return or Trend
p	P'th month

3.10.3 CPI levels

The forward CPI level of the Inflation index is determined from inflation curve ZCIIS quotes:

$$CPI_n = CPI_{Last}(1 + Z_n)^{T_n}$$

Where:

CPI_n	CPI of inflation curve pillar n
CPI_{Last}	Last published fixing
R_n	CPI rate for inflation curve pillar n
T_n	Ratio of number of days from reference date to interpolate over the number of days in a year.

Assuming that CPIs for all pillars are known, Market practice is to use 30/360 month basis convention. The CPI rates are calculated using the following formula:

$$Z_n = \left(\frac{S_{Last}.\,CPI_n}{S_n.\,CPI_{Last}}\right)^{\frac{1}{t'_n}} - 1$$

Where:

CPI_{Last}	Last published index (generally previous month from system date)
CPI_n	CPI for pillar n
S_{Last}	Seasonality for last published index month
S_n	Seasonality for pillar n
t'_n	Time to maturity from fixing date n to last published index date using 30/360 by month basis convention (Diff Years * 12 + Diff Months) / 12

The reference CPI is calculated by interpolating:

$$CPI_0 = CPI_{M-L} + \frac{d-1}{D} * (CPI_{M-L+1} - CPI_{M-L})$$

Where:

M	Current month of the market data date
L	Index lag in month
d	Day of the current month
D	Number of days in the current month

3.10.4 Interpolation

For obtaining the CPI on a date that falls between two pillars, first log linear interpolation between these CPI pillars is used to obtain the monthly CPI values between the pillars. Hence the CPI of the p^{th} month between two pillars that are n months apart,

$$CPI_p = S_p\left[\left(\frac{CPI_1}{S_1}\right)^{1-\frac{p}{n}}\left(\frac{CPI_2}{S_2}\right)^{\frac{p}{n}}\right]$$

where: CPI_1 is the CPI of left pillar closest to month p, CPI_2 is the CPI of right pillar closest to month p, S_p is the Seasonality adjustment for month p, S_1 is the Seasonality factor for left pillar month, S_2 is the Seasonality factor for right pillar month, p is the Number of months from the left pillar to the month for which interpolation is being done, n is the Number of months between left and right curve pillars.

Then, to obtain the CPI on a particular date T, one of two methodologies is used depending on whether the trade is piecewise or linear:

- For a piecewise trade, CPI_T is the CPI of the month in which T falls. Hence all dates in month p have the same CPI, equal to CPI_p.
- For a linear trade, CPI_T is linearly interpolated between the nearest two monthly CPI's that were calculated above.

3.10.5 Extrapolation

For obtaining CPI on a date T before the first pillar of the curve (i.e. where there is no left pillar and therefore interpolation as described above cannot be done), extrapolation of the first pillar's ZCIIS rate is used. i.e. For

T < first pillar,

$$CPI_T = CPI_0(1 + R_{1^{st}})^{t_T}$$

where CPI_0 is the Reference CPI, calculated as described above, R_{1st} is the ZCIIS quote for the 1st pillar, t_T is the time from the reference CPI date to date T, expressed in years. Similarly, for obtaining CPI on a date T after the last pillar of the curve (i.e. where there is no right pillar and therefore interpolation as described above cannot be done), extrapolation of the last pillar's ZCIIS rate is used. i.e. For T > last pillar (30Y in the case of EUR HICPxT, USD CPI and FR CPI and 50Y in the case of UK RPI),

$$CPI_T = CPI_0(1 + R_{last})^{t_T}$$

where CPI_0 is the Reference CPI, calculated as described above, R_{last} is the ZCIIS rate quote for the last pillar, t_T is the time from the reference CPI date to date T, expressed in years.

