

Simplex Duo on forward curve only

Let P be a portfolio of swap trades with n forward deltas on n forward tenors.

$$\bar{\delta}^f = \delta_1^f, \dots, \delta_n^f$$

$$\bar{t}^f = t_1^f, \dots, t_n^f$$

Presuming that each hedge trade is a fixed rate payer we will have h hedge swaps with unit forward deltas below.

$$\bar{\delta}^{f,1} = \delta_1^{f,1}, \dots, \delta_n^{f,1}$$

$$\bar{\delta}^{f,2} = \delta_1^{f,2}, \dots, \delta_n^{f,2}$$

$$\vdots$$

$$\bar{\delta}^{f,h} = \delta_1^{f,h}, \dots, \delta_n^{f,h}$$

Let N_1, \dots, N_h be the notional of each hedge swap.

Let us choose these notionals such that the absolute notional is minimised and that the combination of the new hedge swaps and the original portfolio of swap trades has an aggregated δ that is within a certain tolerated range around zero which will be called T_1^f, \dots, T_n^f .

Introduce for each hedge trade two notionals N^S, N^L which must be zero or positive and represent the notional in fixed rate receiver and notional in fixed rate payer respectively.

Now writing this as a linear programming problem in non standard form; with $\tilde{\cdot}$ indicating a variable :-

$$\min \sum_{i=1}^h (\tilde{N}_i^L + \tilde{N}_i^S)$$

such that

$$0 \leq \tilde{N}_i^S, 0 \leq \tilde{N}_i^L \quad \forall \quad i = 1, \dots, h$$

$$-T_j^f \leq \sum_{i=1}^h (\tilde{N}_i^L - \tilde{N}_i^S) \delta_j^{f,i} + \delta_j^f \leq T_j^f \quad \forall \quad j = 1, \dots, n$$

This problem can be solved where possible with the simplex algorithm.

Addressing the fixing risk

Let us assume that fixing is identified as a daily ladder of net notional being fixed on a given day in the future. That this ladder will have a time horizon H say one year. So for the portfolio P there will be f fixing days ϕ

$$\phi_1, \dots, \phi_f$$

with net notional ν on each of these days

$$\nu_1, \dots, \nu_f$$

and some tolerance θ on each of these days

$$\theta_1, \dots, \theta_f$$

The hedge basket will need to be enhanced to target this fixing risk. For example a selection of trades which are generated as slices of trades in original offset portfolio. Each hedge trade will potentially have unit fixing notional on each of these days.

$$\bar{\nu}^1 = \nu_1^1, \dots, \nu_f^1$$

$$\bar{\nu}^2 = \nu_1^2, \dots, \nu_f^2$$

$$\vdots$$

$$\bar{\nu}^h = \nu_1^h, \dots, \nu_f^h$$

and we want the constraints

$$-\theta_j \leq \sum_{i=1}^l (\tilde{N}_i^L - \tilde{N}_i^S) \nu_j^i + \nu_j \leq \theta_j \quad \forall \quad j = 1, \dots, f$$

to hold.

Now enhancing the simplex phase with these additional constraints. We have the linear problem :-

$$\min \sum_{i=1}^h (\tilde{N}_i^L + \tilde{N}_i^S)$$

such that

$$0 \leq \tilde{N}_i^S, 0 \leq \tilde{N}_i^L \quad \forall \quad i = 1, \dots, h$$

$$-T_j^f \leq \sum_{i=1}^h (\tilde{N}_i^L - \tilde{N}_i^S) \delta_j^{f,i} + \delta_j^f \leq T_j^f \quad \forall \quad j = 1, \dots, n$$

$$-\theta_j \leq \sum_{i=1}^h (\tilde{N}_i^L - \tilde{N}_i^S) \nu_j^i + \nu_j \leq \theta_j \quad \forall \quad j = 1, \dots, f$$

$$i=1$$

This problem can be solved where possible with the simplex algorithm.

Solving for the fixed rates

The simplex phase will have solved for the notional required in each hedge trade.

It is a fact that any linear program that is not infeasible or unbounded has an optimal solution. This solution will have at most n non zero values where n is the number of constraints. Thus the simplex phase will generate at most n hedge trades.