3.10.6 Pricing

The PV of a trade is given by discounting the simple payout back to the valuation date:

$$PV = \phi.N\left[\frac{CPI_{T_M}}{CPI_0} - (1+K)^M\right].DF(T_{M+L})$$

Where:

CPI_{T_M}	Index level observed at the swap fixing date T_M
CPI_0	Reference Index level observed at the fixing date T_0 = Swap start date – Lag
ϕ:	Pay or receive flag related to the Inflation leg: $\phi = \begin{cases} -1, & Pay\ Inflation \\ 1, & Receive\ Inflation \end{cases}$
K	Fixed rate that nullifies the ZCIIS PV at inception
M	Swap length in integer years
N	Swap notional amount
$DF(T_{M+L})$	Discount factor from swap payment date to pricing date

4 IRS Sensitivities

SwapClear calculates the sensitivity of the value of a Swap for a change in Interest Rates. On a Swap, the first derivative of NPV with respect to the Interest Rate is known as Delta. The second derivative of NPV with respect to the Interest Rate is known as Gamma. These sensitivities can be calculated with respect to the Par Curve and with respect to the Zero Coupon Rate Curve.

A Swap has two categories of sensitivities:

1. Forward Sensitivity is the sensitivity of the NPV of a Swap with respect to a change in the forward rates.
2. Discount Sensitivity is the sensitivity of the NPV of a Swap with respect to a change in the discount rate applied on the cashflows of a Swap.

SwapClear calculates Delta and Gamma for both Forward and Discount sensitivities.

SwapClear calculates the sensitivities with respect to the Zero Coupon Curve using an analytical approach.

The *n*-th derivative of $\emptyset_i$ is:

$$\frac{\partial^n \emptyset_i}{\partial z_i^n} = (-1)^n * \delta(VD, i)^n * \emptyset_i$$

In order to the calculate the sensitivities with respect to the Par Curve, SwapClear uses the Jacobian Transformation to convert the Zero Delta into Par Delta, and in addition of the Jacobian Transformation, it uses the Hessian Matrix to convert the Zero Gamma into Par Gamma.

The Jacobean transformation is applied as follows:

$$\begin{bmatrix} \frac{\partial NPV}{\partial m_1} \\ \frac{\partial NPV}{\partial m_2} \\ \cdots \\ \frac{\partial NPV}{\partial m_n} \end{bmatrix} = [dMdZ^{-1}]^T \begin{bmatrix} \frac{\partial NPV}{\partial z_1} \\ \frac{\partial NPV}{\partial z_2} \\ \cdots \\ \frac{\partial NPV}{\partial z_n} \end{bmatrix}$$

Where,

$$dMdZ = \begin{bmatrix} \frac{\partial m_1}{\partial z_1} & \frac{\partial m_1}{\partial z_2} & \cdots & \frac{\partial m_1}{\partial z_n} \\ \frac{\partial m_2}{\partial z_1} & \frac{\partial m_2}{\partial z_2} & \cdots & \frac{\partial m_2}{\partial z_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial m_n}{\partial z_1} & \frac{\partial m_n}{\partial z_2} & \cdots & \frac{\partial m_n}{\partial z_n} \end{bmatrix}$$

Entries in the $dMdZ$ matrix are calculated as below:

$$\frac{\partial m_i}{\partial z_j} = \frac{\partial m_i}{\partial NPV_i} * \frac{\partial NPV_i}{\partial z_j} = \frac{\partial NPV_i}{\partial z_j} \div \frac{\partial NPV_i}{\partial m_i}$$

SwapClear consolidates the Sensitivities of a Swap per Zero Curve, using the Zero Curve Knot Points. In order to consolidate the sensitivities, it is necessary to project the sensitivities from the Sensitivity Dates to the Knot Point Dates of the relevant Curve.

If the sensitivity date is in the same date of a Knot Point Date, then the whole sensitivity is allocated to this Knot Point.

If the sensitivity is between two dates of the curve, then this is linear interpolated to project it into the two neighbouring Knot Points.

5 Piecewise Inflation Sensitivities

In case of a piecewise interpolation, a trade only requests one index level. Using log linear index level interpolation:

$$PV = \phi . N . \left[\frac{CPI_{Last} . S_p}{CPI_0 . S_{Last}} (1+Z_1)^{t'_1\left(1-\frac{p}{n}\right)} . (1+Z_2)^{t'_2\frac{p}{n}} - (1+K)^{t_3} \right] . Df$$

Where
CPI_0: Reference CPI of the deal, calculated from swap start date and using the deal lag
S_1: Seasonality factor for left pillar month
S_2: Seasonality factor for right pillar month
S_p: Seasonality adjustment for month p
p: Number of months from the fixing date to the left pillar
n: Number of months between left and right pillars
K: Fixed rate
N: Nominal
t_1: Time to maturity for the left pillar using 30/360 by month basis convention
t_2: Time to maturity for the right pillar using 30/360 by month basis convention
t_3: Time to maturity for the swap following using 30/360 by month basis convention
$Df = exp^{-Zc*t_3}$: Discount factor from the payment date to the pricing date using ACT/365 basis convention
ϕ: Pay or receive flag related to the Inflation leg: $\phi = \begin{cases} -1, & Pay\ Inflation \\ 1, & Receive\ Inflation \end{cases}$

5.1 Piecewise Par Delta

Par Delta can be calculated as a composition of four derivatives: the derivative of the PV against the CPI observed, composed with the derivative of that CPI against its CPI rate, projected on the CPI rates of the curve and subsequently taking the derivative of the CPI rate with respect to the par rate.

$$\frac{\partial PV}{\partial R_1} = \frac{\partial PV}{\partial CPI_p} . \frac{\partial CPI_p}{\partial Z_p} . \frac{\partial Z_p}{\partial Z_1} . \frac{\partial Z_1}{\partial R_1}$$

$$\frac{\partial PV}{\partial R_2} = \frac{\partial PV}{\partial CPI_p} . \frac{\partial CPI_p}{\partial Z_p} . \frac{\partial Z_p}{\partial Z_2} . \frac{\partial Z_2}{\partial R_2}$$

$$\frac{\partial \mathbf{Z_n}}{\partial \mathbf{R_n}} = \left(\frac{\boldsymbol{CPI_{S0}}}{\boldsymbol{CPI_{Last}}} . \frac{\boldsymbol{S_{Last}}}{\boldsymbol{S_n}} \right)^{\frac{1}{t'_n}} . \frac{\boldsymbol{t_n}}{\boldsymbol{t'_n}} . (\mathbf{1} + \boldsymbol{R_n})^{\frac{t_n}{t'_n} - 1}$$

5.2 Piecewise Par Gamma

Par Gamma is the second order derivative with respect to Par rates. It is a symmetric matrix; where only four terms are not null.

$$\begin{array}{c} \\ Date_1 \\ Date_2 \\ Date_3 \end{array} \begin{pmatrix} Date_1 & Date_2 & Date_3 \\ \frac{\partial^2 PV}{\partial {R_1}^2} & \frac{\partial^2 PV}{\partial R_1 \partial R_2} & 0 \\ \frac{\partial^2 PV}{\partial R_1 \partial R_2} & \frac{\partial^2 PV}{\partial {R_2}^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{\partial^2 Z_n}{\partial {R_n}^2} = \left(\frac{CPI_{S0}}{CPI_{Last}}.\frac{S_{Last}}{S_n}\right)^{\frac{1}{t'_n}}.\frac{t_n}{t'_n}.\left(\frac{t_n}{t'_n} - 1\right).(1 + R_n)^{\frac{t_n}{t'_n}-2}$$

5.3 Piecewise Par/ZC IRS Cross Gamma

Cross Gamma is a matrix that mixes interest rate and inflation Deltas. In case of piecewise interpolation, the matrix has only four non null terms at maximum.

$$\begin{array}{cc} & \begin{array}{ccc} Date_{IRS_1} & Date_{IRS_2} & Date_{IRS_3} \end{array} \\ \begin{array}{c} Date_{INF_1} \\ \\ Date_{INF_2} \\ \\ Date_{INF_3} \end{array} & \begin{pmatrix} \dfrac{\partial^2 \mathrm{PV}}{\partial \mathrm{R}_1\, \partial \mathrm{ZC}_1} & \dfrac{\partial^2 \mathrm{PV}}{\partial \mathrm{R}_1\, \partial \mathrm{ZC}_2} & 0 \\ \dfrac{\partial^2 \mathrm{PV}}{\partial \mathrm{R}_2\, \partial \mathrm{ZC}_1} & \dfrac{\partial^2 \mathrm{PV}}{\partial \mathrm{R}_2\, \partial \mathrm{ZC}_2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

As defined in the first part, there are therefore 6 different cases to calculate Cross Gamma and project it on both curves.