Let assumes that we have a set of NPV ladders bucketed to the forward tenors for the portfolio P .

$$\bar{V}^P = V_1^P, \dots, V_t^P$$

and for the fixed leg (with par fixed rate) and floating leg of each active hedge trade with notional calculated in the first phase.

$$\begin{aligned} \bar{V}^{FIX,1} &= V_1^{FIX,1}, \dots, V_t^{FIX,1} & \bar{V}^{FLOAT,1} &= V_1^{FLOAT,1}, \dots, V_t^{FLOAT,1} \\ &\vdots & &\vdots \\ \bar{V}^{FIX,n} &= V_1^{FIX,n}, \dots, V_t^{FIX,n} & \bar{V}^{FLOAT,n} &= V_1^{FLOAT,n}, \dots, V_t^{FLOAT,n} \end{aligned}$$

as well as the par rate for each active hedge trade

$$\bar{R}^{PAR} = R_1^{PAR}, \dots, R_n^{PAR}$$

Now solve the matrix equation for \tilde{R}

$$\sum_{i=1}^n \frac{\tilde{R}_i}{R_i^{PAR}} V_j^{FIX,i} = V_j^P + \sum_{i=1}^n V_j^{FLOAT,i} \quad \forall \quad j = 1, \dots, t$$

Where there are more hedge trades than tenors (which can happen in the case where we are addressing fixing risk), we must solve for the t sized subset of the n hedge trades and set the remainder to the original par rate.

Solving Bucketed NPV

Starting with the linear problem :-

$$\min \sum_{i=1}^h (\tilde{N}_i^L + \tilde{N}_i^S)$$

With Constraints

$$0 < \tilde{N}_i^S, 0 < \tilde{N}_i^L \quad \forall \quad i = 1 \quad h$$

$$v_j \geq \nu_j, v_j \leq \nu_j \quad \forall \quad j = 1, \dots, f$$

Forward Curve Constraints

$$-T_j^f \leq \sum_{i=1}^h (\tilde{N}_i^L - \tilde{N}_i^S) \delta_j^{f,i} + \delta_j^f \leq T_j^f \quad \forall \quad j = 1, \dots, n$$

Fixing Notional Constraints

$$-\theta_j \leq \sum_{i=1}^h (\tilde{N}_i^L - \tilde{N}_i^S) \nu_j^i + \nu_j \leq \theta_j \quad \forall \quad j = 1, \dots, f$$

we now want to add the constraint for the bucketed NPV rewriting this in term of unit bucketed npv where

$$V_j^{FIX,i} = U_j^{FIX,i} (\tilde{N}_i^L - \tilde{N}_i^S)$$

$$V_j^{FLOAT,i} = U_j^{FLOAT,i} (\tilde{N}_i^L - \tilde{N}_i^S)$$

gives

$$\sum_{i=1}^n \frac{U_j^{FIX,i}}{R_i^{PAR}} (\tilde{R}_i^L \tilde{N}_i^L - \tilde{R}_i^S \tilde{N}_i^S) = V_j^P + \sum_{i=1}^n U_j^{FLOAT,i} (\tilde{N}_i^L - \tilde{N}_i^S) \quad \forall \quad j = 1, \dots, t$$

this is not linear in the variable \tilde{R}_i^L , \tilde{R}_i^S , \tilde{N}_i^L and \tilde{N}_i^S so we introduce a new variables

$$\tilde{Y}_i^L = \tilde{R}_i^L \tilde{N}_i^L \quad \forall \quad i = 1, \dots, h$$

$$\tilde{Y}_i^S = \tilde{R}_i^S \tilde{N}_i^S \quad \forall \quad i = 1, \dots, h$$

which gives

$$\sum_{i=1}^n \frac{U_j^{FIX,i}}{R_i^{PAR}} (\tilde{Y}_i^L - \tilde{Y}_i^S) - \sum_{i=1}^n U_j^{FLOAT,i} (\tilde{N}_i^L - \tilde{N}_i^S) = V_j^P \quad \forall \quad j = 1, \dots, t$$

which is linear in \tilde{Y}_i^L , \tilde{Y}_i^S , \tilde{N}_i^L and \tilde{N}_i^S which finally gives us

$$\min \sum_{i=1}^h (\tilde{N}_i^L + \tilde{N}_i^S)$$

With Constraints

$$0 \leq \tilde{N}_i^S, 0 \leq \tilde{N}_i^L, -\infty \leq \tilde{Y}_i^L, \tilde{Y}_i^S \leq \infty \quad \forall \quad i = 1, \dots, h$$