<u>In between two IRS and INF tenors:</u>

The general formulas when an index level falls in between two curve pillars are the following:

$$\frac{\partial PV}{\partial R_2 \partial ZC_1} = -t_p.(1-\alpha).\phi.\frac{CPI_{S0}}{CPI_0}.N.Df.S_p.\left[\frac{(1+R_1)^{t_1}}{S_1}\right]^{1-\frac{p}{n}}.\frac{t_2\,\frac{p}{n}}{{S_2}^{\frac{p}{n}}}.(1+R_2)^{\frac{p}{n}t_2-1}$$

$$\frac{\partial PV}{\partial R_2 \partial ZC_2} = -t_p.\alpha.\phi.\frac{CPI_{S0}}{CPI_0}.N.Df.S_p.\left[\frac{(1+R_1)^{t_1}}{S_1}\right]^{1-\frac{p}{n}}.\frac{t_2\,\frac{p}{n}}{{S_2}^{\frac{p}{n}}}.(1+R_2)^{\frac{p}{n}t_2-1}$$

Where:

$\alpha = \dfrac{Date_p - Date_1}{Date_2 - Date_1}$

$Date_2$: Date of right tenor

$Date_1$: Date of left tenor

$Date_p$: Swap payment date

<u>Before first tenor of Inflation curve and on an IRS tenor:</u>

When an index level is observed before the first pillar of the curve, the trade is only sensitive to the right pillar as the left is the last published index and is fixed.

$$\frac{\partial PV}{\partial R_2 \partial ZC} = -t_p.\phi.\frac{N.Df.S_p}{CPI_0}.\left(\frac{CPI_{Last}}{S_1}\right)^{1-\frac{p}{n}}.\left(\frac{CPI_{S0}}{S_2}\right)^{\frac{p}{n}}.t_2.\frac{p}{n}.(1+R_2)^{\frac{p}{n}t_2-1}$$

<u>Before first tenor of Inflation curve and in between two IRS tenors:</u>

$$\frac{\partial PV}{\partial R_2 \partial ZC_1} = -t_p.(1-\alpha).\phi.\frac{N.Df.S_p}{CPI_0}.\left(\frac{CPI_{Last}}{S_1}\right)^{1-\frac{p}{n}}.\left(\frac{CPI_{S0}}{S_2}\right)^{\frac{p}{n}}.t_2.\frac{p}{n}.(1+R_2)^{\frac{p}{n}t_2-1}$$

$$\frac{\partial PV}{\partial R_2 \partial ZC_2} = -t_p.\alpha.\phi.\frac{N.Df.S_p}{CPI_0}.\left(\frac{CPI_{Last}}{S_1}\right)^{1-\frac{p}{n}}.\left(\frac{CPI_{S0}}{S_2}\right)^{\frac{p}{n}}.t_2.\frac{p}{n}.(1+R_2)^{\frac{p}{n}t_2-1}$$

On a tenor of Inflation curve and IRS curve:

When an index level falls on a pillar date, the trade is only sensitive to that unique pillar and the delta formula is much simpler.

$$\frac{\partial PV}{\partial R \partial ZC} = -t_p.\phi.\frac{N.CPI_{S0}.Df}{CPI_0}.t_e.(1+R)^{t_e-1}$$

Where:
t_p: Time to maturity from payment date to pricing date using ACT/365 basis convention
t_e: Time to maturity from swap end date using 30/360 by month basis convention

On a tenor of Inflation curve and in between IRS curve tenors:

$$\frac{\partial PV}{\partial R \partial ZC_1} = -t_p.(1-\alpha).\phi.\frac{N.CPI_{S0}.Df}{CPI_0}.t_e.(1+R)^{t_e-1}$$

$$\frac{\partial PV}{\partial R \partial ZC_2} = -t_p.\alpha.\phi.\frac{N.CPI_{S0}.Df}{CPI_0}.t_e.(1+R)^{t_e-1}$$

In between two Inflation tenors and on IRS tenor:

$$\frac{\partial PV}{\partial R_1 \partial ZC} = -t_p.\phi.\frac{CPI_{S0}}{CPI_0}.N.Df.S_p.\left[\frac{(1+R_2)^{t_2}}{S_2}\right]^{\frac{p}{n}}.\frac{t_1\left(1-\frac{p}{n}\right)}{S_1^{1-\frac{p}{n}}}.(1+R_1)^{t_1\left(1-\frac{p}{n}\right)-1}$$

$$\frac{\partial PV}{\partial R_2 \partial ZC} = -t_p.\phi.\frac{CPI_{S0}}{CPI_0}.N.Df.S_p.\left[\frac{(1+R_1)^{t_1}}{S_1}\right]^{1-\frac{p}{n}}.\frac{t_2\frac{p}{n}}{S_2^{\frac{p}{n}}}.(1+R_2)^{\frac{p}{n}t_2-1}$$

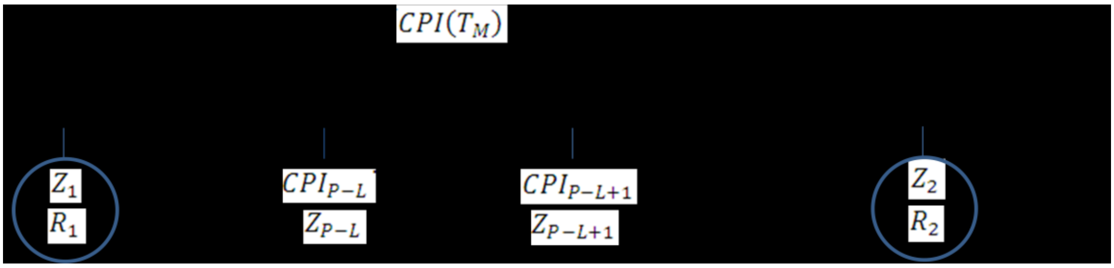
6 Linear Inflation Sensitivities

$$PV = \phi.N.\left[\frac{\alpha.CPI_{P-L} + (1-\alpha).CPI_{P-L+1}}{CPI_0} - (1+K)^{t_3}\right].Df$$

Where
CPI_0: Reference CPI of the deal, calculated from swap start date and using the deal lag
α=$1 - \frac{d-1}{D}$: Coefficient of the linear interpolation
$CPI_0 = \alpha.CPI_{M-L} + (1-\alpha).CPI_{M-L+1}$
CPI_{P-L}: CPI of the current month where the index level fixes
CPI_{P-L+1}: CPI of the month following CPI_{P-L}
M: Current month of the market data date
L: Index lag in month
d: Day of the Swap End date
D: Number of days in the Swap End date month
P: Swap End date
K: Fixed rate
N: Nominal
t_3: Time to maturity for the swap following using 30/360 by month basis convention
$Df = exp^{-Zc*t_3}$: Discount factor from the payment date to the pricing date using ACT/365 basis convention
ϕ: Pay or receive flag related to the Inflation leg: $\phi = \begin{cases} -1, & Pay\ Inflation \\ 1, & Receive\ Inflation \end{cases}$

6.1 Linear Par Delta

Par Delta for linear indices is more complex due to the projection on different dates and link between index levels and par rates.