Foward Curve Constraints

$$-T_j^f \leq \sum_{i=1}^h (\tilde{N}_i^L - \tilde{N}_i^S) \delta_j^{f,i} + \delta_j^f \leq T_j^f \quad \forall \quad j = 1, \dots, n$$

$$\theta_j \geq \sum_{i=1}^h (\nu_i^L - \nu_i^S) \nu_j^i + \nu_j \leq \theta_j \quad \forall \quad j = 1, \dots, f$$

Fixing Notional Constraints

$$-\theta_j \leq \sum_{i=1}^h (\tilde{N}_i^L - \tilde{N}_i^S) \nu_j^i + \nu_j \leq \theta_j \quad \forall \quad j = 1, \dots, f$$

Bucketed NPV Constraints

$$\sum_{i=1}^n \frac{U_j^{FIX,i}}{R_i^{PAR}} (\tilde{Y}_i^L - \tilde{Y}_i^S) - \sum_{i=1}^n U_j^{FLOAT,i} (\tilde{N}_i^L - \tilde{N}_i^S) = V_j^P \quad \forall \quad j = 1, \dots, n$$

Note this formulation has a problem as fixed rate is not forced to zero when the notional of the trade is at zero. The next formulation solves this issue.

Fixed Rate in Range

Let us know assume we wish to constrain the fixed rate of each hedge trade to be within a fixed range.

$$F_i^{MIN} \leq \tilde{R}_i^L \leq F_i^{MAX} \quad \forall \quad i = 1, \dots, h$$

$$F_i^{MIN} \leq \tilde{R}_i^S \leq F_i^{MAX} \quad \forall \quad i = 1, \dots, h$$

Writing in terms of $\tilde{Y}_i^L, \tilde{Y}_i^S$

$$F_i^{MIN} \tilde{N}_i^L \leq \tilde{Y}_i^L \leq F_i^{MAX} \tilde{N}_i^L \quad \forall \quad i = 1, \dots, h$$

$$F_i^{MIN} \tilde{N}_i^S \leq \tilde{Y}_i^S \leq F_i^{MAX} \tilde{N}_i^S \quad \forall \quad i = 1, \dots, h$$

resulting in the full problem

$$\min \sum_{i=1}^h (\tilde{N}_i^L + \tilde{N}_i^S)$$

With Constraints

$$0 \leq \tilde{N}_i^S, 0 \leq \tilde{N}_i^L, -\infty \leq \tilde{Y}_i^L, \tilde{Y}_i^S \leq \infty \quad \forall \quad i = 1, \dots, h$$

Forward Curve Constraints

$$-T_j^f \leq \sum_{i=1}^h (\tilde{N}_i^L - \tilde{N}_i^S) \delta_j^{f,i} + \delta_j^f \leq T_j^f \quad \forall \quad j = 1, \dots, n$$

Fixing Notional Constraints

$$h \quad - \quad -$$

$$-\theta_j \leq \sum_{i=1} (\tilde{N}_i^L - \tilde{N}_i^S) \nu_j^i + \nu_j \leq \theta_j \quad \forall \quad j = 1, \dots, f$$

Bucketed NPV Constraints

$$\sum_{i=1}^n \frac{U_j^{FIX,i}}{R_i^{PAR}} (\tilde{Y}_i^L - \tilde{Y}_i^S) - \sum_{i=1}^n U_j^{FLOAT,i} (\tilde{N}_i^L - \tilde{N}_i^S) = V_j^P \quad \forall \quad j = 1, \dots, t$$

Fixed Rate Range

$$F_i^{MIN} \tilde{N}_i^L \leq \tilde{Y}_i^L \leq F_i^{MAX} \tilde{N}_i^L \quad \forall \quad i = 1, \dots, h$$

$$F_i^{MIN} \tilde{N}_i^S \leq \tilde{Y}_i^S \leq F_i^{MAX} \tilde{N}_i^S \quad \forall \quad i = 1, \dots, h$$