Where
$CPI(T_M)$: CPI of the trade used for pricing and interpolated from CPI_{P-L} and CPI_{P-L+1}
CPI_{P-L}: CPI of the current month of T_M
CPI_{P-L+1}: CPI of month following CPI_{P-L}
Z_{P-L}: CPI rate of the current month of T_M
Z_{P-L+1}: CPI rate of month following CPI_{P-L}
Z_1: CPI rate of left pillar of the current bucket
R_1: Market quote of left pillar of the current bucket (par rate)
Z_2: CPI rate of right pillar of the current bucket
R_2: Market quote of right pillar of the current bucket (par rate)

$$\frac{\partial PV}{\partial R_n} = \left(\frac{\partial PV}{\partial CPI_{P-L}}.\frac{\partial CPI_{P-L}}{\partial Z_{P-L}}.\frac{\partial Z_{P-L}}{\partial Z_1} + \frac{\partial PV}{\partial CPI_{P-L+1}}.\frac{\partial CPI_{P-L+1}}{\partial Z_{P-L+1}}.\frac{\partial Z_{P-L+1}}{\partial Z_1}\right).\frac{\partial Z_1}{\partial R_n} + \left(\frac{\partial PV}{\partial CPI_{P-L+1}}.\frac{\partial CPI_{P-L+1}}{\partial Z_{P-L+1}}.\frac{\partial Z_{P-L+1}}{\partial Z_2} + \frac{\partial PV}{\partial CPI_{P-L}}.\frac{\partial CPI_{P-L}}{\partial Z_{P-L}}.\frac{\partial Z_{P-L}}{\partial Z_2}\right).\frac{\partial Z_2}{\partial R_n}$$

All the terms involved in the Delta calculation are known except $\frac{\partial Z_i}{\partial R_n}$. This term is a part of the Jacobian matrix which helps convert Zero sensitivity into Par sensitivity.

$$\mathrm{R_n} = \left[\frac{CPI_{Last}}{CPI_0.S_{Last}}.\left[a.S_{n_{P-L}}.(1+Z_{n_{P-L}})^{t_{n_{P-L}}} + (1-a).S_{n_{P-L+1}}.(1+Z_{n_{P-L+1}})^{t_{n_{P-L+1}}}\right]\right]^{1/t_n} - 1$$

Where
$\mathrm{R_n}$: Par rate for pillar n
Z_n: CPI rate for pillar n
S_{Last}: Seasonality of last published index month
S_n: Seasonality for pillar n month
t_n: Time to maturity in between the fixing date n and the reference index level date using 30/360 by month basis convention
t_l: Time to maturity in between the fixing date n and the last published index date using 30/360 by month basis convention
α: Coefficient of the linear interpolation $\alpha = 1 - \frac{d-1}{D}$

$$\boldsymbol{\frac{\partial R}{\partial Z} = \frac{\partial R_i}{\partial Z_{i_{P-L+1}}} + \frac{\partial R_i}{\partial Z_{i-1_{P-L}}}.\frac{\partial Z_{i-1_{P-L}}}{\partial Z_{i_{P-L+1}}}}$$

The final matrix to derive a CPI rate regarding Par rates and project the sensitivity on the pillar dates can be calculated by inverting $\frac{\partial R}{\partial Z_{p-L+1}}$.

6.2 Linear Par Gamma

Par Gamma for linear indices is as complex as Par Delta due to the index level dependency to previous Par rates when stripping the Inflation curve. Gamma is calculated using the CPI rate Gamma and then composing it with the Hessian matrix to translate CPI rate Gamma into Par Gamma.

$$\frac{\partial^2 \mathbf{PV(p)}}{\partial \mathbf{R_i} \partial \mathbf{R_j}} = \sum_{\mathbf{k=1,2}} \frac{\partial \mathbf{PV}}{\partial \mathbf{Z_k}} \cdot \frac{\partial^2 \mathbf{Z_k}}{\partial \boldsymbol{R_i} \partial \boldsymbol{R_j}} + \sum_{\mathbf{k,l=1,2}} \frac{\partial^2 \mathbf{PV}}{\partial \mathbf{Z_k} \partial \mathbf{Z_l}} \cdot \frac{\partial \mathbf{Z_k}}{\partial \boldsymbol{R_i}} \cdot \frac{\partial \mathbf{Z_l}}{\partial \boldsymbol{R_j}}$$

$$\frac{\partial^2 \mathbf{PV}}{\partial \mathbf{R_i} \partial \mathbf{R_j}} = \frac{\partial^2 \mathbf{PV(P-L)}}{\partial \mathbf{R_i} \partial \mathbf{R_j}} + \frac{\partial^2 \mathbf{PV(P-L+1)}}{\partial \mathbf{R_i} \partial \mathbf{R_j}}$$

Hessian Matrix:

Second derivative of CPI rate 1 to Par rates

$$\begin{array}{c|c} & \begin{matrix} \boldsymbol{Date_1} & \boldsymbol{Date_2} & \boldsymbol{Date_3} \end{matrix} \\ \begin{matrix} \boldsymbol{Date_1} \\ \boldsymbol{Date_2} \\ \boldsymbol{Date_3} \end{matrix} & \begin{pmatrix} \frac{\partial^2 Z_1}{\partial {R_1}^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Second derivative of CPI rate 2 to Par rates

$$\begin{array}{c|c} & \begin{matrix} \boldsymbol{Date_1} & \boldsymbol{Date_2} & \boldsymbol{Date_3} \end{matrix} \\ \begin{matrix} \boldsymbol{Date_1} \\ \boldsymbol{Date_2} \\ \boldsymbol{Date_3} \end{matrix} & \begin{pmatrix} \frac{\partial^2 Z_2}{\partial {R_1}^2} & \frac{\partial^2 Z_2}{\partial R_1 \partial R_2} & 0 \\ \frac{\partial^2 Z_2}{\partial R_1 \partial R_2} & \frac{\partial^2 Z_2}{\partial {R_2}^2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Second derivative of CPI rate 3 to Par rates

$$\begin{array}{c|c} & \begin{matrix} \boldsymbol{Date_1} & \boldsymbol{Date_2} & \boldsymbol{Date_3} \end{matrix} \\ \begin{matrix} \boldsymbol{Date_1} \\ \boldsymbol{Date_2} \\ \boldsymbol{Date_3} \end{matrix} & \begin{pmatrix} \frac{\partial^2 Z_3}{\partial {R_1}^2} & \frac{\partial^2 Z_3}{\partial R_1 \partial R_2} & \frac{\partial^2 Z_3}{\partial R_1 \partial R_3} \\ \frac{\partial^2 Z_3}{\partial R_1 \partial R_2} & \frac{\partial^2 Z_3}{\partial {R_2}^2} & \frac{\partial^2 Z_3}{\partial R_2 \partial R_3} \\ \frac{\partial^2 Z_3}{\partial R_1 \partial R_3} & \frac{\partial^2 Z_3}{\partial R_2 \partial R_3} & \frac{\partial^2 Z_3}{\partial {R_3}^2} \end{pmatrix} \end{array}$$

7 Appendix

7.1 Accruals for OIS

Accrual is an analytic usually used for accounting purposes. It is the difference between the dirty and clean MV. Accrual for OIS is the sum of fixed leg accrual, plus floating leg accrual.

$$Accrual_{OIS} = Accrual_{OISfix} + Accrual_{OISflt}$$

7.1.1 Fix Leg

The accrual of the Fix Leg of an OIS is calculated in the same way as a standard vanilla Interest Rate Swap.

$$Accrual_{OISfix} = Ntl * R_{Fix} * \delta(SD^P, ED^P) * \theta(SD^P, ED^P, VD)$$

7.1.2 Floating Leg

For an OIS floating leg, accrual is based on a "floating accrual flow" computed based on known and past overnight rates (ON) of the current period and by "crystallizing" the unknown and future ONs of what is remaining of the current period to the **last known Overnight fixing** i.e. ONs falling after the value date are supposed to be all equal to the ONr_{VD} fixed during the EOD of value date. This "Floating accrual flow" is different from the floating flow used in MV computation which is estimated based on the OIS curve.

$$Accrual_{OISflt} = Ntl * \left[\left(AF_{SD^P,VD} * AF_{VD,ED^P}\right) - 1\right] * \theta(SD^P, ED^P, VD)$$

Where

$$AF_{SD^P,VD} = \prod_{i=SD^P}^{VD-1} 1 + ONr_{i+1} * \delta(i, i+1)$$

$$AF_{VD,ED^P} = \prod_{i=VD}^{ED^P} 1 + ONr_{VD} * \delta(i, i+1)$$